Lab 2: Part A: Logistic regression and GDA

We explore logistic regression and GDA in this notebook on several simple data sets.

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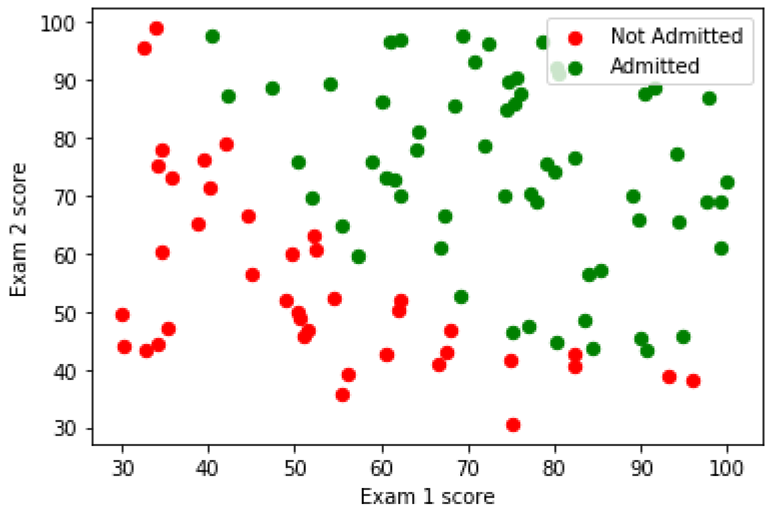
In [1]:
```
import pandas as pd
import random
import numpy as np
import matplotlib.pyplot as plt

import sklearn
from sklearn import linear_model
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import Pipeline
from sklearn.model_selection import cross_val_score
from sklearn.model_selection import learning_curve
from sklearn.model_selection import KFold
from sklearn.model_selection import GridSearchCV


# This is a bit of magic to make matplotlib figures appear inline in the noteb
ook
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipyt
hon
%load_ext autoreload
%autoreload 2
```

Unregularized logistic regression

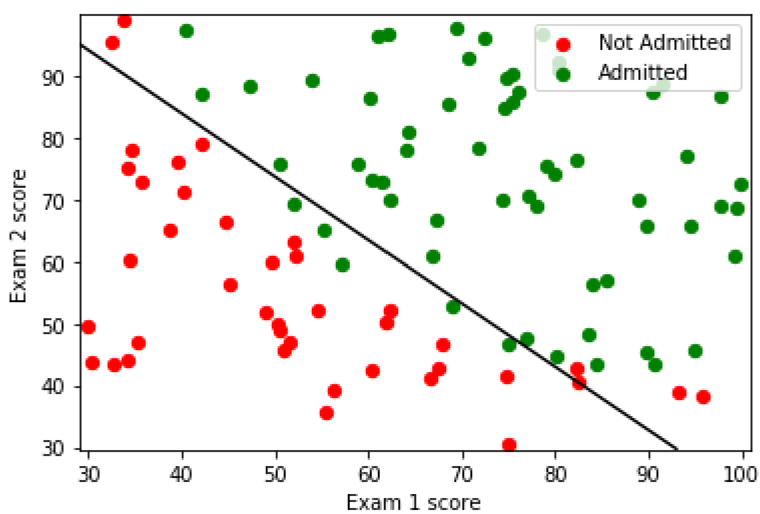
In [2]:
```
data = pd.read_csv('../data/ex1data1.txt')
X = np.vstack([data.x1,data.x2]).T
y = data.y

def plot_twoclass_data(X,y,xlabel,ylabel,legend):
    fig = plt.figure()
    X0 = X[np.where(y==0)]
    X1 = X[np.where(y==1)]
    plt.scatter(X0[:,0],X0[:,1],c='red', s=40, label = legend[0])
    plt.scatter(X1[:,0],X1[:,1],c='green', s = 40, label=legend[1])
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    plt.legend(loc="upper right")

plot_twoclass_data(X,y,'Exam 1 score', 'Exam 2 score',['Not Admitted','Admitte
d'])
```



Fit logistic model and visualize decision boundary

In [3]:
```
alpha = 1e-5
logreg = linear_model.LogisticRegression(C=1.0/alpha,solver='lbfgs',fit_interc
ept=True)
logreg.fit(X,y)

print("Theta found by sklearn: ", logreg.coef_, logreg.intercept_)

def plot_decision_boundary_logistic(X,y,logreg,  xlabel, ylabel, legend):
    plot_twoclass_data(X,y,xlabel,ylabel,legend)

    # create a mesh to plot in
    h = 0.01
    x1_min, x1_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    x2_min, x2_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx1, xx2 = np.meshgrid(np.arange(x1_min, x1_max, h),
                           np.arange(x2_min, x2_max, h))

    # make predictions on this mesh
    Z = np.array(logreg.predict(np.c_[xx1.ravel(), xx2.ravel()]))

    # Put the result into a color contour plot
    Z = Z.reshape(xx1.shape)
    plt.contour(xx1,xx2,Z,cmap=plt.cm.gray,levels=[0.5])

plot_decision_boundary_logistic(X,y,logreg,'Exam 1 score', 'Exam 2 score',['No
t Admitted','Admitted'])
```

```
Theta found by sklearn:  [[0.20623222 0.2014719 ]] [-25.16138457]
```



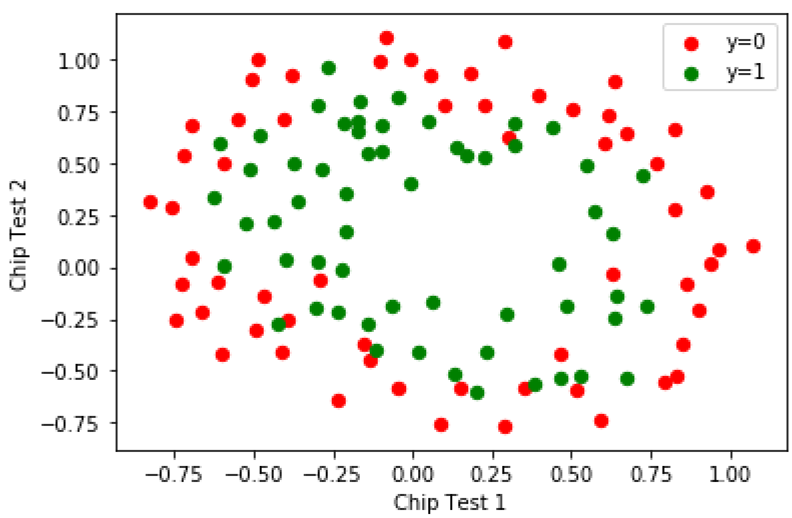
Predicting with a logistic regression model

In [4]:
```
#  calculate the probability of a student being admitted with score of 45,85

pred_prob = logreg.predict_proba(np.array([[45,85]]))[0][1]
print("For a student with 45 on exam 1 and 85 on exam 2, the probability of ad
mission = ", pred_prob)

# compute accuracy on the training set

predy = logreg.predict(X)

# calculate the accuracy of predictions on training set

accuracy = np.mean(predy==y)
print("Accuracy on the training set = ", accuracy)
```

```
For a student with 45 on exam 1 and 85 on exam 2, the probability of admissio
n =  0.7762901213932106
Accuracy on the training set =  0.89
```

Regularized logistic regression

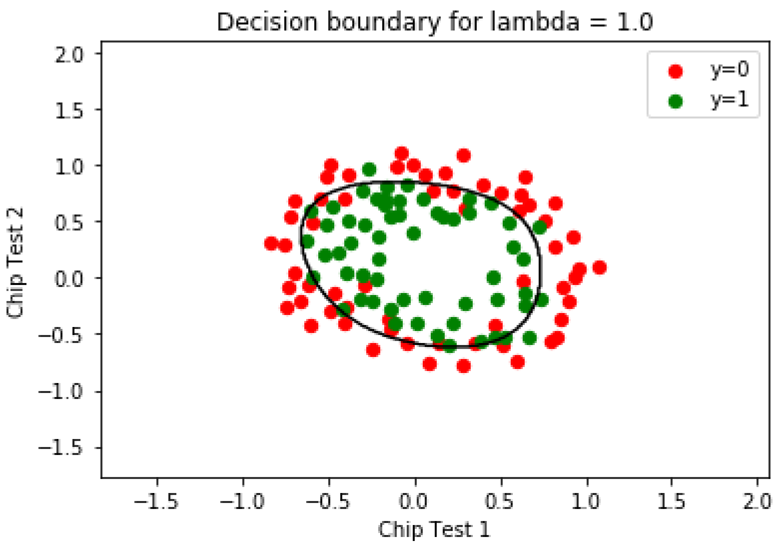
In [5]:
```
data = pd.read_csv('../data/ex2data2.txt')
X = np.vstack([data.x1,data.x2]).T
y = data.y

plot_twoclass_data(X,y,'Chip Test 1', 'Chip Test 2',['y=0','y=1'])
```



Expanding basis functions and regularizing logistic regression

In [6]:
```
# map the features in ex2data2.txt into a pth order polynomial

import sklearn
from sklearn.preprocessing import PolynomialFeatures

alpha = 1.0

polynomial_features = PolynomialFeatures(degree=5,include_bias=False)
logreg = linear_model.LogisticRegression(C=1.0/alpha,solver='lbfgs',fit_interc
ept=True)
pipeline = Pipeline([("polynomial_features", polynomial_features),
                         ("logistic", logreg)])
pipeline.fit(X, y)


print("Theta found by logistic regression with L2 reg: ", logreg.coef_[0], log
reg.intercept_)

# accuracy on training set

predy = pipeline.predict(X)
print("Accuracy on training set for sklearn theta = ", np.mean(predy==y))
score = cross_val_score(pipeline, X, y, scoring='accuracy', cv=10)
print("Accuracy in 10 fold CV = ", np.mean(score), '+/-', np.std(score))

def plot_decision_boundary_logreg_poly(X,y,pipeline,reg,  xlabel, ylabel, lege
nd):
    plot_twoclass_data(X,y,xlabel,ylabel,legend)

    # create a mesh to plot in
    h = 0.01
    x1_min, x1_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    x2_min, x2_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx1, xx2 = np.meshgrid(np.arange(x1_min, x1_max, h),
                         np.arange(x2_min, x2_max, h))

    Z = np.array(pipeline.predict(np.c_[xx1.ravel(), xx2.ravel()]))

    # Put the result into a color contour plot
    Z = Z.reshape(xx1.shape)
    plt.contour(xx1,xx2,Z,cmap=plt.cm.gray,levels=[0.5])
    plt.title("Decision boundary for lambda = " + str(reg))

plot_decision_boundary_logreg_poly(X,y,pipeline,alpha,'Chip Test 1', 'Chip Tes
t 2',['y=0','y=1'])
```

```
Theta found by logistic regression with L2 reg:  [ 0.61094032  1.18617982 -2.
18001135 -0.89229892 -1.55337305  0.04325463
 -0.36397794 -0.34450195 -0.29806854 -1.63380057 -0.041294   -0.626188
 -0.25213732 -1.39663306 -0.34707794 -0.2129194  -0.0452132  -0.28154528
 -0.28321566 -0.64850621] [1.2744877]
Accuracy on training set for sklearn theta =  0.8389830508474576
Accuracy in 10 fold CV =  0.7121212121212122 +/- 0.19087301246392152
```

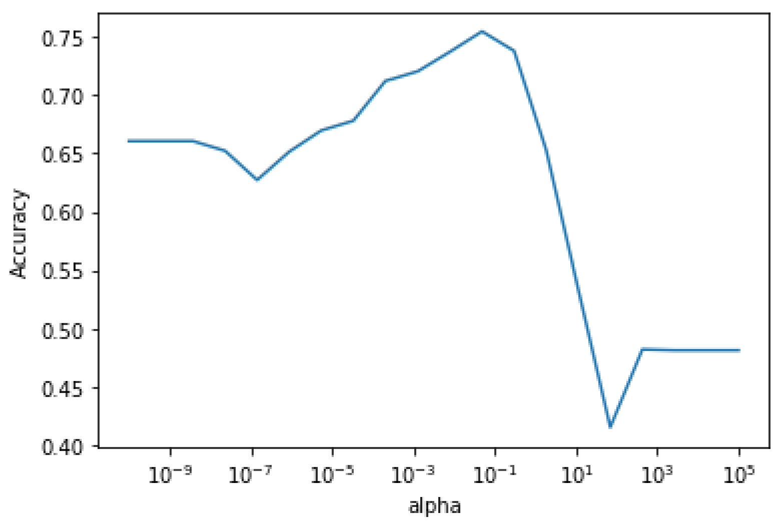


Try this!

Experiment with various values of the regularization parameter in the cell above and comment on accuracies as well as decision boundary shape. Try values of 0.001, 0.01, 0.1, 1.0, 10.0

Selecting best alpha by cross validation

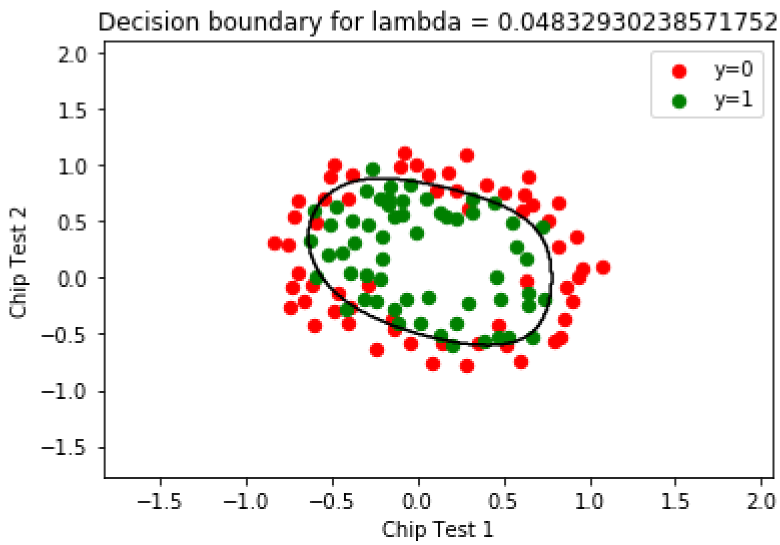
In [7]:
```
# What is the best value for the regularization term?

# to find the optimal value of alpha, select the highest degree you want to wo
rk with
# and then sweep through the alphas on a logarithmic scale

n_alphas = 20
alphas = np.logspace(-10, 5, n_alphas)
scores = []

for alpha in alphas:
    polynomial_features = PolynomialFeatures(degree=6,
                                             include_bias=False)
    logreg = linear_model.LogisticRegression(C=1.0/alpha,solver='liblinear',ma
x_iter = 1000,fit_intercept=True)
    pipeline = Pipeline([("polynomial_features", polynomial_features),
                         ("logreg", logreg)])
    pipeline.fit(X, y)
    score = cross_val_score(pipeline, X, y,
                            scoring='accuracy', cv=10)
    scores.append(np.mean(score))


plt.semilogx(alphas,scores)
plt.xlabel('alpha')
plt.ylabel('Accuracy')
plt.show()
```



Building final model with best alpha

In [8]:
```
# build the final model with the optimal alpha
best_alpha = alphas[scores.index(max(scores))]


polynomial_features = PolynomialFeatures(degree=6,
                                         include_bias=False)
logreg = linear_model.LogisticRegression(C=1.0/best_alpha,solver='liblinear',f
it_intercept=True)
pipeline = Pipeline([("polynomial_features", polynomial_features),
                         ("logreg", logreg)])
pipeline.fit(X, y)
score = cross_val_score(pipeline, X, y, scoring='accuracy', cv=10)
print("Accuracy in 10 fold CV = ", np.mean(score), '+/-', np.std(score))
plot_decision_boundary_logreg_poly(X,y,pipeline,best_alpha,'Chip Test 1', 'Chi
p Test 2',['y=0','y=1'])
```

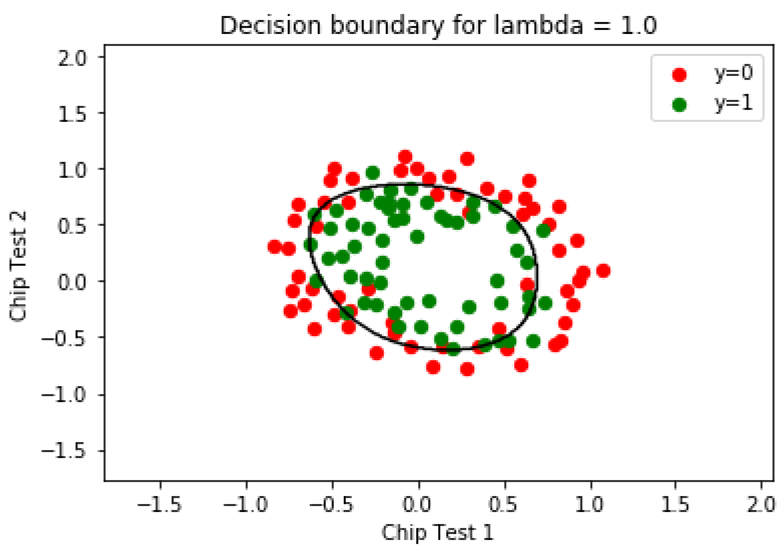
```
Accuracy in 10 fold CV =  0.7545454545454546 +/- 0.16814768968491012
```



Regularizing logistic regression with L1 norm

In [9]:
```
# impose L1 penalty rather than L2; need to use liblinear solver
alpha = 1.0

polynomial_features = PolynomialFeatures(degree=6,
                                         include_bias=False)
logreg = linear_model.LogisticRegression(C=1.0/alpha,solver='liblinear',fit_in
tercept=True,penalty='l1')
pipeline = Pipeline([("polynomial_features", polynomial_features),
                       ("logreg", logreg)])
pipeline.fit(X, y)
print("Theta found by logistic with L1 penalty: ", logreg.coef_, logreg.interc
ept_)
predy = pipeline.predict(X)
print("Accuracy on training set = ", np.mean(predy==y))
score = cross_val_score(pipeline, X, y, scoring='accuracy', cv=10)
print("Accuracy in 10 fold CV = ", np.mean(score), '+/-', np.std(score))



plot_decision_boundary_logreg_poly(X,y,pipeline,alpha,'Chip Test 1', 'Chip Tes
t 2',['y=0','y=1'])
```

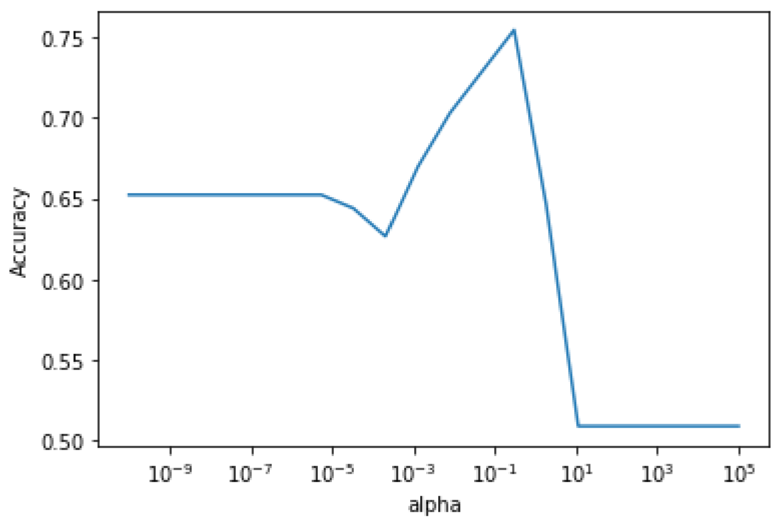
```
Theta found by logistic with L1 penalty:  [[ 0.68645031  1.28025356 -4.862145
81 -1.62149245 -2.34057983  0.
   0.          0.          0.          0.          0.          0.
   0.         -2.36893449  0.          0.          0.          0.
   0.          0.          0.          0.          0.          0.
   0.          0.          0.        ]] [1.86931211]
Accuracy on training set =  0.7966101694915254
Accuracy in 10 fold CV =  0.6621212121212121 +/- 0.2348289336925362
```



Finding the best value for L1 regularization paremeter

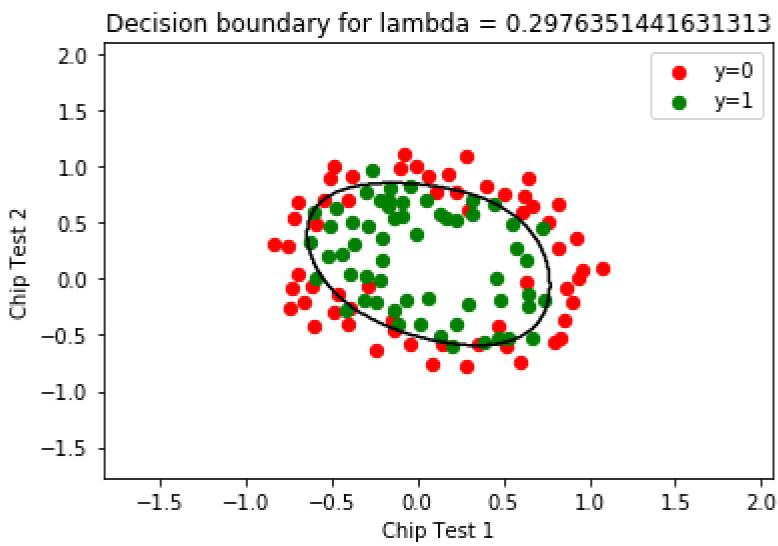
In [10]:

```
#  What is the best value for the regularization term?

# to find the optimal value of alpha, select the highest degree you want to wo
rk with
# and then sweep through the alphas on a logarithmic scale

n_alphas = 20
alphas = np.logspace(-10, 5, n_alphas)
scores = []

for alpha in alphas:
    polynomial_features = PolynomialFeatures(degree=6,
                                             include_bias=False)
    # logreg = linear_model.LogisticRegression(C=1.0/alpha,solver='liblinear',
max_iter = 1000, fit_intercept=True,penalty='l1')
    logreg = linear_model.LogisticRegression(C=1.0/alpha,solver='liblinear',ma
x_iter = 5000, fit_intercept=True,penalty='l1')
    pipeline = Pipeline([("polynomial_features", polynomial_features),
                         ("logreg", logreg)])
    pipeline.fit(X, y)
    score = cross_val_score(pipeline, X, y,
                                 scoring='accuracy', cv=10)
    scores.append(np.mean(score))

plt.semilogx(alphas,scores)
plt.xlabel('alpha')
plt.ylabel('Accuracy')
plt.show()
```



In [11]:
```
# build the final model with the optimal alpha
best_alpha = alphas[scores.index(max(scores))]


polynomial_features = PolynomialFeatures(degree=6,
                                         include_bias=False)
logreg = linear_model.LogisticRegression(C=1.0/best_alpha,solver='liblinear',f
it_intercept=True,penalty='l1')
pipeline = Pipeline([("polynomial_features", polynomial_features),
                     ("logreg", logreg)])
pipeline.fit(X, y)
score = cross_val_score(pipeline, X, y, scoring='accuracy', cv=10)
print("Accuracy in 10 fold CV = ", np.mean(score), '+/-', np.std(score))
plot_decision_boundary_logreg_poly(X,y,pipeline,best_alpha,'Chip Test 1', 'Chi
p Test 2',['y=0','y=1'])
```

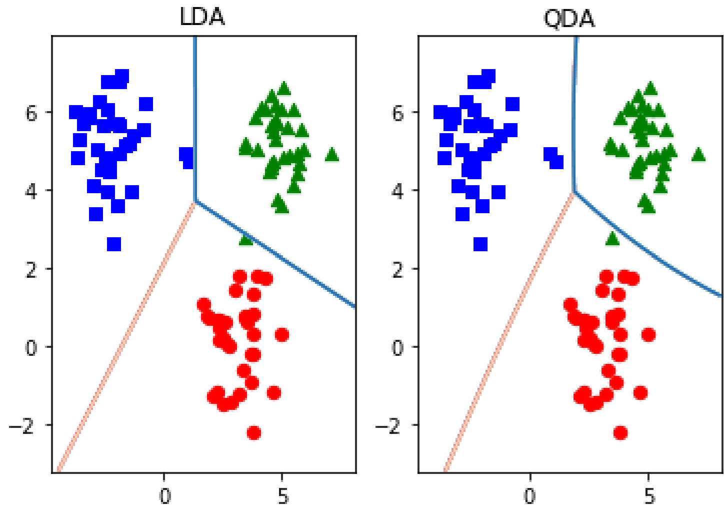
```
Accuracy in 10 fold CV =  0.7545454545454546 +/- 0.17222814646642898
```



LDA and QDA on synthetic data

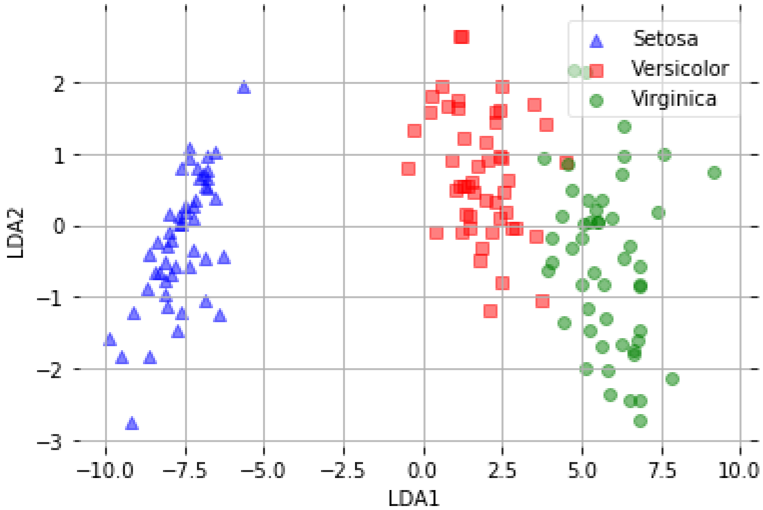
In [12]:
```
# GDA on synthetic data

from sklearn.datasets import make_blobs
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis as QDA

X, y = make_blobs(n_samples=100, n_features=2, centers=[(-2,5),(3,0),(5,5)])

lda = LDA()
lda.fit(X,y)

qda = QDA()
qda.fit(X,y)

def plot_decision_boundary_lda(X,y,lda,   legend):
    plt.scatter(X[y==0,0],X[y==0,1],s=40,marker='s',color='b')
    plt.scatter(X[y==1,0],X[y==1,1],s=40,marker='o',color='r')
    plt.scatter(X[y==2,0],X[y==2,1],s=40,marker='^',color='g')

    # create a mesh to plot in
    h = 0.01
    x1_min, x1_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    x2_min, x2_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx1, xx2 = np.meshgrid(np.arange(x1_min, x1_max, h),
                         np.arange(x2_min, x2_max, h))

    # make predictions on this mesh
    Z = np.array(lda.predict(np.c_[xx1.ravel(), xx2.ravel()]))

    # Put the result into a color contour plot
    Z = Z.reshape(xx1.shape)
    plt.contour(xx1,xx2,Z,cmap='RdBu')

plt.figure()
plt.subplot(1,2,1)
plot_decision_boundary_lda(X,y,lda,['Class 1','Class 2','Class3'])
plt.title('LDA')
plt.subplot(1,2,2)
plot_decision_boundary_lda(X,y,qda,['Class 1','Class 2','Class3'])
plt.title('QDA')
```

Out[12]: Text(0.5, 1.0, 'QDA')



GDA with IRIS data

In [13]:
```
# Gaussian Discriminant Analysis for IRIS data

from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from sklearn import datasets

iris = sklearn.datasets.load_iris()
label_dict = {0: 'Setosa', 1: 'Versicolor', 2:'Virginica'}

X = iris.data
y = iris.target

# LDA
sklearn_lda = LDA(n_components=2)
X_lda_sklearn = sklearn_lda.fit_transform(X, y)


def plot_iris_lda(X):

    ax = plt.subplot(111)
    for label,marker,color in zip(
        range(0,3),('^', 's', 'o'),('blue', 'red', 'green')):

        plt.scatter(x=X[:,0][y == label],
                    y=X[:,1][y == label] * -1, # flip the figure
                    marker=marker,
                    color=color,
                    alpha=0.5,
                    label=label_dict[label]
                    )

    plt.xlabel('LDA1')
    plt.ylabel('LDA2')

    leg = plt.legend(loc='upper right', fancybox=True)
    leg.get_frame().set_alpha(0.5)


    # hide axis ticks
    plt.tick_params(axis="both", which="both", bottom="off", top="off",
            labelbottom="on", left="off", right="off", labelleft="on")

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

    plt.grid()
    plt.tight_layout
    plt.show()

plot_iris_lda(X_lda_sklearn)
```



In [14]:
```
y
```

Out[14]:
```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2])
```

In [ ]: