Sympy_JacobianExample

October 2, 2020

[1]:
```python
from sympy import Matrix, MatrixSymbol, Symbol, derive_by_array



X = Matrix(MatrixSymbol('x', 3,3)); X
W = Matrix(MatrixSymbol('w', 3,2)); W
```

[1]:

$$\begin{bmatrix} w_{0,0} & w_{0,1} \\ w_{1,0} & w_{1,1} \\ w_{2,0} & w_{2,1} \end{bmatrix}$$

[2]:
```python
X*W
```

[2]:

$$\begin{bmatrix} w_{0,0}x_{0,0} + w_{1,0}x_{0,1} + w_{2,0}x_{0,2} & w_{0,1}x_{0,0} + w_{1,1}x_{0,1} + w_{2,1}x_{0,2} \\ w_{0,0}x_{1,0} + w_{1,0}x_{1,1} + w_{2,0}x_{1,2} & w_{0,1}x_{1,0} + w_{1,1}x_{1,1} + w_{2,1}x_{1,2} \\ w_{0,0}x_{2,0} + w_{1,0}x_{2,1} + w_{2,0}x_{2,2} & w_{0,1}x_{2,0} + w_{1,1}x_{2,1} + w_{2,1}x_{2,2} \end{bmatrix}$$

[3]:
```python
derive_by_array(X*W, X)
```

[3]:

$$\begin{bmatrix} \begin{bmatrix} w_{0,0} & w_{0,1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{1,0} & w_{1,1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{2,0} & w_{2,1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{0,0} & w_{0,1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{1,0} & w_{1,1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{2,0} & w_{2,1} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ w_{0,0} & w_{0,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ w_{1,0} & w_{1,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ w_{2,0} & w_{2,1} \end{bmatrix} \end{bmatrix}$$

[4]:
```python
(X*W).diff(X)
```

[4]:

$$\begin{bmatrix} \begin{bmatrix} w_{0,0} & w_{0,1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{1,0} & w_{1,1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{2,0} & w_{2,1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{0,0} & w_{0,1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{1,0} & w_{1,1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{2,0} & w_{2,1} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ w_{0,0} & w_{0,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ w_{1,0} & w_{1,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ w_{2,0} & w_{2,1} \end{bmatrix} \end{bmatrix}$$

[5]:
```
from sympy import diff, sin, exp, symbols, Function
#from sympy.core.multidimensional import vectorize #

#@vectorize(0,1)
#def vdiff(func, arg):
#     return diff(func, arg)




x, y, z = symbols('x y z')
f, g, h = list(map(Function, 'fgh'))

xv = x,y,z
#f(xv).subs({x:1, y:2,z:3})
yv = [f(*xv), g(*xv), h(*xv)]; yv
```

[5]: [f(x, y, z), g(x, y, z), h(x, y, z)]

[6]:
```
Matrix(yv)
```

[6]:

$$\begin{bmatrix} f(x,y,z) \\ g(x,y,z) \\ h(x,y,z) \end{bmatrix}$$

[7]:
```
Matrix(xv)
```

[7]:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

[8]:
```
Matrix(yv).jacobian(xv)
```

[8]:
$$\left[\begin{matrix}\frac{\partial}{\partial x} f{\left(x,y,z \right)} & \frac{\partial}{\partial y} f{\left(x,y,z \right)} & \frac{\partial}{\partial z} f{\left(x,y,z \right)}\\\frac{\partial}{\partial x} g{\left(x,y,z \right)} & \frac{\partial}{\partial y} g{\left(x,y,z \right)} & \frac{\partial}{\partial z} g{\left(x,y,z \right)}\\\frac{\partial}{\partial x} h{\left(x,y,z \right)} & \frac{\partial}{\partial y} h{\left(x,y,z \right)} & \frac{\partial}{\partial z} h{\left(x,y,z \right)}\end{matrix}\right]$$

[9]:
```
derive_by_array(yv, xv)
```

[9]:
$$\left[\begin{matrix}\frac{\partial}{\partial x} f{\left(x,y,z \right)} & \frac{\partial}{\partial x} g{\left(x,y,z \right)} & \frac{\partial}{\partial x} h{\left(x,y,z \right)}\\\frac{\partial}{\partial y} f{\left(x,y,z \right)} & \frac{\partial}{\partial y} g{\left(x,y,z \right)} & \frac{\partial}{\partial y} h{\left(x,y,z \right)}\\\frac{\partial}{\partial z} f{\left(x,y,z \right)} & \frac{\partial}{\partial z} g{\left(x,y,z \right)} & \frac{\partial}{\partial z} h{\left(x,y,z \right)}\end{matrix}\right]$$

[10]:
```
assert Matrix(derive_by_array(yv, xv)).transpose() == Matrix(yv).
↪jacobian(xv)
```

[11]:
```
### TEST 2: substituting values
m = Matrix(yv).jacobian(xv)
m.subs({x:1, y:2, z:3})
```

[11]:
$$\left[\begin{matrix}\left. \frac{d}{d x} f{\left(x,2,3 \right)} \right|_{x=1} & \left. \frac{d}{d y} f{\left(1,y,3 \right)} \right|_{y=2} & \left. \frac{d}{d z} f{\left(1,2,z \right)} \right|_{z=3}\\\left. \frac{d}{d x} g{\left(x,2,3 \right)} \right|_{x=1} & \left. \frac{d}{d y} g{\left(1,y,3 \right)} \right|_{y=2} & \left. \frac{d}{d z} g{\left(1,2,z \right)} \right|_{z=3}\\\left. \frac{d}{d x} h{\left(x,2,3 \right)} \right|_{x=1} & \left. \frac{d}{d y} h{\left(1,y,3 \right)} \right|_{y=2} & \left. \frac{d}{d z} h{\left(1,2,z \right)} \right|_{z=3}\end{matrix}\right]$$

[12]:
```
m.subs({f(*xv):x**2 * y*z, g(*xv):sin(x*y*z*3), h(*xv):y + z*exp(x)})
```

[12]:
$$\left[\begin{matrix}\frac{\partial}{\partial x} x^{2} y z & \frac{\partial}{\partial y} x^{2} y z & \frac{\partial}{\partial z} x^{2} y z\\\frac{\partial}{\partial x} \sin{\left(3 x y z \right)} & \frac{\partial}{\partial y} \sin{\left(3 x y z \right)} & \frac{\partial}{\partial z} \sin{\left(3 x y z \right)}\\\frac{\partial}{\partial x} \left(y + z e^{x}\right) & \frac{\partial}{\partial y} \left(y + z e^{x}\right) & \frac{\partial}{\partial z} \left(y + z e^{x}\right)\end{matrix}\right]$$

[13]:
```
m_subs = m.subs({f(*xv):x**2 * y*z, g(*xv):sin(x*y*z*3), h(*xv):y +␣
↪z*exp(x)})
m_subs.doit()
```

[13]:
$$\left[\begin{matrix}2 x y z & x^{2} z & x^{2} y\\3 y z \cos{\left(3 x y z \right)} & 3 x z \cos{\left(3 x y z \right)} & 3 x y \cos{\left(3 x y z \right)}\\z e^{x} & 1 & e^{x}\end{matrix}\right]$$

[14]:
```
m_subs.doit().subs({x:1, y:2, z:3})
```

[14]:

$$\begin{bmatrix} 12 & 3 & 2 \\ 18\cos(18) & 9\cos(18) & 6\cos(18) \\ 3e & 1 & e \end{bmatrix}$$

[15]: