Sympy_DerivMatrix

October 5, 2020

[1]:

[1]:
```
from sympy import diff, sin, exp, symbols, Function, Matrix,␣
 ↪MatrixSymbol, FunctionMatrix, derive_by_array


from sympy import Symbol


def var(letter: str, i: int, j: int) -> Symbol:
     letter_ij = Symbol('{}_{}{}'.format(letter, i+1, j+1),␣
 ↪is_commutative=True)
     return letter_ij

def func(i, j):
     y_ij = Function('y_{}{}'.format(i+1,j+1))(*X)
     return y_ij


n,m,p = 3,3,2

X = Matrix(n, m, lambda i, j: var('x', i, j)); X
```

[1]:
$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

[2]:
```
#Y = MatrixSymbol(Function('y'), 2, 3); Matrix(Y)
#M = MatrixSymbol('M',2,2); Matrix(M)
```

```
#Y = Matrix(m, p, lambda i,j: Function('y_{}{}'.
    ↪format(i+1,j+1))(X) ); Y

Y = Matrix(m, p, lambda i,j:  func(i, j)); Y
```

[2]:

$$\begin{bmatrix} y_{11}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & y_{12}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ y_{21}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & y_{22}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \\ y_{31}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) & y_{32}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) \end{bmatrix}$$

0.0.1 Derivative of Matrix With Respect a Matrix

Let $X = \{x_{ij}\}$ be a matrix of order $m \times n$ and let

$$y = f(X)$$

be a scalar function of X, so $y \in \mathbb{R}$ and $f : \mathbb{R}^{m \times n} \to \mathbb{R}$,

Also let the matrix $Y = \{y_{ij}(X)\}$ be of size $p \times q$.

Then we can define the derivative of Y with respect to X as the following matrix of order $mp \times nq$:

$$\frac{\partial Y}{\partial X} = \begin{pmatrix} \frac{\partial Y}{\partial x_{11}} & \frac{\partial Y}{\partial x_{12}} & \dots & \frac{\partial Y}{\partial x_{1n}} \\ \frac{\partial Y}{\partial x_{21}} & \frac{\partial Y}{\partial x_{22}} & \dots & \frac{\partial Y}{\partial x_{23}} \\ \vdots & \vdots & & \vdots \\ \frac{\partial Y}{\partial x_{m1}} & \frac{\partial Y}{\partial x_{m2}} & \dots & \frac{\partial Y}{\partial x_{mn}} \end{pmatrix} = \left\{ \frac{\partial y_{ij}}{\partial x_{lk}} \right\}$$

[3]:

```
# GOT IT this is the definition of gradient matrix (matrix of
    ↪partial derivatives or dY/dX)
D = derive_by_array(Y, X); D
```

[3]:

$$\left[\begin{matrix}\left[\begin{matrix}\frac{\partial}{\partial x_{11}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{11}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{11}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{12}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{12}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{12}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{13}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{13}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{13}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\\\left[\begin{matrix}\frac{\partial}{\partial x_{21}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{21}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{21}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{22}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{22}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{22}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{23}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{23}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{23}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\\\left[\begin{matrix}\frac{\partial}{\partial x_{31}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{31}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{31}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{32}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{32}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{32}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{33}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{33}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{33}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\end{matrix}\right]$$

[4]:
```
D.subs({Y[0,0]: X[0,0]**2 + X[1,0]}).doit()
```

[4]:

$$\left[\begin{matrix}\left[\begin{matrix}2 x_{11} & \frac{\partial}{\partial x_{11}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{11}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{11}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}0 & \frac{\partial}{\partial x_{12}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{12}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{12}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}0 & \frac{\partial}{\partial x_{13}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{13}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{13}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\\\left[\begin{matrix}1 & \frac{\partial}{\partial x_{21}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{21}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{21}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}0 & \frac{\partial}{\partial x_{22}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{22}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{22}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}0 & \frac{\partial}{\partial x_{23}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{23}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{23}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\\\left[\begin{matrix}0 & \frac{\partial}{\partial x_{31}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{31}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{31}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}0 & \frac{\partial}{\partial x_{32}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{32}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{32}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}0 & \frac{\partial}{\partial x_{33}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{33}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{33}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\end{matrix}\right]$$

[5]:
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Y.diff(X) ## GOT IT
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[5]:

$$\left[\begin{matrix}\left[\begin{matrix}\frac{\partial}{\partial x_{11}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{11}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{11}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{11}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{12}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{12}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{12}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{12}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{13}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{13}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{13}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{13}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\\\left[\begin{matrix}\frac{\partial}{\partial x_{21}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{21}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{21}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{21}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{22}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{22}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{22}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{22}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{23}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{23}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{23}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{23}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\\\left[\begin{matrix}\frac{\partial}{\partial x_{31}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{31}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{31}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{31}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{32}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{32}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{32}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{32}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right] & \left[\begin{matrix}\frac{\partial}{\partial x_{33}} y_{11}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{12}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{33}} y_{21}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{22}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\\\frac{\partial}{\partial x_{33}} y_{31}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right) & \frac{\partial}{\partial x_{33}} y_{32}\left(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}\right)\end{matrix}\right]\end{matrix}\right]$$

[6]:
```
Yval = Y.subs({Y[0,0]: X[0,0]**2 + X[0,1]*X[1,0] - X[1,1],
         Y[0,1]: X[1,1]**3 + 4* X[0,1] + X[0,0] - X[1,0],
         Y[1,0]: X[1,0] * X[0,0] + 3*X[0,1] * X[1,1],
```

```python
        Y[1,1]: X[1,1] + X[1,0] + X[0,1] + X[0,0],
        Y[2,0]: 2*X[0,0]**2 * X[0,1] * 3*X[1,0] + 4*X[1,1],
        Y[2,1]: 3*X[0,1] - 5*X[1,1] * X[0,0] - X[1,0]**2})

Yval
```

[6]:

$$\begin{bmatrix} x_{11}^{2} + x_{12}x_{21} - x_{22} & x_{11} + 4x_{12} - x_{21} + x_{22}^{3} \\ x_{11}x_{21} + 3x_{12}x_{22} & x_{11} + x_{12} + x_{21} + x_{22} \\ 6x_{11}^{2}x_{12}x_{21} + 4x_{22} & -5x_{11}x_{22} + 3x_{12} - x_{21}^{2} \end{bmatrix}$$

[7]:
```python
DYval = D.subs({Y[0,0]: X[0,0]**2 + X[0,1]*X[1,0] - X[1,1],
        Y[0,1]: X[1,1]**3 + 4* X[0,1] + X[0,0] - X[1,0],
        Y[1,0]: X[1,0] * X[0,0] + 3*X[0,1] * X[1,1],
        Y[1,1]: X[1,1] + X[1,0] + X[0,1] + X[0,0],
        Y[2,0]: 2*X[0,0]**2 * X[0,1] * 3*X[1,0] + 4*X[1,1],
        Y[2,1]: 3*X[0,1] - 5*X[1,1] * X[0,0] - X[1,0]**2})
DYval
```

[7]:

$$\left[\begin{matrix}
\begin{bmatrix} \frac{\partial}{\partial x_{11}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{11}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{11}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{11}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{11}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{11}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} &
\begin{bmatrix} \frac{\partial}{\partial x_{12}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{12}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{12}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{12}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{12}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{12}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} &
\begin{bmatrix} \frac{\partial}{\partial x_{13}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{13}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{13}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{13}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{13}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{13}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} \\
\begin{bmatrix} \frac{\partial}{\partial x_{21}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{21}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{21}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{21}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{21}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{21}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} &
\begin{bmatrix} \frac{\partial}{\partial x_{22}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{22}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{22}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{22}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{22}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{22}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} &
\begin{bmatrix} \frac{\partial}{\partial x_{23}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{23}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{23}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{23}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{23}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{23}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} \\
\begin{bmatrix} \frac{\partial}{\partial x_{31}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{31}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{31}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{31}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{31}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{31}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} &
\begin{bmatrix} \frac{\partial}{\partial x_{32}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{32}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{32}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{32}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{32}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{32}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix} &
\begin{bmatrix} \frac{\partial}{\partial x_{33}}\left(x_{11}^{2} + x_{12}x_{21} - x_{22}\right) & \frac{\partial}{\partial x_{33}}\left(x_{11} + 4x_{12} - x_{21} + x_{22}^{3}\right) \\ \frac{\partial}{\partial x_{33}}\left(x_{11}x_{21} + 3x_{12}x_{22}\right) & \frac{\partial}{\partial x_{33}}\left(x_{11} + x_{12} + x_{21} + x_{22}\right) \\ \frac{\partial}{\partial x_{33}}\left(6x_{11}^{2}x_{12}x_{21} + 4x_{22}\right) & \frac{\partial}{\partial x_{33}}\left(-5x_{11}x_{22} + 3x_{12} - x_{21}^{2}\right) \end{bmatrix}
\end{matrix}\right]$$

[8]:
```python
DYval.doit()
```

[8]:

$$\left[\begin{matrix}
\begin{bmatrix} 2x_{11} & 1 \\ x_{21} & 1 \\ 12x_{11}x_{12}x_{21} & -5x_{22} \end{bmatrix} & \begin{bmatrix} x_{21} & 4 \\ 3x_{22} & 1 \\ 6x_{11}^{2}x_{21} & 3 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
\begin{bmatrix} x_{12} & -1 \\ x_{11} & 1 \\ 6x_{11}^{2}x_{12} & -2x_{21} \end{bmatrix} & \begin{bmatrix} -1 & 3x_{22}^{2} \\ 3x_{12} & 1 \\ 4 & -5x_{11} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\end{matrix}\right]$$

[9]:
```
# ### GOAL: testing the A kronecker B rule for diff of Y = AXB
from sympy import Lambda
l, m, n, q = 3, 5, 4, 2

A = Matrix(l, m, lambda i, j: var('a', i, j))
X = Matrix(m, n, lambda i, j: var('x', i, j))
W = Matrix(n, q, lambda i, j: var('w', i, j))
Y = X*W; Y
```

[9]:
$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} + w_{41}x_{14} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} + w_{42}x_{14} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} + w_{41}x_{24} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} + w_{42}x_{24} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} + w_{41}x_{34} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} + w_{42}x_{34} \\ w_{11}x_{41} + w_{21}x_{42} + w_{31}x_{43} + w_{41}x_{44} & w_{12}x_{41} + w_{22}x_{42} + w_{32}x_{43} + w_{42}x_{44} \\ w_{11}x_{51} + w_{21}x_{52} + w_{31}x_{53} + w_{41}x_{54} & w_{12}x_{51} + w_{22}x_{52} + w_{32}x_{53} + w_{42}x_{54} \end{bmatrix}$$

[10]:
```
from sympy.matrices import zeros
E_12 = zeros(m, n)
E_12[1-1,2-1] = 1
E_12
```

[10]:
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[11]:
```
Y = X*W; Y
```

[11]:
$$\begin{bmatrix} w_{11}x_{11} + w_{21}x_{12} + w_{31}x_{13} + w_{41}x_{14} & w_{12}x_{11} + w_{22}x_{12} + w_{32}x_{13} + w_{42}x_{14} \\ w_{11}x_{21} + w_{21}x_{22} + w_{31}x_{23} + w_{41}x_{24} & w_{12}x_{21} + w_{22}x_{22} + w_{32}x_{23} + w_{42}x_{24} \\ w_{11}x_{31} + w_{21}x_{32} + w_{31}x_{33} + w_{41}x_{34} & w_{12}x_{31} + w_{22}x_{32} + w_{32}x_{33} + w_{42}x_{34} \\ w_{11}x_{41} + w_{21}x_{42} + w_{31}x_{43} + w_{41}x_{44} & w_{12}x_{41} + w_{22}x_{42} + w_{32}x_{43} + w_{42}x_{44} \\ w_{11}x_{51} + w_{21}x_{52} + w_{31}x_{53} + w_{41}x_{54} & w_{12}x_{51} + w_{22}x_{52} + w_{32}x_{53} + w_{42}x_{54} \end{bmatrix}$$

[12]:
```
E_12*W
```

[12]:
$$\begin{bmatrix} w_{21} & w_{22} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

[13]:
```
derive_by_array(Y, X[0,1])
```

[13]:

$$\begin{bmatrix} w_{21} & w_{22} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

[14]:
```
assert Matrix(derive_by_array(Y, X[0,1])) == E_12 * W

assert Matrix(derive_by_array(Y, X[0,1])) == Y.diff(X[0,1])
```