Trigonometric Identities

Basic Identities

$$\sin^{2}(\theta) + \cos^{2}(\theta) = 1$$
$$\sec^{2}(\theta) = 1 + \tan^{2}(\theta)$$
$$\csc^{2}(\theta) = 1 + \cot^{2}(\theta)$$

Sums and Differences

$$\begin{aligned} \cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \\ \cos(\alpha-\beta) &= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \\ \sin(\alpha-\beta) &= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \\ \sin(\frac{\pi}{2}-\theta) &= \cos(\theta) \\ \cos(\frac{\pi}{2}-\theta) &= \sin(\theta) \end{aligned}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$
$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Double Angle

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$\cos(2\theta) = 2\cos^2(\theta) - 1$$
$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$
$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Half Angle

$$\sin^2(\theta) = \frac{1 - \cos(2(\theta))}{2}$$
$$\cos^2(\theta) = \frac{1 + \cos(2(\theta))}{2}$$

Inverse

$$\cos^{-1}(\theta) = \frac{\pi}{2} - \sin^{-1}(\theta)$$
$$\cot^{-1}(\theta) = \frac{\pi}{2} - \tan^{-1}(\theta)$$
$$\csc^{-1}(\theta) = \frac{\pi}{2} - \sec^{-1}(\theta)$$

Product-Sum

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}$$
$$\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$\sin(\alpha)\cos(\beta) = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$
$$\cos(\alpha)\sin(\beta) = \frac{\sin(\alpha+\beta) - \sin(\alpha-\beta)}{2}$$

$$\sin(\alpha - \beta)\sin(\alpha + \beta) = \sin^2(\alpha) - \sin^2(\beta)$$

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Law Of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\omega)}{w}$$

Law Of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$