Lecture 8: Steins paradox and hockey shooting statistics

Skidmore College

Goals

- Stein's Paradox
- Shooting Percentages in hockey
- Tools: Likelihood estimation, bias/variance

Set-up:

We are NHL general managers after the 2012-2013 season. Who are we going to sign? Assume all else is equal (same contract, same stats), here are two players in the 2012-13 season.

Player	Goals
David Krejci	17
Evgeni Malkin	7

Set-up:

We are NHL general managers after the 2012-2013 season. Who are we going to sign?

Player	Goals	Shots	Shooting %
David Krejci	17	106	16.0%
Evgeni Malkin	7	101	6.9%

Why does this information matter?

Set-up:

We are NHL general managers after the 2012-2013 season. Who are we going to sign?

Player	Goals	Shots	Shooting %
David Krejci (C)	17	106	16.0%
Evgeni Malkin (C)	7	101	6.9%

Information we want:

- What shooting percentages can we expect for Krejci and Malkin going forward?

Interlude:

Let's say we are interested in the overall fraction of the Skidmore students that will support a football team, p_0. In a completely randomized survey of 100 students, 22% of the Skidmore campus supports the adoption of a football team.

- Our sample statistic, $\hat{p} = 0.22$, is **unbiased** for p_0 because $E[\hat{p}] = p_0$.
- That is, our best guess as to the true fraction of the Skidmore students that support a football team is 22%. If we had one guess, that's it.
- *Note*: $\hat{p} = 0.22$ is biased for p_0 if $E[\hat{p}] \neq p_0$

Back to hockey

Player	Goals	Shots	Shooting %
David Krejci (C)	17	106	16.0%
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- Let p_K and p_M are the true probabilities that a Krejci or Malkin shot will score a goal, respectively
- What are our estimates of p_K and p_M?
  - $\hat{p}_K = 0.160$ is unbiased for p_K ($E[\hat{p}_K] = p_K$)
  - $\hat{p}_M = 0.069$ is unbiased for p_M ($E[\hat{p}_M] = p_M$)
- *Note*: $\hat{p}_M$ and $\hat{p}_K$ are called maximum likelihood estimators

Back to hockey

Player	Goals	Shots	Shooting %
David Krejci (C)	17	106	16.0%
Evgeni Malkin (C)	7	101	6.9%

What other information could we use?

- League-wide shooting percentage for forwards is 10.6%
- How do we incorporate this information?

James-Stein estimator

Via Efron & Morris, $z = \bar{y} + c(y - \bar{y})$,

- $\bar{y}$ is grand average of averages
- y is average of a single data set
- c is a shrinking factor, $c = \frac{N/0.25}{N/0.25 + 1/\sigma^2}$
  - N is number of observations we have on a player
  - σ^2 is variance of observations from one player to the next

James-Stein estimator, translated

Via Efron & Morris, $\hat{p}_{JS} = \bar{\hat{p}} + c * (\hat{p} - \bar{\hat{p}})$,

- $\bar{\hat{p}}$ is average of each players shooting percentage
- $\hat{p}$ is a single players observation
- c is a shrinking factor, $c = \frac{N/0.25}{N/0.25+1/\sigma^2}$
  - N is number of shooters
  - σ^2 is variance in shooting percentages between players

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- c is a shrinking factor, $c = \frac{N/0.25}{N/0.25 + 1/\sigma^2}$
  - N is number of shooters
  - σ^2 is variance in shooting percentages between players
- Plug in $c = 1$:
- Plug in $c = 0$:

James-Stein estimator, translated

Via Efron & Morris, $\hat{p}_{JS} = \bar{\hat{p}} + c * (\hat{p} - \bar{\hat{p}})$,

- $\bar{\hat{p}}$ is average of each players shooting percentage
- $\hat{p}$ is a single players observation
- c is a shrinking factor, $c = \frac{N/0.25}{N/0.25 + 1/\sigma^2}$
  - N is number of shooters
  - σ^2 is variance in shooting percentages between players
- Decreases in N:
- Increases in N:

James-Stein estimator, implemented

- Initial data: shooting statistics from the 2012-2013 season

```
library(RCurl); library(tidyverse)
url <- getURL("https://raw.githubusercontent.com/statsbylopez/StatsSports/maste
nhl.data <- read_csv(url)
nhl.data <- nhl.data %>% filter(TOI > 500)
nhl.data <- na.omit(nhl.data)
nhl.data$ShP <- nhl.data$Goals/nhl.data$Shots
nhl.data %>% slice(1:2)
```

```
## # A tibble: 2 x 20
##   Name   Position Team  Games Season   Age Salary Goals Assists Goals_Sixty
##   <chr>  <chr>    <chr> <int>  <int> <int>  <dbl> <int>   <int>       <dbl>
## 1 Just~  RL       DET      61 2.01e7    22   0.71     4       4        0.44
## 2 Just~  RL       DET      85 2.01e7    23   0.75     7      11        0.47
## # ... with 10 more variables: Assists_Sixty <dbl>, CF_Percent <dbl>,
## #   PDO <dbl>, CFRel_Percent <dbl>, Corsi <int>, CorsiFor <int>,
## #   CorsiAgainst <int>, Shots <int>, TOI <dbl>, ShP <dbl>
```

James-Stein estimator, implemented

- Initial data: shooting statistics from the 2012-2013 season

```
first.season <- nhl.data %>% filter(Season==20122013)
first.players <- first.season %>%
  group_by(Name) %>%
  filter(Shots <= 106, Shots >= 100, Position !="D") %>%
  select(Name, Position, Goals, Shots, ShP)
dim(first.players)
```

```
## [1] 12  5
```

James-Stein estimator, implemented

```r
head(first.players)
```

```
## # A tibble: 6 x 5
## # Groups:   Name [6]
##   Name              Position Goals Shots    ShP
##   <chr>             <chr>    <int> <int>  <dbl>
## 1 Jason.Chimera     L            4   101 0.0396
## 2 Johan.Franzen     RL           8   105 0.0762
## 3 Brendan.Gallagher R           13   103 0.126
## 4 Taylor.Hall       L           12   106 0.113
## 5 Jarome.Iginla     R           10   103 0.0971
## 6 David.Krejci      C           17   106 0.160
```

12 forwards, each with between 100-106 shots

James-Stein estimator, implemented

```r
p.bar <- mean(first.players$ShP)
p.bar
```

```
## [1] 0.1057114
```

```r
p.hat <- first.players$ShP
p.hat
```

```
##  [1] 0.03960396 0.07619048 0.12621359 0.11320755 0.09708738 0.16037736
##  [7] 0.06930693 0.13725490 0.08571429 0.19417476 0.11000000 0.05940594
```

James-Stein estimator, implemented

```
N <- first.players$Shots
N
```

```
##  [1] 101 105 103 106 103 106 101 102 105 103 100 101
```

```
sigma.sq <- sd(p.hat)^2 ##Rough approximation
sigma.sq
```

```
## [1] 0.001953588
```

James-Stein estimator, implemented

```
c <- (N/0.25)/(N/0.25 + 1/sigma.sq)
c
```

```
##  [1] 0.4411065 0.4507024 0.4459460 0.4530502 0.4459460 0.4530502 0.4411065
##  [8] 0.4435368 0.4507024 0.4459460 0.4386548 0.4411065
```

- Hockey shrinking factor after 100-105 shots: $c = 0.45$
- How to interpret c?

James-Stein estimator, implemented

Calculating the MLE and James-Stein estimates

```
first.players$ShP.MLE <- first.players$ShP
first.players$ShP.JS <- p.bar + c*(p.hat - p.bar)
head(first.players)
```

```
## # A tibble: 6 x 7
## # Groups:   Name [6]
##   Name              Position Goals Shots    ShP ShP.MLE ShP.JS
##   <chr>             <chr>    <int> <int>  <dbl>   <dbl>  <dbl>
## 1 Jason.Chimera     L            4   101 0.0396  0.0396 0.0766
## 2 Johan.Franzen     RL           8   105 0.0762  0.0762 0.0924
## 3 Brendan.Gallagher R           13   103 0.126   0.126  0.115
## 4 Taylor.Hall       L           12   106 0.113   0.113  0.109
## 5 Jarome.Iginla     R           10   103 0.0971  0.0971 0.102
## 6 David.Krejci      C           17   106 0.160   0.160  0.130
```

James-Stein estimator, implemented

How to judge estimation accuracy?

- Let's compare to career shooting percentage through March, 2016
- Each player with at least 200 shots
- In principle, a player's career % represents something closer to the truth (his true %)

Comparing the estimates

Mean absolute error

```
first.players1[1:3,]
```

```
## # A tibble: 3 x 5
## # Groups:   Name [3]
##   Name                ShP ShP.MLE ShP.JS ShP.Career
##   <chr>             <dbl>   <dbl>  <dbl>      <dbl>
## 1 Jason.Chimera     0.04    0.04   0.077      0.076
## 2 Johan.Franzen     0.076   0.076  0.092      0.083
## 3 Brendan.Gallagher 0.126   0.126  0.115      0.095
```

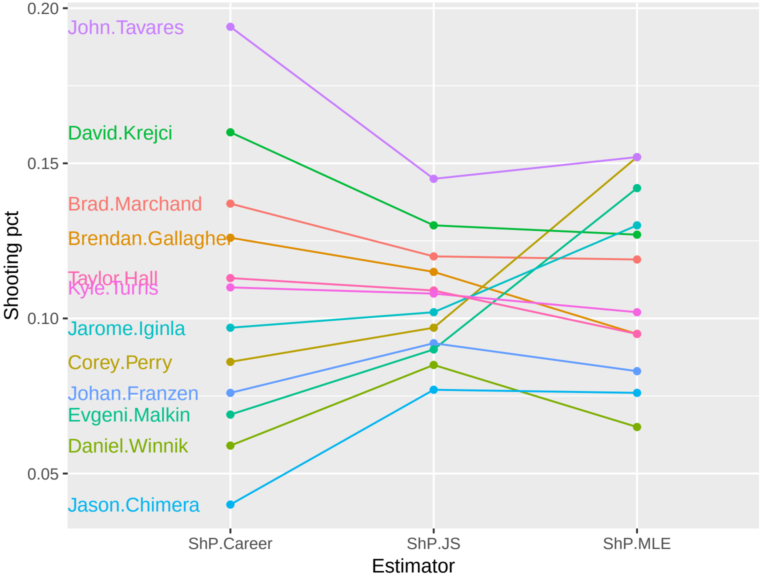
Comparing the estimates

```
first.players1 %>%
  ungroup() %>%
  mutate(abs.error.mle = abs(ShP.MLE - ShP.Career),
         abs.error.js = abs(ShP.JS - ShP.Career)) %>%
  summarise(mae.mle = mean(abs.error.mle),
            mae.js = mean(abs.error.js))
```

```
## # A tibble: 1 x 2
##   mae.mle mae.js
##     <dbl>  <dbl>
## 1  0.0309  0.018
```

How'd we do? How to interpret these numbers?

Visualizing the J-S estimator



Summary:

1. **Stein's Paradox**: Circumstances in which there are estimators better than the arithmetic average

- `better` defined by accuracy
- `better` estimators use combination of individual ones

2. Bias/Variance trade-off: $\hat{p}_{JS}$ versus $\hat{p}$

Article is here: `https://baseballwithr.wordpress.com/2016/02/15/revisiting-efron-and-morriss-baseball-study/`