```
from IPython.display import Markdown

from sympy import Matrix, Symbol, derive_by_array, Lambda, symbols,
Derivative, diff
from sympy.abc import x, y, i, j, a, b
```

```
# Defining variable-element matrices $X \in \mathbb{R}^{n \times m}$ and $W
\in \mathbb{R}^{m \times p}$:
```

```
def var(letter: str, i: int, j: int) -> Symbol:
    letter_ij = Symbol('{}_{}{}'.format(letter, i+1, j+1),
is_commutative=True)
    return letter_ij


n,m,p = 3,3,2

X = Matrix(n, m, lambda i,j : var('x', i, j)); X
```

```
$\displaystyle \left[\begin{matrix}x_{11} & x_{12} & x_{13}\\x_{21} &
x_{22} & x_{23}\\x_{31} & x_{32} & x_{33}\end{matrix}\right]$
```

```
W = Matrix(m, p, lambda i,j : var('w', i, j)); W
```

```
$\displaystyle \left[\begin{matrix}w_{11} & w_{12}\\w_{21} & w_{22}\\w_{31}
& w_{32}\end{matrix}\right]$
```

```
# Defining $N = \nu(X, W) = X \times W$
#
# * $\nu : \mathbb{R}^{(n \times m) \times (m \times p)} \rightarrow
\mathbb{R}^{n \times p}$
# * $N \in \mathbb{R}^{n \times p}$
```

```
v = Lambda((a,b), a*b); v
```

```
$\displaystyle \left( \left( a, \  b\right) \mapsto a b \right)$
```

```
N = v(X, W); N
```

```
$\displaystyle \left[\begin{matrix}w_{11} x_{11} + w_{21} x_{12} + w_{31}
x_{13} & w_{12} x_{11} + w_{22} x_{12} + w_{32} x_{13}\\w_{11} x_{21} +
w_{21} x_{22} + w_{31} x_{23} & w_{12} x_{21} + w_{22} x_{22} + w_{32}
x_{23}\\w_{11} x_{31} + w_{21} x_{32} + w_{31} x_{33} & w_{12} x_{31} +
w_{22} x_{32} + w_{32} x_{33}\end{matrix}\right]$
```

```
# Defining $S = \sigma_{\text{apply}}(N) = \sigma_{\text{apply}}(\nu(X,W))
= \sigma_\text{apply}(X \times W) = \Big \{ \sigma(XW_{ij}) \Big\}$.
#
#
# Assume that $\sigma_{\text{apply}} : \mathbb{R}^{n \times p} \rightarrow
\mathbb{R}^{n \times p}$ while $\sigma : \mathbb{R} \rightarrow
\mathbb{R}$, so the function $\sigma_{\text{apply}}$ takes in a matrix and
returns a matrix while the simple $\sigma$ acts on the individual elements
$N_{ij} = XW_{ij}$ in the matrix argument $N$ of $\sigma_{\text{apply}}$.
#
# * $\sigma : \mathbb{R} \rightarrow \mathbb{R}$
# * $\sigma_\text{apply} : \mathbb{R}^{n \times p} \rightarrow
\mathbb{R}^{n \times p}$
# * $S \in \mathbb{R}^{n \times p}$
```

```
from sympy import Function

# Nvec = Symbol('N', commutative=False)

sigma = Function('sigma')
sigma(N[0,0])
```

```
$\displaystyle \sigma{\left(w_{11} x_{11} + w_{21} x_{12} + w_{31} x_{13}
\right)}$
```

```
# way 1 of declaring S
S = N.applyfunc(sigma); S
```

```
$\displaystyle \left[\begin{matrix}\sigma{\left(w_{11} x_{11} + w_{21}
x_{12} + w_{31} x_{13} \right)} & \sigma{\left(w_{12} x_{11} + w_{22}
x_{12} + w_{32} x_{13} \right)}\\\sigma{\left(w_{11} x_{21} + w_{21} x_{22}
+ w_{31} x_{23} \right)} & \sigma{\left(w_{12} x_{21} + w_{22} x_{22} +
w_{32} x_{23} \right)}\\\sigma{\left(w_{11} x_{31} + w_{21} x_{32} + w_{31}
x_{33} \right)} & \sigma{\left(w_{12} x_{31} + w_{22} x_{32} + w_{32}
x_{33} \right)}\end{matrix}\right]$
```

```
# way 2 of declaring S (better way)
sigmaApply = lambda matrix:  matrix.applyfunc(sigma)

sigmaApply(N)
```

```
$\displaystyle \left[\begin{matrix}\sigma{\left(w_{11} x_{11} + w_{21}
x_{12} + w_{31} x_{13} \right)} & \sigma{\left(w_{12} x_{11} + w_{22}
x_{12} + w_{32} x_{13} \right)}\\\sigma{\left(w_{11} x_{21} + w_{21} x_{22}
+ w_{31} x_{23} \right)} & \sigma{\left(w_{12} x_{21} + w_{22} x_{22} +
w_{32} x_{23} \right)}\\\sigma{\left(w_{11} x_{31} + w_{21} x_{32} + w_{31}
x_{33} \right)} & \sigma{\left(w_{12} x_{31} + w_{22} x_{32} + w_{32}
x_{33} \right)}\end{matrix}\right]$
```

```
sigmaApply(X**2) # can apply this function to any matrix argument.
```

```
$\displaystyle \left[\begin{matrix}\sigma{\left(x_{11}^{2} + x_{12} x_{21}
+ x_{13} x_{31} \right)} & \sigma{\left(x_{11} x_{12} + x_{12} x_{22} +
x_{13} x_{32} \right)} & \sigma{\left(x_{11} x_{13} + x_{12} x_{23} +
x_{13} x_{33} \right)}\\\sigma{\left(x_{11} x_{21} + x_{21} x_{22} + x_{23}
x_{31} \right)} & \sigma{\left(x_{12} x_{21} + x_{22}^{2} + x_{23} x_{32}
\right)} & \sigma{\left(x_{13} x_{21} + x_{22} x_{23} + x_{23} x_{33}
\right)}\\\sigma{\left(x_{11} x_{31} + x_{21} x_{32} + x_{31} x_{33}
\right)} & \sigma{\left(x_{12} x_{31} + x_{22} x_{32} + x_{32} x_{33}
\right)} & \sigma{\left(x_{13} x_{31} + x_{23} x_{32} + x_{33}^{2}
\right)}\end{matrix}\right]$
```

```
S = sigmaApply(v(X,W)) # composing
S
```

```
$\displaystyle \left[\begin{matrix}\sigma{\left(w_{11} x_{11} + w_{21}
x_{12} + w_{31} x_{13} \right)} & \sigma{\left(w_{12} x_{11} + w_{22}
```

```
x_{12} + w_{32} x_{13} \right)}\\\sigma{\left(w_{11} x_{21} + w_{21} x_{22}
+ w_{31} x_{23} \right)} & \sigma{\left(w_{12} x_{21} + w_{22} x_{22} +
w_{32} x_{23} \right)}\\\sigma{\left(w_{11} x_{31} + w_{21} x_{32} + w_{31}
x_{33} \right)} & \sigma{\left(w_{12} x_{31} + w_{22} x_{32} + w_{32}
x_{33} \right)}\end{matrix}\right]$
```