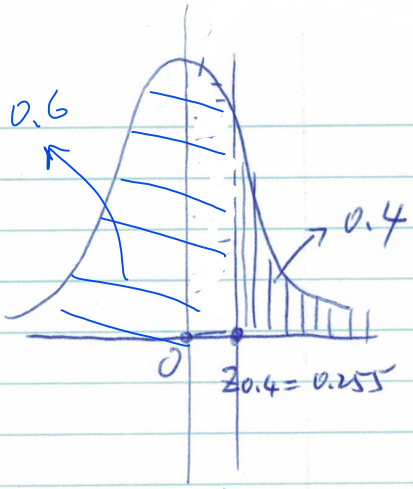
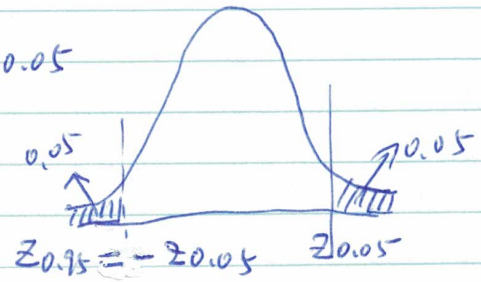
Q1. 8 pts

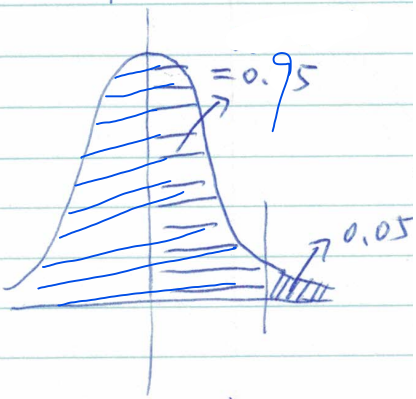
1. $P(0 \le Z \le z_{0.4}) = 0.5 - 0.4 = 0.1$

2pts $z_{0.4} = 0.254$



$z_{0.95} = -z_{0.05}$





$P(0 \le Z \le z_{0.05}) = 0.5 - 0.05 = 0.45$

$z_{0.95} = -z_{0.05} = -1.645$

2. $t_{0.4}^{(5)} = 0.267$, $t_{0.4}^{(10)} = 0.26$, $t_{0.4}^{(40)} = 0.255$, $t_{0.4}^{(120)} = 0.254$

4pts $\nu \nearrow \Rightarrow t_{0.4}^{(\nu)} \to z_{0.4}$ do not request

3. $F_{0.05}^{(2,4)} = 6.94$

2pts $F_{0.01}^{(1,6)} = 13.75$

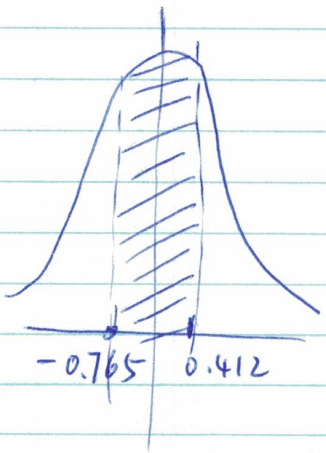
Hilroy

Q2 ① $X \sim N(2.65, 0.85)$

4pts $P(2 \le x \le 3) = P\left(\frac{2-2.65}{0.85} \le Z \le \frac{3-2.65}{0.85}\right)$

$= P(-0.765 \le Z \le 0.412)$

$= P(Z \le 0.412) - P(Z \le -0.765)$

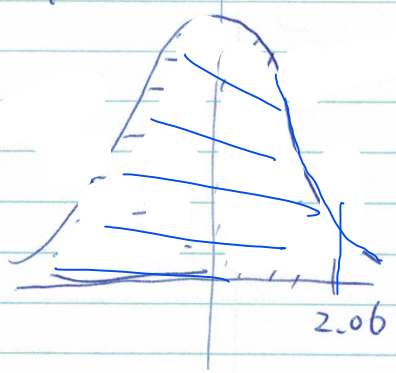


$= 0.4379$

2pts ② $\bar{X} \sim N\left(2.65, \left(\frac{0.85}{\sqrt{25}}\right)^2\right) = N(2.65, 0.17)$

4pts ③ $P(\bar{x} \le 3) = P\left(Z \le \frac{3-2.65}{0.85/\sqrt{25}}\right) = P(Z \le 2.06)$

$= 0.9803$



Hilroy

Q3.

3 pts ① $\bar{x} = (3.3 + 6.3 + 9.5 + 7.4 + 4.0 + 9.6)/6 = 6.68$

3 pts ② $s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(3.3-6.68)^2 + (6.3-6.68)^2 + (9.5-6.68)^2 + \cdots}{5}}$

$= 2.67$

4 pts ③ normal population, n small, σ unknown $\Rightarrow \bar{x} \pm t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}$

$\alpha = 0.1 \quad t_{\alpha/2}^{(n-1)} = t_{0.05}^{5} = 2.015$

$\left[6.68 \pm 2.015 \cdot \frac{2.67}{\sqrt{6}}\right] = [4.48, 8.88]$

4 pts ④ $s_p = 2.67 \quad n_p = 6$

$n_p - 1 = 5 < 30 \quad B = 1$

$n = \left(t_{\alpha/2}^{(n_p-1)} \frac{s_p}{B}\right)^2 = \left(2.015 \cdot \frac{2.67}{1}\right)^2 = 28.9$

$n = 30$

Q4

$H_0: \mu = 65$ $\quad H_a: \mu > 65$

normal, $n=10$ small, σ unknown $\Rightarrow t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$, $\left[\bar{x} \pm t_\alpha^{(n-1)} \frac{s}{\sqrt{n}}\right]$

3pts

$\bar{x} = 67.6$ $\quad s = 7.95$

$t_\alpha^{(n-1)} = t_{0.05}^{(9)} = 1.833$

$t = \frac{67.6 - 65}{7.95/\sqrt{10}} = 1.03$

① P-value $= P(T > 1.03)$. $T \sim t(n-1) = t(9)$

3pts

$\Rightarrow 0.15 > \alpha$

$\therefore$ not reject H_0

② CI: $\left[\bar{x} - t_\alpha^{(n-1)} \frac{s}{\sqrt{n}}, \infty\right] = \left[67.6 - 1.833 \times \frac{7.95}{\sqrt{10}}, \infty\right) = [62.99, \infty)$

3pts

$65 \in CI \Rightarrow$ not reject H_0

③ reject point: $t_{0.05}^{(n-1)} = t_{0.05}^{9} = 1.833$

$P(T > 1.833) = 0.05$ $\quad T \sim t(9)$

3pts.

$t = 1.03 < 1.833 \Rightarrow$ not reject H_0