L1 and L2 Penalty and Sparsity in Logistic Regression

Experiment with MNIST digits. Comparison of the sparsity (percentage of zero coefficients) of solutions when L1 and L2 penalty are used for different values of C. We can see that large values of C give more freedom to the model. Conversely, smaller values of C constrain the model more. In the L1 penalty case, this leads to sparser solutions. We classify 8x8 images of digits into two classes: 0-4 against 5-9. The visualization shows coefficients of the models for varying C.

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        # License: BSD 3 clause
        import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.linear model import LogisticRegression
        from sklearn import datasets
        from sklearn.preprocessing import StandardScaler
        digits = datasets.load_digits()
        X, y = digits.data, digits.target
        X = StandardScaler().fit_transform(X)
        # classify small against large digits
        y = (y > 4).astype(np.int)
        # Set regularization parameter
        for i, C in enumerate((100, 1, 0.01)):
            # turn down tolerance for short training time
            clf_l1_LR = LogisticRegression(C=C, penalty='l1', solver='liblinear', tol=
        0.01)
            clf 12 LR = LogisticRegression(C=C, penalty='12', solver='liblinear', tol=
        0.01)
            clf l1 LR.fit(X, y)
            clf 12 LR.fit(X, y)
            coef 11 LR = clf 11 LR.coef .ravel()
            coef 12 LR = clf 12 LR.coef .ravel()
            # coef_l1_LR contains zeros due to the
            # L1 sparsity inducing norm
            sparsity l1 LR = np.mean(coef l1 LR == 0) * 100
            sparsity 12 LR = np.mean(coef 12 LR == 0) * 100
            print("lambda= ", 1./C)
            print("Sparsity with L1 penalty: %.2f%%" % sparsity_l1_LR)
            print("score with L1 penalty: %.4f" % clf_l1_LR.score(X, y))
            print("Sparsity with L2 penalty: %.2f%" % sparsity 12 LR)
            print("score with L2 penalty: %.4f" % clf l2 LR.score(X, y))
            11_{plot} = plt.subplot(3, 2, 2 * i + 1)
            12 plot = plt.subplot(3, 2, 2 * (i + 1))
            if i == 0:
                l1_plot.set_title("L1 penalty")
                12 plot.set title("L2 penalty")
            11_plot.imshow(np.abs(coef_l1_LR.reshape(8, 8)), interpolation='nearest',
                            cmap='binary', vmax=1, vmin=0)
            12_plot.imshow(np.abs(coef_12_LR.reshape(8, 8)), interpolation='nearest',
                            cmap='binary', vmax=1, vmin=0)
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plt.text(-8, 3, "reg = " + str(1./C))
l1_plot.set_xticks(())
l1_plot.set_yticks(())
12_plot.set_xticks(())
12_plot.set_yticks(())
```

lambda= 0.01 Sparsity with L1 penalty: 6.25% score with L1 penalty: 0.9115 Sparsity with L2 penalty: 4.69% score with L2 penalty: 0.9098 lambda= 1.0 Sparsity with L1 penalty: 9.38% score with L1 penalty: 0.9110 Sparsity with L2 penalty: 4.69% score with L2 penalty: 0.9093 lambda= 100.0 Sparsity with L1 penalty: 85.94%

score with L1 penalty: 0.8620 Sparsity with L2 penalty: 4.69% score with L2 penalty: 0.8915

In [0]: