

The illusory promise of the Aligned Rank Transform

A systematic study of rank transformations

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Abstract



Introduction

The Aligned Rank Transform (ART) is a popular statistical method, commonly used when the statistical assumptions of parametric procedures, such as normality assumptions, are not met. Its popularity stems from several Monte Carlo experiments demonstrating that the method is robust and powerful. However, contrary experimental findings suggest that ART may be less robust compared to simpler rank transformation methods, failing to adequately control Type I error rates across various scenarios, including discrete data, skewed distributions, and distributions with unequal variances. These results have not gained attention within the Human-Computer Interaction community, where ART is routinely used.

Objectives

This research aims to clarify the contradictory findings from previous studies by reevaluating the robustness and effectiveness of ART. Its complementary goal is to investigate the performance of alternative nonparametric methods.

Methods

We conducted a series of Monte Carlo experiments to systematically evaluate ART across various data distributions and experimental designs. Our comparative analysis includes two alternative transformation methods: the simple rank transformation and the inverse normal transformation, along with parametric ANOVA without transformation. Additionally, we present supplementary experiments involving other competitive nonparametric methods in the [appendix](#).

Our experimental methodology builds upon a latent variable modeling approach, enabling us to simulate main and interaction effects across diverse distributions and address interpretation issues related to these effects. In addition to Monte Carlo experiments, we explore the behavior of ART through illustrative examples and reanalyze data from previous user studies.

Results

Our results indicate that ART fails under a range of discrete distributions, including binomial distributions, and Likert-type data with 5, 7, and 11 ordinal levels. In addition, ART systematically confounds effects in skewed distributions or distributions with unequal variances. We report results for various experimental designs, including between-subjects, within-subjects, and mixed designs with two or three factors.

Surprisingly, ART functions as intended only under the strict assumption of a linear model with continuous responses, equal variances, and no missing data. In all other scenarios, it performs worse than simpler rank transformation methods and, in many cases — such as with ordinal data — significantly worse than parametric ANOVA. Our analysis of real-world data further demonstrates that using ART often leads to incorrect conclusions. By contrast, the inverse normal transformation proves to be the most versatile and robust nonparametric method, though we also identify scenarios where it inflates errors or distort effects.

In addition, our results show that testing interactions in the presence of multiple main effects remains a challenge for all methods. We raise awareness about previous warnings regarding the interpretation of *removable interactions*.

Conclusion

We conclude that ART is a flawed statistical method and should be abandoned. We recommend prioritizing parametric methods, including linear models and their generalizations. While we propose the inverse normal transformation as a generic nonparametric alternative, we caution researchers about risks and limitations of rank transformation methods.

Materials

We provide [supplementary materials](#) with experimental code, raw experimental results, and detailed analyses of examples and case studies presented throughout the article. Source files of our article, appendix, and supplementary materials can also be found on OSF at <https://osf.io/hs7j8>.

Materials, Authorship, License, Conflicts



1 Introduction

In Human-Computer Interaction (HCI) and various fields within the behavioral sciences, researchers often gather data through user studies. Such data are typically messy, have small sample sizes, and may violate common statistical assumptions, such as the normality assumption. To address these challenges, researchers commonly employ nonparametric tests, which require fewer assumptions about the data. However, while nonparametric tests for simple one-factorial designs are well-established, researchers face challenges in selecting appropriate methods when dealing with multifactorial designs that require testing for both main effects and interactions. The Aligned Rank Transform or ART ([Higgins, Blair, and Tashtoush 1990](#); [Salter and Fawcett 1993](#); [Wobbrock et al. 2011](#)) addresses this problem by bridging the gap between nonparametric tests and ANOVAs. Its popularity in HCI research has surged, in part due to the ARTool toolkit ([Wobbrock et al. 2011](#); [Kay et al. 2021](#)), which makes the method easy to use. Oppenlaender and Hosio ([2025](#)) identify the paper by Wobbrock et al. ([2011](#)) as major milestone in HCI research, ranking third in terms of citations in the ACM CHI Proceedings (1981-2024), behind Braun and Clark's ([2006](#)) article on thematic analysis and Hart and Staveland's ([1988](#)) article on the NASA-TXL index.

However, given their substantial impact on research outcomes in our community, these widely adopted methods deserve closer scrutiny. As the first author previously argued ([Tsandilas 2018](#)), the HCI community lacks the necessary expertise to critically assess the validity of statistical techniques. Our findings suggest that the widespread use of ART largely stems from a lack of awareness of its underlying assumptions. Many researchers treat ART as a general-purpose nonparametric alternative when ANOVA's assumptions are violated, overlooking the fact that ART's alignment mechanism imposes even stricter assumptions — including a linear relationship between the response and predictors, equal variances, and continuous data.

Early Monte Carlo experiments in the 1990s ([Salter and Fawcett 1993](#); [Mansouri and Chang 1995](#)) and more recent studies ([Elkin et al. 2021](#)) suggested that ART is a robust alternative to ANOVA when normality assumptions are violated. These results have contributed to ART's reputation as a well-established method. However, other research ([Lüpsen 2017, 2018](#)) raised concerns about the robustness of the method, demonstrating that ART fails to control the Type I error rate in many scenarios, such as when data are ordinal or are drawn from skewed distributions. Unfortunately, these warnings have not received sufficient attention, and many authors still rely on Wobbrock et al.'s ([2011](#)) assertion that "*The ART is for use in circumstances similar to the parametric ANOVA, except that the response variable may be continuous or ordinal, and is not required to be normally distributed.*"

This paper aims to clarify the severity of these issues and understand the potential risks of the method. We present the outcomes of a series of Monte Carlo experiments, following a distinctive simulation methodology grounded in latent variable modeling. This approach enables us to simulate effects consistently across a broad spectrum of distributions, both discrete and continuous, and allows us to address issues in interpreting main and interaction effects. To ensure the clarity of our findings and facilitate their reproducibility, we divide the experimental process into multiple smaller experiments, where each experiment focuses on a distinct variable (e.g., sample size, experimental design, variance ratio) or measure (Type I error rate, power, precision of effect size estimates).

Our findings corroborate Lüpsen's alarming conclusions. We provide overwhelming evidence that ART confounds effects and raises Type I error rates at very high levels across a diverse array of non-normal distributions, including skewed, binomial, and ordinal distributions, as well as distributions with unequal variances. These issues persist for both main and interaction effects. Our results further show that simpler rank transformation methods outperform ART, while parametric ANOVA generally poses fewer risks than ART when distributions deviate from normal. Given these new insights, we conclude that ART is not a viable analysis method and advocate for its abandonment. We provide recommendations for alternative analysis methods, while we also raise warnings about the interpretation of interaction effects.

Illustrative example

We will begin with an illustrative example to demonstrate how the aligned rank transform can lead to an increase in false positives and a significant inflation of observed effects. This example will also serve as a brief introduction to the key concepts and methods employed throughout the paper.

Suppose a team of HCI researchers conduct an experiment to compare the performance of three user interface techniques (A, B, and C) that help users complete image editing tasks of four different difficulty levels. The experiment follows a fully balanced 4×3 repeated-measures factorial design, where each participant ($N = 12$) performs 12 tasks in a unique order. The researchers measure the time that it takes participants to complete each task. In addition, the participants rate their perceived efficiency completing each task, using an ordinal scale with five levels: (1) *inefficient*, (2) *somewhat inefficient*, (3) *neutral*, (4) *somewhat efficient*, and (5) *efficient*.

The following table presents the experimental results:

Example dataset: Time (in minutes) spent by 12 participants and their perceived efficiency (ordinal scale) for four difficulty levels and three user interface techniques. Scroll down for full results.

Participant	Difficulty	Technique	Time	Perceived_Efficiency
P01	Level1	A	0.20	efficient
P01	Level1	B	0.17	somewhat efficient
P01	Level1	C	0.14	neutral
P01	Level2	A	0.45	efficient

The experiment is hypothetical but has similarities with real-world experiments, e.g., see the experiments of Fruchard et al. (2023).

Time performance. Time measurements have been randomly sampled from a population in which: (1) *Difficulty* has a large main effect; (2) *Technique* has no main effect; and (3) there is no interaction effect between the two factors. To generate time values, we drew samples from a log-normal distribution. The log-normal distribution is often a good fit for real-world measurements that are bounded by zero and have low means but large variance (Limpert, Stahel, and Abbt 2001). Task-completion times are good examples of such measurements (Sauro and Lewis 2010).

[Figure 1](#) presents two boxplots that visually summarize the main effects observed through the experiment. We plot medians to account for the fact that distributions are skewed. We observe that differences in the overall time performance of the three techniques are not visually clear, although the overall median time is somewhat higher for Technique B. In contrast, time performance clearly deteriorates as task difficulty increases. We also observe that for the most difficult tasks (Level 4), the median time for Technique C is lower than the median time for Techniques A and B, so we may suspect that *Difficulty* interacts with *Technique*. However, since the spread of observed values is extremely large and the number of data points is small, such differences could result from random noise.

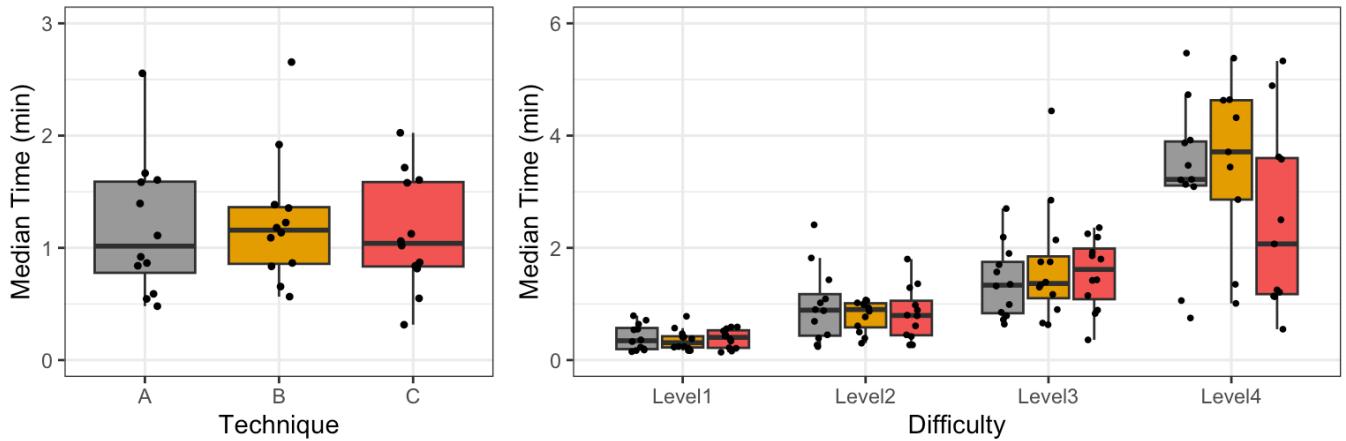


Figure 1: Boxplots summarizing time performance results for our illustrative example. Dots represent the median time for each individual participant.

We opt for a multiverse analysis ([Dragicevic et al. 2019](#)) to analyze the data, where we conduct a repeated-measures ANOVA with four different data-transformation methods:

1. *Log transformation (LOG)*. Data are transformed with the logarithmic function. For our data, this is the most appropriate method as we drew samples from a log-normal distribution.
2. *Aligned rank transformation (ART)*. Data are transformed and analyzed with the ARTool ([Wobbrock et al. 2011](#); [Elkin et al. 2021](#)).
3. *Pure rank transformation (RNK)*. Data are transformed with the original rank transformation ([Conover and Iman 1981](#)), which does not perform any data alignment.
4. *Inverse normal transformation (INT)*. The data are transformed by using their normal scores. This rank-based method is simple to implement and has been commonly used in some disciplines. However, it has also received criticism ([Beasley, Erickson, and Allison 2009](#)).

For comparison, we also report the results of the regular parametric ANOVA with no transformation (*PAR*). For each ANOVA analysis, we use a linear mixed-effects model, treating the participant identifier as a random effect. To simplify our analysis and like Elkin et al. ([2021](#)), we consider random intercepts but no random slopes. For example, we use the following R code to create the model for the log-transformed response:

```
m.log <- lmer(log(Time) ~ Difficulty*Technique + (1|Participant), data = df)
```

The table below presents the *p*-values for the main effects of the two factors and their interaction:

p-values for main and interaction effects

	PAR	LOG	ART	RNK	INT
Difficulty	1.8×10^{-26}	8.1×10^{-47}	9.0×10^{-43}	4.3×10^{-46}	4.4×10^{-44}
Technique	.10	.18	.00061	.38	.17
Difficulty × Technique	.056	.10	.0017	.24	.23

The disparity in findings between ART and the three alternative transformation methods is striking. ART suggests that all three effects are statistically significant. What adds to the intrigue is the fact that ART's *p*-values for *Technique* and its interaction with *Difficulty* are orders of magnitude lower than the *p*-values obtained from all other methods. We will observe similar discrepancies if we conduct contrast tests with the ART procedure ([Elkin et al. 2021](#)), though we leave this as an exercise for the reader.

We also examine effect size measures, which are commonly reported in scientific papers. The table below presents results for partial η^2 , which describes the ratio of variance explained by a variable or an interaction:

partial η^2 and its 95% confidence interval

	PAR	LOG	ART	RNK	INT
Difficulty	.64 [.55, 1.0]	.83 [.79, 1.0]	.80 [.76, 1.0]	.83 [.79, 1.0]	.81 [.77, 1.0]
Technique	.04 [.00, 1.0]	.03 [.00, 1.0]	.11 [.03, 1.0]	.02 [.00, 1.0]	.03 [.00, 1.0]
Difficulty × Technique	.10 [.00, 1.0]	.08 [.00, 1.0]	.16 [.04, 1.0]	.06 [.00, 1.0]	.06 [.00, 1.0]

We observe that ART exaggerates both the effect of *Technique* and its interaction with *Difficulty*. This example demonstrates a more general problem with ART's alignment mechanism under long-tailed distributions. ART tends to confound effects: a strong effect on a single factor often leads the method to detect spurious effects on other factors or to identify non-existent interactions.

Perceived efficiency. The perceived efficiency ratings were randomly sampled from a population with no main effect on any of the two factors and no interaction effect. [Figure 2](#) shows that participants' ratings are concentrated at the higher end of the scale but with no clear trends.

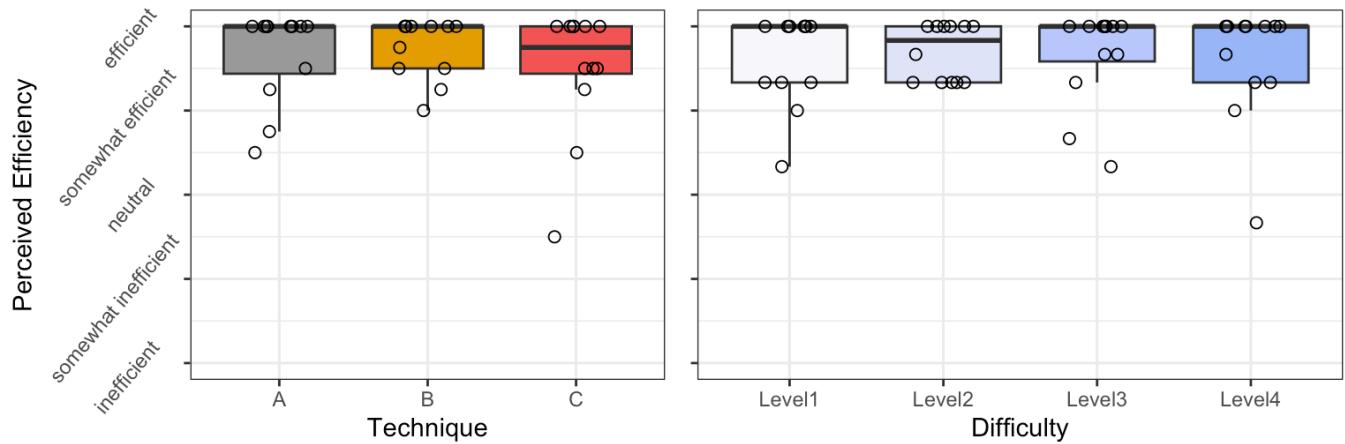


Figure 2: Boxplots summarizing the results on efficiency as perceived by participants. Circles represent the mean rating of each individual participant.

For our analysis, we compare the results of PAR, ART, RNK, and INT, since LOG is not relevant for this type of data. We also report the results of the Friedman test ([Friedman 1940](#)) for each factor, where we first aggregate the data by taking the mean over the other factor. Results are as follows:

p-values for main and interaction effects

	PAR	ART	RNK	INT	Friedman Test
Difficulty	.93	.0011	.86	.88	.83
Technique	.18	.00016	.32	.23	.24
Difficulty × Technique	.27	.10	.33	.34	

ART's p-values for the two main effects are surprisingly low, while no other method shows any substantial evidence for such effects. We also present the methods' effect size estimates:

partial η^2 and its 95% confidence interval

	PAR	ART	RNK	INT

	PAR	ART	RNK	INT
Difficulty	.004 [.00, 1.0]	.12 [.03, 1.0]	.006 [.00, 1.0]	.005 [.00, 1.0]
Technique	.03 [.00, 1.0]	.13 [.05, 1.0]	.02 [.00, 1.0]	.02 [.00, 1.0]
Difficulty × Technique	.06 [.00, 1.0]	.04 [.00, 1.0]	.05 [.00, 1.0]	.05 [.00, 1.0]

The discrepancies between ART's estimates for the two main effects and those of the other methods are large. As we demonstrate in this paper, such discrepancies stem from ART's inability to handle discrete distributions. The issue becomes more pronounced with larger sample sizes. Lüpsen (2017) previously warned researchers about this problem, but his warnings have largely been ignored.

Overview

The above example does not reflect a rare phenomenon. We will show that ART's error inflation is systematic for a range of distributions that deviate from normality, both continuous and discrete.

The paper is structured as follows. [Section 2](#) introduces nonparametric tests and rank transformations, and explains how ART is constructed. It also provides a summary of previous experimental results regarding the robustness of ART and other related rank-based procedures, along with a survey of recent studies using ART. [Section 3](#) clarifies the method's distributional assumptions. [Section 4](#) investigates issues regarding effect interpretation and introduce our distribution simulation approach. [Section 5](#) outlines our experimental methodology, while [Section 6](#) presents our findings. [Section 7](#) revisits results of previous studies employing ART and illustrates how its application can lead to erroneous conclusions. [Section 8](#) offers recommendations for researchers. Finally, [Section 9](#) concludes the paper.

In addition to the main sections of the paper, we provide an [appendix](#) with results from additional Monte Carlo experiments.

2 Background

We provide background on the history, scope, construction, and use of rank transformation methods, with a particular focus on ART. Additionally, we summarize the results of previous evaluation studies and position our own work within this context.

Nonparametric statistical tests

A common assumption of ANOVAs and linear regression is that the distribution of residuals is normal. More technically, the normality assumption concerns the sampling distribution of the mean. Therefore, when sample sizes are sufficiently large, these methods are robust to departures from the normality assumption because the sampling distribution of the mean is asymptotically normal, regardless of the shape of the population distributions (Central Limit Theorem). However, their accuracy drops if samples are relatively small and populations markedly deviate from normal.

The goal of *nonparametric* procedures is to address this problem by making fewer or no assumptions about the underlying distributions. The original idea of replacing the data values by their ranks belongs to Spearman (1904). The key advantage of ranks is that they preserve the monotonic relationship of values while also allowing the derivation of an exact probability distribution. This probability distribution can then be used for statistical inference, such as calculating a *p*-value. The idea of using ranks was adopted by other researchers, leading to the development of various nonparametric tests commonly used today. Representative examples include the Mann-Whitney U test ([Mann and Whitney 1947](#)) and the Kruskal-Wallis test ([Kruskal and Wallis 1952](#)) for independent samples, and the Wilcoxon sign-rank test ([Wilcoxon 1945](#)) and the Friedman test ([Friedman 1940](#)) for paired samples and repeated measures.

Rank transformations

The scope of nonparametric tests is limited, as they only support simple experimental designs with a single independent variable. Rank transformations aim to address this limitation by bridging the gap between nonparametric statistics and ANOVA.

The rank transform. Conover (2012) provides a comprehensive introduction to the simple rank transform, which consists of sorting the raw observations and replacing them by their ranks. In case of ties, tied observations are assigned *fractional ranks* equal to their average order position in the ascending sequence of values. For example, the two 0.3 instances of the Y responses in Figure 3 (a), which are the lowest values in the dataset, receive a rank of 1.5 (see $\text{rank}(Y)$), while the next value (a 0.4) in the sequence receives a rank of 3.

Conover and Iman (1981) showed that for one-factor designs, using ANOVAs on these ranks produces tests that are asymptotically equivalent or good replacements of traditional nonparametric tests (see Section 3). However, a series of studies in the 80s raised caution flags on the use of the method, showing that it may confound main and interaction effects in two-factor and three-factor experimental designs. For an extensive review of these studies, we refer readers to Sawilowsky (1990).

The aligned rank transform. In response to these negative results, many researchers turned to aligned (or adjusted) rank transformations. Sawilowsky (1990) discusses several variations of aligned rank-based transformations (ART) and tests, while Higgins, Blair, and Tashtoush (1990) describe the specific ART method that we evaluate in this paper for two-way designs. Wobbrock et al. (2011) generalize it to more than two factors, and a decade later, Elkin et al. (2021) show how to apply it to multifactor contrast tests. Figure 3 explains the general method for a design with two factors, A and B .

(a) Example dataset

S	A	B	Y	$\text{rank}(Y)$	ART_A	ART_B	ART_{AB}	$INT(Y)$
s_1	a_1	b_1	0.4	3	7.5	3	3	-0.49
s_1	a_1	b_2	3.0	7	5.5	6	6	0.89
s_1	a_2	b_1	0.8	4	3	4	8	-0.16
s_1	a_2	b_2	1.6	5	1	5	1	0.16
s_2	a_1	b_1	0.3	1.5	5.5	2	2	-1.15
s_2	a_1	b_2	3.1	8	7.5	7	7	1.53
s_2	a_2	b_1	0.3	1.5	2	1	5	-1.15
s_2	a_2	b_2	2.3	6	4	8	4	0.49

(b) ART calculation

$$ART_A(y_{sij}) = \text{rank}(y_{sij} - \bar{y}_{ij} + \bar{y}_{a_i} - \bar{y}) \text{ estimated main effect } a_i$$

$$ART_B(y_{sij}) = \text{rank}(y_{sij} - \bar{y}_{ij} + \bar{y}_{b_j} - \bar{y}) \text{ estimated main effect } b_j$$

$$ART_{AB}(y_{sij}) = \text{rank}(y_{sij} - \bar{y}_{ij} + \bar{y}_{a_i b_j} - \bar{y} - (\bar{y}_{a_i} - \bar{y}) - (\bar{y}_{b_j} - \bar{y}))$$

residual estimated interaction effect $a_i b_j$

(c) Mean terms

$\bar{y} = \sum_{sij} y_{sij}$	$\bar{y}_{a_i} = \sum_{sj} y_{sij}$	$\bar{y}_{b_j} = \sum_{si} y_{sij}$	$\bar{y}_{a_i b_j} = \bar{y}_{ij} = \sum_s y_{sij}$
grand mean	mean for $A = a_i$	mean for $B = b_j$	cell mean mean for $A = a_i$ & $B = b_j$

Figure 3: Example showing the use of the rank, ART, and INT transformation methods (a). Construction of ART for a two-factor design (b,c)

The key intuition behind the transform is that responses are ranked after they are *aligned*, such that effects of interest are separated from other effects. This implies that for each effect, either main or interaction, a separate ranking is produced. This is clearly illustrated in the example dataset shown in [Figure 3](#) (a), where each aligned ranking (ART_A , ART_B , and ART_{AB}) is for testing a different effect (A , B , and $A \times B$, respectively).

Even in this very small dataset ($n = 2$), the three ART rankings are distinct from one another and also differ significantly from the ranking produced by a simple rank transformation. Interestingly, identical responses (e.g., the two values of 0.3) can be assigned very different ranks, while responses that differ substantially (e.g., 0.3 and 3.0) may receive the same rank. More surprisingly, larger values can receive lower ranks — revealing that ART is a non-monotonic transformation. For instance, for subject s_1 under condition b_1 , the responses are 0.4 and 0.8 for a_1 and a_2 , yet their respective ART_A ranks are 7.5 and 3.0. In this case, ART significantly distorts the actual difference between the two groups, a_1 and a_2 .

As we will show later, this behavior can lead to unstable results and misleading conclusions, and it represents a central flaw of the method. For example, running a repeated-measures ANOVA on these ART ranks yields a p -value of .044 for the effect of A . In contrast, using a repeated-measures ANOVA on the ranks of simple rank transformation produces a p -value of .46, while applying a log-transformation — more appropriate for this type of data — yields a p -value of .85.

[Figure 3](#) (b-c) details the calculation of the transformation, where we highlight the following terms: (i) residuals (in yellow) that represent the unexplained error (due to individual differences in our example); (ii) main effects (in green and pink) estimated from the observed means of the individual levels of the two factors; and (iii) interaction effect estimates (in blue). Observe that the estimates of the two main effects are subtracted from the interaction term. The objective of this approach is to eliminate the influence of main effects when estimating interaction effects.

This is not the only alignment technique discussed in the literature. Sawilowsky (1990) suggests that, at least for balanced designs, interactions could also be removed when aligning main effects, in the same way main effects are removed when aligning interactions. This approach is also taken by Leys and Schumann (2010), who derived a common ranking for both main effects after subtracting the interaction term. We do not evaluate these alternative alignment methods in this paper, as they are, to the best of our knowledge, not commonly used in practice.

The inverse normal transform. A third transformation method we evaluate is the rank-based inverse normal transformation (INT). INT has been in use for over 70 years (Waerden 1952) and exists in several variations (Beasley, Erickson, and Allison 2009; Solomon and Sawilowsky 2009). Its general formulation is as follows:

$$INT(Y) = \Phi^{-1}\left(\frac{rank(Y) - c}{N - 2c + 1}\right) \quad (1)$$

where N is the total number of observations and Φ^{-1} is the standard normal quantile function, which transforms a uniform distribution of ranks into a normal distribution. Different authors have used a different parameter c . In our experiments, we use the *Rankit* formulation (Bliss, Greenwood, and White 1956), where $c = 0.5$, since past simulation studies (Solomon and Sawilowsky 2009) have shown that it is more accurate than alternative formulations. However, as Beasley, Erickson, and Allison (2009) report, the choice of c is of minor importance. For our experiments, we implement the INT method in R as follows:

```
INT <- function(x){
  qnorm((rank(x) - 0.5)/length(x))
}
```

[Figure 3](#) (a) shows how this function transforms the responses for our example dataset.

Other non-parametric rank-based methods. Several other rank-based statistical tests handle interactions, with the ANOVA-type statistic (ATS) (Brunner and Puri 2001) being the most representative one. Kaptein, Nass, and

Markopoulos (2010) introduced this method to the HCI community, advocating its use for analyzing Likert-type data as a viable alternative to parametric ANOVA. In addition to ATS, Lüpsen (2017, 2018, 2023) investigated several other multifactorial nonparametric methods. In particular, the author evaluated the hybrid ART+INT technique proposed by Mansouri and Chang (1995), which applies INT on the ranks of ART. He also tested multifactorial generalizations of the van der Waerden test (Waerden 1952) and the Kruskal-Wallis and Friedman tests (Kruskal and Wallis 1952; Friedman 1940). The former is based on INT, but instead of using F-tests on the transformed values as part of ANOVA, it computes χ^2 ratios over sums of squares. These two methods are not widely available, but implementations in R can be downloaded from Lüpsen's website (Lüpsen 2021).

Experimental evaluations

Previous studies have evaluated ART and related procedures using various Monte Carlo simulations, often producing conflicting results and conclusions.

Results in support of ART. A number of experiments conducted during the 80s and 90s suggested that ART is robust for testing interaction effects. Noteworthy instances include studies, such as those by Salter and Fawcett (1993), which compared the method to parametric ANOVA. The authors found that ART remains robust even in the presence of outliers or specific non-normal distributions, such as the logistic, exponential, and double exponential distributions. Their findings indicated only a marginal increase in error rates (ranging from 6.0% to 6.3% instead of the expected 5%) when applied to the exponential distribution. Furthermore, ART demonstrated superior statistical power compared to parametric ANOVA. Mansouri and Chang (1995) evaluated ART under a different set of non-normal distributions (normal, uniform, log-normal, exponential, double exponential, and Cauchy) in the presence of increasing main effects. Except for the Cauchy distribution, ART maintained Type I error rates close to nominal levels across all scenarios, irrespective of the magnitude of main effects. In contrast, the error rates of the rank transformation reached very high levels (up to 100%) as the magnitude of main effects increased, even under the normal distribution. ART only failed under the Cauchy distribution, which is well-known to be pathological.

More recently, Elkin et al. (2021) compared ART to parametric t-tests for testing multifactor contrasts under six distributions: normal, log-normal, exponential, double exponential, Cauchy, and Student's t-distribution ($\nu = 3$). Their results confirmed that ART keeps Type I error rates close to nominal levels across all distributions, except for the Cauchy distribution. In addition, they found that ART exhibits a higher power than the t-test.

While most evaluation studies have focused on continuous distributions, Payton et al. (2006) have also studied how various transformations (rank, ART, log-transform, and squared-root transform) perform under the Poisson distribution, focusing again on interaction effects when main effects were present. The authors found that ART and parametric ANOVA (no transformation) performed best, keeping Type I error rates close to nominal levels. All other transformations inflated error rates.

Warnings. While the above results indicate that ART is a robust method, other studies have identified some serious issues. The second author of this paper has observed that, in certain cases, ART seems to detect spurious effects that alternative methods fail to identify (Casiez 2022). Such informal observations, conducted with both simulated and real datasets, motivated us to delve deeper into the existing literature.

Carletti and Claustraix (2005) report that “*aligned rank transform methods are more affected by unequal variances than analysis of variance especially when sample sizes are large.*” Years later, Lüpsen (2018) conducted a series of Monte Carlo experiments, comparing a range of rank-based transformations, including the rank transformation, ART, INT, a combination of ART and INT (ART+INT), and ATS. His experiments focused on a 2×4 balanced between-subjects design and a 4×5 severely unbalanced design and tested normal, uniform, discrete uniform (integer responses from 1 to 5), log-normal, and exponential distributions, with equal or unequal variances. Furthermore, they tested both interaction and main effects when the magnitude of other effects increased. The results revealed that ART inflates error rates beyond acceptable levels in several configurations: right-skewed distributions (log-normal and exponential), discrete responses, unequal variances, and unbalanced designs. Lüpsen (2018) also found that using INT

in combination with ART (ART+INT) is preferable to the pure ART technique. However, as the method still severely inflated error rates in many settings, Lüpsen (2018) concluded that both ART and ART+INT are “*not recommendable*.”

Another notable finding by Lüpsen (2018) was that the simple rank transformation “*appeared not as bad as it is often described*” (Lüpsen 2018), outperforming ART in many scenarios, such as discrete and skewed distributions, or distributions with unequal variances. These results are in full contradiction with the findings of Mansouri and Chang (1995).

The same author conducted an additional series of experiments (Lüpsen 2017), focusing on two discrete distributions (uniform and exponential) with a varying number of discrete levels: 2, 4, and 7 levels with integer values for the uniform distribution, and 5, 10, and 18 levels with integer values for the exponential distribution. Again, the Type I error rates of both ART and ART+INT reached very high levels, but error rates were especially pronounced when the number of discrete levels became small and the sample size increased. Given these results, the author’s conclusion was the following:

“the ART as well as the ART+INT cannot be applied to Likert and similar metric or ordinal scaled variables, e.g. frequencies like the number of children in a family or the number of goals, or ordinal scales with ranges from 1 to 5” (Lüpsen 2017).

Results on other rank-based methods. We are also interested in understanding how ART compares with INT, which is frequently used in some research domains, such as genetics research (Beasley, Erickson, and Allison 2009). Beasley, Erickson, and Allison (2009) conducted an extensive evaluation of the method and reached the conclusion that “*INTs do not necessarily maintain proper control over Type 1 error rates relative to the use of untransformed data unless they are coupled with permutation testing.*” Lüpsen (2018) included INT in his evaluation and found that in most cases, it maintained better control of Type I error rates compared to ART and the pure rank transformation, while also presenting a higher statistical power. However, he also identified several cases where INT failed to sufficiently control for Type I errors, such as design configurations with unequal variances in unbalanced designs or skewed distributions, and when testing interactions in the presence of non-null main effects.

In addition to INT, Lüpsen (2018) evaluated the ATS method but found it to suffer from low power while presenting similar challenges as the rank transformation with regard to Type I error rates. Amongst all evaluated methods, the author identified the generalized van der Waerden test as the method that provided the best overall control of Type I error rates. More recently, Lüpsen (2023) conducted a new series of experiments testing rank-based nonparametric methods on split-plot designs with two factors. While the author reported on various tradeoffs of the methods, he concluded that overall, the generalized van der Waerden test and the generalized Kruskal-Wallis and Friedman tests were the best performing methods.

The use of ART in experimental research

To get a sense of how frequently ART is used in experimental research, we examined the citations of ARTTool (Wobbrock et al. 2011) indexed by Google Scholar. As shown in [Figure 4](#), the rate of citations to the original CHI 2011 paper introducing the method to the HCI community is steadily increasing, with over 400 citations in 2023 alone.

Cited by 2295

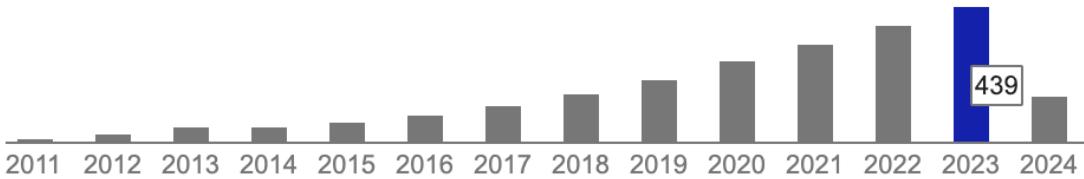


Figure 4: Citations of the CHI 2011 paper on ARTTool ([Wobbrock et al. 2011](#)) as shown in Google Scholar on 28 April 2024. We analyzed the 439 citing bibliographic entries of 2023 in more detail. Note: *Citations have continued to increase throughout the 2024–2025 period.*

[Figure 5](#) presents the most frequently citing venues for 2023. We observe that the HCI, Augmented Reality (AR), and Virtual Reality (VR) venues dominate these citations. However, we found that ARTTool is used in other scientific disciplines. For example, among the papers published in 2023, we counted nine *Nature Scientific Reports* and a total of 25 articles appearing in journals with the words “neuro” or “brain” in their title, including the *Journal of Neuropsychology* (three citations), the *Journal of Neuroscience* (two citations), the *Journal of Cognitive Neuroscience* (two citations), *Neuropsychologia* (two citations), *Nature Neuroscience*, *Neuropharmacology*, *Brain*, and *NeuroImage*.

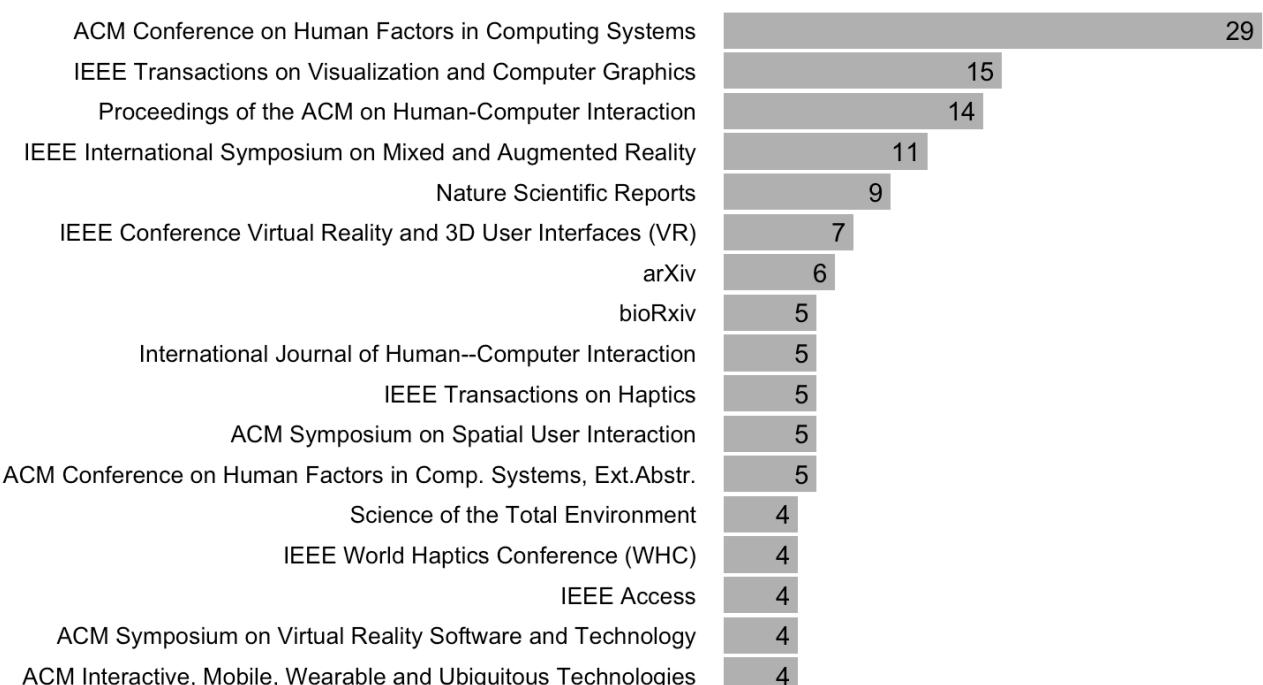


Figure 5: Publication venues with the highest number of citations to ARTTool ([Wobbrock et al. 2011](#)) in 2023. Note that among the articles published in the IEEE Transactions on Visualization and Computer Graphics, eight articles appeared at the IEEE International Symposium on Mixed and Augmented Reality (IEEE ISMAR’23), and three articles appeared at the IEEE Conference on Virtual Reality and 3D User Interfaces (IEEE VR’23). So the total number of citing articles appearing at these two venues is 19 for IEEE ISMAR’23 and 10 for IEEE VR’23.

To gain insights into the prevalent experimental designs using ART, we examined a subset of citing papers comprising the 39 most recent English publications as of November 2023. Among these, 25 report using a within-participants design, three papers report using a between-participants design, while 11 papers report using a mixed design that involved both between- and within-participants factors. The number of factors ranges from one to five, with two factors representing approximately 64% of the experimental designs. Participant counts in these studies vary between 12 and 468, with a median of 24 participants. Within-cell sample sizes (n) range from 8 to 48, with a median of 20.

Furthermore, we explored the types of data analyzed using ART. Among the 39 papers, 21 use ART for analyzing ordinal data, often in conjunction with other data types. Ordinal data include responses to Likert scales or other scales

of subjective assessment, such as the NASA Task Load Index (NASA TLX) ([Hart and Staveland 1988](#)). Furthermore, several authors apply ART to individual ordinal items, including Likert items with five levels (two papers), seven levels (seven papers), and 11 levels (three papers). Other types of data for which the method is used include task-completion or response times, counts, and measures of accuracy, such as target hit rates or recall scores. A common rationale for employing ART for such measures is the detection of departures from normality assumptions, often identified through tests like the Shapiro-Wilk test. Interestingly, several authors use ART only for ratio data, opting for nonparametric tests such as the Friedman test when analyzing ordinal data.

Out of the 39 papers examined, 38 identify at least one statistically significant main effect using ART. Additionally, 30 papers conduct tests for interactions, with 24 of them reporting at least one statistically significant interaction effect. Only five papers have publicly available data. We revisit the results of three of these papers in [Section 7](#).

Positioning

We observe that ART is frequently used for the analysis of experimental results. Interestingly, the cautions raised by Lüpsen ([2017, 2018](#)) have been widely overlooked, and ART is commonly applied to datasets, including Likert data with five or seven levels, where the method has been identified as particularly problematic. Given the contradictory findings in the existing literature, researchers grapple with significant dilemmas regarding which past recommendations to rely on.

Our goal is to verify Lüpsen's findings by employing an alternative set of experimental configurations and a distinctly different methodology. In particular, we adopt a latent variable modeling approach that establishes a unified framework for data generation across all distributions. This approach allows us to identify interpretation issues regarding the definition of the null hypothesis, especially for interaction effects, and better explain when and why each method fails. Moreover, it facilitates the assessment of the methods through more suitable generation procedures for ordinal data, as suggested by Liddell and Kruschke ([2018](#)). Our simulation strategy is similar to that of Elkin et al. ([2021](#)) but employs a simpler method for generating data from a latent parameter space. In addition, we evaluate critical scenarios that Elkin et al. ([2021](#)) did not consider, particularly the influence of no-null effects and discrete data.

With a focus on clarity, we chose to exclusively examine the three rank-based transformations presented earlier: the pure rank transformation (RNK), ART, and INT. While we do not elaborate on the performance of ATS in the main paper, additional experimental results are available in the [appendix](#). These findings indicate that its performance is comparable to the rank transformation but seems to be inferior to the simpler and more versatile INT. The [appendix](#) features additional results regarding the performance of the generalized van der Waerden test, as well as the generalized Kruskal-Wallis and Friedman tests. Our findings show that, at least in balanced designs, these methods suffer from low power issues in certain situations without demonstrating any clear benefits compared to INT.

Finally, we chose to limit our investigation to balanced experimental designs. The rationale behind this decision is that we have not encountered any prior claims about ART's suitability for unbalanced data, and the ARTTool ([Kay et al. 2021](#)) issues a warning in such situations. However, the [appendix](#) presents additional results on experimental designs with missing data, which confirm that in such situations, ART's accuracy may deteriorate further.

3 Revisiting ART's distributional assumptions

ART is often regarded as a nonparametric method, but its alignment mechanism relies on a set of strict assumptions. For a two-way experimental design with independent observation, Higgins, Blair, and Tashtoush ([1990](#)) assume the following model for the responses:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \quad (2)$$

where μ is the grand mean, α_i is the main effect of the first factor for level i , β_j is the main effect of the second factor for level j , $(\alpha\beta)_{ij}$ is the interaction effect, and ϵ_{ijk} is the residual error term, assumed to have a common variance. The index $k = 1, \dots, n$ denotes the subject number.

This formulation makes several key assumptions:

1. A linear relationship between effects and responses;
2. A common variance among all experimental conditions; and
3. Continuous responses.

The proof by Mansouri and Chang (1995) assessing the limiting distributional properties of ART for interaction effects is based on these same assumptions.

It is worth noting that ANOVA relies on similar assumptions, with the added requirement that errors be normally distributed. One might therefore argue that ART imposes fewer assumptions than ANOVA. Additionally, since ART is rank-based, it may appear reasonable to expect some degree of robustness to violations of the linear model.

Unfortunately, this line of reasoning is not correct. ART's assumptions pertain specifically to its alignment mechanism, which is unique to this method. The rank transformation is applied only after alignment, and the authors of ART have not provided any theoretical or empirical justification for their alignment method under non-linear models, non-equal variances, or discrete data. We explain the severity of these assumptions in more detail.

Violations of the linearity assumption

Suppose we studied image-editing tasks (as in our illustrative example), measuring the time participants needed to complete them for two difficulty levels: *easy* and *hard*. If we assume that observed times follow log-normal distributions, the model of Higgins, Blair, and Tashtoush (1990) will generate distributions as the ones in Figure 6. We observe that the distribution for the hard task is simply shifted to the right, while their shape conveniently remains identical. Mansouri and Chang (1995) have shown that at least for interaction effects, ART remains robust under such distributions. However, how realistic are these distributions?

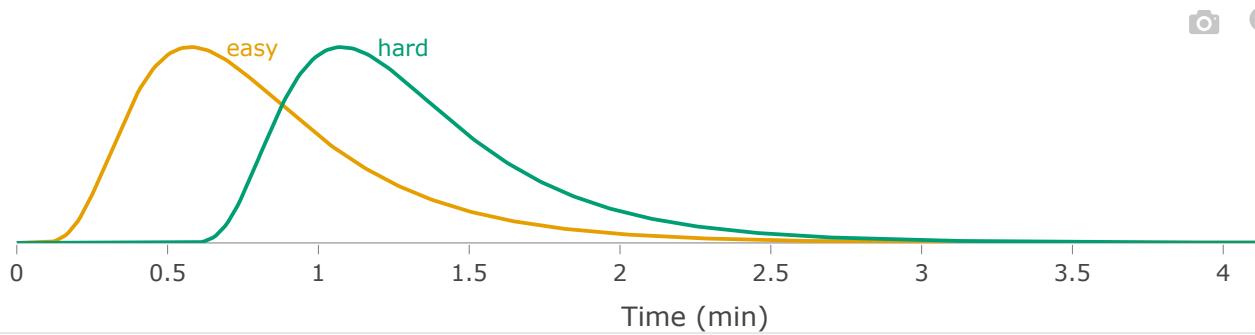


Figure 6: The model of Higgins, Blair, and Tashtoush (1990) assumes that distributions have an identical shape across all conditions. Here, the log-normal distribution of the time measure is simply shifted to the right as the task difficulty increases.

Heavy-tailed distributions, such as the log-normal distribution here, commonly arise in nature when measurements cannot be negative or fall below a certain threshold (Limpert, Stahel, and Abbt 2001), e.g., the minimum time needed to react to a visual stimulus. In most experimental research, however, this threshold does not shift across conditions while preserving the distribution's shape. Instead, distributions are more likely to resemble the ones in Figure 7.

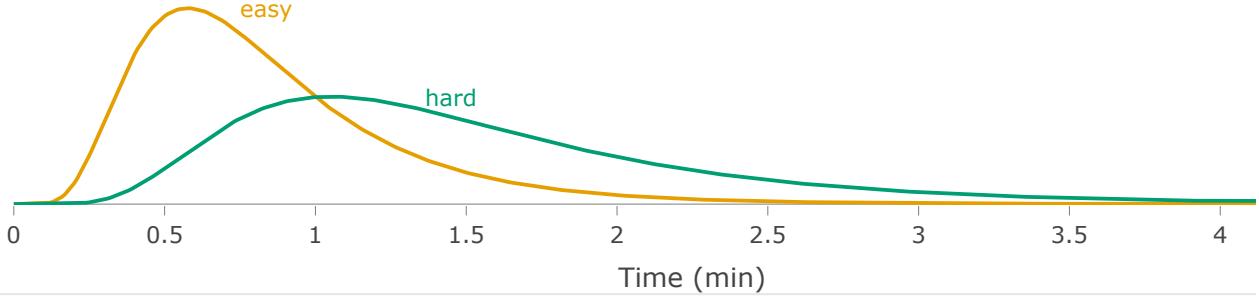


Figure 7: Log-normal distributions produced using a generalized linear model. As the mean of the distributions increases, their overall shape changes.

In these distributions, the mean for hard tasks also increases, but this increase is not reflected as a simple global shift in the distribution. Instead, the overall shape of the distribution changes. The model we used to generate these distributions is structured as follows:

$$\log(Y_{ijk} - \theta) = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \quad (3)$$

where ϵ_{ijk} is normally distributed, and θ represents a threshold below which response values cannot occur. Unfortunately, ART's alignment mechanism is not designed to handle such cases. The alignment calculations shown in [Figure 3](#) include various mean terms that are sensitive to distribution differences across the levels of factors. In particular, the values of a random sample that appear in the tail of the right distribution can disproportionately influence the within-cell mean. This, in turn, leads to the exaggeration of all ranks within that cell, causing the method — as our results demonstrate — to confound effects.

We acknowledge that Elkin et al. (2021) diverged from the modeling approach described by Higgins, Blair, and Tashtoush (1990) and Mansouri and Chang (1995) and considered nonlinear models to evaluate ART. Yet, their experiments did not reveal that ART confounds effects, simply because they assessed Type I error rates only in scenarios where all main effects were null.

Heteroscedasticity issues

Myers et al. (2012) explain that “if the underlying distribution of the response variable is not normal, but is a continuous, skewed distribution, such as the lognormal, gamma, or Weibull distribution, we often find that the constant variance assumption is violated, and in such cases, “the variance is a function of the mean” (Pages 54-55). This pattern frequently occurs in studies measuring task-completion times, whether the task is visual, motor, or cognitive. As tasks become more difficult and prolonged, variance tends to increase. Similarly, slower participants generally exhibit greater variance across tasks compared to faster, more practiced users. This suggests that ART’s assumption of constant variance in skewed distributions is largely unrealistic.

Wagenmakers and Brown (2007) conducted an analysis of nine independent experiments to empirically investigate this pattern in response time distributions. They confirmed that standard deviations were proportional to the mean. The model we presented earlier (see [Equation 3](#)) is in line with these observations. Even when the standard deviation of the error term ϵ_{ijk} is constant, the standard deviation of observed log-normal distributions increases linearly with their mean. [Figure 1](#) (right) shows this pattern in our illustrative example, where the standard deviation of time responses increases proportionally with task difficulty.

However, a question remains: Is ART’s failure in handling such models due solely to heteroscedasticity issues? For example, consider the two time distributions in [Figure 8](#), which — despite having different shapes — share the same standard deviation. Would ART perform correctly in this case?

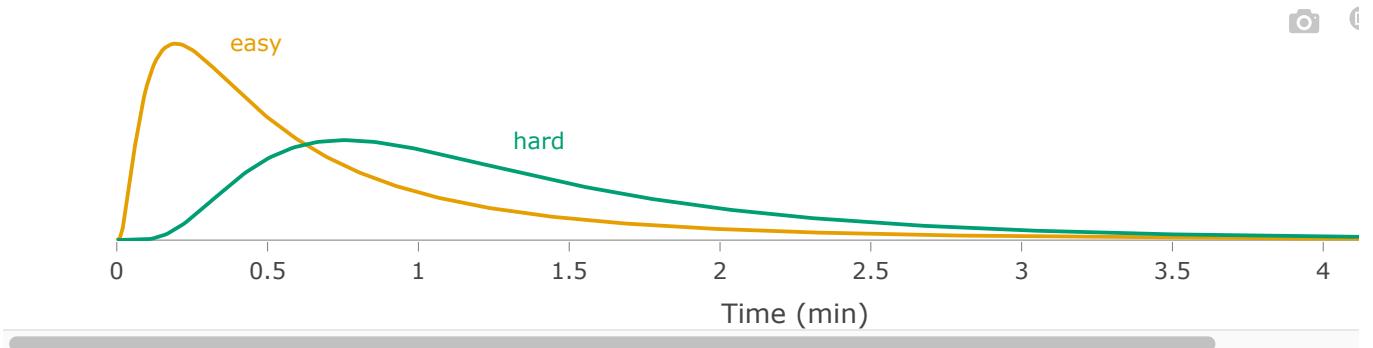


Figure 8: Log-normal distributions with equal variances but different means. Notice that their shapes are distinct.

While we do not directly address this question in the main paper, we include additional results in the [appendix](#) showing that as long as distribution shapes differ across levels, ART tends to confound effects. Naturally, as the shapes become more similar, the problem diminishes. In conclusion, heteroscedasticity may exacerbate the problem, but it is not its sole cause.

Discrete distributions

As discussed earlier, Lüpsen (2017) found that ART frequently fails when responses are discrete. The problem becomes more pronounced for variables with few discrete levels and as the sample size increases. The author provides a detailed analysis of why this happens, summarized as follows:

“There is an explanation for the increase of the type I error rate when the number of distinct values gets smaller or the sample size larger: due to the subtraction of the other effects – a linear combination of the means – from the observed values even tiny differences between the means lead to large differences in the ranking” (Lüpsen 2017).

[Figure 9](#) illustrates this problem using a simple dataset with a binary variable. In our example, all but one value are zero. Notice how dramatically ART alters the ranks and exaggerates the difference between the two levels of B (b_1 vs. b_2). This issue persists even when a large number of participants are added and there is only a single value of 1 among thousands of 0s, causing the method to produce arbitrarily low p -values. Then, a single value variation (e.g., turning this 1 to 0) will cause ART’s results to drastically change. Although this example illustrates an extreme case, ART’s instability can manifest in various forms, inflating error rates even when responses span a wider range of discrete values.

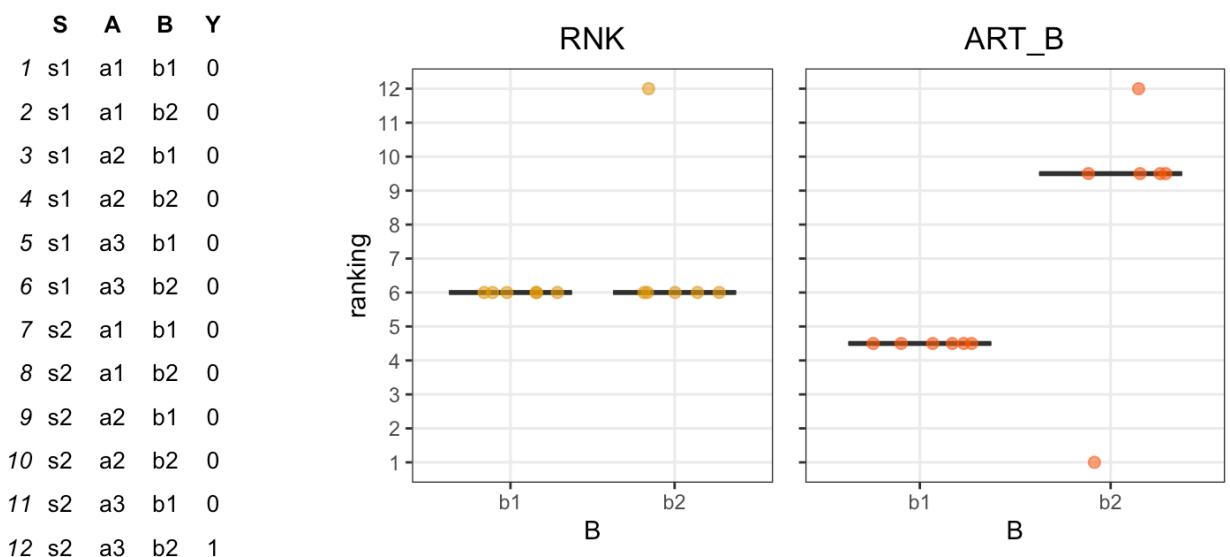


Figure 9: Results from an experiment following a 3×2 design, where the response variable Y is binary and indicates the presence of an error. The plots show the rankings produced by the simple rank transformation (RNK) and by ART for the second factor. The solid lines represent group medians.

4 Defining the null hypothesis of interest

Before comparing different statistical methods, it is essential to assess whether they are actually comparable. If two methods are not designed to test the same null hypothesis, then direct comparisons between them can be misleading. In what follows, we clarify the interpretation of main and interaction effects and explain how we address potential issues in our analysis.

Interpreting main effects

The traditional ANOVA is used to test differences between two or more means. However, nonparametric tests often target other population parameters. For example, the Wilcoxon sign-rank test is commonly described as a test of medians for paired samples (McDonald 2014) and is used when population means are not of interest, e.g., when population distributions are skewed. The Mann-Whitney U and the Kruskal-Wallis tests are used, instead, to assess whether two or more independent samples come from the same population, or more technically, whether the mean ranks of the groups are the same. They can be only interpreted as tests of medians under the strict assumption that the population distributions of all groups have identical shapes and scales (George W. Divine and Juarez-Colunga 2018).

Defining the null hypothesis of interest of a rank transformation is more challenging. Conover and Iman (1981) show that the simple rank transformation procedure (R NK) is equivalent to the Mann-Whitney U and Kruskal-Wallis tests for independent samples. For paired samples, however, it results in a new test, which is different from the Wilcoxon sign-rank test and the Friedman test. Defining the null hypothesis of interest of ART is even more challenging because of the hybrid nature of the method. In particular, while ART is a rank-based transformation procedure, it aligns data with respect to means, where alignment is performed independently for each group.

Dealing with interpretation issues. To avoid such interpretation issues, we focus — unless otherwise stated — on effects that apply monotonic transformations to population distributions. This also ensures a monotonic relationship between different measures of central tendency such as medians and means (with the exception of the Cauchy distribution, where the mean is undefined). In other words, if a treatment increases the population mean, it will also increase the population median. We present an example in [Figure 10](#). The figure shows two population distributions corresponding to the two intermediate levels of difficulty of our illustrative example (see [Figure 1](#)). We observe that the increased difficulty of the task translates both the population mean and the median to the right. In this case, we expect a statistical test to reject the null hypothesis, no matter whether it tests the population mean, the median, or the overall distribution shape.

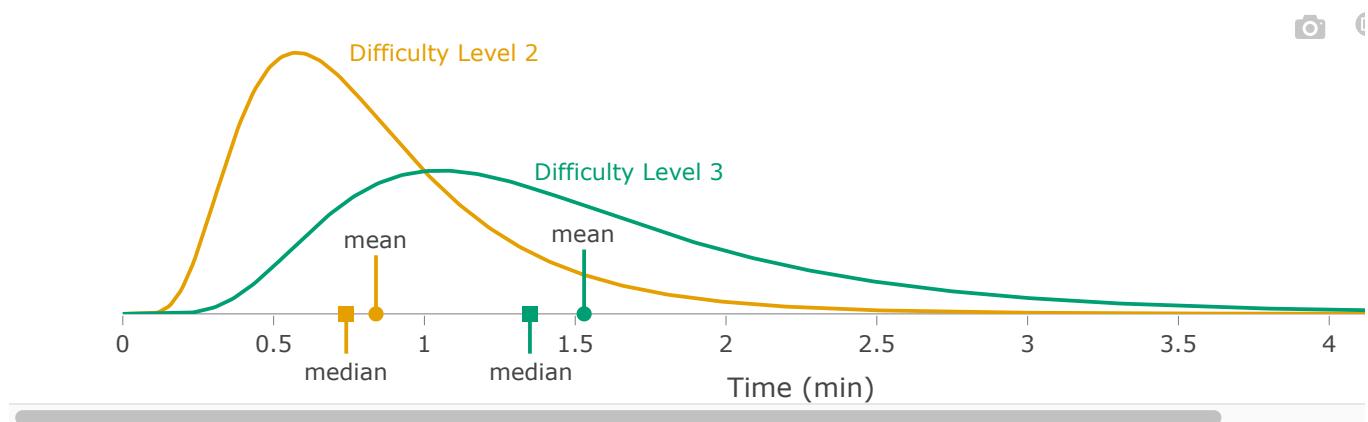


Figure 10: Time distributions for two task populations with difficulty levels 2 and 3 (see [Figure 1](#)).

Interpreting interaction effects

The ART procedure was proposed as an alternative to the rank transformation (Conover and Iman 1981) for testing interactions. As Higgins, Blair, and Tashtoush (1990) explained, the rank transformation is non-linear and, as a result,

it changes the structure of interactions. Therefore, “*interaction may exist in the transformed data but not in the original data, or vice versa*” (Higgins, Blair, and Tashtoush 1990). [Figure 11](#) demonstrates the problem. In this example, the data have been sampled from perfectly normal distributions with equal variances. We observe that while no interaction effect appears in the original data (lines are parallel), the rank transformation deforms the trend. In particular, differences are more pronounced for the middle points of the three-level factor (“medium difficulty”). The figure also shows that the inverse normal transformation also deforms the interaction but to a lesser extent. Note that the problem emerges when the main effect is strong on all interacting factors.

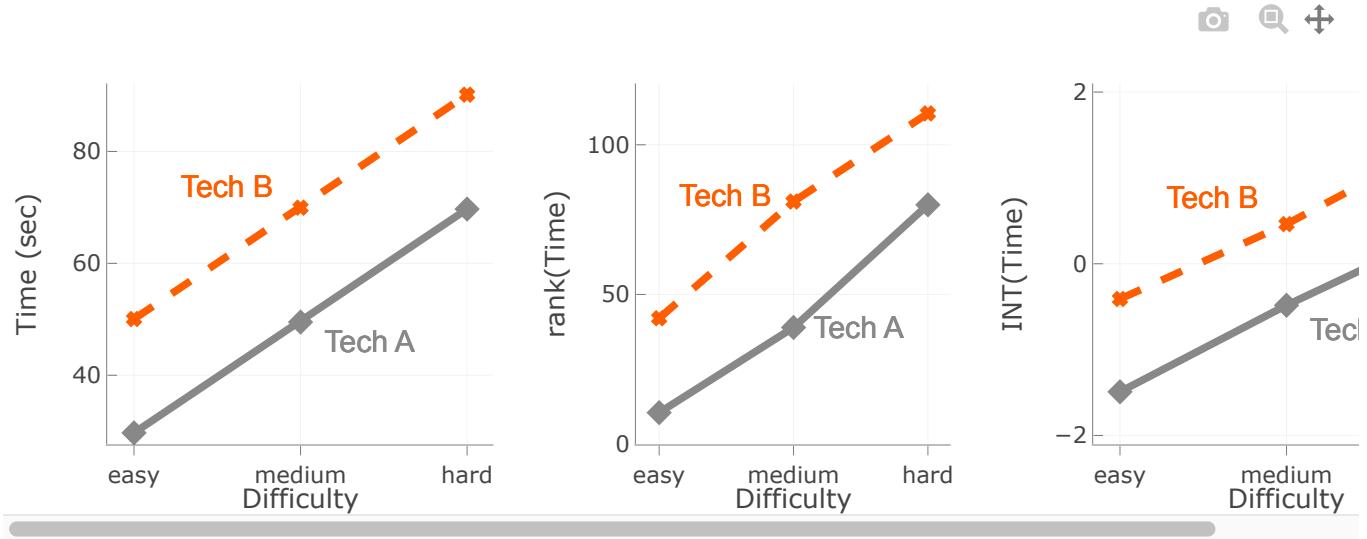


Figure 11: Visualization of interaction effect for a 3×2 experimental design before and after applying the rank transformation and the inverse normal transformation (INT) on an [example dataset](#) (within-participants design, $n = 20$). All data points represent means.

ART aims to correct this problem. However, non-linear transformations come into place in various ways in experimental designs (Loftus 1978; Wagenmakers et al. 2012). They can deform distributions, making the interpretation of observed effects especially challenging. Before presenting our experimental method, we discuss these problems and explain how our approach takes them into consideration.

Removable interactions. Let us take a different [dataset](#) from a fictional experiment (within-participants design with $n = 24$) that evaluates the performance of two techniques (*Tech A* and *Tech B*) under two task difficulty levels (*easy* vs. *hard*). The experiment, for example, could test a mobile typing task, where the levels of difficulty correspond to texts of different lengths (*short* vs. *long*) under two typing techniques (*with* vs. *without auto-completion*). We assume that the researchers measure two dependent variables: task-completion time and perceived performance, which is measured through a five-level ordinal scale (from “very quick” to “very slow”). In this example, the main effects of task difficulty and technique are large. It is less clear, however, whether there is also an interaction between the two factors.

[Figure 12](#) visualizes the means for each combination of the levels of the factors and highlights the possible interactions. Let us first concentrate on the first two plots that present results for the time measure. The trends in the left plot indicate an interaction effect, since the two lines seem to diverge as the task difficulty increases.

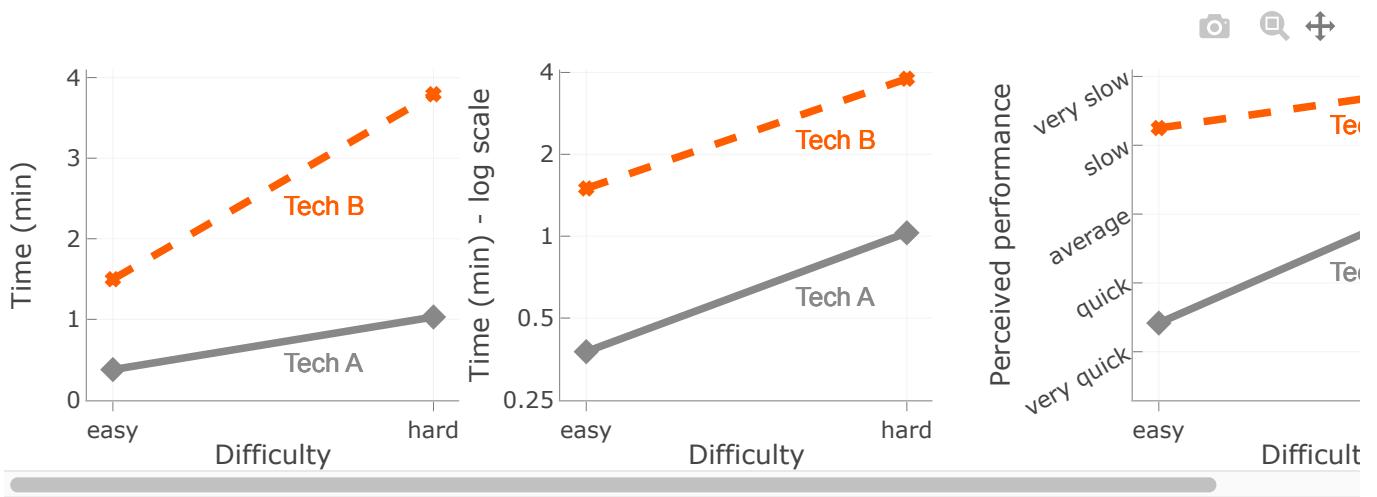


Figure 12: The line charts visualize the effects of task difficulty (easy vs. hard) and technique (Tech A vs. Tech B) for two measures: task completion time (left and middle) and perceived performance (right). All data points represent group means.

But how meaningful is this interpretation of interaction? As we discussed earlier, time measurements are often taken from skewed distributions where the variance is not constant. Therefore, large effects are harder to observe in quick tasks than in slow ones. However, such trends do not necessarily reveal any real interactions, because they are simply due to observations at different time scales. [Figure 12](#) (middle) visualizes the effects using a logarithmic scale. Notice that the lines in the plot are now almost parallel, suggesting no interaction effect.

The concept of *removable* or *uninterpretable* interactions, that is, interactions that disappear after applying a monotonic non-linear transformation, was introduced by Loftus (1978). Over three decades later, Wagenmakers et al. (2012) revisited this work and found that psychology researchers are largely unaware of the concept, drawing incorrect conclusions about psychological effects on the basis of meaningless interactions.

This issue also extends to data collected from questionnaires. The right plot in [Figure 12](#) shows results for perceived performance. Again, the line trends suggest an interaction effect. Unfortunately, the scale is ordinal, which means that distances between the five levels of the scale may not be perceived as equal by people. Furthermore, the scale is bounded, so the reason that the two lines are not parallel might be simply due to the absence of additional levels beyond the extreme “very slow” ranking. Concluding that there is a meaningful interaction here could be incorrect. Liddell and Kruschke (2018) extensively discuss how ordinal scales deform interactions.

Formal testing. We now formally test the above interactions by using ANOVA with different transformation methods. Below, we present the p -value returned by each method for task-completion time:

p -values for interaction effect on task-completion time

PAR	LOG	ART	RNK	INT
.023	.67	.00073	.66	.67

We observe that *RNK* and *INT* lead to p -values very close to the p -value of *LOG*, which suggests a similar interpretation of interaction effects. In contrast, *ART* returns a very low p -value (lower than the p -value of the regular ANOVA), indicating a different interpretation.

We also test the interaction effect on the ordinal dependent variable:

p -values for interaction effect on perceived performance

PAR	ART	RNK	INT	ATS
.0020	.00075	.0067	.0037	.0081

Notice that we omit the log-transformation method (*LOG*), as it is not relevant in this context. Instead, we conduct an analysis with the nonparametric ATS method (Brunner and Puri 2001), as implemented in the R package *nparLD* (Noguchi et al. 2012). All *p*-values are low, suggesting that an interaction effect exists. However, if we conduct a more appropriate analysis using an ordered probit model (Bürkner and Vuorre 2019; Christensen 2023), we find no supportive evidence for such an effect (check our analysis in the supplementary material).

In conclusion, focusing solely on the issues with the rank transformation illustrated in [Figure 11](#) is akin to missing the forest for the trees. When parallel main effects are present, interpreting interactions can be challenging for all methods — and ART offers no solution to this problem.

Disagreement on the interpretation of effects in GLMs

We further discuss interpretation issues in the broader context of generalized linear models (GLMs).

GLMs are defined by a link function g that connects a variable — linearly defined by the predictors — to a response variable Y , which follows an arbitrary distribution. For an experimental design with two factors, x_1 and x_2 , the expected value (or mean) of Y conditional on x_1 and x_2 is expressed as follows:

$$E(Y|x_1, x_2) = g^{-1}(a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2) \quad (4)$$

where g^{-1} is the inverse of the link function.

Statistical procedures based on GLMs define interaction in terms of the interaction coefficient a_{12} in the linear component of the model. That is, the null hypothesis should be rejected when the coefficient is non-zero. However, there is no consensus among researchers on this approach. We clarify the debate and explain how we address it.

Defining interactions on the scale of the response variable. Ai and Norton (2003) and, more recently, McCabe et al. (2022) show that the interaction coefficient alone is not sufficient to describe the actual interaction trend on the natural scale of the response variable. They argue that an interaction effect should instead be defined as the observed change in the marginal effect of x_1 as a function of x_2 . Thus, the trends shown in [Figure 12](#) do represent interactions, since the difference between *Tech A* and *Tech B* changes as a function of *Difficulty* on the original reponse scale (time or perceived performance).

Using this definition, the authors show that a zero interaction coefficient ($a_{12} = 0$) does not necessarily imply the absence of an interaction effect. In fact, for continuous predictors, they demonstrate that when $a_{12} = 0$, the interaction effect can still emerge and is proportional to a_1 and a_2 .

Responding to Ai and Norton (2003). Greene (2010) responds that the interpretation of Ai and Norton (2003) “produces generally uninformative and sometimes contradictory and misleading results” and instead advocates for using the model’s coefficients as the basis for inference:

“Statistical testing about the model specification is done at this step. Hypothesis tests are about model coefficients and about the structural aspects of the model specifications. Partial effects are neither coefficients nor elements of the specification of the model. They are implications of the specified and estimated model” (Greene 2010, 295).

Our position is fully aligned with Greene’s argument. As our earlier example (see [Figure 12](#)) demonstrates, interactions assessed on the scale of the response variable can be misleading about the true underlying effects that generated the data. How meaningful is it to declare interaction effects that arise solely from parallel main effects and lack any theoretical interpretation? It is worth noting that Ai and Norton (2003), as well as McCabe et al. (2022), overlook Loftus’ (1978) critique of uninterpretable interactions.

When the interpretation of effects becomes ambiguous. Issues with interpreting interaction effects arise only when there are parallel main effects — that is, when the coefficients of all interacting factors are non-zero. Interpretation issues may also extend to main effects when the interaction coefficient a_{12} is non-zero. In such cases, we may observe a change in the marginal effect of a factor (e.g., x_1), even if its associated model coefficient (e.g., a_1) is zero. We discuss such scenarios in more depth in the [appendix](#).

By contrast, interpretation is unambiguous in these situations:

1. *Main effects*: when the interaction coefficient a_{12} is zero.
2. *Interaction effect*: when either a_1 or a_2 is zero.

For example, the divergence in ART’s results in our illustrative example (see [Figure 1](#)) cannot be attributed to interpretation issues, since the sample was drawn from a population with no effect of *Technique* ($a_1 = 0$) and no interaction ($a_{12} = 0$).

Throughout the remainder of the article, we make every effort to clarify when the interpretation of an effect is ambiguous. Because different methods may rely on different assumptions about the null hypothesis, we analyze such situations separately. However, it is important to note that the existing literature offers no guidance on how ART is expected to interpret main and interaction effects under these conditions. Early evaluations of ART did not consider nonlinear models, and therefore this issue was not raised. While Elkin et al. (2021) do examine such models, they do not address interpretation issues.

Our approach

Responses in a generalized linear model can follow arbitrary distributions. A key challenge is how to simulate such distributions while maintaining control over the parameters of the linear submodel. We address this by adopting a latent variable modeling approach, in which a normally distributed latent variable is transformed into the observed response variable.

Latent variables. We assume there is a single latent variable \mathcal{Y} that is distinct from the variable Y we observe. For example, the latent variable may represent a population’s performance potential, working memory capacity, the perceived utility of a new technology, or quality of life. For convenience, we assume that this latent variable is linear, continuous, and normally distributed. This is a common assumption in latent variable modeling, e.g., in diffusion-decision models that predict response time and error in two-choice decision tasks ([Ratcliff and McKoon 2008](#)), and in ordinal models ([Liddell and Kruschke 2018](#)). This assumption allows us to define null hypotheses within a shared, normalized space, regardless of the scale of the observed data.

Observed variables. Then, the response variable Y is derived from this latent variable through a monotonic transformation, thus $Y = \mathcal{T}(\mathcal{Y})$, where $\mathcal{T}(y_1) \leq \mathcal{T}(y_2)$ if and only if $y_1 \leq y_2$. A transformation for example occurs when study participants perform a selection task or respond to a Likert-scale item through a questionnaire. [Figure 13](#) shows how we transform normal distributions to log-normal, binomial, and ordinal scales.

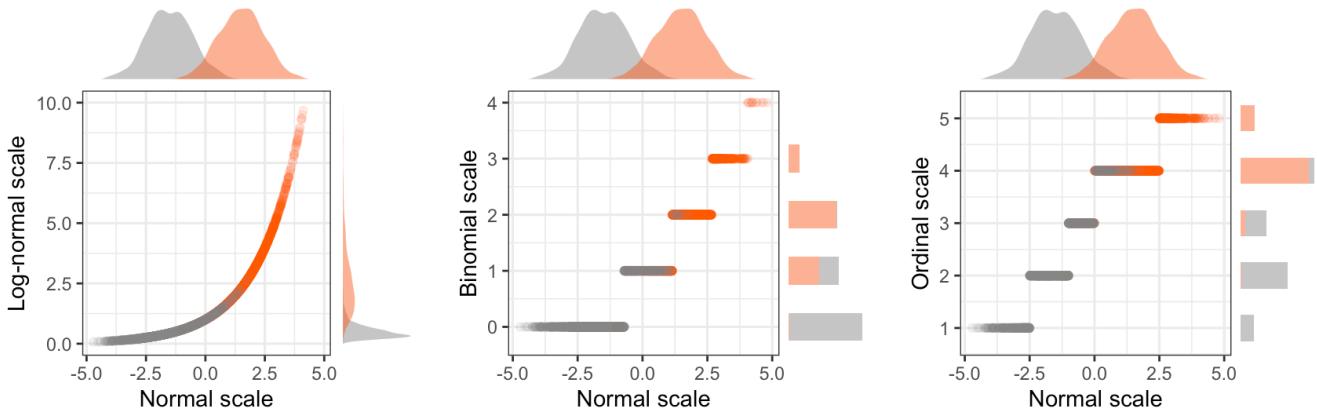


Figure 13: Transformation of normal latent variables to other continuous or discrete scales: log-normal (left), binomial (middle), and ordinal (right).

To transform the latent variable to a ratio scale (e.g., a log-normal and binomial scale), we adopt the distribution conversion approach of *faux* v1.2.1 (DeBruine 2023), an R package for experimental simulations. We first derive the cumulative density distribution of the latent variable. We then use the inverse quantile function of the target distribution to derive the observed variable. For example, in [Figure 13](#) (left), where we transform the latent variable to a log-normal scale with parameters $\mu = 0$ and $\sigma = 1$, we use the following R function:

```
norm2lnorm <- function(x, meanlog = 0, sdlog = 1, mu = mean(x), sd = sd(x), ...) {
  p <- pnorm(x, mu, sd)
  qlnorm(p, meanlog, sdlog, ...)
}
```

For the binomial scale of [Figure 13](#) (left), we use instead the inverse quantile function of the binomial distribution `qbinom(p, size, prob)` with parameters `size = 10` and `prob = .1`, which respectively represent the number of Bernoulli trials and their success probability.

To transform the latent variable to an ordinal scale, we implement an ordered-probit model, as explained by Liddell and Kruschke (2018). According to this model, we discretize the latent variable with thresholds that determine the ordinal levels of interest. For our example in [Figure 13](#) (right), we use as threshold the values $(-2.5, -1, 0, 2.5)$, defining an ordinal scale of five levels. Observe that these thresholds are not equidistant.

Interpreting effects. Our approach allows us to simulate main and interaction effects on the latent variable \mathcal{Y} and observe how these effects manifest on the transformed variable $Y = \mathcal{T}(\mathcal{Y})$.

As discussed earlier, we make clear distinction between the following two scenarios:

1. The definition of the null hypothesis remains the same regardless of whether we evaluate effects on the latent variable \mathcal{Y} or the response variable Y .
2. The definition of the null hypothesis changes with the scale: a null effect on the latent variable \mathcal{Y} may not remain null when evaluated on the response variable Y .

Unless we explicitly mention interpretation issues, we focus on the first scenario. Note that in the case of discrete response variables, the function \mathcal{T} is monotonic but not *strictly* monotonic, since it involves a loss of information. Consequently, an effect on the latent variable may not be observable in the responses, or the observed magnitude of the effect may appear reduced.

5 Experimental method

We can now detail our experimental method. We evaluate the standard parametric approach (*PAR*) and the three rank-transformation methods (*RNK*, *INT*, and *ART*) that we introduced earlier. We conduct a series of Monte Carlo experiments that assess their performance under a variety of experimental configurations:

1. We evaluate *ratio* and *ordinal* data. For ratio data, we examine four representative *continuous* distributions (normal, log-normal, exponential, and Cauchy distribution) and two *discrete* distributions (Poisson and binomial distribution). For ordinal data, we examine distributions for 5-level, 7-level, and 11-level Likert item responses.
2. We present results for five experimental designs. To simplify our presentation, we start with a 4×3 repeated-measures factorial design. We then show how our conclusions generalize to four additional designs: (i) a 2×3 between-subjects design; (ii) a 2×4 mixed design, with a between-subjects factor and a repeated-measures factor; (iii) a $2 \times 2 \times 2$ repeated-measures design, and (iv) a $3 \times 3 \times 3$ repeated-measures design.
3. We focus on three sample sizes, $n = 10$, $n = 20$, and $n = 30$, where n represents the cell size in an experimental design. In within-subjects designs, where all factors are treated as repeated measures, n corresponds to the number of subjects N (commonly human participants in HCI research). In contrast, in a 2×3 between-subjects design, a cell size of $n = 20$, implies a total of $N = 120$ subjects. For ordinal data only, we also report results for a larger range of sample sizes, up to $n = 512$.
4. We test the robustness of the methods when the variances on the latent variable are unequal.
5. In addition to Type I error rates, we compare the statistical power of the methods and assess the accuracy of their effect size estimates.
6. We analyze both main and interaction effects, examining how increasing the effect size of one or two factors influences the Type I error rates of other factors and their interactions.

Previous evaluations of rank transformation methods ([Beasley, Erickson, and Allison 2009](#); [Lüpsen 2018](#)) have also examined unbalanced designs, where cell sizes vary across the levels of a factor. When combined with unequal variances, such designs often pose challenges for both parametric procedures ([Blanca et al. 2018](#)) and rank transformation methods ([Beasley, Erickson, and Allison 2009](#); [Lüpsen 2018](#)). As noted earlier, we do not include unbalanced designs in our evaluation. However, we provide additional experimental results on missing data in the [appendix](#).

Statistical modeling

To model the latent variable \mathcal{Y} , we use a two-way (two factors) or a three-way (three factors) mixed-effects model. For simplicity, we explain here the model for two factors. Its extension to three factors is straightforward. The model has the following form:

$$y_{ijk} = \mu + s_k + a_1 x_{1i} + a_2 x_{2j} + a_{12} x_{1i} x_{2j} + \epsilon_{ijk} \quad (5)$$

- μ is the grand mean
- s_k is the random intercept effect of the k -th subject, where $k = 1..n$
- x_{1i} is a numerical encoding of the i -th level of factor X_1 , where $i = 1..m_1$
- x_{2j} is a numerical encoding of the j -th level of factor X_2 , where $j = 1..m_2$
- a_1 , a_2 , and a_{12} express the magnitude of main and interaction effects
- ϵ_{ijk} is the experimental error effect

To encode the levels of the two factors $x_{1i} \in X_1$ and $x_{2j} \in X_2$ we proceed as follows:

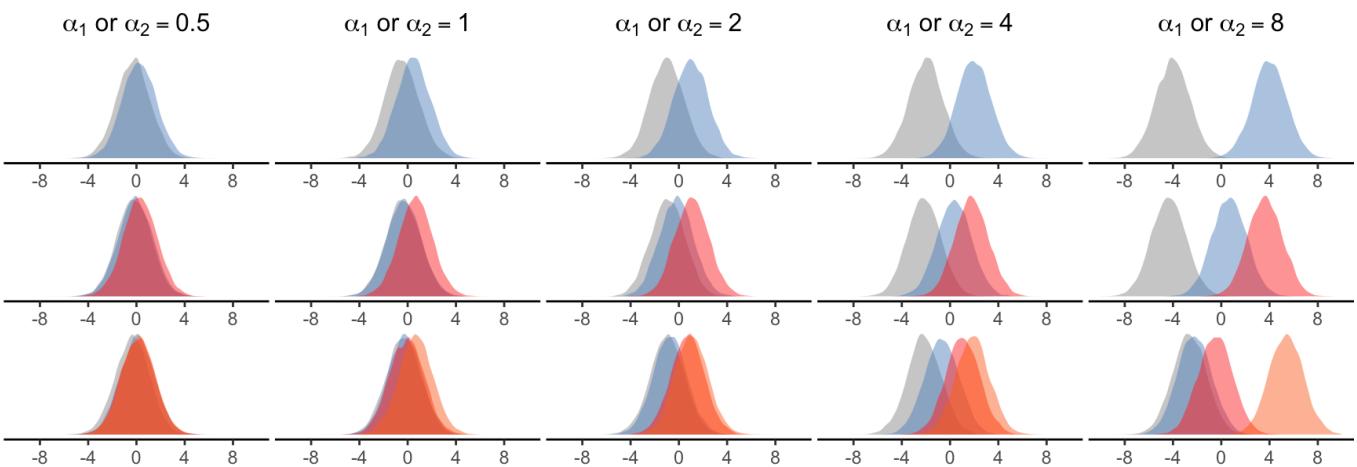
1. We normalize the distance between their first and their second levels such that $x_{12} - x_{11} = 1$ and $x_{22} - x_{21} = 1$. This approach enables us to conveniently control for the main and interaction effects by simply varying the parameters a_1 , a_2 , and a_{12} .
2. For the remaining levels, we randomly sample from a uniform distribution that spans the range between these two extreme levels, i.e., between x_{11} and x_{12} for X_1 , and between x_{21} and x_{22} for X_2 . This approach allows us to generate and evaluate a substantial variety of configurations, each representing different relative effects between levels.
3. We require all levels to sum up to 0, or $\sum_{i=1}^{m_1} x_{1i} = 0$ and $\sum_{j=1}^{m_2} x_{2j} = 0$, which ensures that the grand mean is μ .

For example, we can encode a factor with four levels as $\{-.6, .4, .1, .1\}$ or as $\{-.5, .5, .3, -.3\}$.

While random slope effects can have an impact on real experimental data ([Barr et al. 2013](#)), we do not consider them here for two main reasons: (1) to be consistent with previous evaluations of the ART procedure ([Elkin et al. 2021](#)); and (2) because mixed-effects procedures with random slope effects are computationally demanding, adding strain to simulation resources. However, there is no good reason to believe that adding random slope effects would impact our findings and conclusions.

Population control and distribution conversions

To simplify our simulations, we fix the following population parameters: $\mu = 0$, $\sigma = 1$, and $\sigma_s = 1$. We then control the magnitude of effects by varying a_1 , a_2 , and, for some experiments a_{12} . [Figure 14](#) presents the range of values that we test for a_1 and a_2 . We also visualize their effects on the latent variable for factors with two, three, and four categorical levels.



[Figure 14](#): Varying a_1 or a_2 to control the magnitude of main effects. The plots show examples of population distributions on the latent variable \mathcal{Y} for factors (X_1 or X_2) with two (top), three (middle), or four categorical levels (bottom).

We follow the approach of DeBruine and Barr ([2021](#)) and use the R package *faux* v1.2.1 ([DeBruine 2023](#)) to simulate data for our mixed-effects models. We assume that the distributions of random intercepts and errors are normal, or $s_k \sim N(0, \sigma_s)$ and $\epsilon_{ijk} \sim N(0, \sigma)$. To simulate the observed variable Y , we then transform the response values y_{ijk} as described in [Section 3](#). A key advantage of this method is that we can generate responses for any distribution, while we control effects on the latent variable in the exact same way.

Implementation of rank transformation methods

For the aligned rank transformation (ART), we use the R implementation of ARTTool v0.11.1 ([Kay et al. 2021](#)). For the pure rank transformation (RNK), we use R's *rank()* function. We use the *Rankit* formulation ([Bliss, Greenwood, and White 1956](#)) for the inverse normal transformation (INT), as explained earlier. Unless explicitly mentioned otherwise,

we use the formula $Y' \sim X1*X2 + (1|S)$ with R's *lmer* function, except for the between-subjects design where we use the formula $Y' \sim X1*X2 + Error(S)$ with R's *aov* function.

Evaluation measures

Significance tests have two types of errors. *Type I errors*, or false positives, are mistaken rejections of the null hypothesis. Type II errors, of false negatives, are failures to reject a null hypothesis that is actually true. In our illustrative example in [Figure 1](#), a Type I error is finding that there is an effect of the choice of the technique on time performance. A *Type II error*, in turn, is finding that the task difficulty has no effect on time performance.

Statistical significance testing requires setting a significance threshold known as significance or α (alpha) level, with typical values $\alpha = .05$ and $\alpha = .01$. The Type I error rate of a well-behaved significance test should be close to this nominal alpha level. An error rate clearly above this level suggests that the significance test is too liberal, while an error rate clearly below this level suggests that the test is too conservative. Four of our experiments specifically assess the Type I error rate of the methods. We test two significance levels: $\alpha = .05$ and $\alpha = .01$. For brevity, we only report results for $\alpha = .05$ in the main paper and include additional results in our supplementary material.

We do not directly evaluate Type II errors. Instead, we report on statistical power defined as $Power = 1 - \beta$, where β is the rate of Type II errors. Significance tests do not provide any power guarantees. However, we can compare the power of different methods to evaluate their relative performance. In addition to power, we assess effect size estimates, where we use as ground truth the estimates of a parametric ANOVA conducted on the latent variable \mathcal{Y} . While partial η^2 is the most commonly used effect size measure, we also evaluate Cohen's f , as its expected value is proportional to the real magnitude of effect. However, η^2 can be directly derived from Cohen's f as follows:

$$\eta^2 = \frac{f^2}{1 + f^2} \quad (6)$$

As explained earlier, interaction effects are distorted when the latent variable \mathcal{Y} is transformed to produce the observed responses. These transformations also influence the Type I error rates and statistical power we observe. In this analysis, we focus on evaluating these measures with respect to the effects applied to the latent variable \mathcal{Y} . We explicitly make this distinction whenever it is relevant to our discussion.

Hardware platform and iterations

Our experimental R code is available in our supplementary material. We ran our experiments separately in a cluster of 8 machines Dell R640 Intel Xeon Silver 4112 2.6GHz with 4 cores and 64 GB memory. Our R code was parallelized to use all four cores of each machine. Some experiments took a few hours to complete, while others took several days.

To estimate the power and Type I error rates of the four methods with enough precision, we ran 5000 iterations for each population configuration and each sample size.

6 Results

Each experiment concentrated on a unique combination of distributions, experimental designs, and evaluation measures. We organize our results into several subsections, addressing both main and interaction effects.

Type I error rates in ratio scales

We first evaluate Type I error rates for ratio scales. We test a 4×3 repeated-measures design, where we refer to the 4-level factor as X_1 and the 3-level factor as X_2 . For the observed response variable $Y = \mathcal{T}(\mathcal{Y})$, we evaluate transformations to four continuous and two discrete distributions:

1. No transformation, or $Y = \mathcal{Y}$. Distributions are normal with a grand mean $\mu = 0$ (e.g., see [Figure 14](#)).

2. Log-normal distribution $\text{Log}N(\mu, \sigma)$ with global parameters $\mu = 0$ and $\sigma = 1$. As discussed in [the introduction](#), the log-normal distribution is a good model for various measures bounded by zero, such as task-completion times.

3. Exponential distribution $\text{Exp}(\lambda)$ with a global parameter $\lambda = 2$. The exponential distribution naturally emerges when describing the time elapsed between events. For example, we could use it to model the time a random person spends with a public display, or the waiting time before a new person approaches to interact with the display, when the average waiting time is $\frac{1}{\lambda}$.

4. Cauchy distribution $\text{Cauchy}(x_0, \gamma)$ with global parameters $x_0 = 0$ and $\gamma = 1$. The Cauchy distribution is the distribution of the ratio of two independent normally distributed random variables. It rarely emerges in practice. However, it is commonly used in statistics to test the robustness of statistical procedures because both its mean and variance are undefined. As we discussed earlier, past evaluations of ART ([Mansouri and Chang 1995](#); [Elkin et al. 2021](#)) show that the method fails under the Cauchy distribution.

5. Poisson distribution $\text{Pois}(\lambda)$ with a single parameter $\lambda = 3$. It expresses the probability of a given number of events in a fixed interval of time. For example, we could use it to model the number of people who interact with a public display in an hour, when the average rate is $\lambda = 3$ people per hour.

6. Binomial distribution $B(k, p)$ with parameters $k = 10$ and $p = .1$. It frequently appears in HCI research, as it can model the number of successes and failures in a series of experimental tasks. For example, we could use it to model the number of errors that participants make in a series of $k = 10$ repetitions of a memorization task, when the average error rate is 10%, thus the average error probability is $p = .1$.

Specifically for the log-normal and binomial distributions, we present results for a wider range of their parameters in the [appendix](#).

Main effects. [Figure 15](#) presents Type I error rates for the main effect of X_2 as the magnitude of the main effect of X_1 increases.

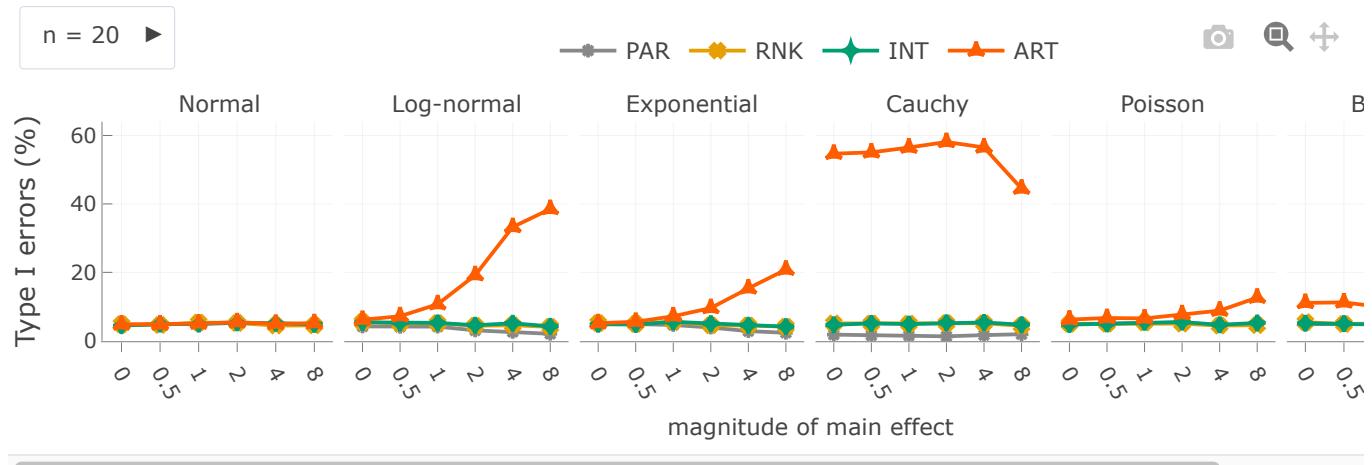


Figure 15: Type I error rates ($\alpha = .05$) for the **main effect of X_2** as a function of the magnitude a_1 of the main effect of X_1

The results show a very good behavior for RNK and INT across all distributions. The regular parametric ANOVA (PAR) keeps error rates below 5%. However, error rates become extremely low for some distributions, suggesting a loss in statistical power. We confirm previous results that ART fails to control the Type I error rate under the Cauchy distribution ([Mansouri and Chang 1995](#); [Elkin et al. 2021](#)) but we also show that the method is problematic with other non-normal distributions. As the main effect on the first factor X_1 increases, Type I errors on the second factor X_2 grow and reach high levels. This pattern is particularly pronounced under the log-normal distribution. We also observe that for the binomial distribution, error rates are high ($\approx 11\%$ for $n = 20$) even when effects on X_1 are zero. In addition, error rates further grow when the sample size increases.

Contrasts. The same problems appear when we run ART's procedure for contrasts (Elkin et al. 2021). [Figure 16](#) shows our results, where we report average error rates for three pairwise comparisons (since X_2 has three levels). In the rest of the paper, we only show results for overall effects, since results for contrasts exhibit the same patterns.

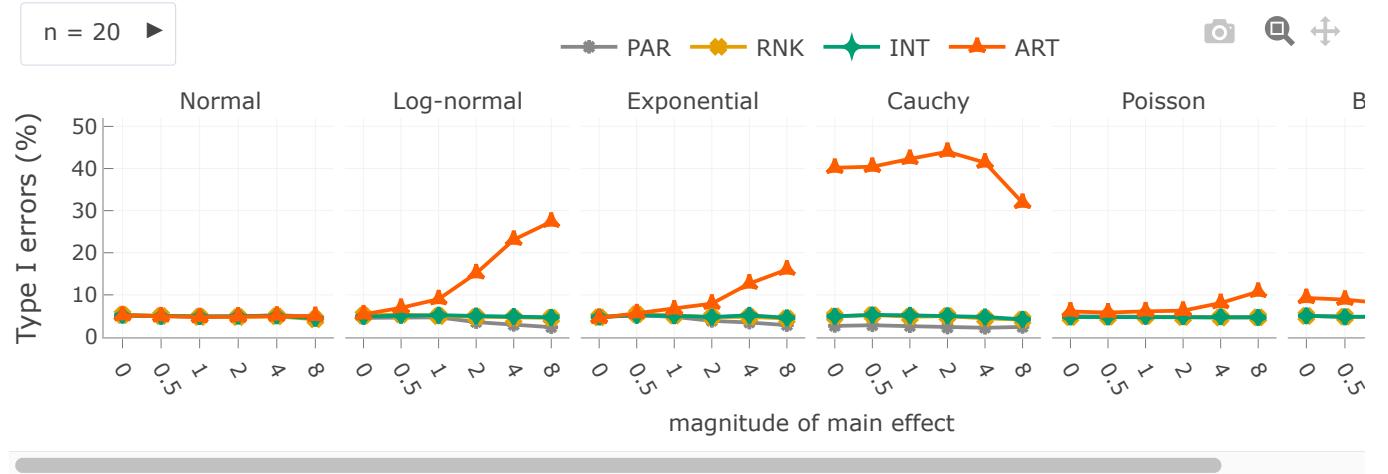


Figure 16: Average Type I error rates ($\alpha = .05$) for **contrasts on factor X_2** as a function of the magnitude a_1 of the main effect of X_1

Interaction effects. [Figure 17](#) presents Type I error rates for the interaction effect $X_1 \times X_2$, when the main effect on X_2 is zero while the main effect on X_1 increases. Overall, we observe the same trends as for main effects. Again, ART fails in similar ways, although its error rates are now slightly lower.

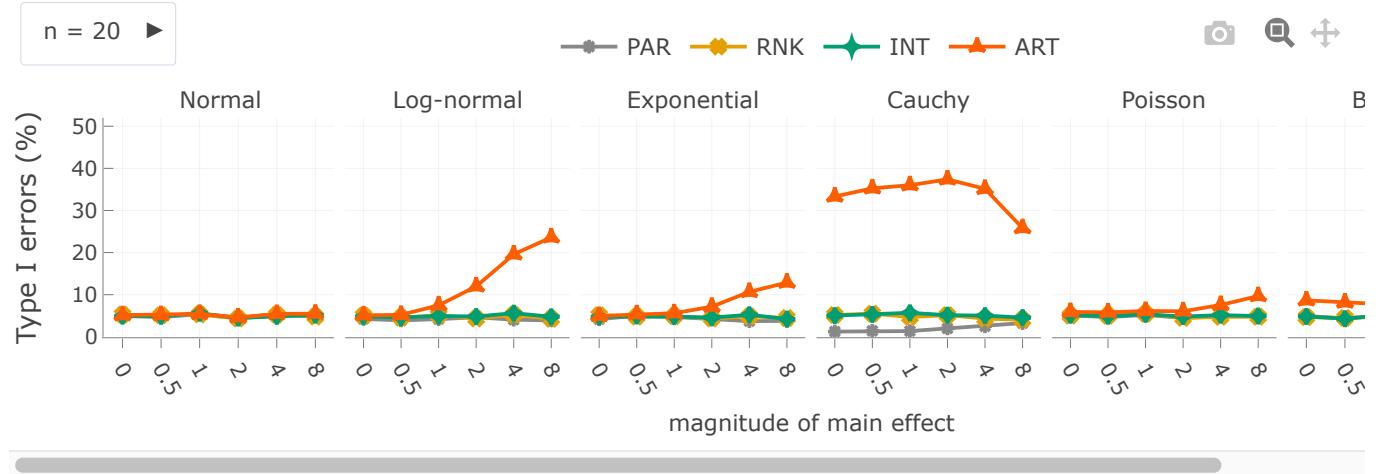


Figure 17: Type I error rates ($\alpha = .05$) for the **interaction $X_1 \times X_2$** as a function of the magnitude a_1 of the main effect of X_1

The scenarios above do not capture the failure cases of the rank transformation discussed earlier (see [Figure 11](#)), as those arise only when both coefficients a_1 and a_2 are non-zero. Since there can be different possible definitions of an interaction effect in these cases, we address them in a separate subsection.

Type I error rates in ordinal scales

We also evaluate Type I error rates for ordinal scales. We test again a 4×3 repeated-measures design. We focus on individual Likert items levels and implement an ordered-probit method (Liddell and Kruschke 2018) to discretize the latent variable Y into 5, 7, or 11 ordinal levels. To derive the discretization thresholds, we first consider the range $[-2SD, 2SD]$, where SD is the overall standard deviation of the responses y_{ijk} . We then divide this range into 5, 7, or 11 intervals, following two different strategies: (i) setting thresholds to be *equidistant*; or (ii) considering *flexible* thresholds, randomly drawing their position in the above range. [Figure 18](#) presents examples of equidistant and flexible thresholds for a 5-level scale when the magnitude of main effect of X_1 is either $a_1 = 2$ or $a_1 = 8$, while all other effects are zero.

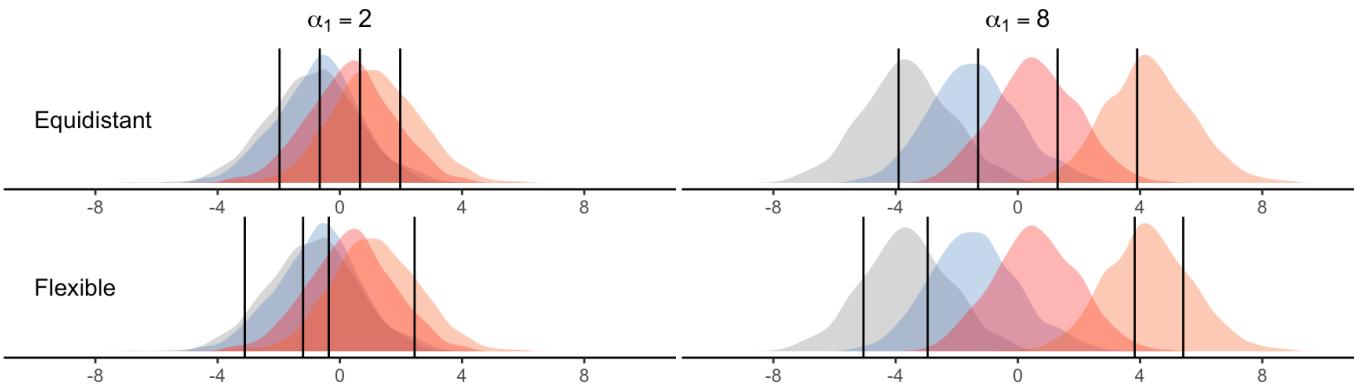


Figure 18: The four vertical lines in each plot represent thresholds defining 5-level ordinal scales. Thresholds are either equidistant (top) or flexible (bottom) within a range of ± 2 standard deviations around the grand mean $\mu = 0$.

Main effects. Figure 19 present Type I errors for X_2 's main effect, as we vary X_1 's magnitude of main effect.

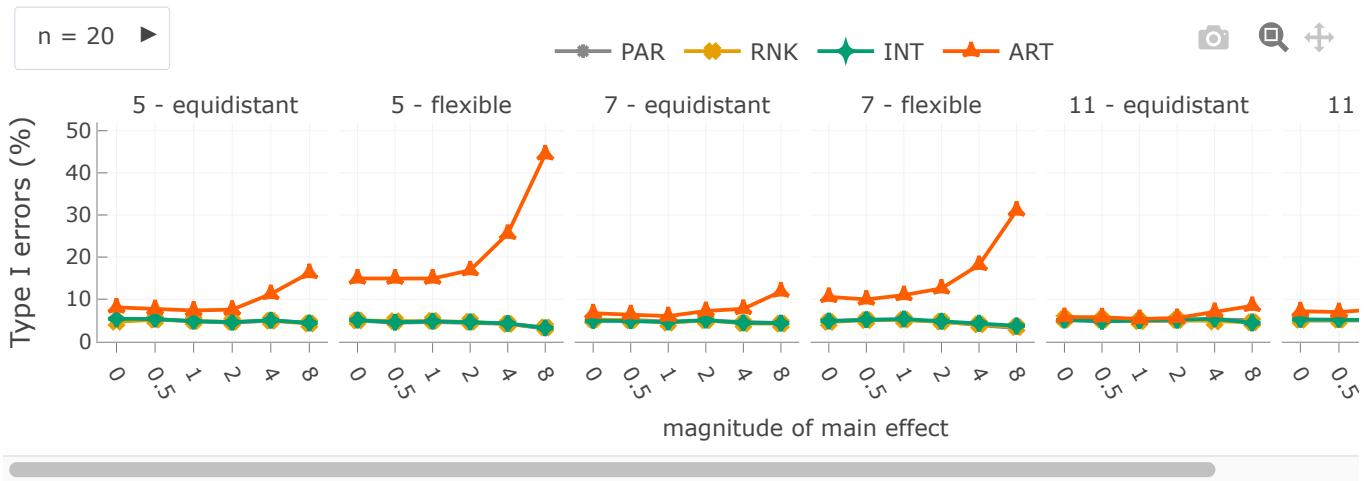


Figure 19: Type I error rates ($\alpha = .05$) for the **main effect of X_2** as a function of the magnitude a_1 of the main effect of X_1

Our results indicate that PAR, RNK, and INT consistently maintain error rates close to 5% across all tested ordinal scales. In contrast, ART exhibits a significant inflation of error rates, although this issue becomes less severe when the number of levels increases. Notably, error rates are more pronounced for flexible thresholds and tend to increase with sample size.

Interaction effects. Figure 20 displays error rates for the interaction $X_1 \times X_2$ with a single main effect applied to X_1 . The observed patterns align with our previous findings; ART consistently inflates error rates.

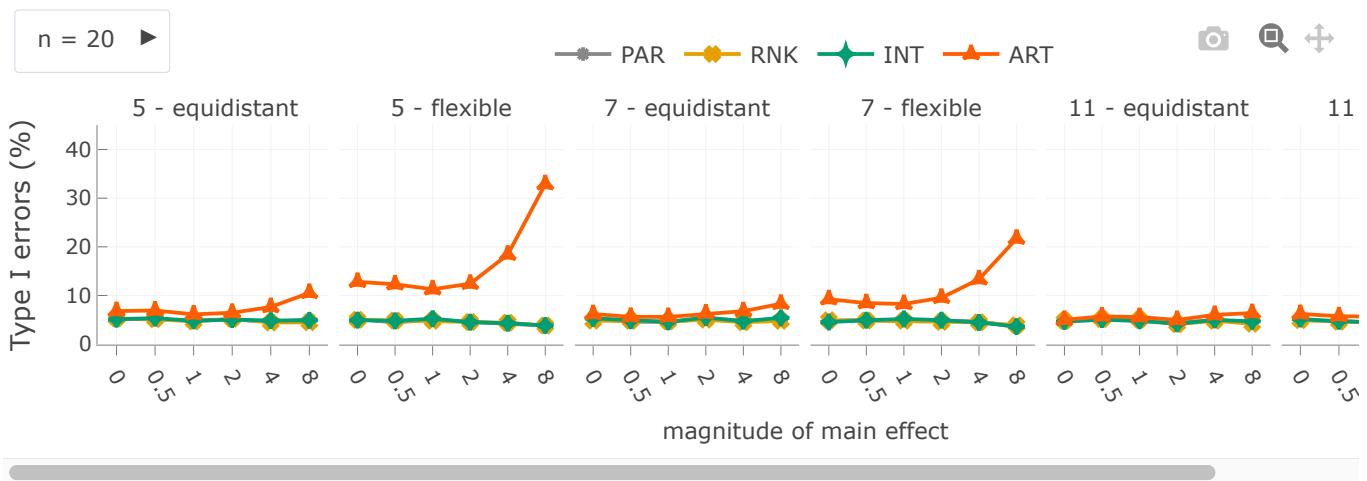


Figure 20: Type I error rates ($\alpha = .05$) for the **interaction $X_1 \times X_2$** as a function of the magnitude a_1 of the main effect of X_1

Analysis for large samples. Let us further explore ART's error rates when sample sizes become larger. As shown in [Figure 21](#), error rates explode even when all population effects are null.

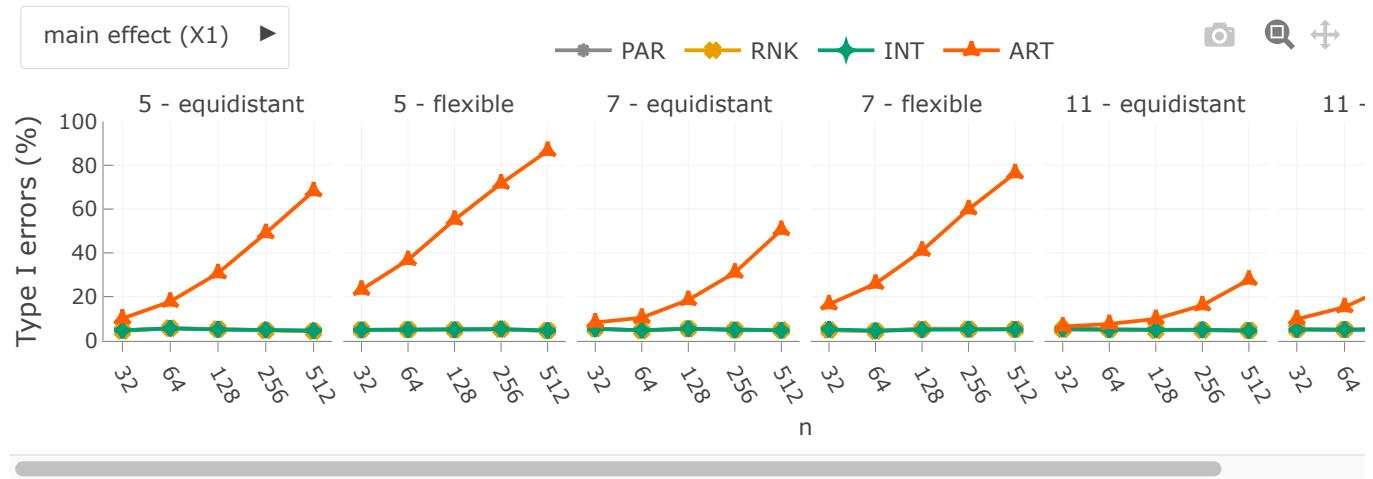


Figure 21: Type I error rates ($\alpha = .05$) as a function of the size n of the sample, where all population effects are null.

We will observe similar trends if we test other discrete distributions, such as binomial distributions. These results confirm that ART is inappropriate for discrete data.

Type I errors across experimental designs

We investigate the generalizability of the aforementioned results to other experimental designs involving two or three factors. We assess five out of the six ratio scales examined earlier. We exclude the Cauchy distribution and replace it by a 5-level ordinal scale with flexible thresholds.

Main effects. [Figure 22](#) illustrates Type I errors for the main effect of X_2 while varying the magnitude of the main effect of X_1 , with a focus on $n = 20$. Across all cases, RNK and INT consistently maintain error rates close to 5%. PAR's error rate remains overall close to 5% but reaches notably low levels for specific combinations of designs ($2 \times 2 \times 2$ and $3 \times 3 \times 3$ repeated-measures) and distributions (log-normal and exponential). In contrast, ART inflates error rates for all non-normal distributions, with discrepancies across different designs. We observe that ART is particularly problematic in discrete distributions when applied to the $2 \times 2 \times 2$ and $3 \times 3 \times 3$ designs.

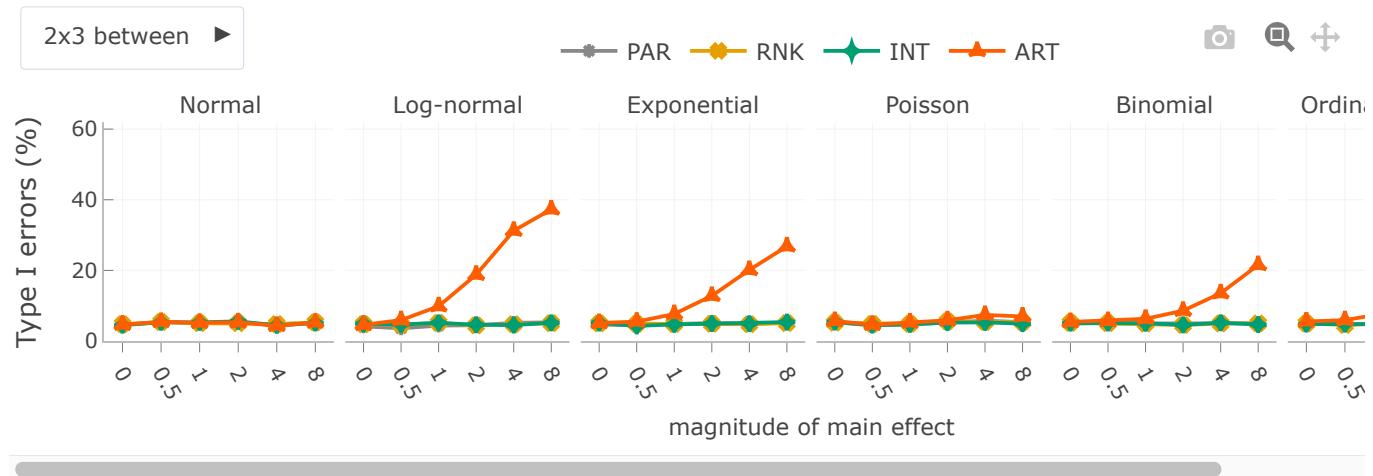


Figure 22: Type I error rates ($\alpha = .05$) for the **main effect of X_2** as a function of the magnitude a_1 of the main effect of X_1 ($n = 20$)

We provide additional results for the between-subjects design in [Figure 23](#), where we vary the effect of X_2 and measure Type I error rates on X_1 . ART's error rates for the three discrete distributions appear higher than those observed in [Figure 22](#), indicating a potential dependence on the number of levels of the factors. Again, we observe a consistent trend where error rates in all discrete distributions increase with the sample size.

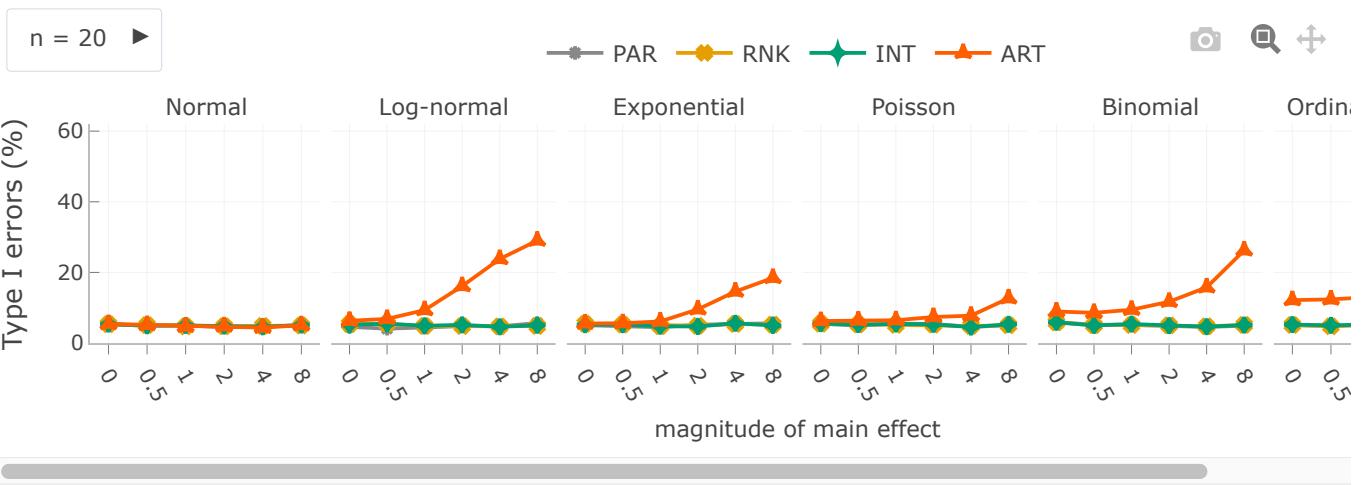


Figure 23: Additional results for the 2×3 between-subjects design, where we measure Type I error rates ($\alpha = .05$) for the **main effect of X_1** . We now vary the magnitude a_2 of the main effect of X_2 .

Interaction effects. [Figure 24](#) displays Type I error rates for the interaction effect $X_1 \times X_2$ while varying the effect of X_1 ($n = 20$). We observe consistent trends in line with our previous findings. For additional insights, we direct readers to our raw experimental data, which demonstrate that these trends persist across other interaction terms, namely $X_1 \times X_3$, $X_2 \times X_3$, and $X_1 \times X_2 \times X_3$, within the two 3-factor designs.

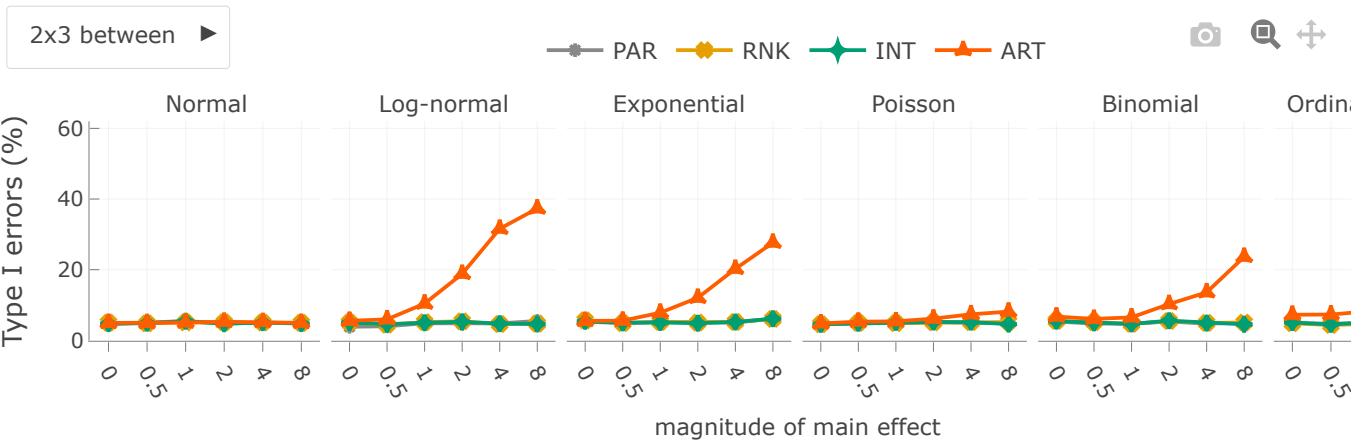


Figure 24: Type I error rates ($\alpha = .05$) for the **interaction $X_1 \times X_2$** as a function of the magnitude a_1 of the main effect of X_1 ($n = 20$)

Interactions under parallel main effects

Let us now examine interaction effects when both coefficients a_1 and a_2 increase in parallel. In this scenario, the interpretation of an interaction effect can become ambiguous. Here, we define the null hypothesis in terms of the coefficients of the latent linear model, following the recommendation of Greene (2010) — that is, $a_{12} = 0$.

Ratio scales. [Figure 25](#) presents Type I error rates for ratio scales.

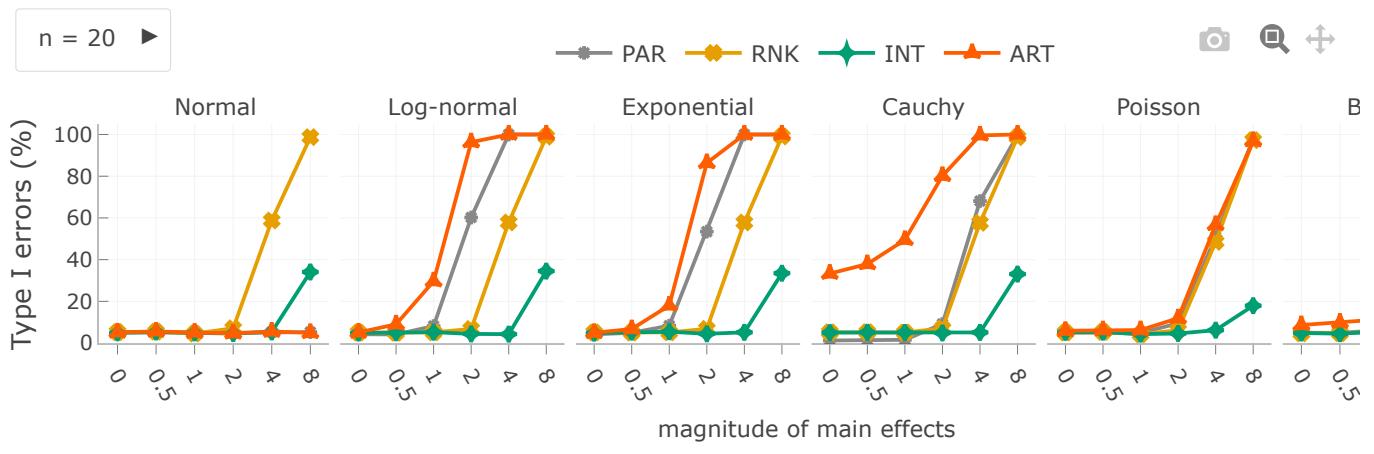


Figure 25: Type I error rates ($\alpha = .05$) for the **interaction** $X_1 \times X_2$ as a function of the magnitudes $a_1 = a_2$ of the main effects of X_1 and X_2 . Note: Type I errors are defined based on the null hypothesis $a_{12} = 0$. A different definition may lead to different results.

Error rates become exceptionally high in some cases, reaching up to 100%. However, these results require careful interpretation. As discussed in [Section 4](#), interaction effects in nonlinear models can be interpreted in two distinct ways, and the results from the different methods are not consistent with our definition. Let us examine the outcomes for each method:

- **PAR.** Error rates are high for all non-normal distributions and become even higher when the sample size increases. However, this behavior clearly stems from the method's sensitivity to the scale of the observations, rather than to the model parameters.
- **RNK.** Error rates explode when both main effects exceed a certain level (e.g., when $a_1, a_2 \geq 4$ and $n = 20$). This problem arises from the way the rank transformation distorts interaction effects (see [Figure 11](#)). For all continuous distributions, we observe that error rates exhibit the exact same patterns.
- **INT.** It performs better than RNK. Errors begin to increase across all distributions only when the main effects become large. An exception is the binomial distribution, for which the error rates of INT and RNK are similar.
- **ART.** It maintains correct error rates as long as population distributions are normal. For all other distributions, error rates increase rapidly as effect sizes grow. Two factors explain this behavior: (i) a lack of statistical robustness to violation of the method's assumptions, as also observed in our previous experiments; and (ii) interaction interpretation issues. These results confirm that ART is not scale-invariant, interpreting interactions relative to the scale of the observed data.

Ordinal scales. [Figure 26](#) presents our results for ordinal data. No method maintains low error rates under all conditions. We note that ART consistently performs worse than PAR. INT performs worse than ART and PAR in one specific case ($a_1, a_2 = 8$ in a scale with 11 equidistant levels), but overall, it exhibits a better behavior than all other methods. Once again, the high error rates can be partly explained by insufficient statistical robustness and ambiguities in how the methods interpret interaction effects.

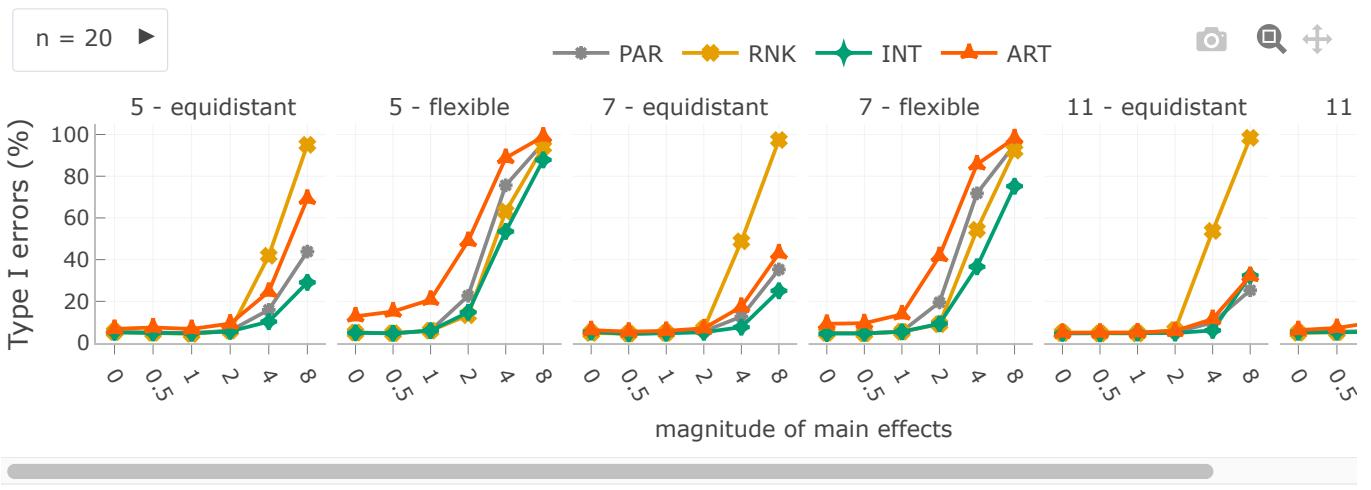


Figure 26: Type I error rates ($\alpha = .05$) for the **interaction** $X_1 \times X_2$ as a function of the magnitudes $a_1 = a_2$ of the main effects of X_1 and X_2 . Note: Type I errors are defined based on the null hypothesis $a_{12} = 0$. A different definition may lead to different results.

Other designs. Our findings in [Figure 27](#) further demonstrate that all methods struggle to accurately detect interactions for different reasons. Interestingly, INT shows unusually low error rates in certain configurations (e.g., 2×3 and $2 \times 2 \times 2$), indicating potential difficulty in detecting subtle interaction effects when main effects are large. RNK follows a similar trend for the $2 \times 2 \times 2$ design. Both RNK and INT also exhibit higher error rates under ordinal and binomial scales, though these remain lower than those observed for PAR and ART.

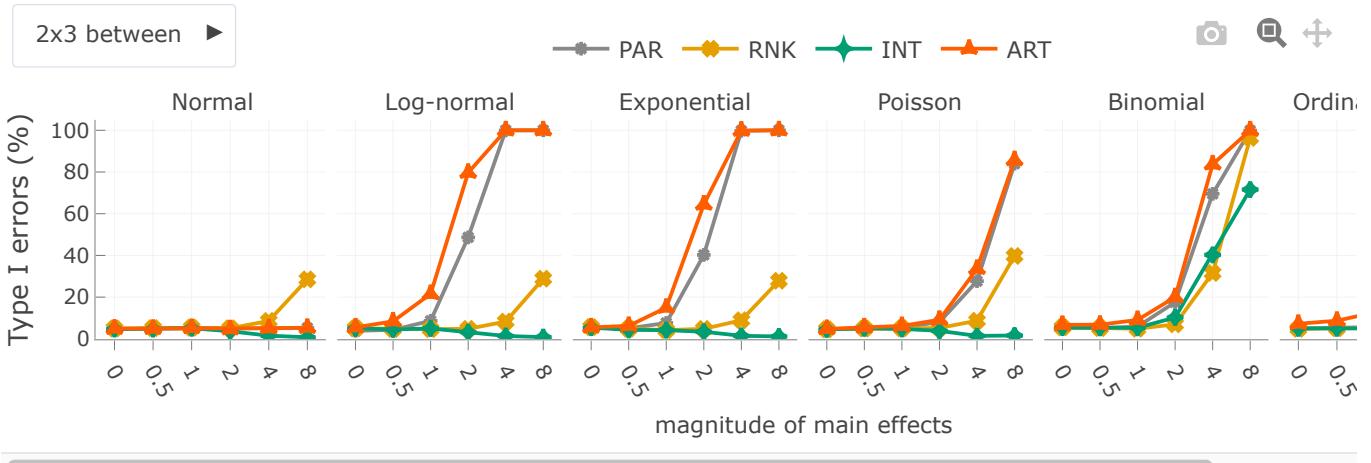


Figure 27: Type I error rates ($\alpha = .05$) for the **interaction** $X_1 \times X_2$ as a function of the magnitudes $a_1 = a_2$ of the main effects of X_1 and X_2 ($n = 20$). Note: Type I errors are defined based on the null hypothesis $a_{12} = 0$. A different definition may lead to different results.

Some readers may argue that evaluating PAR and ART using an interaction definition that accounts for the scale of the observed variable might be more appropriate. We offer three responses to this criticism:

1. It is unclear how to reliably control the null hypothesis at the level of the response scale when both main effects are varied.
2. Our earlier experiments demonstrate that ART fails to maintain nominal Type I error rates for interactions even when only a single main effect is present. Introducing a second main effect could only exacerbate the problem.
3. The purpose of these results is not to provide precise benchmarks, but rather to illustrate the complexity and pitfalls of interpreting interaction effects under assumption violations — regardless of the method used. Many researchers are unaware of these issues and often misinterpret the p -value when testing interactions. Note that Elkin et al. (2021) compared ART to ANOVA on log-transformed data under the belief that ART tests null hypotheses about parameters of the latent linear model. Our findings clearly show that this is not the case.

Type I errors under unequal variances

Many statistical procedures assume equal variances among all levels of each independent variable, or more strictly, among all possible pairs of independent variable levels (known as sphericity assumption in repeated-measures ANOVA). Nonparametric tests are often mistakenly considered to be free of such assumptions, but as we discussed earlier, this is generally incorrect.

Our next experiment evaluates the behavior of the four methods when populations on the latent variable \mathcal{Y} have unequal variances. As we previously discussed, even when variances are equal at the latent space, they may appear as unequal on the response scale if the response distributions are skewed. This is the case, for example, with the log-normal and the exponential distributions. So, in this experiment, we explore more extreme violations of the equal-variance assumption for these distributions.

We examine three 2-factor designs: (i) a 4×3 within-subjects design; (ii) a 2×3 between-subjects design; and (iii) a 2×4 mixed design. We set all effects to zero and then vary the ratio r_{sd} of standard deviations between the levels of the first factor X_1 . [Figure 28](#) shows the five ratios that we test when X_1 has two levels. When X_1 has additional levels (4×3 design), their standard deviation is randomly drawn in a range between the standard deviations of the first two levels.

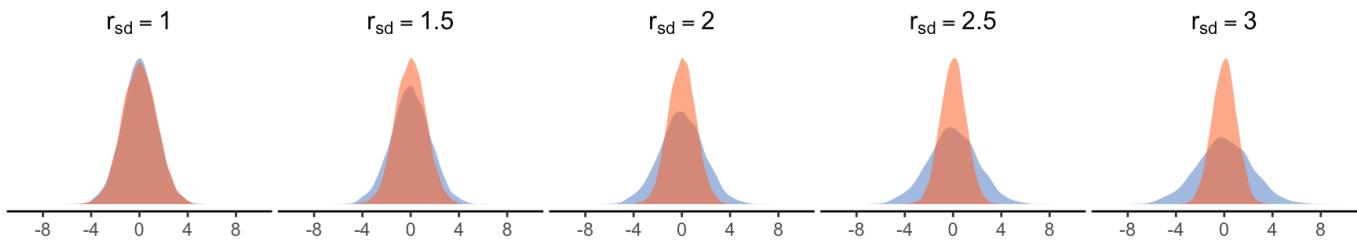


Figure 28: Varying the ratio r_{sd} of standard deviations between the levels of X_1 . The plots show examples of population distributions for factors with two categorical levels.

Main effects. We begin by investigating how the four methods detect main effects on X_1 . The interpretation of results in this scenario is challenging due to differences in the original populations — although they share the same means (and medians), their variances differ. Depending on the null hypothesis of interest, conclusions may vary. Complicating matters further, non-linear transformations can result in distributions with differing means and medians. Consequently, the choice of statistical method may lead to different outcomes; one method may be sensitive to mean differences while others to median differences.

[Figure 29](#) presents the percentage of times each method rejects the null hypothesis (% positives) at a significance level of $\alpha = .05$. If the null hypothesis of interest posits that “ X_1 has no effect on the mean of the latent variable \mathcal{Y} ,” then these instances should be considered as Type I errors. Under normal distributions, all methods either moderately inflate or deflate the positives ratio as r_{sd} increases. For the 4×3 within-subjects design ($r_{sd} \geq 2.5$), the ratio for PAR, RNK, and ART reaches levels of around 7 – 8%, while for INT, it drops below 3% under the 2×4 mixed design ($r_{sd} \geq 2.5$).

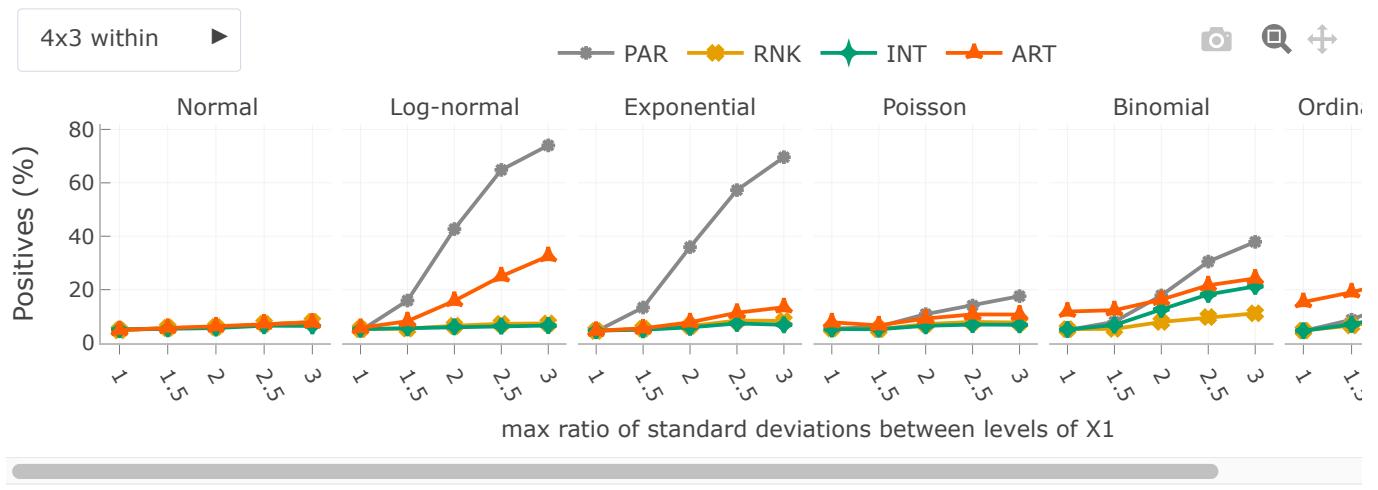


Figure 29: The percentage of positives (i.e., rejecting the null hypothesis) for the **main effect of X_1** as the ratio of standard deviations r_{sd} on X_1 increases ($n = 20$). Depending on the hypothesis of interest, these percentages can be interpreted as Type I error rates ($\alpha = .05$).

The performance of RNK and INT remains unchanged across continuous distributions, a result expected due to the preservation of medians in such scales. Conversely, mean differences grow with increasing r_{sd} , which explains why PAR's ratio of positives reaches high levels under these distributions. However, this trend is not consistent across all three designs. The behavior of ART is less clear, further supporting our argument that its null hypothesis of interest is ill-defined. Under discrete distributions, the trends of positives vary across designs, with PAR and ART exhibiting the highest rates.

We also investigate the influence of unequal variances among the levels of X_1 on positive rates for X_2 , as depicted in [Figure 30](#). These can be reliably considered as Type I errors because source populations defined by X_2 are identical. The error rates of PAR, RNK, and INT do not seem to be affected by an increase of r_{sd} . In contrast, ART yields high error rates, surpassing acceptable levels even in the case of normal distributions (check results for the 2×4 mixed design). Thus, the problem of unequal variance is more serious for ART than for other methods, as the method's sensitivity propagates across factors. This further demonstrates that ART's alignment procedure causes the method to confound effects.

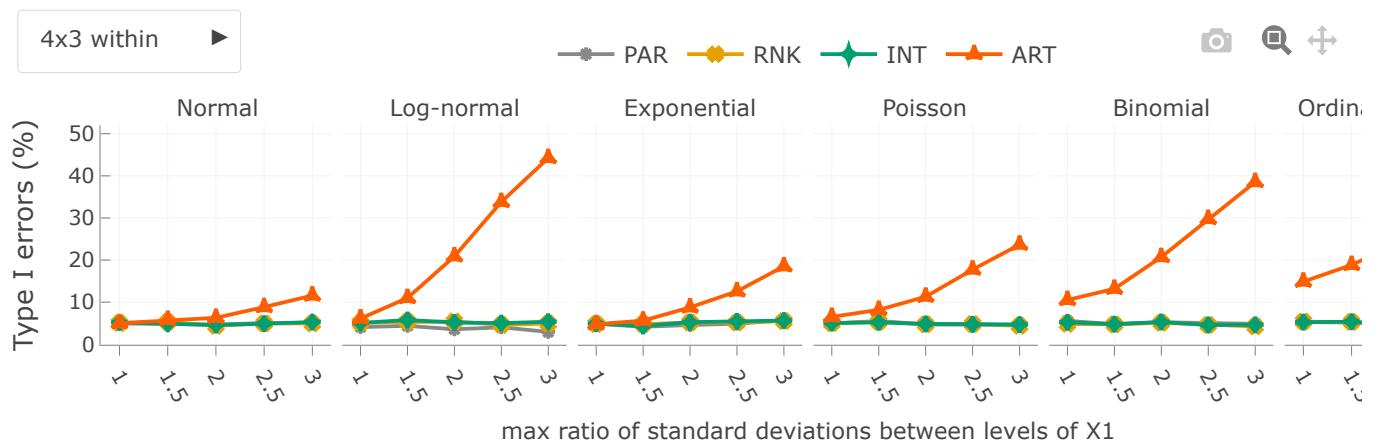


Figure 30: Type I error rates ($\alpha = .05$) for the **main effect of X_2** as the ratio of standard deviations r_{sd} on X_1 increases ($n = 20$)

Interaction effects. Finally, we measure Type I error rates for the interaction effect $X_1 \times X_2$, shown in [Figure 31](#). The error rates of ART exhibit similar trends, with a somewhat lower level under the 4×3 within-subjects design (compared to [Figure 30](#)). Additionally, for this design, we observe error inflation for the three other methods, albeit to a significantly lesser degree.

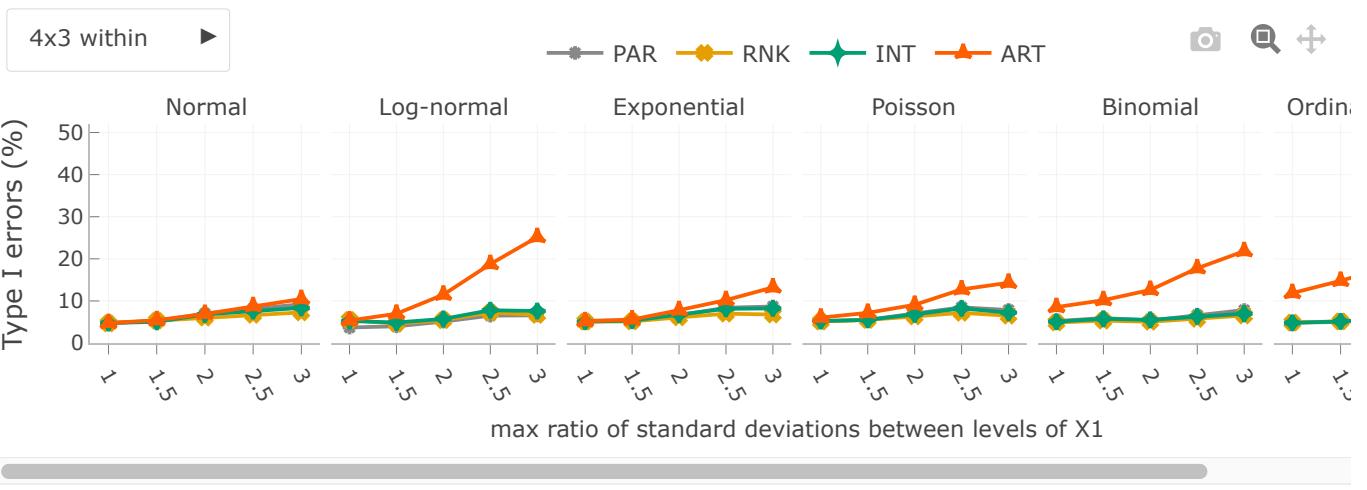


Figure 31: Type I error rates ($\alpha = .05$) for the **interaction effect** $X_1 \times X_2$ as the ratio of standard deviations r_{sd} on X_1 increases ($n = 20$)

Statistical power

Our sixth experiment compares the statistical power of the four methods on the three 2-factor designs. Because there is a tradeoff between Type I and Type II errors, high power can simply be the result of a high Type I error rate. Since parallel effects can inflate errors (see our previous results), we focus here on single effects, both main or interaction effects. We only report results for $n = 20$ (see supplementary materials for additional results).

Main effects. We individually vary the magnitude of effect on X_1 and X_2 and observe the power of each method to detect this effect ($\alpha = .05$). Specifically, we manipulate the parameters a_1 or a_2 within the range of 0.4 to 1.0. Depending on the experimental design, this range allows us to simulate both low-power experiments (e.g., with less than 30% power) and high-power experiments (e.g., with more than 80% power).

[Figure 32](#) presents our results for X_1 . In several configurations, the differences in power among the methods are marginal. To enhance clarity in our comparisons, we also present the method rankings. Readers are encouraged to interact with the graphs for precise power values. PAR exhibits the highest power under the normal distribution, closely followed by INT. INT is also consistently the winning method across all non-normal distributions. PAR's power is notably low under the log-normal and exponential distribution.

ART initially appears advantageous under the three discrete distributions for small effects, likely due to inflated Type I errors even without an effect on X_2 (as observed in [Figure 23](#)). However, as effects grow larger, its relative power diminishes, and ART eventually becomes the least effective method.

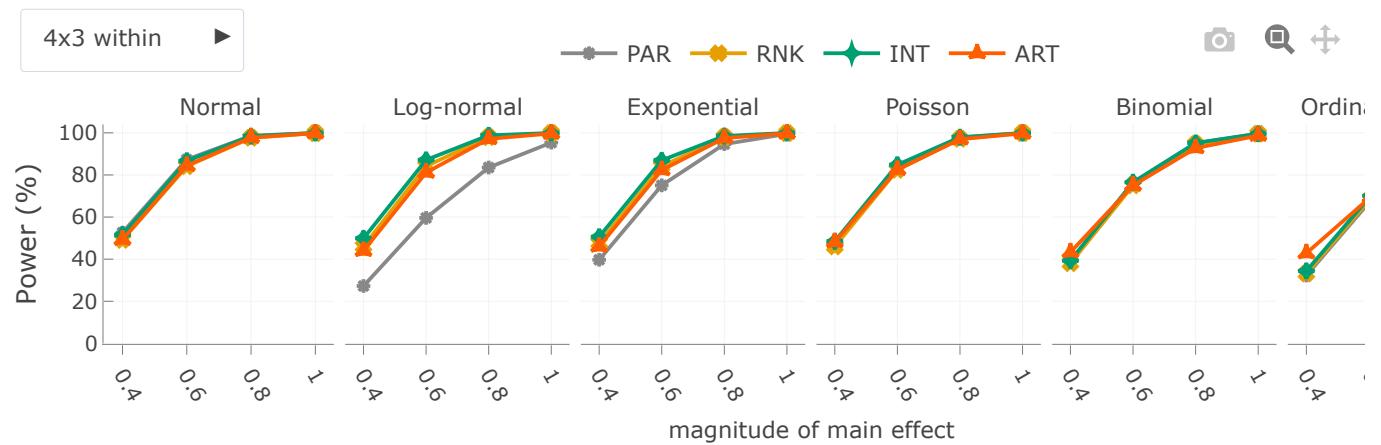


Figure 32: Power ($\alpha = .05$) for detecting the **main effect of X_1** as a function of its magnitude a_1 , where $n = 20$.

[show ranking](#)

[Figure 33](#) presents our results for X_2 . Given the different number of levels for this factor, we now assess a different range of power for each design. Nevertheless, we observe similar patterns. INT stands out as the most powerful

method. Although ART surpasses PAR under the log-normal and exponential distributions, it does not demonstrate any other notable advantages compared to the other methods.

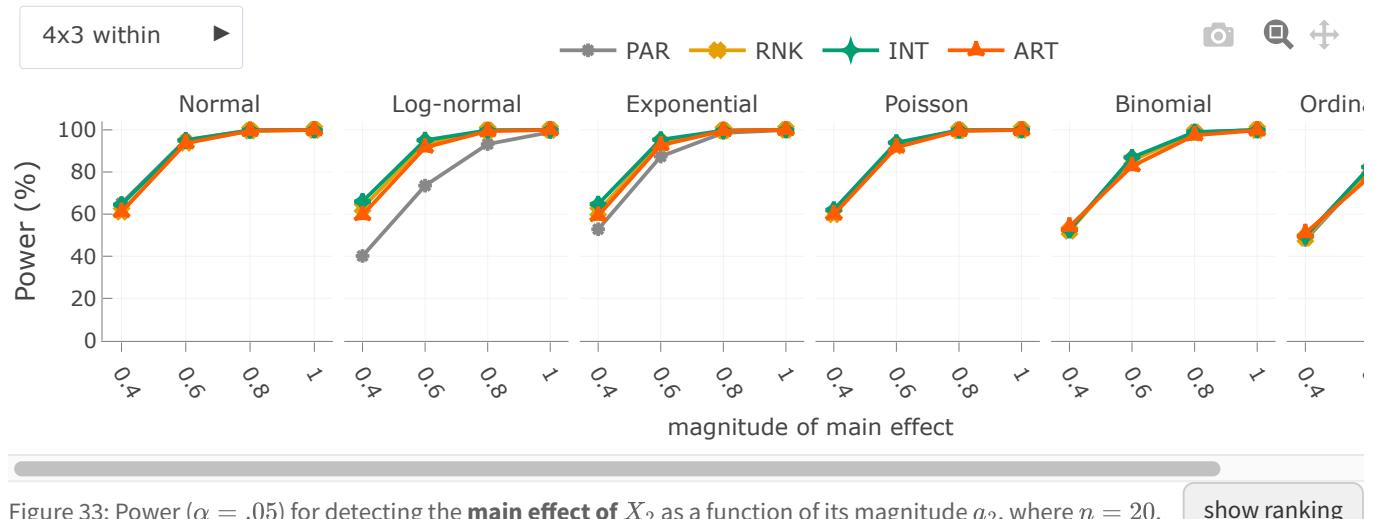


Figure 33: Power ($\alpha = .05$) for detecting the **main effect of X_2** as a function of its magnitude a_2 , where $n = 20$. show ranking

Interaction effects. We also vary the magnitude of the interaction effect $X_1 \times X_2$ by manipulating the parameter a_{12} across the range of 0.5 to 2.0. We then measure the power to detect this interaction effect. Our results are summarized in [Figure 34](#), aligning with findings for main effects. ART's relative advantage diminishes later under the binomial and ordinal distributions. Once again, the method's inflated Type I error rates could account for this observed trend.

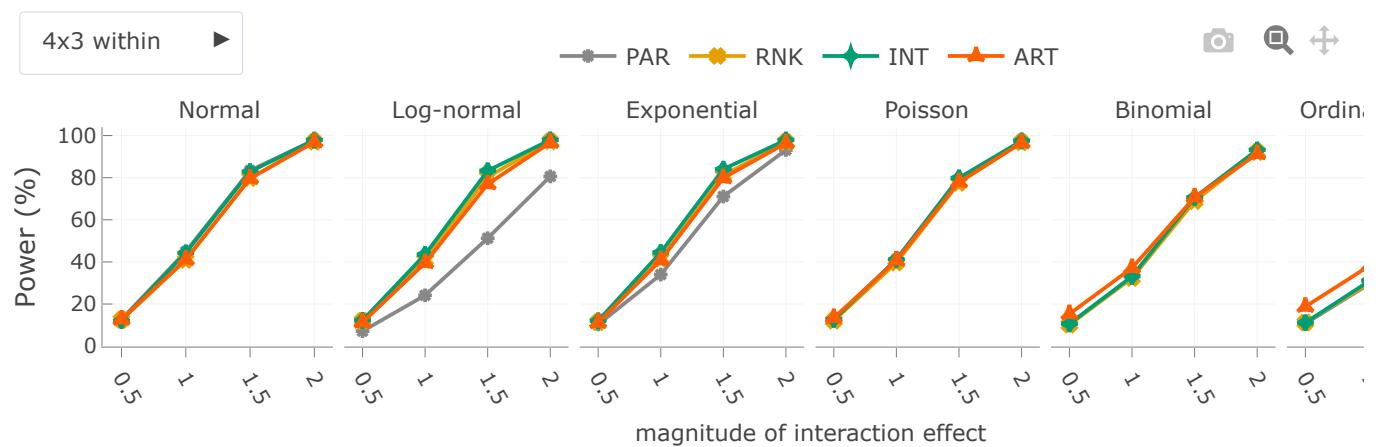


Figure 34: Power ($\alpha = .05$) for detecting the **interaction effect $X_1 \times X_2$** as a function of its magnitude a_{12} , where $n = 20$. show ranking

Effect size estimates

We expect that discrepancies in the p -values produced by different methods will also be reflected in their corresponding effect size estimates. We present results from various experiments, concentrating on either partial η^2 estimates or Cohen's f estimates.

Effect sizes estimates in log-normal distributions. Consider a 4×3 repeated-measures design with $n = 20$, where the effect of the first factor X_1 is null ($a_1 = 0$) and there is no interaction ($a_{12} = 0$). We assume that the responses follow log-normal distributions. [Figure 35](#) displays the partial η^2 estimated by each method for the effect of X_1 as the magnitude of the effect on the second factor increases. Each cell contains 300 data points. The ground truth uses ANOVA directly on the latent variable, where distributions are normal.

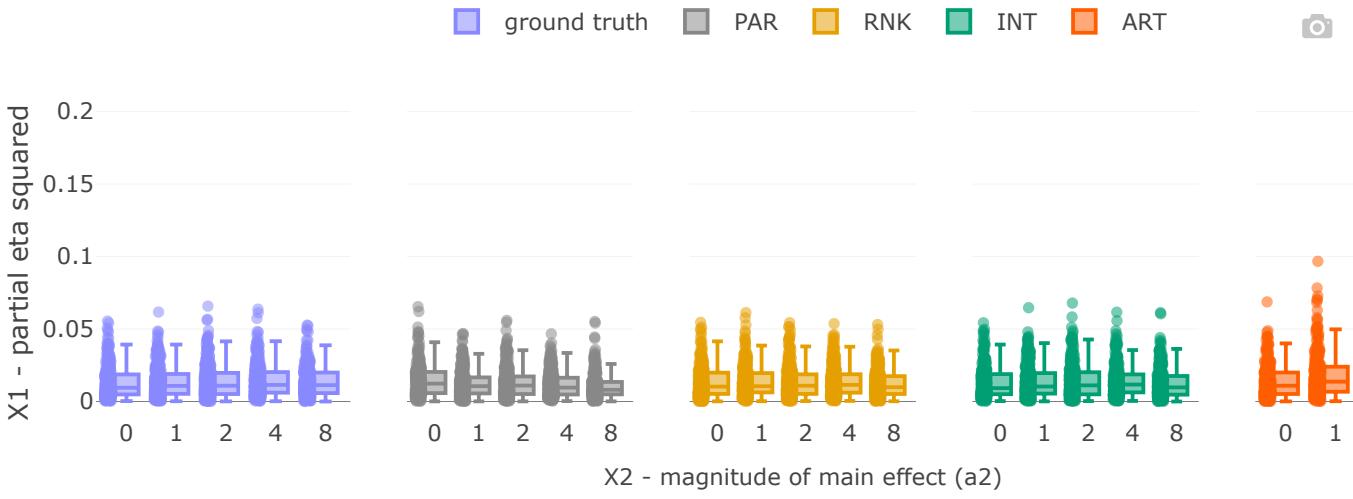


Figure 35: Boxplots summarizing the η^2 estimates obtained with each method for 300 data points ($n = 20$). The population effect of X_1 is null, and responses follow **log-normal distributions**. Hover over the plots to compare the methods.

ART is the only method whose effect size estimates for X_1 are affected by the magnitude of the effect on X_2 . The results clearly demonstrate that the method's alignment mechanism fails in nonlinear models with skewed distributions and confounds effects.

[Figure 36](#) presents η^2 estimates when the effect of X_1 is no longer null ($a_1 = 1$). We observe that PAR is not ideal for such data, as it consistently underestimates effect sizes. RNK also slightly underestimates effects. ART tends to underestimate effects when $a_2 = 0$, but as a_2 increases, its estimates become more variable and show an overall tendency to increase. INT provides the most precise estimates but is also slightly affected when $a_2 = 8$.

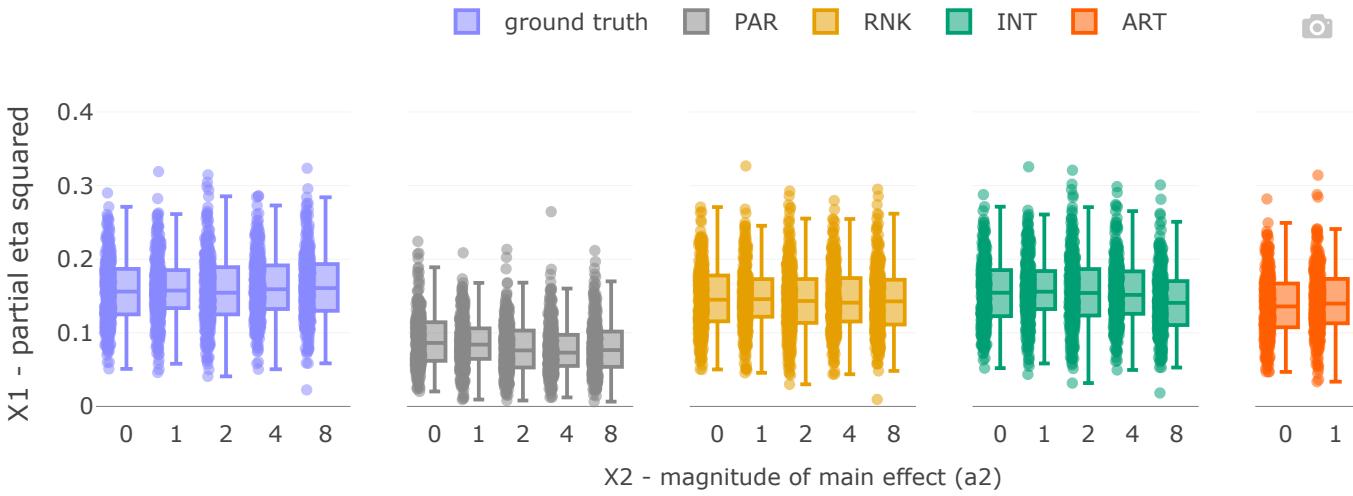


Figure 36: Boxplots summarizing the η^2 estimates obtained with each method for 300 data points ($n = 20$). The population effect of X_1 is now non-null ($a_1 = 1$). Responses follow again **log-normal distributions**. Hover over the plots to compare the methods.

Our final experiment assesses the precision of the effect size estimates (Cohen's f) of each method.

Relationship to ground truth estimates. We evaluate the precision of effect size estimates for wider range of effects. We now use Cohen's f as a measure of effect size, which is expected to be proportional to the real effect. Without loss of generality, we focus on the main effect of X_1 . We also limit our analysis to three distributions: normal, log-normal, and ordinal with five flexible levels.

[Figure 37](#) illustrates the relationship between the Cohen's f obtained with each method and the f of the ground-truth method, when a_1 and a_2 vary within the range $[-2, 2]$. The proximity of data points to the black diagonal indicates

the closeness of estimates to the ground truth. Points above the line are likely to overestimate the effect size, while points under the line are likely to underestimate it.

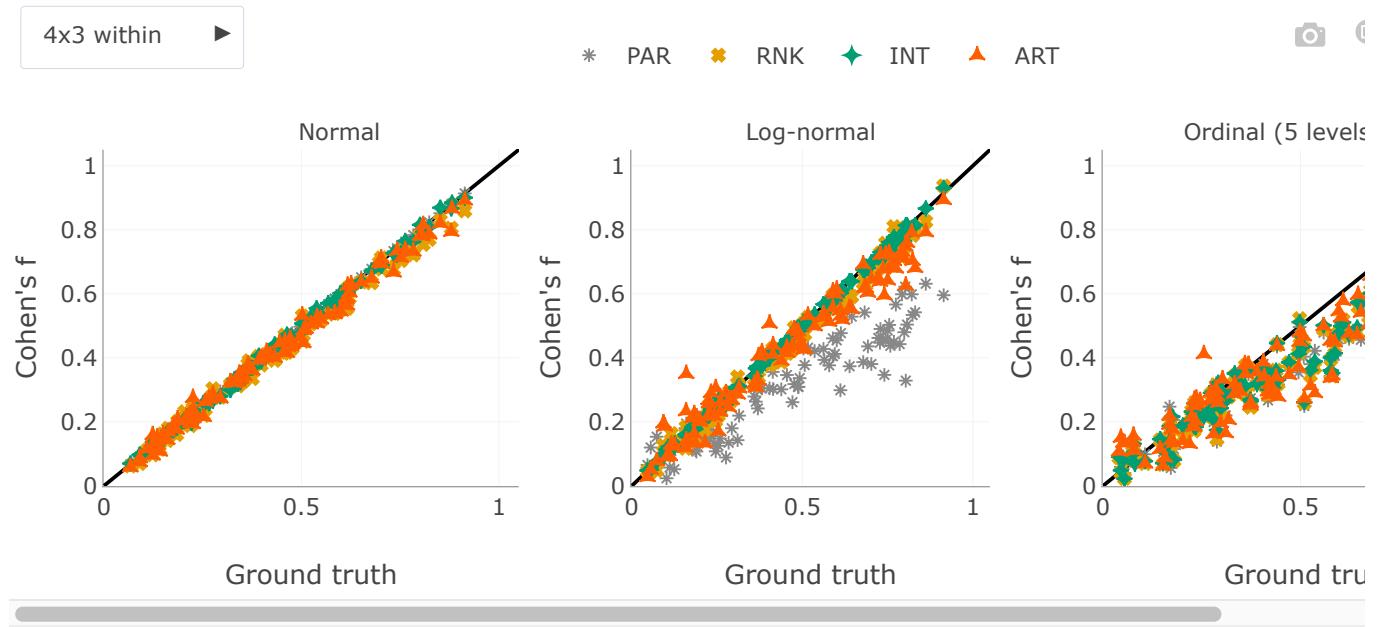


Figure 37: Scatterplots showing the relationship between effect size estimates (Cohen's f) of each method and estimates of the ground-truth method for the main effect of X_1 , when a_1 and a_2 vary within the range $[-2, 2]$.

We observe that PAR tends to underestimate effect sizes under log-normal distributions, in consistency with its low power (see [Figure 32](#)). ART's estimates are instead spread across both sides. We invite the reader to zoom in on the lower range of values ($f < 0.4$), where the method frequently exaggerates effects. INT demonstrates the highest precision, followed by RNK. However, their performance deteriorates under the ordinal scale. Still, ART performs worse than all other methods.

[Figure 38](#) shows the same relationship but for a larger range of effects $[-8, 8]$.

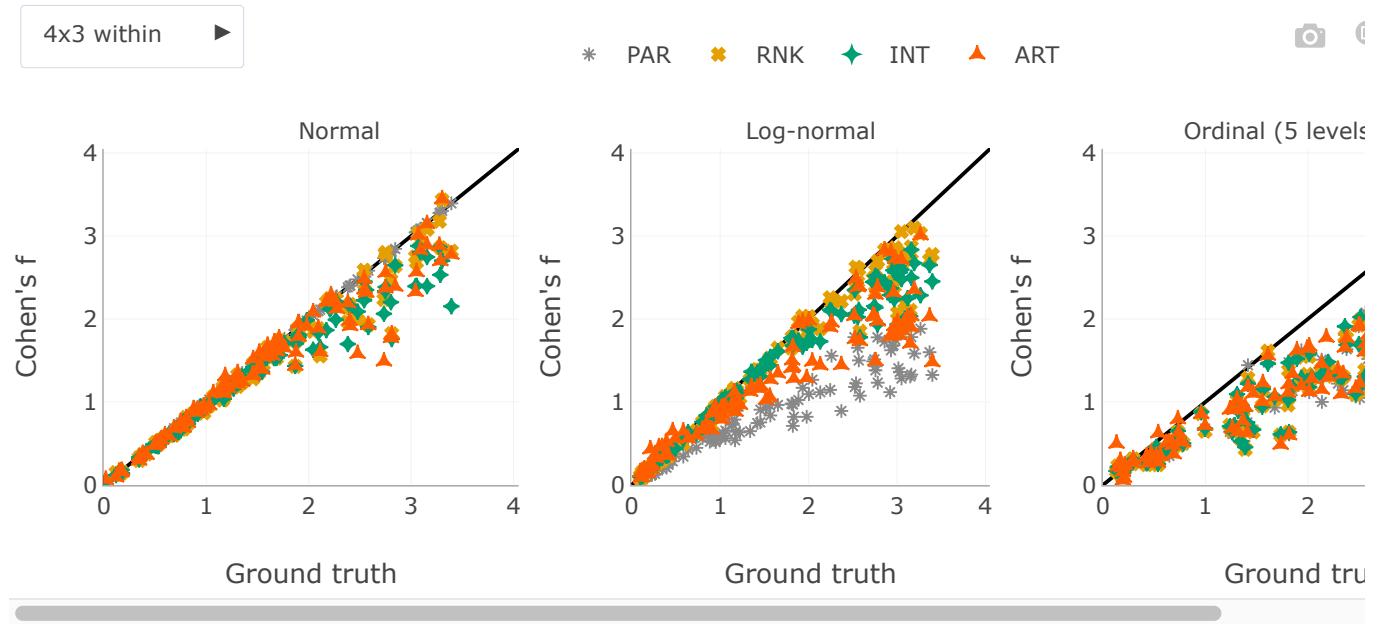


Figure 38: Scatterplots showing the relationship between effect size estimates (Cohen's f) of each method and estimates of the ground-truth method for the main effect of X_1 , when a_1 and a_2 vary within the range $[-8, 8]$.

We observe that all three rank-based methods struggle to detect large effects, even when the underlying distribution is normal. This result reflects a fundamental limitation of ranks. Consider, for example, two sets of numbers $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. The mean difference between them is $\bar{B} - \bar{A} = 3$, whether calculated from the row

values or their ranks. However, if the set B becomes $\{9, 10, 11\}$, the mean difference of raw values increases to 8, while the mean difference in ranks remains 3. As effects become larger, this problem becomes more apparent.

In the case of ordinal data, all methods yield highly variable estimates. This variability is largely due to the limited resolution of the ordinal scale itself: with only five discrete levels, it cannot adequately represent the full range of variation in the latent variable. The systematic underestimation of large effects across methods is also due to the limited range of the ordinal scale's extremes, which masks the true magnitude of underlying differences.

Interactions. We also provide additional results illustrating how rank-based methods can fail to accurately estimate interaction effects. [Figure 39](#) shows the estimated values of partial η^2 for the interaction term under normal distributions, as both main effects increase. Once again, we observe that RNK can substantially distort the estimated interaction. While INT reduces this distortion, the problem reappears as the main effects grow larger.

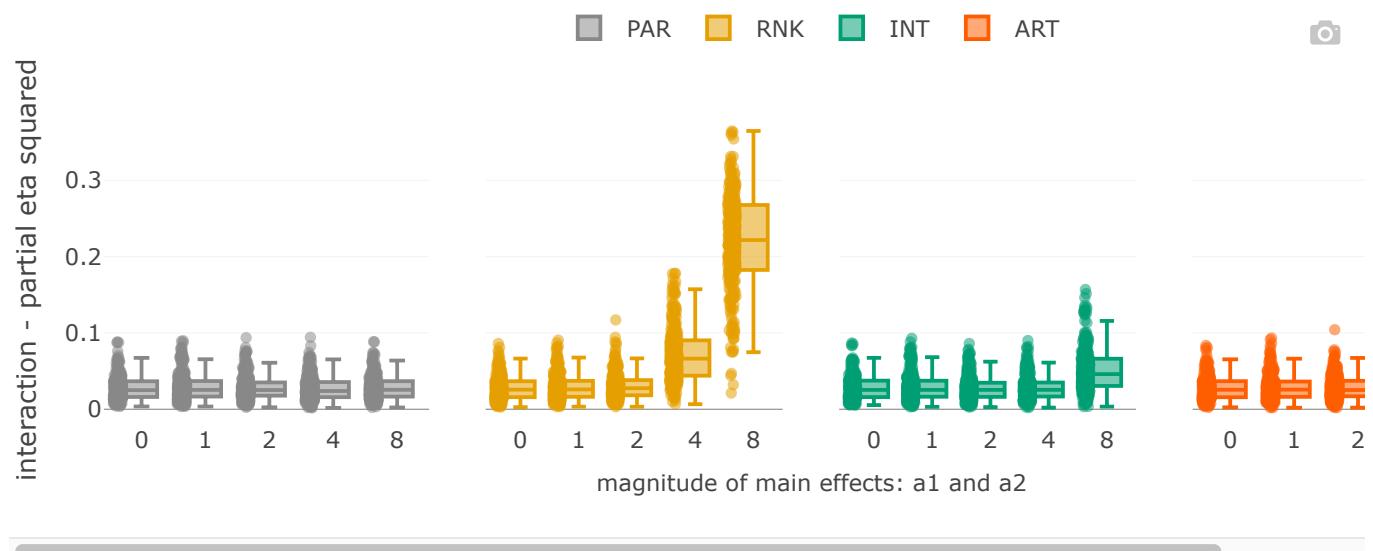


Figure 39: Boxplots summarizing the η^2 estimates for interactions obtained with each method for 300 data points ($n = 20$). The population interaction effect is null, and responses follow **normal distributions**. Hover over the plots to compare the methods.

If the data take a discrete form, effect size estimates become even more distorted across all methods. [Figure 40](#) presents results for an ordinal scale with 5 flexible levels.

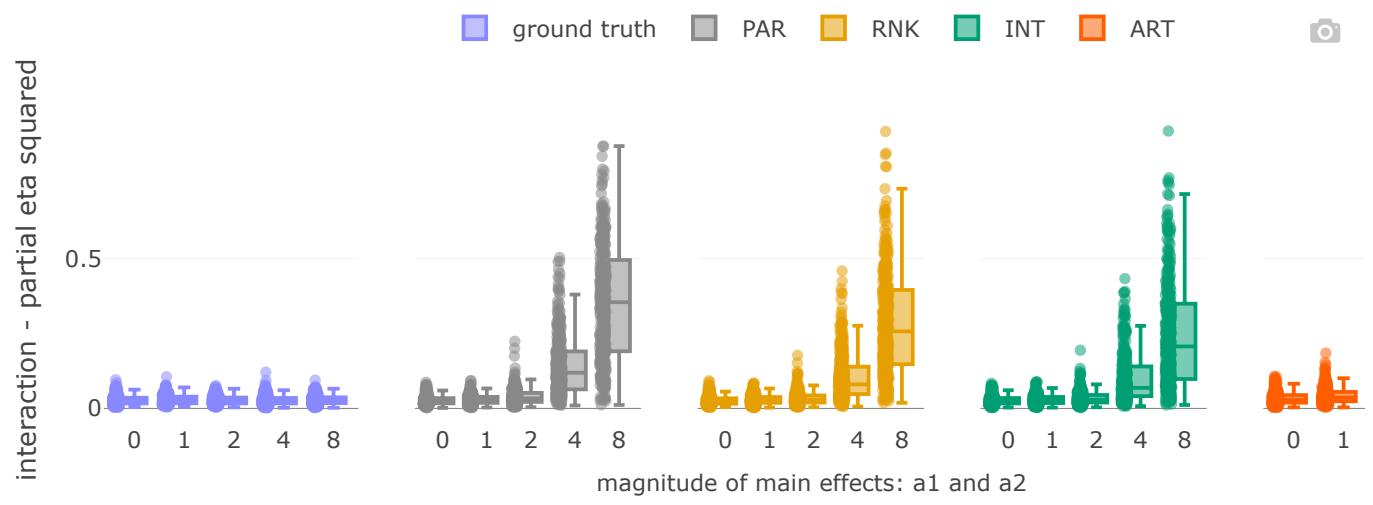


Figure 40: Boxplots summarizing the η^2 estimates for interactions obtained with each method for 300 data points ($n = 20$). The population interaction effect is null, and responses are **ordinal with 5 flexible levels**. Hover over the plots to compare the methods.

As discussed extensively earlier, such ordinal scales can distort interaction trends when main effects are non-zero, leading to interpretation issues. Although the methods may detect strong interaction effects under these conditions,

the effects may not be intrinsic or meaningful.

7 Case studies

In this section, we examine some specific types of data through concrete examples. We focus on recent work (published in 2023) that has used ART and made data publicly available. We identify problems and evaluate possible risks. We also present alternative methods for conducting the analysis and compare their outcomes.

Binary responses

Martin et al. (2023) ran two experiments to investigate the psychological phenomenon of change blindness, that is, how humans fail to detect certain changes in a scene. We focus here on their first experiment, which evaluated how 22 participants succeeded in observing object manipulations in an immersive environment. The authors studied the influence of four different factors, namely the Type of manipulation (4 levels), its Distance from the observer (2 levels), the Complexity of the manipulated item (2 levels), and the field of view (not examined here). To analyze the effect of the first three factors, the authors used a $4 \times 2 \times 2$ repeated-measures ANOVA using ART.

Their design has two particularities. First, the response variable is binary (Detected vs. Not Detected). Second, the design is unbalanced; each participant did not complete the same tasks, so their types and complexities did not appear with the same frequency among participants. The authors pointed to these issues to justify the use of ART:

“Since we are dealing with nonparametric, unbalanced data and multiple factors, we apply the Aligned Rank Transform (ART) for nonparametric factorial ANOVA” (Martin et al. 2023).

In their ART model, the authors treated the participant identifier and the task identifier as random effects. We could re-run their analysis to replicate the results presented in Table 1 (Martin et al. 2023). However, we extended the analysis by fitting a linear mixed-effects model (LMER function) without any transformation (PAR). When responses are binary, RNK and INT do not alter the data, leading to results that coincide with those of PAR.

Below, we present the p -values obtained from each method:

p-values keeping two significant digits - Type III hypothesis tests

	ART	PAR
Type	.92	.016
Distance	.031	.12
Complexity	.023	.0027
Type \times Distance	.71	.13
Type \times Complexity	.00077	.0039
Distance \times Complexity	.0011	.0029
Type \times Distance \times Complexity	.00055	.00016

There are notable differences between the results obtained from the two methods, but it is important to note that the aforementioned p -values are outcomes of Type III hypothesis tests. While these tests are deemed more suitable for unbalanced data when interaction effects emerge, their use remains a subject of debate (Smith and Cribbie 2022), and some experts argue that interpreting main effects in the presence of interactions may not be meaningful (Hector, Von Felten, and Schmid 2010). Thus, the presence of interactions in the above unbalanced design complicifies the detection and interpretation of main effects. In this specific case, the authors decided to create partial models with two factors each time to further evaluate contrasts, and interestingly, they now used t-tests instead of ART.

We emphasize that the ARTool raises a warning for such unbalanced designs:

F values of ANOVAs on aligned responses not of interest are not all ~0. ART may not be appropriate.

Our tests in the [appendix](#) show that ART's Type I error rates for main effects increase further in unbalanced designs with missing data.

Monte Carlo simulation. What if the experimental design were balanced, and such issues did not emerge? What risks would arise from using ART with such binary responses? To delve into this issue, we conduct a new Monte Carlo experiment.

Like Martin et al. (2023), we investigate a $4 \times 2 \times 2$ repeated-measures design and set the number of participants to $n = 22$. However, we now test a perfectly balanced design. We transform the latent variable to a Bernoulli distribution with a success probability parameter $p = .46$ — average success rate found by Martin et al. (2023) — and then assess the Type I error rate of ART in the absence of any effect. We also assess the Type I error rate of regular ANOVA (LMER model) with no transformation (PAR). Our results over 5000 iterations are as follows:

Type I error rates (%) for $\alpha = .05$ (nominal error rate = 5%) and $p = .46$

	X_1	X_2	X_3	$X_1 \times X_2$	$X_1 \times X_3$	$X_2 \times X_3$	$X_1 \times X_2 \times X_3$
PAR/LMER	4.4	4.5	4.6	5.5	5.1	5.2	4.9
ART	22.4	19.9	20.2	22.6	21.8	19.8	21.1

Type I error rates (%) for $\alpha = .01$ (nominal error rate = 1%) and $p = .46$

	X_1	X_2	X_3	$X_1 \times X_2$	$X_1 \times X_3$	$X_2 \times X_3$	$X_1 \times X_2 \times X_3$
PAR/LMER	0.7	1.1	0.9	1.1	1.0	1.1	1.0
ART	10.0	8.8	9.7	10.6	9.7	9.6	9.9

We observe that ART exhibits significant challenges with such binary data, as its error rates escalate to very high levels for all main and interaction effects. In contrast, PAR (LMER) consistently maintains error rates close to nominal levels. We extend our simulation to Bernoulli distributions with a low success (or error) probability parameter $p = .06$. The results are now as follows:

Type I error rates (%) for $\alpha = .05$ (nominal error rate = 5%) and $p = .06$

	X_1	X_2	X_3	$X_1 \times X_2$	$X_1 \times X_3$	$X_2 \times X_3$	$X_1 \times X_2 \times X_3$
PAR/LMER	4.9	4.8	6.0	4.4	4.0	4.6	4.6
ART	78.1	68.6	68.7	71.9	71.3	68.8	70.5

Type I error rates (%) for $\alpha = .01$ (nominal error rate = 1%) and $p = .06$

	X_1	X_2	X_3	$X_1 \times X_2$	$X_1 \times X_3$	$X_2 \times X_3$	$X_1 \times X_2 \times X_3$
PAR/LMER	1.0	1.0	1.2	0.8	0.7	1.1	0.9
ART	68.6	60.6	60.2	61.6	60.1	60.6	59.9

Clearly, ART is completely inappropriate when responses are binary. In contrast, parametric ANOVA demonstrates an excellent behavior even when the success (or error) rate is as low as 6%.

A more adapted approach for this type of data would be to use a generalized linear model (GLMER) with a *binomial link function*, which essentially performs a logistic regression. Such models are in line with the latent variable approach, where the link function establishes a connection between a normally distributed latent psychological variable (e.g., the human ability to detect changes in a virtual environment) and the observed variable (e.g., successes or failures in sequential change detection tests). Consequently, we expect them to exhibit stronger predictive power than regular linear models and support a more appropriate interpretation of interaction effects. However, inference with such models requires additional steps, such as comparing alternative models using various criteria (Bolker et al. 2009), so we do not include them in our evaluation here.

Likert items

Rosso et al. (2023) conducted a body-swap experiment, where pairs participants (“dyads”) performed a joint-finger tapping task under four different conditions, structured under two different factors:

- *Coupling*. Participants saw either their own hand (*Coupled*) or their partner’s hand (*Uncoupled*).
- *Perspective*. Participants saw the hand from a first person (*1P*) perspective or from a second person perspective (*2P*).

Overall, the experiment was structured as a 2×2 within-participants design with 38 participants (19 dyads). The authors investigated numerous measures. Here, we only revisit their analysis of the sense of ownership of the visually perceived hand, for which they used ART (Page 10). The variable was measured through a self-reported scale with five levels (1 to 5). The authors report:

Aligned rank transform (ART) ANOVA revealed significant main effects of Coupling (Df residual = 147, F = 104.353, p < 0.001) and Perspective (Df residual = 147, F = 8.983, p < 0.01) on the self-reported ownership ratings. The former indicates that participants were capable of telling apart their own hand from the partner’s regardless of the visual perspective, whilst the latter indicates that perceiving a hand in 1st person generally resulted in a stronger sense of ownership. Crucially, the interaction effect between Coupling and Perspective (Df residual = 147, F = 5.232, p < 0.05) revealed that the increase in ownership relative to the 2nd person perspective was significantly stronger when participants were coupled. (Rosso et al. 2023)

Thus, the authors found supportive statistical evidence for both main effects and the interaction of the two factors. We replicated their analysis and found that the author did not conduct a repeated-measures ANOVA, i.e., they did not treat ratings from the same participant as independent. We re-analyzed the data with a mixed-effects *lmer* model, treating the participant identifier as a random effect nested under the identifier of participant pairs (dyads). Conclusions do not change since our analysis is more powerful, so *p*-values are now even smaller for all three effects. However, we also conducted the analysis using the three other methods (PAR, RANK, INT), and unfortunately, conclusions are very different:

p-values keeping two significant digits

	ART	PAR	RNK	INT
Coupling	3.2×10^{-19}	4.9×10^{-18}	9.5×10^{-20}	5.4×10^{-19}
Perspective	.0016	.20	.25	.29
Coupling × Perspective	.015	.78	.90	.95

All four methods agree that there is strong evidence about the effect of Coupling. Nevertheless, the results of PAR, RANK, and INT for the effects of Perspective and Coupling × Perspective do not support the authors’ conclusions. The discrepancy between the *p*-values returned by these methods and the *p*-values of ART is striking.

One may notice that a single entry (out of 152 entries) in the above dataset is missing, so the design is slightly unbalanced, and the ARTool again outputs a warning. Could this imbalance explain the above differences? The answer is negative. If we complete the missing row with a neutral value (e.g., a 3 in the 5-level ordinal scale) to dismiss the warning, the resulting p -values will still be very similar to the ones reported above.

Ordered probit models. We conducted complementary analyses with ordered probit models, where we used two different methods: (1) ordinal regression as implemented in the *ordinal* R package ([Christensen 2023](#)); and (2) a Bayesian statistics framework with the *brms* R package ([Bürkner and Vuorre 2019](#)).

For the first (frequentist) analysis, we use the following R code:

```
library(ordinal)
# We make sure that the response variable is treated as ordinal
ownership_data$ownership <- factor(ownership_data$ownership, ordered = TRUE)

fit_frequentist <- clmm(
  ownership ~ Coupling + Perspective + Coupling:Perspective + (1|Dyad/Participant),
  link = "probit",
  threshold = "flexible",
  data = ownership_data
)
```

We can build a simpler regression model by setting `threshold = "equidistant"` and compare the two models by conducting a likelihood-ratio test. For this specific example, the flexible thresholds do not improve the model's predictive power, so the simpler model is more promising. However, both models lead to almost identical parameter estimates.

For the Bayesian formulation, we use the following code in R:

```
library(brms)
fit_ownership <- brm(
  formula = ownership ~ Coupling + Perspective + Coupling:Perspective
  + (1|Dyad/Participant),
  data = ownership_data,
  family = cumulative("probit")
)
```

Notice that we do not use informative priors in the above model, although this is possible. For an extensive discussion about why using a Bayesian framework for this type of analysis, we refer interested readers to Liddell and Kruschke ([2018](#)). Bürkner and Vuorre ([2019](#)) provide an excellent introductory tutorial to this approach. Its main shortcoming is the substantial computational resources it demands, whereas fitting a model with the frequentist approach is extremely fast.

Below, we present parameter estimates from the two methods, where brackets represent either 95% confidence intervals (frequentist method), or 95% credible intervals (Bayesian method):

Effect estimates and their 95% confidence or credible intervals. They are expressed as standard deviations of the latent variable

	Frequentist (Christensen 2023)	Bayesian (Bürkner and Vuorre 2019)
Coupling (Coupled – Uncoupled)	−1.85 [−2.46, −1.24]	−1.95 [−2.58, −1.35]
Perspective (1P – 2P)	0.29 [−0.37, 0.94]	0.28 [−0.39, 0.95]
Coupling × Perspective	−0.11 [−0.93, 0.70]	−0.09 [−0.89, 0.75]

Interpreting these results requires special attention, as the above estimates do not refer to the observed ordinal scale of measurements. All effects are expressed in standard deviation units over the latent variable, so they are analogous to Cohen's d standardized effect sizes. The above results suggest that participants' sense of ownership was 1.85 (or 1.95) standard deviations lower in the Coupled condition, where standard deviations refer to the continuous latent scale of sense of ownership.

The whole 95% CI for both methods is far below zero, which means that there is overwhelming evidence that this effect is strong. In contrast, we observe that the intervals for Perspective and the interaction term extend from negative to positive values. Thus, there is no sufficient statistical evidence for these effects. If we subsequently conduct a likelihood-ratio test with alternative models, the simpler model with `formula = ownership ~ Coupling + (1|Dyad/Participant)` will be the winning one.

These results are consistent with the results of our *lmer* models using PAR, RNK, and INT. In conclusion, using ART to analyze these data is problematic. Here, the presence of a strong effect on the first factor seems to make ART sensitive in detecting other non-existent effects (or perhaps in amplifying tiny effects).

Likert scales

Other authors have used ART to analyze Likert scales that group several items together. For example, Karolus et al. (2023) investigate different methods (referred to as “gamification types”) for communicating feedback on users’ task proficiency, such as during a writing task. The authors used a 3×2 between-participants design to structure the study, testing the following two factors:

- *Gamification type*. Each participant was presented with one of the following elements providing feedback: (i) a *progress bar* (ii) an *Emoji*, or (iii) *none*.
- *Feedback type*. Feedback was either *continuous* or *revision-based* (provided at the end of the task to help participants revise).

The authors collected responses from 147 participants through the Amazon Mechanical Turk Service. They analyzed a range of measures, but here we focus on their analysis of participants' responses to the Situational Motivation Scale (SIMS) (Blanchard, Guay, and Vallerand 2000). The scale consists of four subscales (intrinsic motivation, identified regulation, external regulation, and amotivation), where each contains four 7-level Likert items. For each subscale, the authors conducted an ANOVA using ART on the average score. Below, we present the results of the same analysis with all four methods for intrinsic motivation and identified regulation:

Intrinsic motivation: p -values keeping two significant digits (partial η^2 in parentheses when $p < .05$)

	ART	PAR	RNK	INT
Gamification	.0057 (0.07)	.010 (0.06)	.0058 (0.07)	.0062 (0.07)
Feedback	.77	.59	.56	.55
Gamification \times Feedback	.94	.69	.94	.86

Identified regulation: p -values keeping two significant digits (partial η^2 in parentheses when $p < .05$)

	ART	PAR	RNK	INT
Gamification	.019 (0.05)	.011 (0.06)	.013 (0.06)	.0085 (0.07)
Feedback	.55	.74	.42	.55
Gamification \times Feedback	.99	.86	.98	.93

Overall, differences between the results of the four methods are reasonably small and support the authors' conclusions. We omit results for external regulation and amotivation for which all p -values are greater than .05 — again, results are consistent among the four methods.

In many situations, the above methods will behave correctly and produce similar results. However, it is difficult to know in advance when problems will arise. In a different experiment, Siestrup and Schubotz (2023) investigated how different types of modifications in video sequences describing a story affect people's episodic memory. The experiment included a rating task, asking participants to rate how much the storyline of modified episodes deviated from an original version (from 1 = 0% different to 6 = 100% different). The authors used ART to analyze aggregated scores from multiple ratings per condition. Below, we present the results of our re-analysis with the four different methods:

p-values keeping two significant digits				
	ART	PAR	RNK	INT
Version	4.5×10^{-20}	2.7×10^{-30}	4.7×10^{-20}	2.0×10^{-17}
Modification	.034	.11	.23	.40
Version × Modification	.19	.17	.18	.058

We see now that results from different methods can lead to different conclusions. In particular, the authors' statement that “modified videos that had already been presented during the fMRI session received lower story-change ratings than those that had been presented in the original version before” (Siestrup and Schubotz 2023), based on the observed effect of the second factor (Modification), is only supported by their analysis with ART. Effect size estimates for this factor also vary among methods, although their wide confidence intervals indicate that those estimates are highly uncertain:

effect size estimates for Modification and their 95% confidence intervals				
	ART	PAR	RNK	INT
partial η^2	0.12 [.01, 1.0]	0.07 [.00, 1.0]	0.04 [.00, 1.0]	0.02 [.00, 1.0]

Ordered probit models. Alternatively, we can use ordered probit models (Bürkner and Vuorre 2019; Christensen 2023), as we did for the analysis of individual Likert items. To this end, we do not average the individual Likert items of the scale but treat instead items and participants as a random effects. For example, to analyze participants' responses for intrinsic motivation (Karolus et al. 2023), we can use the following formula:

```
formula = intrinsic_motivation ~ 1 + Gamification:Feedback + (1|Item) + (1|Participant)
```

Interested readers can refer to our supplementary material for additional details.

8 Recommendations

Reflecting on our findings, we formulate a set of recommendations that we hope will be helpful to researchers.

Abandoning ART

Our results corroborate Lüpsen's (2017, 2018) warnings, overwhelmingly demonstrating that the ART method is defective. ART's advantages over the simple rank transformation (RNK) are marginal and only applicable to normal distributions with equal variances. However, the method introduces additional risks for a range of non-normal distributions. Specifically, it inflates the Type I error rate of both main effects and interactions. Furthermore, the risks associated with ART are more severe compared to those associated with parametric ANOVA. While skewed

distributions lead to a loss of statistical power with parametric ANOVA, ART consistently inflates Type I error rates. It is also important to emphasize that parametric ANOVA is consistently superior to ART for the analysis of ordinal data, irrespective of whether the thresholds of the ordinal scale are equidistant or not. In light of these findings, we conclude that ART is an outdated procedure and urge researchers to discontinue its use.

Some researchers may be tempted to seek alternative formulations of ART. In particular, Salter and Fawcett (1993) report that employing ART with median-based alignment (i.e., using medians instead of means to align values) for interactions corrects the method's instability under the Cauchy distribution. We present results regarding the method's performance for the 4×3 repeated-measures design in our [appendix](#). We observe that indeed, using a median-based alignment for interactions brings error rates close to nominal levels, provided that main effects remain below a certain threshold. This trend holds across all distributions tested. However, despite this improvement, the method still lags behind INT in performance, and compared to parametric ANOVA, its advantages are evident only for the Cauchy distribution. Additionally, median-based alignment proves inappropriate for main effects, as in most cases, it yields even higher Type I error rates than mean-based alignment.

Adopting other nonparametric methods

Given the widespread adoption of ART, many researchers may feel perplexed about which alternative nonparametric method to use. Our results indicate that INT performs well across various configurations and emerges as the top performer among rank transformations. However, like other rank-based methods, INT faces challenges in accurately inferring interactions when parallel main effects are present. It may also lead to inflated error rates or reduced power in cases of unequal variances, and tends to underestimate large effect sizes. These issues are exacerbated when dealing with discrete distributions. Beasley, Erickson, and Allison (2009) identify additional situations where INT may fail, such as in highly imbalanced designs.

Despite these limitations, INT outperforms parametric ANOVA across various scenarios that we investigated. This is especially evident when distributions are skewed. In these cases, parametric ANOVA experiences a significant loss of power and becomes sensitive to scale interpretation issues, which typically emerge when testing interactions or when variances are unequal. Finally, while it is still feasible to use common one-factor nonparametric tests (e.g., the Kruskal–Wallis test, the Wilcoxon sign-rank test, and the Friedman test) independently on each factor of a multifactorial design, this approach necessitates data aggregation, which compromises the statistical power of these tests. Nevertheless, we found that even for one-factor designs, INT turns out to be more powerful than these nonparametric tests.

Consequently, our recommendation aligns with Gelman's (2015) advice to use INT as a general nonparametric method, with careful consideration of potential cases of failure and a cautious interpretation of interaction effects.

Privileging parametric methods

The scope of rank-based nonparametric statistics is very limited. Baguley (2012) summarizes a fundamental concern regarding their use as follows:

“rank transformations don't just transform the data, they do so in a way that makes it impossible to recover the original scores ... This won't matter if the sole aim is to construct a significance test, but if you want to make predictions or obtain CIs, use of ranks introduces a potentially unnecessary obstacle.” (Baguley 2012, 356)

We also explained that nonparametric tests are not free of assumptions, while rank transformations complicate the interpretation of the null hypothesis. Therefore, there are valid reasons to prioritize parametric methods and reserve rank transformations as last-resort solutions. Merely obtaining a p -value $< .05$ from a normality test (or a test of the homogeneity of variance assumption) is insufficient justification for switching to a nonparametric procedure. According to Baguley (2012), using a significance test to evaluate the statistical assumptions “*is asking completely the*

wrong question” because “*It is not the presence of a violation, but the degree of departure from the assumption (i.e., effect size) that matters*” ([Baguley 2012, 324](#)).

Several papers we reviewed, citing ARTTool ([Wobbrock et al. 2011](#)), justify their use of ART based on the presence of “*nonparametric data*.” However, it is important to clarify that terms like “*parametric*” or “*nonparametric*” refer to properties of the statistical model, not inherent characteristics of the collected data. Even when ANOVA is clearly unsuitable, there are often alternative parametric methods for addressing violations of common statistical assumptions and still ensure that results can be easily communicated and interpreted. For example, one can log-transform positively skewed data following log-normal distributions or use generalized mixed-effects models, which provide support for a broader set of data distributions. Finally, employing simulations, as we did in this article, is an excellent method for evaluating the impact and risks of assumption violations and determine which data analysis procedures to use.

Analyzing Likert-type data

The analysis of Likert-type data requires special attention due to the contradicting research literature on this topic. 20 years ago, Jamieson ([2004](#)) alarmed researchers regarding the frequent use of analyses that treat Likert scales as interval rather than ordinal data. Jamieson’s article provoked a number of vivid responses. Carifio and Perla ([2008](#)) replied that “*the root of many of the logical problems with the ordinalist position*” is the fact that it makes no distinction between “*a Likert response format, a Likert (graded valence) question (or stem) or a Likert scale (collection of items)*.” Based on “*a wide array of additional supporting empirical evidence*”, the authors concluded that:

“*It is, therefore, as the intervalists contend, perfectly appropriate to summarise the ratings generated from Likert scales using means and standard deviations, and it is perfectly appropriate to use parametric techniques like Analysis of Variance to analyse Likert scales*” ([Carifio and Perla 2008](#)).

However, Carifio and Perla ([2008](#)) also stated that analyzing individual Likert items “*is a practice that should only occur very rarely*.” This statement was criticized by Norman ([2010](#)), who argued that it does not convincingly refute the “*ordinalist*” position. According to the author, since ANOVA and other parametric statistics are robust enough to deal with Likert data, even with small sizes and unequal variances, there is “*no fear of coming to the wrong conclusion*.” In the same line, Harpe ([2015](#)) came up with a number of recommendations about these issues, including the following two: (1) “*Individual rating items with numerical response formats at least five categories in length may generally be treated as continuous data*;” and (2) “*Consider nonparametric or categorical data analysis approaches for individual rating items with numerical response formats containing four or fewer categories*.” However, Liddell and Kruschke ([2018](#)) identified several situations in which standard parametric methods fail, even when applied to averages of multiple items in Likert scales. In particular, the authors demonstrated how they can deform interaction effects or inflate errors when assumptions of equal variances are not met.

Our experimental results on individual Likert items with 5, 7, and 11 levels show that parametric ANOVA is robust for the analysis of main effects, as long as variances are equal. On the other hand, we observed that parametric ANOVA method often fails to correctly infer interactions when main effects become large, and the problem becomes more apparent when ordinal thresholds are not equidistant. INT appears to reduce the impact of this problem but without offering a satisfying solution. We also found that at least for 5-level items with flexible thresholds, all methods may lead to imprecise effect size estimates. In conclusion, while ANOVA seems to be a valid method for hypothesis testing even based on the analysis of individual Likert-item responses, we identified situations that require special attention. Unfortunately, nonparametric approaches may not necessarily work well in these situations.

In our prior research ([Fages, Fleury, and Tsandilas 2022](#)), we applied ordered probit models ([Bürkner and Vuorre 2019; Christensen 2023](#)) for analyzing ordinal responses in two experimental studies. Since the HCI community is not familiar with these models, we found it necessary to provide motivation for their use and offer additional explanations to reviewers and readers on how to interpret results. These models offer numerous advantages, including support for

interval estimation and prediction in addition to null hypothesis testing. However, researchers are required to acquaint themselves with concepts of generalized mixed-effects models or Bayesian inference in order to use them correctly. These methods also come with certain limitations — for example, the need to carefully specify priors to ensure convergence in certain scenarios, or their reliance on strict assumptions about population thresholds.

We hope that our examples in the paper and the supplementary materials can inspire the community to give these methods more thoughtful consideration.

Interpreting interactions

We discussed how the scale of the data affects the interpretation of interactions ([Loftus 1978](#)). In particular, when distributional assumptions are violated, a positive test for interaction may not indicate a genuine interaction. We demonstrated that nonparametric methods do not eliminate this problem, as they also struggle to control the Type I error rate of interactions in the presence of strong parallel main effects. Larger sample sizes do not mitigate this issue; instead, the ratio of false positives increases with sample size because the test becomes more sensitive.

A useful method for diagnosing such issues is to plot interactions (e.g., see [Figure 41](#)) and visually inspect the trends of the effects. Loftus ([1978](#)) notes that interactions that do not cross over are removable under monotonic transformations, and therefore they cannot be interpreted. In contrast, interactions that cross over are non-removable. [Figure 41](#) presents examples of a removable and a non-removable interaction. It is noteworthy that the crossover of the non-removable interaction is only discernible on the second chart (in orange-red), where each line shows the effect of Technique.

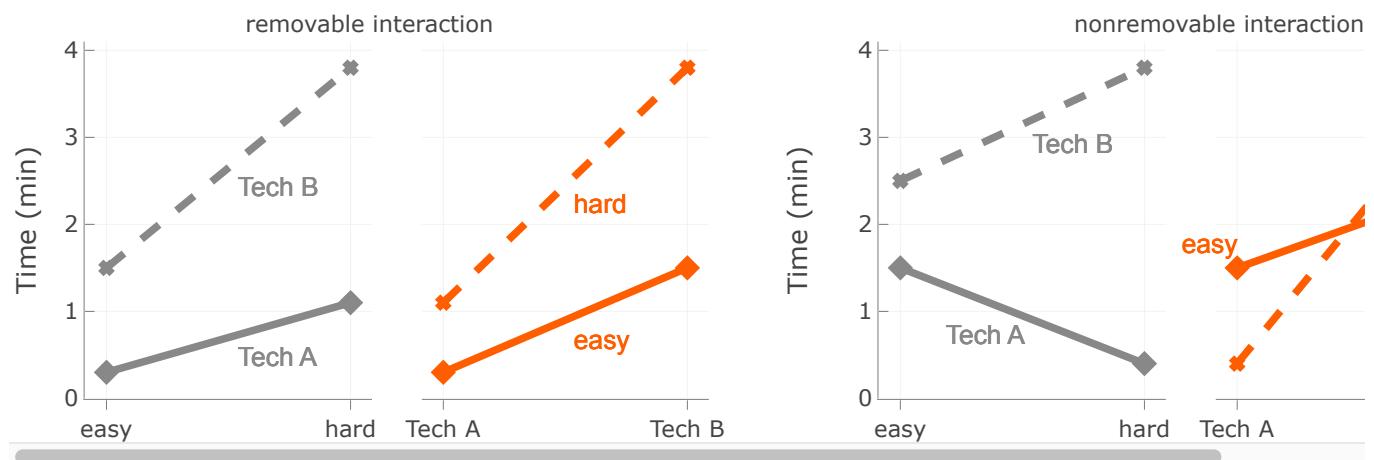


Figure 41: Examples of a removable (left) and a non-removable interaction (right). A removable interaction does not cross over, regardless of how it is plotted.

Research planning and transparency

Planning analysis methods before data collection is a sensible practice, as it helps focus on measures that are easy to analyze and communicate, and anticipate the types of responses they may generate. We also encourage authors to open their datasets so that other researchers can replicate their findings and evaluate the robustness of their analysis methods. Although ART has been used in a large number of studies, very few datasets from these studies are openly available. Therefore, the extent to which the use of this method has contaminated research outcomes remains unclear and unfortunately difficult to evaluate.

9 Conclusions

The Align Rank Transformation (ART) was introduced as a remedy for interaction effect distortions caused by the simple rank transformation ([Conover and Iman 1981](#)). Early simulation experiments demonstrated ART's robustness

for 2-factorial designs, while subsequent tools such as ARTTool ([Wobbrock et al. 2011](#); [Elkin et al. 2021](#)) extended its application to multifactorial designs, for testing main effects, interactions, and contrasts. The ARTTool has gained significant traction, particularly within the fields of Human-Computer Interaction, Augmented Reality, and Virtual Reality, and has been used in the analysis of over a thousand user studies.

However, our experimental findings and examples reveal that ART is defective, performing adequately only under the strict assumptions of linearity, continuous responses with equal variances among groups, and balanced designs. These results corroborate previous warnings regarding ART's performance ([Lüpsen 2017, 2018](#)), leading us to conclude that the method should be abandoned. We recommend prioritizing parametric methods. While we propose the inverse normal transformation (INT) as a generic nonparametric alternative, we identify situations in which this method may also fail. Finally, we caution researchers about the interpretation of interactions.

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