More models with multiple predictors

Data Science in a Box

datasciencebox.org

Modified by Tyler George

Two numerical predictors

The data

```r
pp <- read_csv(
  "data/paris-paintings.csv",
  na = c("n/a", "", "NA")
) %>%
  mutate(log_price = log(price))
```

Multiple predictors

- Response variable: `log_price`
- Explanatory variables: Width and height

```
pp_fit <- linear_reg() %>%
  set_engine("lm") %>%
  fit(log_price ~ Width_in + Height_in, data = pp)
tidy(pp_fit)
```

```
## # A tibble: 3 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   4.77     0.0579      82.4  0
## 2 Width_in      0.0269   0.00373      7.22 6.58e-13
## 3 Height_in    -0.0133   0.00395     -3.36 7.93e- 4
```

Linear model with multiple predictors

```
## # A tibble: 3 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   4.77     0.0579      82.4  0
## 2 Width_in      0.0269   0.00373      7.22 6.58e-13
## 3 Height_in    -0.0133   0.00395     -3.36 7.93e- 4
```

$$\widehat{log\_price} = 4.77 + 0.0269 \times width - 0.0133 \times height$$

Visualizing models with multiple predictors

Plot Code

Visualizing models with multiple predictors

Plot Code

```r
p <- plot_ly(pp,
  x = ~Width_in, y = ~Height_in, z = ~log_price,
  marker = list(size = 3, color = "lightgray", alpha = 0.5,
                line = list(color = "gray", width = 2))) %>%
  add_markers() %>%
  plotly::layout(scene = list(
    xaxis = list(title = "Width (in)"),
    yaxis = list(title = "Height (in)"),
    zaxis = list(title = "log_price")
  )) %>%
  config(displayModeBar = FALSE)
frameWidget(p)
```

Numerical and categorical predictors

Price, surface area, and living artist

- Explore the relationship between price of paintings and surface area, conditioned on whether or not the artist is still living
- First visualize and explore, then model
- But first, prep the data

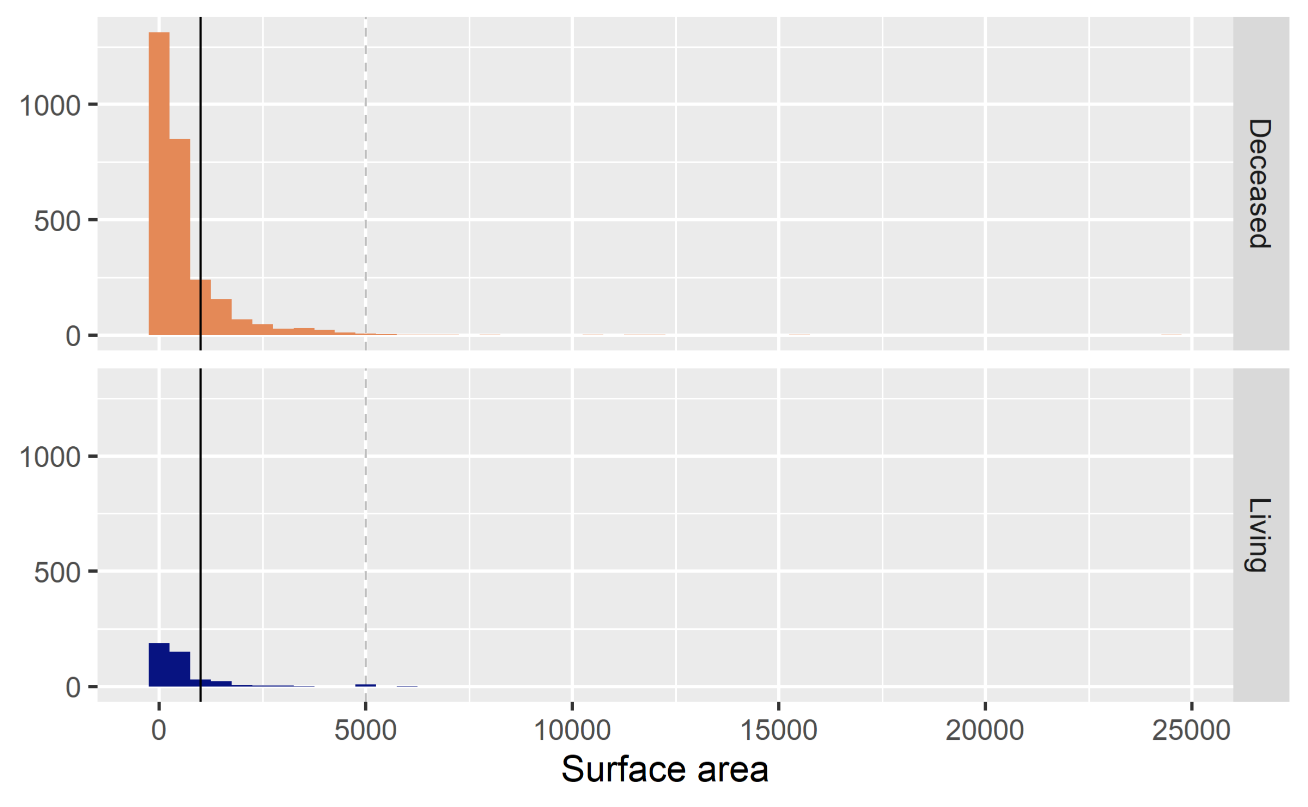
```r
pp <- pp %>%
  mutate(artistliving = if_else(artistliving == 0, "Deceased", "Living"))

pp %>%
  count(artistliving)
```

```
## # A tibble: 2 x 2
##   artistliving     n
##   <chr>        <int>
## 1 Deceased      2937
## 2 Living         456
```

Typical surface area

Plot Code

Typical surface area appears to be less than 1000 square inches (~ 80cm x 80cm). There are very few paintings that have surface area above 5000 square inches.



Typical surface area

Plot Code

```r
ggplot(data = pp, aes(x = Surface, fill = artistliving)) +
  geom_histogram(binwidth = 500) +
  facet_grid(artistliving ~ .) +
  scale_fill_manual(values = c("#E48957", "#071381")) +
  guides(fill = FALSE) +
  labs(x = "Surface area", y = NULL) +
  geom_vline(xintercept = 1000) +
  geom_vline(xintercept = 5000, linetype = "dashed", color = "gray")
```

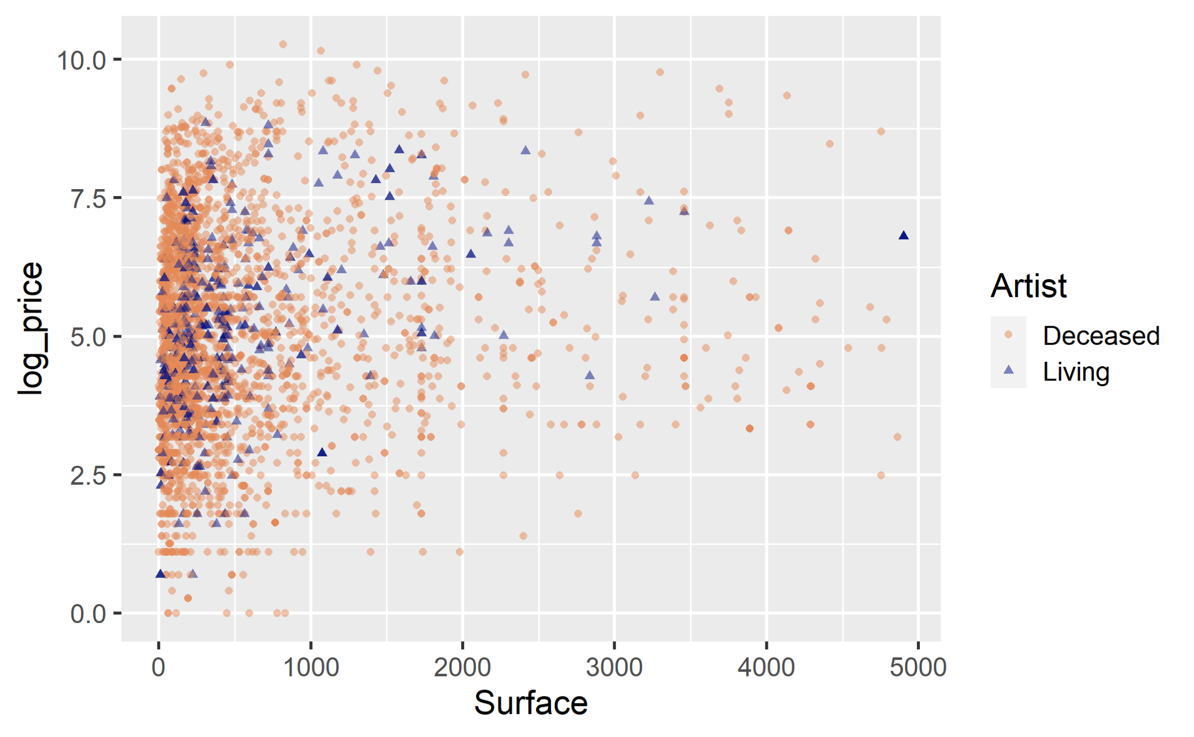
```
## Warning: `guides(<scale> = FALSE)` is deprecated. Please use
## `guides(<scale> = "none")` instead.

## Warning: Removed 176 rows containing non-finite values
## (stat_bin).
```

Narrowing the scope

Plot

Code

For simplicity let's focus on the paintings with `Surface < 5000`:



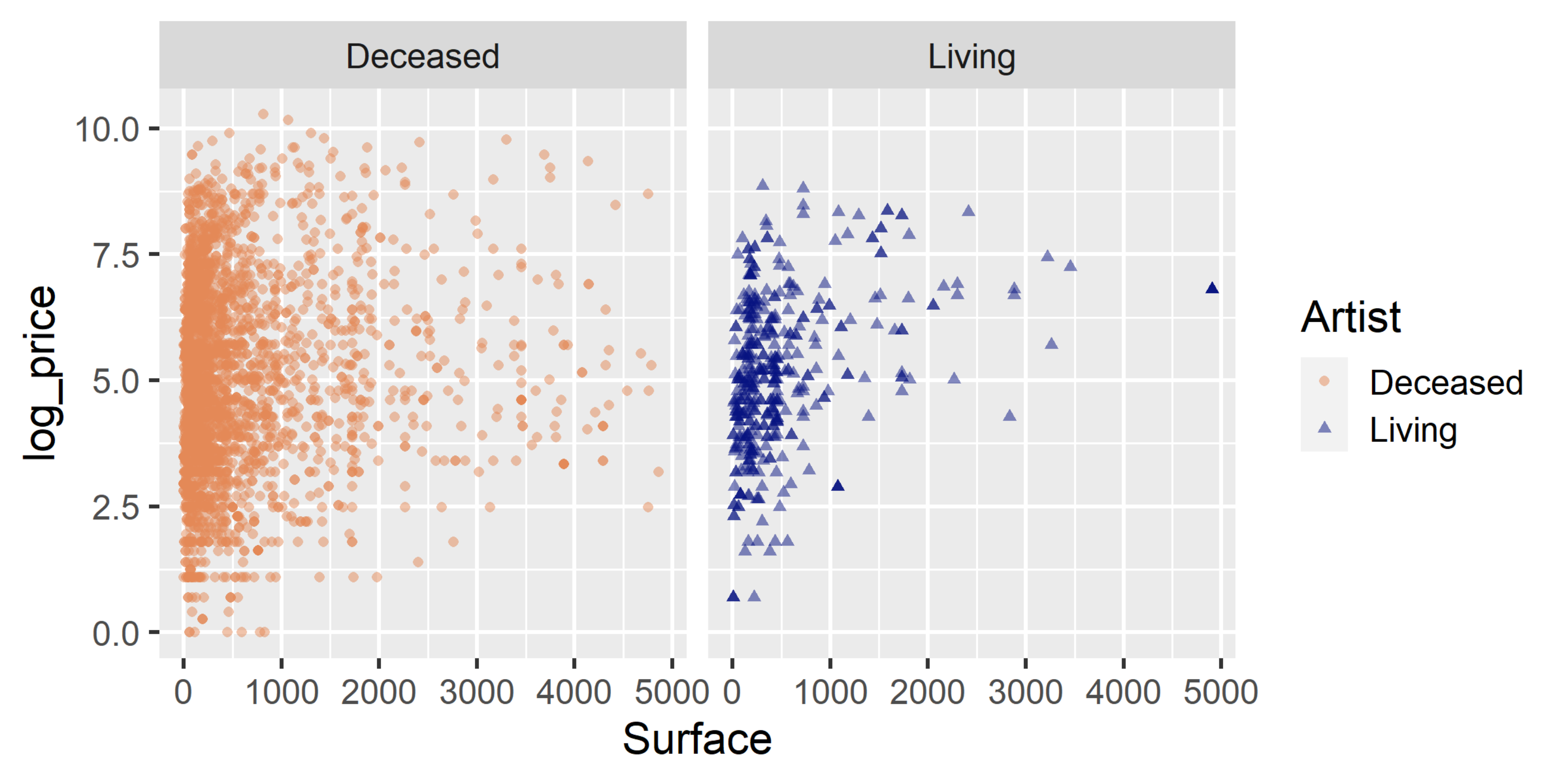
Narrowing the scope

Plot Code

```r
pp_Surf_lt_5000 <- pp %>%
  filter(Surface < 5000)

ggplot(data = pp_Surf_lt_5000,
       aes(y = log_price, x = Surface, color = artistliving, shape = artistliving)) +
  geom_point(alpha = 0.5) +
  labs(color = "Artist", shape = "Artist") +
  scale_color_manual(values = c("#E48957", "#071381"))
```

Facet to get a better look

Plot Code



Facet to get a better look

Plot Code

```r
ggplot(data = pp_Surf_lt_5000,
       aes(y = log_price, x = Surface, color = artistliving, shape = artistliving)) +
  geom_point(alpha = 0.5) +
  facet_wrap(~artistliving) +
  scale_color_manual(values = c("#E48957", "#071381")) +
  labs(color = "Artist", shape = "Artist")
```

Two ways to model

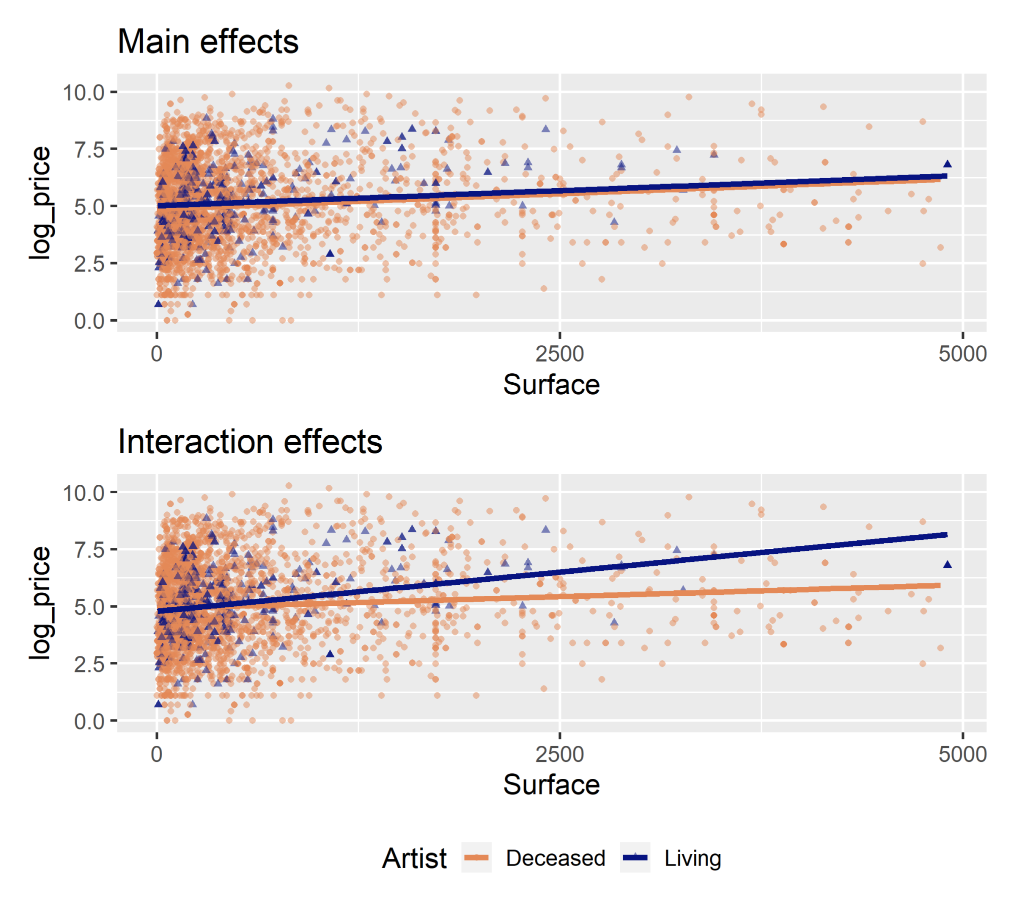
- **Main effects:** Assuming relationship between surface and logged price **does not vary** by whether or not the artist is living.
- **Interaction effects:** Assuming relationship between surface and logged price **varies** by whether or not the artist is living.

Interacting explanatory variables

- Including an interaction effect in the model allows for different slopes, i.e. nonparallel lines.
- This implies that the regression coefficient for an explanatory variable would change as another explanatory variable changes.
- This can be accomplished by adding an interaction variable: the product of two explanatory variables.

Two ways to model

- **Main effects:** Assuming relationship between surface and logged price **does not vary** by whether or not the artist is living
- **Interaction effects:** Assuming relationship between surface and logged price **varies** by whether or not the artist is living



Fit model with main effects

- Response variable: `log_price`
- Explanatory variables: `Surface` area and `artistliving`

```
pp_main_fit <- linear_reg() %>%
  set_engine("lm") %>%
  fit(log_price ~ Surface + artistliving, data = pp_Surf_lt_5000)
tidy(pp_main_fit)
```

```
## # A tibble: 3 x 5
##   term               estimate std.error statistic  p.value
##   <chr>                 <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)        4.88     0.0424       115.   0
## 2 Surface            0.000265 0.0000415      6.39 1.85e-10
## 3 artistlivingLiving 0.137    0.0970         1.41 1.57e- 1
```

Fit model with main effects

- Response variable: `log_price`
- Explanatory variables: `Surface` area and `artistliving`

```r
pp_main_fit <- linear_reg() %>%
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## # A tibble: 3 x 5
##   term               estimate std.error statistic  p.value
##   <chr>                 <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)        4.88     0.0424       115.   0       
## 2 Surface            0.000265 0.0000415      6.39 1.85e-10
## 3 artistlivingLiving 0.137    0.0970         1.41 1.57e- 1
```

$$\widehat{log\_price} = 4.88 + 0.000265 \times surface + 0.137 \times artistliving$$

Solving the model

- Non-living artist: Plug in 0 for `artistliving`

$$\begin{aligned}\widehat{log\_price} &= 4.88 + 0.000265 \times surface + 0.137 \times 0 \\ &= 4.88 + 0.000265 \times surface\end{aligned}$$

Solving the model

- Non-living artist: Plug in 0 for `artistliving`

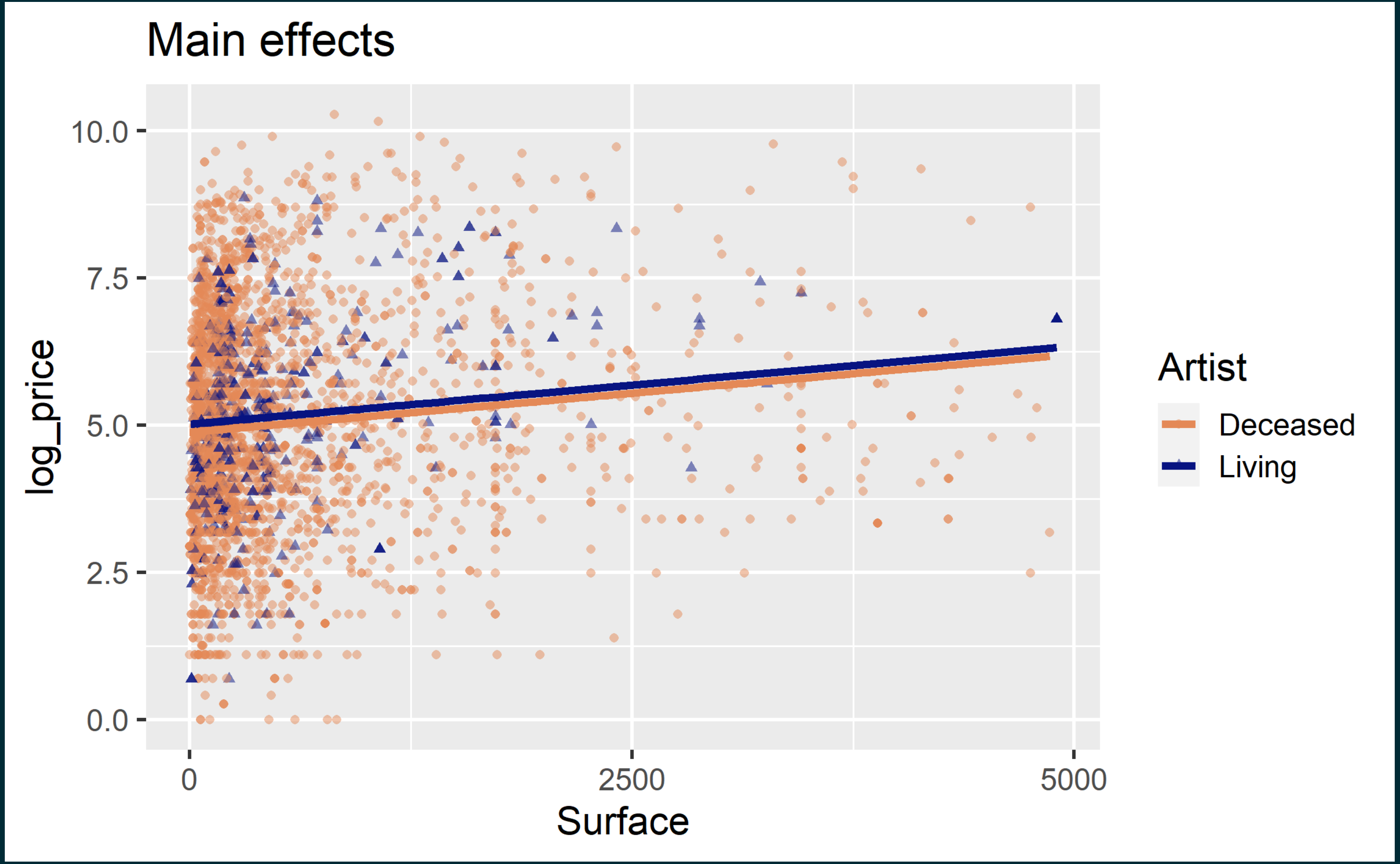
$$\widehat{log\_price} = 4.88 + 0.000265 \times surface + 0.137 \times 0$$
$$= 4.88 + 0.000265 \times surface$$

- Living artist: Plug in 1 for `artistliving`

$$\widehat{log\_price} = 4.88 + 0.000265 \times surface + 0.137 \times 1$$
$$= 5.017 + 0.000265 \times surface$$

Visualizing main effects

- **Same slope:** Rate of change in price as the surface area increases does not vary between paintings by living and non-living artists.
- **Different intercept:** Paintings by living artists are consistently more expensive than paintings by non-living artists.



Interpreting main effects

```
tidy(pp_main_fit) %>%
  mutate(exp_estimate = exp(estimate)) %>%
  select(term, estimate, exp_estimate)
```

```
## # A tibble: 3 x 3
##   term               estimate exp_estimate
##   <chr>                 <dbl>        <dbl>
## 1 (Intercept)        4.88          132.
## 2 Surface            0.000265        1.00
## 3 artistlivingLiving 0.137           1.15
```

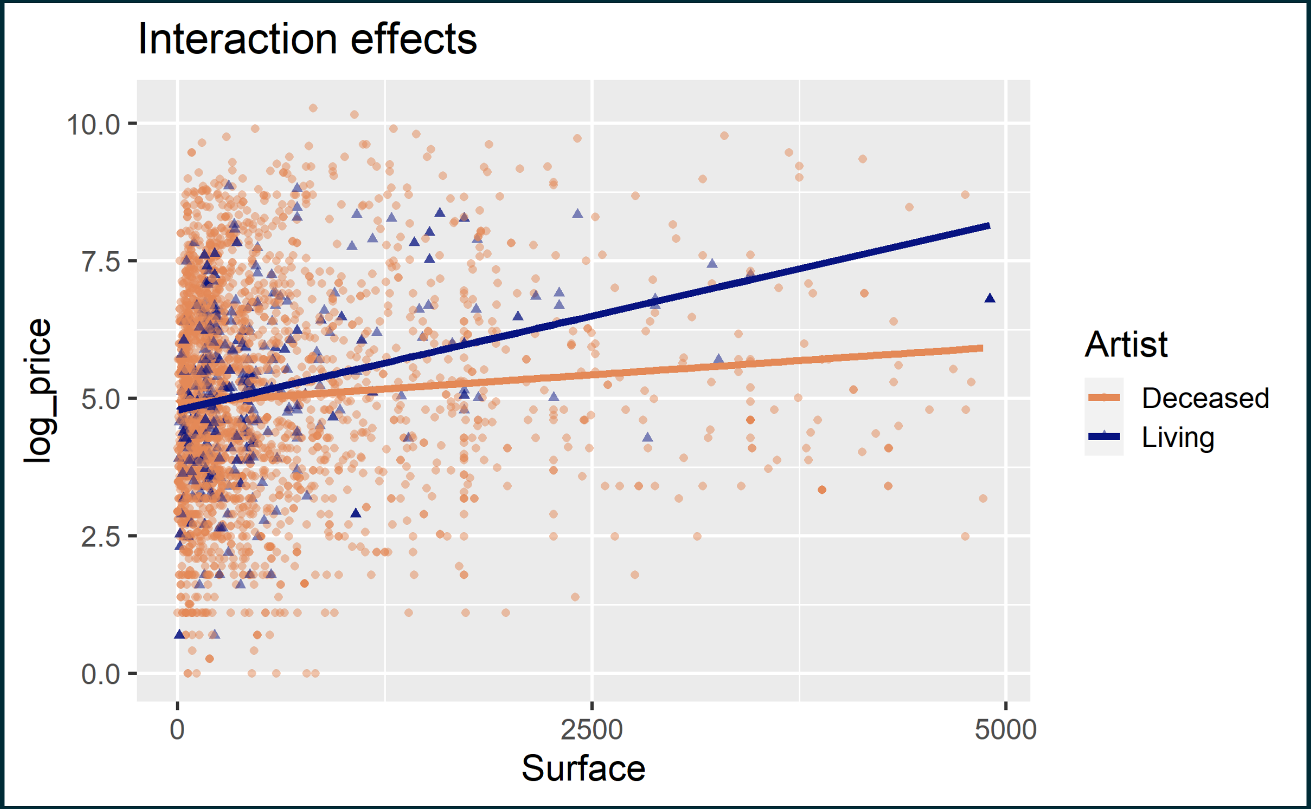
- All else held constant, for each additional square inch in painting's surface area, the price of the painting is predicted, on average, to be higher by a factor of 1.
- All else held constant, paintings by a living artist are predicted, on average, to be higher by a factor of 1.15 compared to paintings by an artist who is no longer alive.
- Paintings that are by an artist who is not alive and that have a surface area of 0 square inches are predicted, on average, to be 132 livres.

Main vs. interaction effects

- The way we specified our main effects model only lets `artistliving` affect the intercept.
- Model implicitly assumes that paintings with living and deceased artists have the *same slope* and only allows for *different intercepts*.

> What seems more appropriate in this case?
>
> - Same slope and same intercept for both colours
> - Same slope and different intercept for both colours
> - Different slope and different intercept for both colours

Interaction: `Surface * artistliving`



Fit model with interaction effects

- Response variable: log_price
- Explanatory variables: Surface area, artistliving, and their interaction

```
pp_int_fit <- linear_reg() %>%
  set_engine("lm") %>%
  fit(log_price ~ Surface * artistliving, data = pp_Surf_lt_5000)
tidy(pp_int_fit)
```

```
## # A tibble: 4 x 5
##   term                       estimate std.error statistic p.value
##   <chr>                         <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)                 4.91e+0 0.0432       114.   0
## 2 Surface                     2.06e-4 0.0000442      4.65 3.37e-6
## 3 artistlivingLiving         -1.26e-1 0.119         -1.06 2.89e-1
## 4 Surface:artistlivingLiving  4.79e-4 0.000126       3.81 1.39e-4
```

Linear model with interaction effects

```
## # A tibble: 4 x 5
##   term                         estimate std.error statistic p.value
##   <chr>                           <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)                   4.91e+0 0.0432       114.   0      
## 2 Surface                       2.06e-4 0.0000442      4.65 3.37e-6
## 3 artistlivingLiving           -1.26e-1 0.119         -1.06 2.89e-1
## 4 Surface:artistlivingLiving    4.79e-4 0.000126       3.81 1.39e-4
```

$$\widehat{log\_price} = 4.91 + 0.00021 \times surface - 0.126 \times artistliving$$

$$+ 0.00048 \times surface * artistliving$$

Interpretation of interaction effects

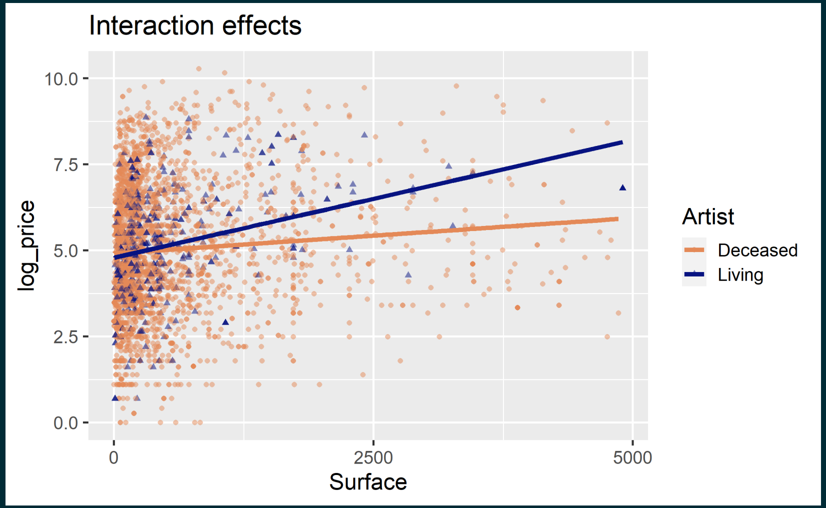
- Rate of change in price as the surface area of the painting increases does vary between paintings by living and non-living artists (different slopes),
- Some paintings by living artists are more expensive than paintings by non-living artists, and some are not (different intercept).

- Non-living artist:

$$\widehat{log\_price} = 4.91 + 0.00021 \times surface$$
$$-0.126 \times 0 + 0.00048 \times surface \times 0$$
$$= 4.91 + 0.00021 \times surface$$

- Living artist:

$$\widehat{log\_price} = 4.91 + 0.00021 \times surface$$
$$-0.126 \times 1 + 0.00048 \times surface \times 1$$
$$= 4.91 + 0.00021 \times surface$$
$$-0.126 + 0.00048 \times surface$$
$$= 4.784 + 0.00069 \times surface$$



Comparing models

It appears that adding the interaction actually increased adjusted R^2, so we should indeed use the model with the interactions.

```r
glance(pp_main_fit)$adj.r.squared
```

```
## [1] 0.01258977
```

```r
glance(pp_int_fit)$adj.r.squared
```

```
## [1] 0.01676753
```

Third order interactions

- Can you? Yes
- Should you? Probably not if you want to interpret these interactions in context of the data.