

```

from sympy import Matrix, Symbol, derive_by_array, Lambda, Function, MatrixSymbol, Derivative
from sympy import var
from sympy.abc import x, i, j, a, b

def myvar(letter: str, i: int, j: int) -> Symbol:
    letter_ij = Symbol('{}_{}_{}'.format(letter, i+1, j+1), is_commutative=True)
    return letter_ij

n,m,p = 3,3,2

X = Matrix(n, m, lambda i,j : myvar('x', i, j)); X

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

W = Matrix(m, p, lambda i,j : myvar('w', i, j)); W

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

A = MatrixSymbol('X',3,3); Matrix(A)
B = MatrixSymbol('W',3,2)

v = lambda a,b: a*b

vL = Lambda((a,b), a*b)

n = Function('v') #, Lambda((a,b), a*b))

vN = lambda mat1, mat2: Matrix(mat1.shape[0], mat2.shape[1], lambda i, j: Symbol("n_{}_{}".format(i+1, j+1)))

Nelem = vN(X, W); Nelem

$$\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix}$$

n(X,W)

$$v \left( \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}, \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \right)$$

n(A,B)

```

$v(X, W)$

`n(X,W).replace(n, v) # replace works when v = python lambda`

$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix}$$

`n(X,W).subs({n: vL}) # subs works when v = sympy lambda`

$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix}$$

`n(X,W).replace(n, vL)`

$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix}$$

`n(X,W).subs({n: v}) # subs() doesn't work when v is python lambda`

$$v \left(\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}, \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \right)$$

`Matrix(n(A,B).subs({n: vL}))`

$$\begin{bmatrix} W_{0,0}X_{0,0} + W_{1,0}X_{0,1} + W_{2,0}X_{0,2} & W_{0,1}X_{0,0} + W_{1,1}X_{0,1} + W_{2,1}X_{0,2} \\ W_{0,0}X_{1,0} + W_{1,0}X_{1,1} + W_{2,0}X_{1,2} & W_{0,1}X_{1,0} + W_{1,1}X_{1,1} + W_{2,1}X_{1,2} \\ W_{0,0}X_{2,0} + W_{1,0}X_{2,1} + W_{2,0}X_{2,2} & W_{0,1}X_{2,0} + W_{1,1}X_{2,1} + W_{2,1}X_{2,2} \end{bmatrix}$$

`#N = v(X, W); N`

`N = n(A,B); N`

$v(X, W)$

`N.replace(n, v)`

XW

`N.replace(n, v).subs({A: X, B:W}) # replacing variable values after doing function doesn't m`

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

`N.subs({n: vL, A:X, B:W})`

$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix}$$

`Nspec = N.subs({A:X, B:W}).replace(n, v); Nspec`

$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix}$$

`N.diff(N)`

1

`N.diff(X)`

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

way 2 of declaring S (better way)

`sigma = Function('sigma')`

`sigmaApply = Function("sigma_apply") #lambda matrix: matrix.applyfunc(sigma)`

`sigmaApply_ = lambda matrix: matrix.applyfunc(sigma)`

`sigmaApply(A)`

$\sigma_{apply}(X)$

`sigmaApply(A).subs({A: X})`

$$\sigma_{apply} \left(\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \right)$$

`sigmaApply_(A)`

$(d \mapsto \sigma(d))_{\circ}(X)$

`sigmaApply(A).subs({A: X}).replace(sigmaApply, sigmaApply_) # NOTE: subs of functions doesn`

$$\begin{bmatrix} \sigma(x_{11}) & \sigma(x_{12}) & \sigma(x_{13}) \\ \sigma(x_{21}) & \sigma(x_{22}) & \sigma(x_{23}) \\ \sigma(x_{31}) & \sigma(x_{32}) & \sigma(x_{33}) \end{bmatrix}$$

`S = sigmaApply(N); S`

$\sigma_{apply}(v(X, W))$

`Derivative(S, S)`

$$\frac{\partial}{\partial \sigma_{apply}(v(X, W))} \sigma_{apply}(v(X, W))$$

```

Derivative(S, S).doit()

1

Derivative(S, n(A,B)).doit()


$$\frac{\partial}{\partial v(X, W)} \sigma_{\text{apply}}(v(X, W))$$


#lambda = Function("lambda")
#Lagain = lambda(sigmaApply(n(A))); Lagain

# diff(Lagain, A) # never execute
#

S.replace(A,X).replace(B,W)


$$\sigma_{\text{apply}} \left( v \left( \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}, \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \right) \right)$$


S.replace(n, v)


$$\sigma_{\text{apply}}(XW)$$


S.subs({A:X, B:W}).replace(n, v)


$$\sigma_{\text{apply}} \left( \begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix} \right)$$


Sspec = S.subs({A:X, B:W}).replace(n, v).replace(sigmaApply, sigmaApply_)
Sspec


$$\begin{bmatrix} \sigma(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}) & \sigma(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13}) \\ \sigma(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23}) & \sigma(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23}) \\ \sigma(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33}) & \sigma(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33}) \end{bmatrix}$$


S.replace(n, vN) #.replace(sigmaApply, sigmaApply_)


$$\sigma_{\text{apply}} \left( \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix} \right)$$


SeleM = S.replace(n, vN).replace(sigmaApply, sigmaApply_); SeleM


$$\begin{bmatrix} \sigma(n_{11}) & \sigma(n_{12}) \\ \sigma(n_{21}) & \sigma(n_{22}) \\ \sigma(n_{31}) & \sigma(n_{32}) \end{bmatrix}$$


import itertools

elemToSpecD = dict(itertools.chain(*[[Nelem[i, j], Nspec[i, j]] for j in range(2)] for i in range(3)))

```

```
elemToSpec = list(elemToSpecD.items())
```

```
Matrix(elemToSpec)
```

$$\begin{bmatrix} n_{11} & w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} \\ n_{12} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} \\ n_{21} & w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} \\ n_{22} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} \\ n_{31} & w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} \\ n_{32} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} \end{bmatrix}$$

```
elemToSpecFuncD = dict(itertools.chain(*[(Nelem[i, j], Function("n_{i}{j}".format(i + 1, j + 1, j + 1))
```

```
elemToSpecFunc = list(elemToSpecFuncD.items())
```

```
Matrix(elemToSpecFunc)
```

$$\begin{bmatrix} n_{11} & n_{11}(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}) \\ n_{12} & n_{12}(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13}) \\ n_{21} & n_{21}(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23}) \\ n_{22} & n_{22}(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23}) \\ n_{31} & n_{31}(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33}) \\ n_{32} & n_{32}(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33}) \end{bmatrix}$$

```
elemToSpecFuncArgsD = dict(itertools.chain(*[(Nelem[i, j], Function("n_{i}{j}".format(i + 1, j + 1, j + 1))
```

```
elemToSpecFuncArgs = list(elemToSpecFuncArgsD.items())
```

```
Matrix(elemToSpecFuncArgs)
```

$$\begin{bmatrix} n_{11} & n_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \\ n_{12} & n_{12}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \\ n_{21} & n_{21}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \\ n_{22} & n_{22}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \\ n_{31} & n_{31}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \\ n_{32} & n_{32}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \end{bmatrix}$$

```
Selelem
```

$$\begin{bmatrix} \sigma(n_{11}) & \sigma(n_{12}) \\ \sigma(n_{21}) & \sigma(n_{22}) \\ \sigma(n_{31}) & \sigma(n_{32}) \end{bmatrix}$$

```
Selelem.subs(elemToSpecD)
```

$$\begin{bmatrix} \sigma(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}) & \sigma(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13}) \\ \sigma(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23}) & \sigma(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23}) \\ \sigma(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33}) & \sigma(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33}) \end{bmatrix}$$

```
Selelem[0,1].diff(Nelem[0,1])
```

```


$$\frac{d}{dn_{12}}\sigma(n_{12})$$

Sele $[0,1]$ .diff(Nelem $[0,1]$ ).subs({Nelem $[0,1]$  : Nspec $[0,1]$ })
#Sele $[0,1]$ .diff(Nelem $[0,1]$ ).subs(dict([Nelem $[0,1]$  : Nspec $[0,1]$ ]))


$$\left.\frac{d}{dn_{12}}\sigma(n_{12})\right|_{n_{12}=w_{12}x_{11}+w_{22}x_{12}+w_{32}x_{13}}$$

Sele $[0,1]$ .diff(Nelem $[0,1]$ ).subs({Nelem $[0,1]$  : Nspec $[0,1]$ }).subs({Nspec $[0,1]$  : 23})


$$\left.\frac{d}{dn_{12}}\sigma(n_{12})\right|_{n_{12}=23}$$

Sele $[0,1]$ .diff(Nelem $[0,1]$ ).subs({Nelem $[0,1]$  : Nspec $[0,1]$ }).replace(sigma, lambda x: 8*x**3)


$$\left.\frac{d}{dn_{12}}8n_{12}^3\right|_{n_{12}=w_{12}x_{11}+w_{22}x_{12}+w_{32}x_{13}}$$

Sele $[0,1]$ .diff(Nelem $[0,1]$ ).replace(sigma, lambda x: 8*x**3)


$$\frac{d}{dn_{12}}8n_{12}^3$$

Sele $[0,1]$ .diff(Nelem $[0,1]$ ).replace(sigma, lambda x: 8*x**3).doit()


$$24n_{12}^2$$

# ### GOT IT: can replace now with expression and do derivative with respect to that expres.
Sele $[0,1]$ .diff(Nelem $[0,1]$ ).subs({Nelem $[0,1]$  : Nspec $[0,1]$ }).replace(sigma, lambda x: 8*x**3)


$$24(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13})^2$$

Sele $[0,1]$ .subs({Nelem $[0,1]$  : Nspec $[0,1]$ }).diff(X $[0,1]$ )#.subs({Nelem $[0,1]$  : Nspec $[0,1]$ })


$$w_{22} \left.\frac{d}{d\xi_1}\sigma(\xi_1)\right|_{\xi_1=w_{12}x_{11}+w_{22}x_{12}+w_{32}x_{13}}$$

Sele $m$ 


$$\begin{bmatrix} \sigma(n_{11}) & \sigma(n_{12}) \\ \sigma(n_{21}) & \sigma(n_{22}) \\ \sigma(n_{31}) & \sigma(n_{32}) \end{bmatrix}$$

nt = Nelem.subs(elemToSpecFunc); nt


$$\begin{bmatrix} n_{11}(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}) & n_{12}(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13}) \\ n_{21}(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23}) & n_{22}(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23}) \\ n_{31}(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33}) & n_{32}(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33}) \end{bmatrix}$$

st = Sele $m$ .subs(elemToSpecFunc); st


$$\begin{bmatrix} \sigma(n_{11}(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})) & \sigma(n_{12}(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13})) \\ \sigma(n_{21}(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23})) & \sigma(n_{22}(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23})) \\ \sigma(n_{31}(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33})) & \sigma(n_{32}(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})) \end{bmatrix}$$


```

`st.diff(nt)`

$$\begin{bmatrix} \left[\begin{array}{c} \frac{\partial}{\partial n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})} \sigma(n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})) \\ 0 \\ 0 \\ 0 \end{array} \right] & 0 \\ \left[\begin{array}{c} \frac{\partial}{\partial n_{21} (w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23})} \sigma(n_{21} (w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23})) \\ 0 \\ 0 \\ 0 \end{array} \right] & 0 \\ \left[\begin{array}{c} \frac{\partial}{\partial n_{31} (w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33})} \sigma(n_{31} (w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33})) \\ 0 \\ 0 \\ 0 \end{array} \right] & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{\partial}{\partial n_{12} (w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13})} \sigma(n_{12} (w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13})) \\ 0 \\ 0 \\ 0 \\ \frac{\partial}{\partial n_{22} (w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23})} \sigma(n_{22} (w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23})) \\ 0 \\ 0 \\ 0 \\ \frac{\partial}{\partial n_{32} (w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})} \sigma(n_{32} (w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})) \end{bmatrix}$$

`st[0,0].diff(st[0,0].args[0])`

$$\frac{\partial}{\partial n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})} \sigma(n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}))$$

`st[0,0].diff(X[0,0])`

$$w_{11} \frac{\partial}{\partial n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})} \sigma(n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})) \frac{d}{d\xi_1} n_{11}(\xi_1) \Big|_{\xi_1 = w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}}$$

`st[0,0].diff(st[1,0].args[0])`

0

`Selem.diff(Nelem)`

$$\begin{bmatrix} \left[\begin{array}{c} \frac{d}{dn_{11}} \sigma(n_{11}) \\ 0 \\ 0 \\ 0 \end{array} \right] & \left[\begin{array}{c} \frac{d}{dn_{12}} \sigma(n_{12}) \\ 0 \\ 0 \\ 0 \end{array} \right] \\ \left[\begin{array}{c} \frac{d}{dn_{21}} \sigma(n_{21}) \\ 0 \\ 0 \\ 0 \end{array} \right] & \left[\begin{array}{c} \frac{d}{dn_{22}} \sigma(n_{22}) \\ 0 \\ 0 \\ 0 \end{array} \right] \\ \left[\begin{array}{c} \frac{d}{dn_{31}} \sigma(n_{31}) \\ 0 \\ 0 \\ 0 \end{array} \right] & \left[\begin{array}{c} \frac{d}{dn_{32}} \sigma(n_{32}) \\ 0 \\ 0 \\ 0 \end{array} \right] \end{bmatrix}$$

`Selem.diff(Nelem).subs(elemToSpecFunc)`

$$\begin{bmatrix} \left[\begin{array}{c} \frac{\partial}{\partial n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})} \sigma(n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})) \\ 0 \\ 0 \\ 0 \end{array} \right] & 0 \\ \left[\begin{array}{c} \frac{\partial}{\partial n_{21} (w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23})} \sigma(n_{21} (w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23})) \\ 0 \\ 0 \\ 0 \end{array} \right] & 0 \\ \left[\begin{array}{c} \frac{\partial}{\partial n_{31} (w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33})} \sigma(n_{31} (w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33})) \\ 0 \\ 0 \\ 0 \end{array} \right] & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{\partial}{\partial n_{12} (w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13})} \sigma(n_{12} (w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13})) \\ 0 \\ 0 \\ 0 \\ \frac{\partial}{\partial n_{22} (w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23})} \sigma(n_{22} (w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23})) \\ 0 \\ 0 \\ 0 \\ \frac{\partial}{\partial n_{32} (w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})} \sigma(n_{32} (w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})) \end{bmatrix}$$

*# CAN even replace elements after have done an operation on them!!! replacing $n_{21} * 2$ with*
`Sspec.subs({Nspec[0, 0]: 3}).replace(sigma, lambda x: 2 * x).replace(Nspec[2, 1] * 2, 4)`

$$\begin{bmatrix} 6 & 2w_{12}x_{11} + 2w_{22}x_{12} + 2w_{32}x_{13} \\ 2w_{11}x_{21} + 2w_{21}x_{22} + 2w_{31}x_{23} & 2w_{12}x_{21} + 2w_{22}x_{22} + 2w_{32}x_{23} \\ 2w_{11}x_{31} + 2w_{21}x_{32} + 2w_{31}x_{33} & 4 \end{bmatrix}$$

`lambd = Function("lambda")`

`lambd_ = lambda matrix : sum(matrix)`

`vN(X, W)`

$$\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix}$$

`vN(A, B)`

$$\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix}$$

`L = lambd(S); L`

$\lambda(\sigma_{apply}(v(X, W)))$

`Nelem`

$$\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix}$$

`L.replace(n, vN)`

$$\lambda\left(\sigma_{apply}\left(\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix}\right)\right)$$

`L.replace(n, vN).replace(sigmaApply, sigmaApply_)`

$$\lambda\left(\begin{bmatrix} \sigma(n_{11}) & \sigma(n_{12}) \\ \sigma(n_{21}) & \sigma(n_{22}) \\ \sigma(n_{31}) & \sigma(n_{32}) \end{bmatrix}\right)$$

`L.replace(n, v)`

$\lambda(\sigma_{apply}(XW))$

`L.replace(n, v).replace(sigmaApply, sigmaApply_)`

$\lambda((d \mapsto \sigma(d))_{\circ}(XW))$


```

L.subs({A:X, B:W}).replace(n, vL).replace(sigmaApply, sigmaApply_)


$$\lambda \left( \begin{bmatrix} \sigma(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}) & \sigma(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13}) \\ \sigma(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23}) & \sigma(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23}) \\ \sigma(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33}) & \sigma(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33}) \end{bmatrix} \right)$$


L.replace(n, vN)


$$\lambda \left( \sigma_{apply} \left( \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \end{bmatrix} \right) \right)$$


L.replace(n, vN).subs({A:X, B:W}).replace(sigmaApply, sigmaApply_).replace(lambd, lambd_)

```

```

σ(n11) + σ(n12) + σ(n21) + σ(n22) + σ(n31) + σ(n32)

from sympy import symbols, Derivative

x, y, r, t = symbols('x y r t') # r (radius), t (angle theta)
f, g, h = symbols('f g h', cls=Function)
h = g(f(x))

```

```
Derivative(h, f(x)).doit()
```

$$\frac{d}{df(x)}g(f(x))$$

```

# Never do this gives recursion ERROR (max depth exceeded)
# h = g(f(A))
# Derivative(h, A).doit()

```

```
from sympy.abc import a, b
```

```
Llower = lambd(sigmaApply(n(a, b)))
Llower
```

```
λ(σapply(v(a, b)))
```

```
Derivative(Llower, a).doit()
```

$$\frac{\partial}{\partial \sigma_{apply}(v(a, b))} \lambda(\sigma_{apply}(v(a, b))) \frac{\partial}{\partial v(a, b)} \sigma_{apply}(v(a, b)) \frac{\partial}{\partial a} v(a, b)$$

```

# ### WAY 1: of substituting to differentiate with respect to expression:
n_ij = Function('n_ij')

```

```

n_ij(A,B) # (N[0,0]); n_ij
n_ij(X,W)
n_ij(A,B).args
(X, W)

# sigma(n_ij).diff(n_ij).replace(n_ij, N[0,0]) # ERROR cannot deriv wi.r.t to the expression

sigma(n_ij(A,B)).diff(n_ij(A,B))


$$\frac{\partial}{\partial n_{ij}(X,W)} \sigma(n_{ij}(X,W))$$


sigma(n_ij(*X,*W)).diff(X[0,0])


$$\frac{\partial}{\partial x_{11}} n_{ij}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}) \frac{\partial}{\partial n_{ij}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32})}$$


nab_ij = n_ij(A,B)
sigma(nab_ij).diff(nab_ij)#.subs({nab_ij : Nspec[0, 0]})


$$\frac{\partial}{\partial n_{ij}(X,W)} \sigma(n_{ij}(X,W))$$


sigma(nab_ij).diff(nab_ij).subs({nab_ij : Nspec[2, 1]})


$$\left. \frac{d}{d\xi} \sigma(\xi) \right|_{\xi=w_{12}x_{31}+w_{22}x_{32}+w_{32}x_{33}}$$


sigma(nab_ij).diff(nab_ij).subs({nab_ij : Nspec[2,1]}).subs({X[2,1]:77777})


$$\left. \frac{d}{d\xi} \sigma(\xi) \right|_{\xi=w_{12}x_{31}+77777w_{22}+w_{32}x_{33}}$$


sigma(nab_ij).diff(nab_ij).subs({nab_ij : 23}) # ERROR if using replace() since it says can


$$\left. \frac{d}{d\xi} \sigma(\xi) \right|_{\xi=23}$$


sigma(nab_ij).diff(nab_ij).subs({nab_ij : Nspec[2,1]}).doit()


$$\left. \frac{d}{d\xi} \sigma(\xi) \right|_{\xi=w_{12}x_{31}+w_{22}x_{32}+w_{32}x_{33}}$$


sigma(nab_ij).subs({nab_ij : Nspec[2,1]})#.diff(X[2,1])


$$\sigma(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})$$


# Substituting the value of the function n_ij first, and THEN differentiating with respect to
sigma(nab_ij).subs({nab_ij : Nspec[2,1]}).diff(X[2,1])


$$w_{22} \left. \frac{d}{d\xi_1} \sigma(\xi_1) \right|_{\xi_1=w_{12}x_{31}+w_{22}x_{32}+w_{32}x_{33}}$$


```

```
Selelem[2,1].subs({Nelem[2,1] : Nspec[2,1]}).diff(X[2,1])
```

$$w_{22} \frac{d}{d\xi_1} \sigma(\xi_1) \Big|_{\xi_1=w_{12}x_{31}+w_{22}x_{32}+w_{32}x_{33}}$$

```
# ### WAY 2:
```

```
n_11 = Function('n_11')(Nspec[0, 0]); n_11
```

```
n_11 (w_11x_11 + w_21x_12 + w_31x_13)
```

```
sigma(n_11)
```

```
sigma(n_11 (w_11x_11 + w_21x_12 + w_31x_13))
```

```
assert Nspec[0,0] == n_11.args[0]
```

```
sigma(n_11).subs({n_11 : n_11.args[0]})
```

```
sigma(w_11x_11 + w_21x_12 + w_31x_13)
```

```
sigma(n_11).diff(n_11) #.replace(n_ij, n_ij.args[0])
```

$$\frac{\partial}{\partial n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})} \sigma(n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}))$$

```
sigma(n_11).diff(n_11).subs({n_11 : n_11.args[0]}).subs({X[0,0]:77777})
```

$$\frac{d}{d\xi} \sigma(\xi) \Big|_{\xi=77777w_{11}+w_{21}x_{12}+w_{31}x_{13}}$$

```
sigma(n_11).diff(n_11).subs({n_11 : n_11.args[0]}).replace(n_11.args[0], 23) # same as subs
```

$$\frac{d}{d\xi} \sigma(\xi) \Big|_{\xi=23}$$

```
sigma(n_11).diff(X[0,0])
```

$$w_{11} \frac{\partial}{\partial n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})} \sigma(n_{11} (w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13})) \frac{d}{d\xi_1} n_{11} (\xi_1) \Big|_{\xi_1=w_{11}x_{11}+w_{21}x_{12}+w_{31}x_{13}}$$

```
id = Lambda(x, x)
```

```
sigma(n_11).diff(X[0,0]).subs({n_11 : id})
```

$$w_{11} \frac{d}{d\xi_1} n_{11} (\xi_1) \Big|_{\xi_1=w_{11}x_{11}+w_{21}x_{12}+w_{31}x_{13}} \frac{d}{d\xi} \sigma(\xi) \Big|_{\xi=(x \mapsto x)}$$

```
# NOTE: so I don't think WAY 2 is correct because here it doesn't simplify the derivative d
```

```
sigma(n_11).diff(X[0,0]).subs({n_11 : Nspec[0,0]})
```

$$w_{11} \frac{d}{d\xi_1} n_{11} (\xi_1) \Big|_{\xi_1=w_{11}x_{11}+w_{21}x_{12}+w_{31}x_{13}} \frac{d}{d\xi} \sigma(\xi) \Big|_{\xi=w_{11}x_{11}+w_{21}x_{12}+w_{31}x_{13}}$$

```

# CORRECT WAY 1
sigma(n_11).subs({n_11 : Nspec[0,0]}).diff(X[0,0])


$$w_{11} \frac{d}{d\xi_1} \sigma(\xi_1) \Big|_{\xi_1 = w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}}$$


# CORRECT WAY 2

sigma(nab_ij).subs({nab_ij : Nspec[0,0]}).diff(X[0,0])


$$w_{11} \frac{d}{d\xi_1} \sigma(\xi_1) \Big|_{\xi_1 = w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}}$$


# CORRECT WAY 3
Selelem[2,1].subs({Nelem[2,1] : Nspec[2,1]}).diff(X[2,1])


$$w_{22} \frac{d}{d\xi_1} \sigma(\xi_1) \Big|_{\xi_1 = w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33}}$$


sigma(n_11) # WAY 1: sigma argument is already hardcoded
sigma(n_11(w_11x_11 + w_21x_12 + w_31x_13))
sigma(nab_ij) # Way 2: sigma argument is function of matrixsymbol (better than 1)
sigma(n_ij(X,W))
Selelem[2,1] # WAY 3: sigma argument is just symbol and we replace it as function with argument

sigma(n_32)
L
lambda(sigma_apply(v(X,W)))
assert Selelem == S.replace(n, vN).replace(sigmaApply, sigmaApply_)

Selelem

$$\begin{bmatrix} \sigma(n_{11}) & \sigma(n_{12}) \\ \sigma(n_{21}) & \sigma(n_{22}) \\ \sigma(n_{31}) & \sigma(n_{32}) \end{bmatrix}$$

L.replace(n, vN).replace(sigmaApply, sigmaApply_)
lambda(
$$\begin{bmatrix} \sigma(n_{11}) & \sigma(n_{12}) \\ \sigma(n_{21}) & \sigma(n_{22}) \\ \sigma(n_{31}) & \sigma(n_{32}) \end{bmatrix}$$
)
#L.replace(n, vN).replace(sigmaApply, sigmaApply_).diff(Nelem[0,0])
Lsum = L.replace(n, vN).replace(sigmaApply, sigmaApply_).replace(lambda, lambda_)
Lsum

```

$$\sigma(n_{11}) + \sigma(n_{12}) + \sigma(n_{21}) + \sigma(n_{22}) + \sigma(n_{31}) + \sigma(n_{32})$$

`Lsum.diff(Nelem)`

$$\begin{bmatrix} \frac{d}{dn_{11}}\sigma(n_{11}) & \frac{d}{dn_{12}}\sigma(n_{12}) \\ \frac{d}{dn_{21}}\sigma(n_{21}) & \frac{d}{dn_{22}}\sigma(n_{22}) \\ \frac{d}{dn_{31}}\sigma(n_{31}) & \frac{d}{dn_{32}}\sigma(n_{32}) \end{bmatrix}$$

`Lsum.subs(elemToSpec)#.diff(X[2,1])`

$$\sigma(w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13}) + \sigma(w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23}) + \sigma(w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33}) + \\ \sigma(w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13}) + \sigma(w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23}) + \sigma(w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33})$$

`Lsum.subs(elemToSpec).diff(X)`

$$\begin{bmatrix} w_{11} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{11}x_{11}+w_{21}x_{12}+w_{31}x_{13}} & + w_{12} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{12}x_{11}+w_{22}x_{12}+w_{32}x_{13}} & w_{21} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{11}x_{11}+w_{21}x_{12}+w_{31}x_{13}} \\ w_{11} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{11}x_{21}+w_{21}x_{22}+w_{31}x_{23}} & + w_{12} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{12}x_{21}+w_{22}x_{22}+w_{32}x_{23}} & w_{21} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{11}x_{21}+w_{21}x_{22}+w_{31}x_{23}} \\ w_{11} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{11}x_{31}+w_{21}x_{32}+w_{31}x_{33}} & + w_{12} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{12}x_{31}+w_{22}x_{32}+w_{32}x_{33}} & w_{21} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=w_{11}x_{31}+w_{21}x_{32}+w_{31}x_{33}} \end{bmatrix}$$

`specToElemD = {v : k for k, v in elemToSpecD.items() }`

`Lsum.subs(elemToSpecD).diff(X).subs(specToElemD)`

$$\begin{bmatrix} w_{11} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{11}} & + w_{12} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{12}} & w_{21} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{11}} & + w_{22} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{12}} & w_{31} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{11}} & + \\ w_{11} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{21}} & + w_{12} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{22}} & w_{21} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{21}} & + w_{22} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{22}} & w_{31} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{21}} & + \\ w_{11} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{31}} & + w_{12} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{32}} & w_{21} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{31}} & + w_{22} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{32}} & w_{31} \frac{d}{d\xi_1}\sigma(\xi_1) \Big|_{\xi_1=n_{31}} & + \end{bmatrix}$$