Sympy_DerivScalarMatrix

October 2, 2020

[1]:
```
from sympy import diff, sin, exp, symbols, Function, Matrix,
    ↪MatrixSymbol, FunctionMatrix, derive_by_array
```

[2]:
```
from sympy import Symbol

def var(letter: str, i: int, j: int) -> Symbol:
     letter_ij = Symbol('{}_{}{}'.format(letter, i+1, j+1),
    ↪is_commutative=True)
     return letter_ij

def func(i, j):
     y_ij = Function('y_{}{}'.format(i+1,j+1))(*X)
     return y_ij


n,m,p = 3,3,2

X = Matrix(n, m, lambda i, j: var('x', i, j)); X
```

[2]:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

[3]:
```
#Y = MatrixSymbol(Function('y'), 2, 3); Matrix(Y)
#M = MatrixSymbol('M',2,2); Matrix(M)
#Y = Matrix(m, p, lambda i,j: Function('y_{}{}'.
    ↪format(i+1,j+1))(X) ); Y

Y = Matrix(m, p, lambda i,j:  func(i, j)); Y
```

[3]:

$$\begin{bmatrix} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & y_{12}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ y_{21}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & y_{22}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ y_{31}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & y_{32}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \end{bmatrix}$$

0.0.1 Derivative of Scalar Function of a Matrix with Respect to the Matrix

Let $X = \{x_{ij}\}$ be a matrix of order $m \times n$ and let

$$y = f(X)$$

be a scalar function of X, so $y \in \mathbb{R}$ and $f : \mathbb{R}^{m \times n} \to \mathbb{R}$, Then we can define the derivative of y with respect to X as the following matrix of order $m \times n$:

$$\frac{\partial y}{\partial X} = \begin{pmatrix} \frac{\partial y}{\partial x_{11}} & \frac{\partial y}{\partial x_{12}} & \cdots & \frac{\partial y}{\partial x_{1n}} \\ \frac{\partial y}{\partial x_{21}} & \frac{\partial y}{\partial x_{22}} & \cdots & \frac{\partial y}{\partial x_{2n}} \\ \vdots & \vdots & & \vdots \\ \frac{\partial y}{\partial x_{m1}} & \frac{\partial y}{\partial x_{m2}} & \cdots & \frac{\partial y}{\partial x_{mn}} \end{pmatrix} = \left\{ \frac{\partial y}{\partial x_{ij}} \right\}$$

The matrix $\frac{\partial y}{\partial X}$ is called the gradient matrix.

[4]:
```
derive_by_array(Y[0,0], X)
```

[4]:

$$\begin{bmatrix} \frac{\partial}{\partial x_{11}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{12}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{13}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ \frac{\partial}{\partial x_{21}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{22}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{23}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ \frac{\partial}{\partial x_{31}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{32}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{33}} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \end{bmatrix}$$

0.0.2 Derivative of Matrix With Respect to Scalar Element of Matrix

Let $X = \{x_{ij}\}$ be a matrix of order $m \times n$ and let

$$y = f(X)$$

be a scalar function of X, so $y \in \mathbb{R}$ and $f : \mathbb{R}^{m \times n} \to \mathbb{R}$,

Also let the matrix $Y = \{y_{ij}(X)\}$ be of size $p \times q$.

Then we can define the derivative of Y with respect to an element

x in X as the following matrix of order $p \times q$:

$$\frac{\partial Y}{\partial x} = \begin{pmatrix} \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial x} & \cdots & \frac{\partial Y}{\partial x} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial x} & \cdots & \frac{\partial Y}{\partial x} \\ \vdots & \vdots & & \vdots \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial x} & \cdots & \frac{\partial Y}{\partial x} \end{pmatrix} = \left\{ \frac{\partial y_{ij}}{\partial x} \right\}$$

[5]:
```
derive_by_array(Y, X[1-1,2-1])
```

[5]:

$$\begin{bmatrix} \frac{\partial}{\partial x_{12}} \mathrm{y}_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{12}} \mathrm{y}_{12}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ \frac{\partial}{\partial x_{12}} \mathrm{y}_{21}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{12}} \mathrm{y}_{22}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ \frac{\partial}{\partial x_{12}} \mathrm{y}_{31}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & \frac{\partial}{\partial x_{12}} \mathrm{y}_{32}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \end{bmatrix}$$