# Quiz 3

Stats 60/160

August 16, 2020

Please show your work. Answers without explanation may not be given full credit.

## 1 True or False

Indicate whether the following statements are true or false and briefly explain your answer.

**(a)** If the population is skewed, then we cannot use the normal distribution to construct confidence interval for the mean.

**Answer** False. When the sample size is large, the sample mean is approximately normal even when the population is skewed. If the population distribution is more skewed then it takes more samples for the sample mean to be approximately normal approximation.

**(b)** A researcher computes a test statistics equal to 2.35 and the two-sided p-value is 1%. This means, if the null hypothesis is correct, then there is only 1% chance of getting a test statistics larger than 2.35 in magnitude.

**Answer** True. This is the correct interpretation of p-values. (here larger than 2.35 in magnitude means either greated than 2.35 or less than -2.35).

**(c)** If a 95% confidence interval for a population parameter is $(10, 15)$, then this means that there is a 95% chance that the true parameter is in the interval $(10, 15)$.

**Answer** False. The interval $(10, 15)$ either contains the true parameter or not, so it does not make sense to assign a probability to this event. However, it is true that if we repeat the experiment, each time deriving a different 95% confidence interval, then 95% of the time, the interval contains the true parameter.

## 2 Drug clearance rate

It was reported that the clearance rate of zolpidem, a popular sleeping aid, is different between men and women. In order to test this claim, you measure the apparent oral clearance rate (in 10 mg (ml/min/kg)) of zolpidem for 20 male and 20 female volunteers. The average for male voluneteers is 6.5 (standard deviation is 4.5); the average for female volunteers is 3.5 (standard deviation is 1.2). Can you reject the null hypothesis that the clearance rate is the same for male and female?

**Answer** We use a two sample t-test. The t-statistics is

$$t = \frac{6.5 - 3.5}{\sqrt{\frac{4.5^2}{20} + \frac{1.2^2}{20}}} = 2.88,$$

and its degree of freedom is 38. The p-value to test the null hypothesis versus two-sided alternative is

$$\Pr(|T_{38}| \geq 2.88) = \Pr(T_{38} \geq 2.88) + \Pr(T_{38} \leq -2.88) = 0.0065,$$

so there is strong evidence against the null hypothesis that the clearance rate is the same for men and women (p-value = 0.0065).

## 3 Surveying number of children

Assume the average number of children per household in San Jose is 2.1 with a standard deviation of 1.2 children. For a class project, 30 students in a class select 50 households in San Jose at random and record the number of children in each household. Approximately how many students would report an average number of children more than 2.3 children per household?

**Answer** The average number of children from 50 households is approximately normal with mean 2.1 and standard deviation $1.2/\sqrt{50} = 0.17$. The probability that one student reports an average higher than 2.3 is

$$\Pr\left(Z \geq \frac{2.3 - 2.1}{0.17}\right) = 0.12.$$

We expect

$$30 \times 0.12 = 3.6$$

students, which is about 4, to report an average number of children higher than 2.3.

## 4 Potential coronary disease drug

A pilot study is conducted to assess effectiveness of a potential drug to treat coronary disease. The researchers recruited a group of 50 subjects who have high risk of heart attack in the treatment group, and they carefully matched 50 subjects in the control group (subjects in the treatment group were given the drug and subjects in the control group were given placebos).

**(a)** Over the next 5 years, 14 people in the treatment group had an heart attack, compared to 22 in the control group. Is the drug effective in terms of reducing rate of heart attack?

**Answer** We want to test the null hypothesis that the rate of heart attack is the same between the treatment and control group. The alternative hypothesis that the drug reduces rate of heart attack (you can also use two-sided alternative). Here there are 50 observations from the treatment group and 50 from the control group, each observation is either 0 or 1 depending on whether the subject has a heart attack.

Answer 1: You can use a two-sample t-test. The t-statistics is

$$\frac{0.28 - 0.44}{\sqrt{0.36 \times (1 - 0.36)(1/50 + 1/50)}} = -1.667,$$

and it is approximately from a t-distribution with 98 degree of freedom. Under the null hypothesis, whether a subject has a heart attack is from a binomial distribution with size 1 and a common probability, which can estimated by the aggregate proportion. Thus in the denominator we use a common variance for two populations. The p-value for one-tailed alternative is 0.050, which provides some evidence against the null hypothesis.

Answer 2: You can use a permutation test, and the p-value for the one-tailed alternative is 0.0307.

```
x <- rep(c(0, 1),c(36, 64)) # result for each subject in the study
B <- 10000
stat_perm <- numeric(B)
for(b in 1:B){
  a <- sample(1:100, 50, replace = FALSE) # randomly assign treatment label
  # compare heart attack rate
  stat <- mean(x[a]) - mean(x[-a])
  stat_perm[b] <- stat
}
mean(stat_perm < -0.16)
```

**(b)** Due to concern that the drug may also increase the risk of depression, a different study was conducted. In this study a simple random sample of 10 subjects was given the drug for two years and then tested using a standard psychological exam for depression. The exam gives each person a score between 0 (no signs of depression) and 20 (crippling depression). The general population has an average score of 5.6. The sample had an average score of 8.7 with SD 6.3. Is there enough evidence here to reject the hypothesis that there is no difference between the average depression score for people on the new drug versus the depression rate in the general population? Test at the 5% level.

**Answer** We use a 1-sample t-test for this problem. The t-statistics is

$$t = \frac{8.7 - 5.6}{6.3/\sqrt{10}} = 1.556$$

Under the null hypothesis, it is approximately from a t distribution with 9 degrees of freedom. The one-sided p-value is

$$\Pr\left(T_9 \geq 1.556\right) = 0.077,$$

therefore we cannot reject the null hypothesis that the drug does not increase the level of depression.

**(c)** Given the analyses you performed in parts (a) and (b) above, what is your conclusion about this drug? What are the arguments for and against giving this drug to patients? Be sure to discuss the significance of the tests and the size of the differences that were observed.

**Answer** There's some evidence that the drug is effective (the observed reduction in heart attack rate is 0.16, and one-sided p-value is about 0.05). While we haven't concluded statistically that it makes people depressed, there is numerical evidence (i.e. a larger sample may very well show significance).

## 5 Microarray test

A DNA microarray is an invention by Stanford professor Pat Brown to simultaneously observe thousands of genes. In essence, the analysis of a microarray boils down to testing thousands of hypothesis.

A biologist performs 500 tests (independently of each other) and applies to each test the standard rejection rule for a 5% significance level. Even if all the null hypotheses are true, the biologist can expect about how many type 1 errors? Give or take how many or so?

You can use the following information (but not required to): If $X$ is from a binomial distribution ($n$ trials and probability of success on a single trial is $p$), then the mean of $X$ is $n \cdot p$ and the standard deviation of $X$ is $\sqrt{n \cdot p \cdot (1-p)}$.

**Answer** The biologist can expect $500 \times 0.05 = 25$ type 1 errors. The standard deviation of it is

$$\sqrt{500 \times 0.05 \times 0.95} = 4.87,$$

so give or take 5 or so.