Lecture 10: Boosting

STATS 202: Statistical Learning and Data Science

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Announcements

- Midterm grades will be out soon
- Homework 3 due today
  - Homework 4 is posted
- Final predictions due in 2 weeks
- Section this Friday goes into more final project details

Outline

- Boosting introduction
- Boosting vs bagging
- Boosting remarks
- AdaBoost
- Boosting training error
- Gradient boosting
- Regularization
- Random tips

Recall

- Decision trees partition our feature space and make predictions within each partitioned region.
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Question: Is there another way of improving the performance of decision trees?

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3. Output prediction, e.g.
   $$\hat{f}_n(x) = \sum_{b=1}^{B} \lambda_b \hat{f}_n^b(x). \tag{3}$$

Boosting

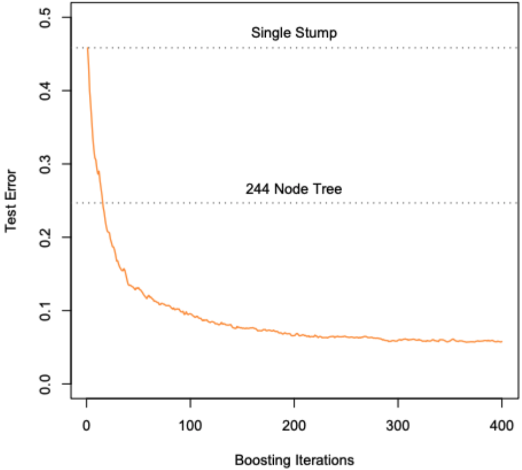

Figure 10.2

Boosting hyperparameters

Hyper-parameters to consider when applying a boosting model:

- The number of learners (aka trees) B to use.
- The shrinkage parameter λ_b.
- The parameters of the learner (e.g. splits in each tree).

Typically, these are found via *cross-validation*.

Boosting vs bagging

Bagging: For $b = 1, \ldots, B$:

1. Created a bootstrapped sample, P_n^b.
2. Get estimate $\hat{f}_n^b(x)$ using P_n^b.

Average the estimates, i.e.

$$\hat{f}_n^{\text{bag}}(x) = \frac{1}{B} \sum_{b=1}^{B} \hat{f}_n^b(x).$$

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Boosting: For $b = 1, \ldots, B$:

1. Get estimate $\hat{f}_n^b(x)$ for the residuals r^{b-1}.
2. Update residuals $r_i^b = r_i^{b-1} - \lambda_b \hat{f}_n^b(x_i)$.

Sum the estimates, i.e.

$$\hat{f}_n^{\text{boost}}(x) = \sum_{b=1}^{B} \lambda_b \hat{f}_n^b(x).$$

- 'Y' is varied for each fit.
- Designed to reduce bias.

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- Boosting can give zero training error, but rarely overfits.
- Can be thought of as fitting a model on multiple data sets.

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3. Output $G_B(x) = \text{sign}\left(\sum_{b=1}^{B} \alpha_b G^b(x)\right)$.

Boosting

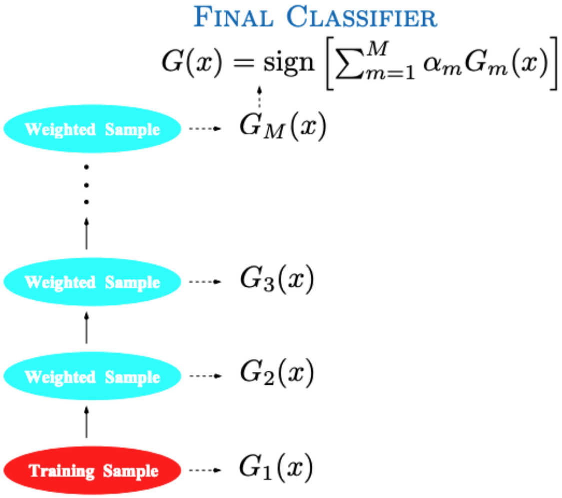


Figure 10.1

AdaBoost example

Sonar Data (n=208)

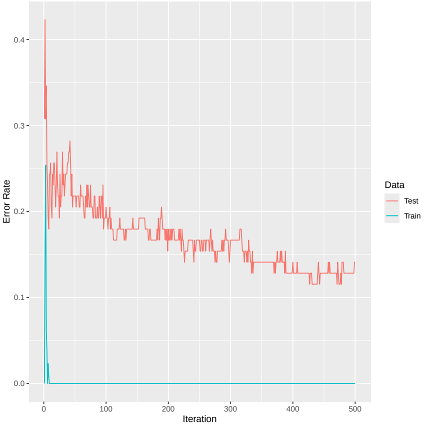
Features

- Measured energy level within one of 60 frequency bands

Label

- Indicator object is a Mine (1) vs Rock (0)

AdaBoost example



AdaBoost applied to the *Sonar Data*.

Training error

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Define:

$$G_B^*(x) = \frac{\sum_{b=1}^{B} \alpha_b G^b(x)}{\sum_{b=1}^{B} \alpha_b} \tag{5}$$

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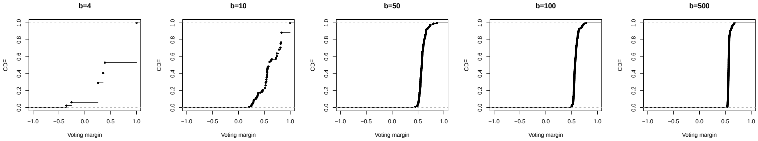
We can look at *voting margins* for our training data, i.e.

$$margin(x) = y * G_B^*(x) \tag{6}$$



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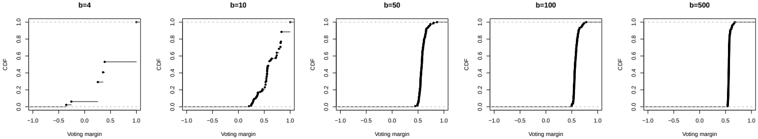


n.b. Letting $err_b \leq 1/2 - \gamma$, then $\text{Error}_{train} \leq (\sqrt{1 - 4\gamma^2})^B$

Weak learners

Question: How “weak” should our learners be?

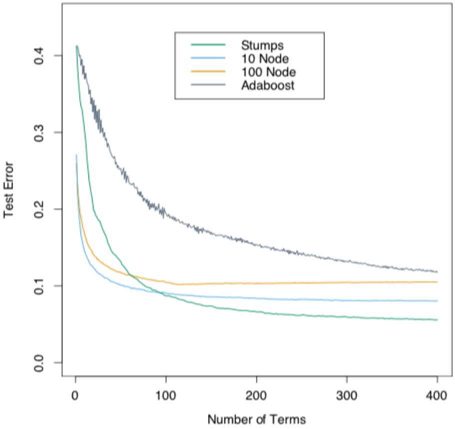
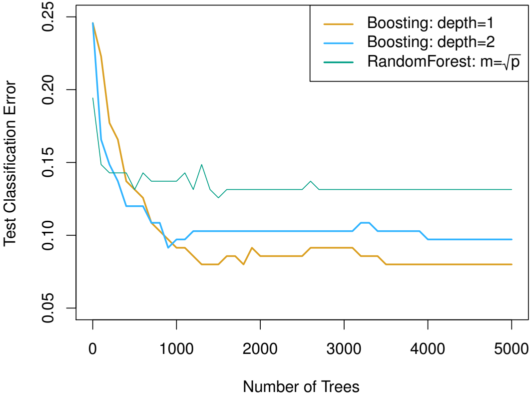


Figure 10.9

Example: Applied to 15-class gene expression data (Figure 8.11).



Forward Stagewise Additive Modeling

Boosting can be viewed as an additive model, i.e.

$$f(\mathbf{X}) = \sum_{m=1}^{M} \beta_m b(\mathbf{X}; \gamma_m) \tag{7}$$

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Algorithm 10.2 *Forward Stagewise Additive Modeling.*

1. Initialize $f_0(x) = 0$.
2. For $m = 1$ to M:
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Problem: RSS isn't a good loss for classification

Define the following loss:

$$L(y, f(x)) = \exp(-yf(x)) \tag{8}$$

Applying the forward stagewise additive representation to Adaboost:

$$\begin{aligned}(\beta_m, G_m) &= \arg\min_{\beta, G} \sum_{i=1}^{n} \exp\left(-y_i(f_{m-1}(x_i) + \beta G(x_i))\right) \\ &= \arg\min_{\beta, G} \sum_{i=1}^{n} w_i^{(m)} \exp\left(-\beta y_i G(x_i)\right)\end{aligned}$$

where $w_i^{(m)} = \exp(-y_i f_{m-1}(x_i))$

It can be shown that

$$w_i^{(m+1)} = w_i^{(m)} \cdot \exp(\alpha_m \mathbb{I}(y_i \neq G_m(x_i))) \cdot \exp(-\beta_m)$$

Gradient Boosting

Gradient boosting generalizes $L(y, f(x))$ to any smooth loss function.

Some common loss functions:

TABLE 10.2. *Gradients for commonly used loss functions.*

Setting	Loss Function	$-\partial L(y_i, f(x_i))/\partial f(x_i)$
Regression	$\frac{1}{2}[y_i - f(x_i)]^2$	$y_i - f(x_i)$
Regression	$\lvert y_i - f(x_i)\rvert$	$\text{sign}[y_i - f(x_i)]$
Regression	Huber	$y_i - f(x_i)$ for $\lvert y_i - f(x_i)\rvert \leq \delta_m$ $\delta_m \text{sign}[y_i - f(x_i)]$ for $\lvert y_i - f(x_i)\rvert > \delta_m$ where $\delta_m = \alpha\text{th-quantile}\{\lvert y_i - f(x_i)\rvert\}$
Classification	Deviance	kth component: $I(y_i = \mathcal{G}_k) - p_k(x_i)$

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   c For $j = 1, ..., J^b$, compute

$$\gamma_i^b = \arg\min_{\gamma} \sum_{x_i \in R_j^b} L(y_i, f^{b-1}(x_i) + \gamma) \tag{10}$$

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3. Output $\hat{f}_n(x) = f^B(x)$.

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- Random splitting: at each iteration a subsample of the training data is drawn at random (without replacement).
- Penalized learning: Apply $L1$ or $L2$ regularization to the terminal nodes.

Example: Shrinkage (ν)

$$f^b(x) = f^{b-1}(x) + \nu \sum_{j=1}^{J^b} \gamma_j^b \mathbb{I}(x \in R_j^b) \tag{11}$$

Further trades off slower / longer learning.

Example: Comparing shrinkage vs not.

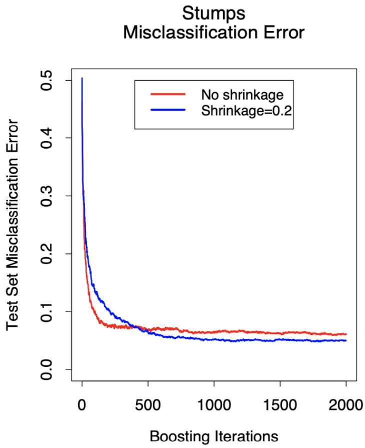

Figure 10.11

Sub-sampling

Example: Shrinkage with sub-sampling.

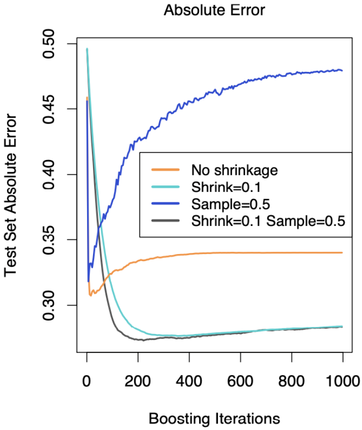

Figure 10.12

Gradient boosting tips

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Some tips from Kaggle master *Owen Zhang*:

GBDT Hyper Parameter Tuning

Hyper Parameter	Tuning Approach	Range	Note
# of Trees	Fixed value	100-1000	Depending on datasize
Learning Rate	Fixed => Fine Tune	[2 - 10] / # of Trees	Depending on # trees
Row Sampling	Grid Search	[.5, .75, 1.0]	
Column Sampling	Grid Search	[.4, .6, .8, 1.0]	
Min Leaf Weight	Fixed => Fine Tune	3/(% of rare events)	Rule of thumb
Max Tree Depth	Grid Search	[4, 6, 8, 10]	
Min Split Gain	Fixed	0	Keep it 0

Best GBDT implementation today: https://github.com/tqchen/xgboost
by **Tianqi Chen** (U of Washington)

DataRobot

References

[1] ISL. Chapter 8

[2] ESL. Chapter 10

[3] Schapire, RE. The Boosting Approach to Machine Learning An Overview. Nonlinear Estimation and Classification, Springer, 2003.