TIME-SERIES ANALYSIS, MODELLING AND FORECASTING USING SAS SOFTWARE

Ramasubramanian V.
IA.S.R.I., Library Avenue, Pusa, New Delhi – 110 012
ramsub@iasri.res.in

1. Introduction

Time series (TS) data refers to observations on a variable that occurs in a time sequence. Mostly these observations are collected at equally spaced, discrete time intervals. The TS movements of such chronological data can be resolved or decomposed into discernible components as trend, periodic (say, seasonal), cyclical and irregular variations. A basic assumption in any TS analysis/modeling is that some aspects of the past pattern will continue to remain in the future. Here it is tacitly assumed that information about the past is available in the form of numerical data. Ideally, at least 50 observations are necessary for performing TS analysis/ modeling, as propounded by Box and Jenkins who were pioneers in TS modeling.

Decomposition models are among the oldest approaches to TS analysis *albeit* a number of theoretical weaknesses from a statistical point of view. These were followed by the crudest form of forecasting methods called the moving averages method. As an improvement over this method which had equal weights, exponential smoothing methods came into being which gave more weights to recent data. Exponential smoothing methods have been proposed initially as just recursive methods without any distributional assumptions about the error structure in them, and later, they were found to be particular cases of the statistically sound AutoRegressive Integrated Moving Average (ARIMA) models.

A detailed discussion regarding various TS components has been done by Croxton *et al.* (1979). A good account on exponential smoothing methods is given in Makridakis *et al.* (1998). A practical treatment on ARIMA modeling along with several case studies can be found in Pankratz (1983). A reference book on ARIMA and related topics with a more rigorous theoretical flavour is by Box *et al.* (1994).

2. Time Series Components

An important step in analysing TS data is to consider the types of data patterns, so that the models most appropriate to those patterns can be utilized. Four types of TS components can be distinguished. They are

(i) Horizontal – when data values fluctuate around a constant value
(ii) Trend – when there is long term increase or decrease in the data
(iii) Seasonal – when a series is influenced by seasonal factor and recurs on a regular periodic basis
(iv) Cyclical – when the data exhibit rises and falls that are not of a fixed period

Note that many data series include combinations of the preceding patterns. After separating out the existing patterns in any TS data, the pattern that remains unidentifiable form the 'random' or 'error' component. Time plot (data plotted over time) and seasonal plot (data plotted against individual seasons in which the data were observed) help in visualizing these patterns while exploring the data.

Many techniques such as time plots, auto-correlation functions, box plots and scatter plots abound for suggesting relationships with possibly influential factors. For long and erratic series, time plots may not be helpful. Alternatives could be to go for smoothing or averaging methods like moving averages, exponential smoothing methods etc. In fact, if the data contains considerable error, then the first step in the process of trend identification is smoothing.

3. Classical time series decomposition methods

Decomposition methods are among the oldest approaches to TS analysis. A crude yet practical way of decomposing the original data (including the cyclical pattern in the trend itself) is to go for a seasonal decomposition either by assuming an additive or multiplicative model such as

$$Y_t = T_t + S_t + E_t \quad \text{or} \quad Y_t = T_t \,.\, S_t \,.\, E_t$$

where Y_t - Original TS data
T_t - Trend component
S_t – Seasonal component
E_t – Error/ Irregular component

If the seasonal variations of a TS increases as the level of the series increases then one has to go for a multiplicative model else an additive model. Alternatively, an additive decomposition of the logged values of the TS can also be done. The decomposition methods may enable one to study the TS components separately or will allow workers to de-trend or to do seasonal adjustments if needed for further analysis.

4. Creating Time variables using SAS Date function

In practice, time series data (say, foodgrain production over years or monthly price data of onion) are available in various formats. In order that, SAS software understands such data with appropriate and corresponding time period reference, it provides functions to create such time variables. One frequently used function in INTNX. This function advances a date value by a given interval.

For example, suppose for a given data, the date value begins on January 1, 1990, and is incremented by one month each time the DATA step executes. For this, consider the following SAS syntax.

```
data electric;
input  elecprod @@;
date =intnx('month', '1jan90'd,_n_-1);
format date monyy.;
datalines;
200005 188715 187464 168720
175734 189430 216776 215393
…
233991 248165 271492 267698
233897 223180 221029
;
proc print data=electric;
id date;
run;
```

Here, in the INTNX function, the first argument specifies the time interval. The second argument specifies a SAS expression representing a date, time or datetime value that serves as a starting point. The third argument specifies a negative or positive integer representing the number of time intervals to increment the SAS data value each time the function executes. The _N_-1 argument specifies that the date value is not incremented on the first execution of the DATA step.

The output of the above SAS program codes can be obtained as under:

```
date     elecpro
JAN90    200005
FEB90    188715
MAR90    187464
...
OCT01    223180
NOV01    221029
```

5. Exponential smoothing model fitting using SAS software

Depending upon whether the data is horizontal, or has trend or has also seasonality the following methods are employed for forecasting purposes.

(i) Simple Exponential smoothing (SES)

For non-seasonal time series data in which there is no trend, simple exponential smoothing (SES) method can be employed. Let the observed time series up to time period t upon a variable Y be denoted by $y_1, y_2, \ldots, y_t$. Suppose the forecast, say, $y_t(1)$, of the next value y_{t+1} of the series (which is yet to be observed) is to be found out. Given data up to time period t-1, the forecast for the next time t is denoted by $y_{t-1}(1)$. When the observation y_t becomes available, the forecast error is found to be $y_t - y_{t-1}(1)$. Thereafter method of simple or single exponential smoothing takes the forecast for the previous period and adjusts it using the forecast error to find the forecast for the next period, i.e.

Level: $y_t(1) = y_{t-1}(1) + \alpha\,[y_t - y_{t-1}(1)]$

Forecast: $y_t(h) = y_t(1);\ h=2,3,\ldots$

where α is the smoothing constant (weight) which lies between 0 and 1. Here $y_t(h)$ is the forecast for h periods ahead i.e. the forecast of y_{t+h} for some subsequent time period t+h based on all the data up to time period t.

(ii) Double exponential smoothing (Holt)

The above SES method was then extended to linear exponential smoothing and has been employed to allow for forecasting non-seasonal time series data with trends. The forecast for Holt's linear exponential smoothing is found by having two equations to deal with –. one for level and one for trend. The forecast is found using two smoothing constants, α and β (with values between 0 and 1), and three equations:

Level: $l_t = \alpha\, y_t + (1-\alpha)(l_{t-1} + b_{t-1}),$

Trend: $b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1},$

Forecast: $y_t(h) = l_t + b_t h.$

Here l_t denotes the level of the series at time t and b_t denotes the trend (additive) of the series at time t. The optimal combination of smoothing parameters α and β should be chosen by minimizing the MSE over observations of the model data set.

(iii) Triple exponential smoothing (Winters)

If the data have no trend or seasonal patterns, then SES is appropriate. If the data exhibit a linear trend, Holt's method is appropriate. But if the data are seasonal, these methods, on their own, cannot handle the problem well. Holt's method was later extended to capture seasonality directly. The Winters' method is based on three smoothing equations—one for the level, one for trend, and one for seasonality. It is similar to Holt's method, with one additional equation to deal with seasonality. In fact there are two different Winters' methods, depending on whether seasonality is modelled in an additive or multiplicative way. The basic equations for Winters' multiplicative method are as follows.

Level: $l_t = \alpha \dfrac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1})$

Trend: $b_t = \beta\,(l_t - l_{t-1}) + (1-\beta)b_{t-1}$

Seasonal: $s_t = \gamma\, y_t / (l_{t-1} + b_{t-1}) + (1-\gamma)s_{t-m}$

Forecast: $y_t(h) = (l_t + b_t h)s_{t-m+h}$

where m is the length of seasonality (e.g., number of, say, months or 'seasons' in a year), l_t represents the level of the series, b_t denotes the trend of the series at time t, s_t is the seasonal component, and $y_t(h)$ is the forecast for h periods ahead. As with all exponential smoothing methods, we need initial values of the components and parameter values. The basic equations for Winters' additive method are as follows.

Level: $l_t = \alpha\left(\mathbf{y_t} - s_{t-m}\right) + (1-\alpha)(l_{t-1} + b_{t-1})$

Trend: $b_t = \beta\,(l_t - l_{t-1}) + (1-\beta)b_{t-1}$

Seasonal: $s_t = \gamma\,(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)\, s_{t-m}$

Forecast: $y_t(h) = (l_t + b_t h) + s_{t-m+h}$

In SAS, exponential smoothing models can be fitted by using the procedure "proc FORECAST" with the METHOD=EXPO option specified. The default METHOD is STEPAR, a simple time trend model combined with an autoregressive model i.e. it first fits a trend model with ordinary least squares regression, then uses a stepwise procedure to select the lags to fit an autoregressive model to the residuals of the trend model. You specify the model with the TREND= option as follows:

- TREND=1 specifies single exponential smoothing (a constant model)
- TREND=2 specifies double exponential smoothing (a linear trend model)
- TREND=3 specifies triple exponential smoothing (a quadratic trend model)

Exponential smoothing forecasts are forecasts for an integrated moving-average process; however, the weighting parameter is specified by the user rather than estimated from the data. Experience has shown that good values for the WEIGHT= option are between 0.05 and 0.3. As a general rule, smaller smoothing weights are appropriate for series with a slowly changing trend, while larger weights are appropriate for volatile series with a rapidly changing trend. This produces defaults of WEIGHT=0.2 for TREND=1, WEIGHT=0.10557 for TREND=2, and WEIGHT=0.07168 for TREND=3.

The SAS syntax is as follows:

```
data past;
input date:monyy. sales @@;
format date monyy.;
cards;
```

```
 JUL89      9.5161
 AUG89      9.6994
 SEP89      9.2644
 OCT89      9.6837
 NOV89     10.0784
 DEC89      9.9005
 JAN90     10.2375
 FEB90      10.694
 MAR90      10.629
 APR90     11.0332
 MAY90      11.027
 JUN90     11.4165
 JUL90     11.2918
 AUG90     11.3475
 SEP90     11.2913
 OCT90     11.3771
 NOV90     11.5457
 DEC90     11.6433
 JAN91     11.9293
 FEB91     11.9752
 MAR91     11.9283
 APR91     11.8985
 MAY91     12.0419
 JUN91     12.3537
 JUL91     12.4546
run;

proc forecast data=past method=stepar trend=2 lead=10 out=pred outlimit outfull
outresid
outest=est outfitstats;
   var sales;
  id date;
run;
proc print data=pred;
run;

proc print data=est;
run;

proc forecast data=past interval=month lead=10
            method=expo weight=0.1 trend=2
            out=pred outfull outresid
            outest=est outfitstats;
  var sales;
  id date;
run;

proc print data=pred;
run;
```

proc print data=est;
run;

The output for the above SAS codes when only the codes related to METHOD=EXPO are selected and run are as under:

```
Obs    date      _TYPE_     _LEAD_      sales
   1    JUL89    ACTUAL        0        9.5161
   2    JUL89    FORECAST      0        9.3326
   3    JUL89    RESIDUAL      0        0.1835
   4    AUG89    ACTUAL        0        9.6994
   5    AUG89    FORECAST      0        9.5269
   6    AUG89    RESIDUAL      0        0.1725
        ....
 97    MAR92    FORECAST      8       13.6183
 98    MAR92    L95           8       13.0146
 99    MAR92    U95           8       14.2219
100    APR92    FORECAST      9       13.7512
101    APR92    L95           9       13.1394
102    APR92    U95           9       14.3631

Obs    _TYPE_      date         sales
   1    N           JUL91           25
   2    NRESID      JUL91           25
   3    DF          JUL91           23
   4    WEIGHT      JUL91          0.1
        ...
  27    AIC         JUL91    -65.75409
  28    SBC         JUL91    -63.31634
  29    CORR        JUL91    0.9772283
```

6. ARIMA model fitting using SAS software

6.1 Stationarity of a TS process

A TS is said to be stationary if its underlying generating process is based on a constant mean and constant variance with its autocorrelation function (ACF) essentially constant through time. Thus, if we consider different subsets of a realization (TS 'sample') the different subsets will typically have means, variances and autocorrelation functions that do not differ significantly.

6.2 Autocorrelation functions

(i) Autocorrelation

Autocorrelation refers to the way the observations in a TS are related to each other and is measured by the simple correlation between current observation (Y_t) and observation from p periods before the current one (Y_{t-p}).

(ii) Partial autocorrelation

Partial autocorrelations are used to measure the degree of association between y_t and y_{t-p} when the y-effects at other time lags 1,2,3,…,p-1 are removed.

(iii) Autocorrelation function(ACF) and partial autocorrelation function(PACF)

Theoretical ACFs and PACFs (Autocorrelations versus lags) are available for the various models chosen (say, see Pankratz, 1983) for various values of orders of autoregressive and moving average components i.e. p and q. Thus compare the correlograms (plot of sample ACFs versus lags) obtained from the given TS data with these theoretical ACF/PACFs, to find a reasonably good match and tentatively select one or more ARIMA models. The general characteristics of theoretical ACFs and PACFs are as follows:- (here 'spike' represents the line at various lags in the plot with length equal to magnitude of autocorrelations)

Model	**ACF**	**PACF**
AR	Spikes decay towards zero	Spikes cutoff to zero
MA	Spikes cutoff to zero	Spikes decay to zero
ARMA	Spikes decay to zero	Spikes decay to zero

6.3 Description of ARIMA representation

(i) ARIMA modelling

In general, an ARIMA model is characterized by the notation ARIMA (p,d,q) where, p, d and q denote orders of auto-regression, integration (differencing) and moving average respectively. In ARIMA parlance, TS is a linear function of past actual values and random shocks. A stationary ARMA (p, q) process is defined by the equation

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} + \ldots - \theta_q \varepsilon_{t-q} + \varepsilon_t$$

where ε_t's are independently and normally distributed with zero mean and constant variance σ^2 for t = 1,2,...n. Note here that the values of p and q, in practice lie between 0 and 3. The degree of differencing of main variable y_t will be discussed in section 7 (i)).

6.4 Model building

(i) Identification

The foremost step in the process of modeling is to check for the stationarity of the series, as the estimation procedures are available only for stationary series. There are two kinds of stationarity, viz., stationarity in 'mean' and stationarity in 'variance'. A cursory look at the graph of the data and structure of autocorrelation and partial correlation coefficients may provide clues for the presence of stationarity. Another way of checking for stationarity is to fit a first order autoregressive model for the raw data and test whether the coefficient 'ϕ_1' is less than one. If the model is found to be non-stationary, stationarity could be achieved mostly by differencing the series. This is applicable for both seasonal and non-seasonal stationarity.

Thus, if 'X_t' denotes the original series, the non-seasonal difference of first order is

$$Y_t = X_t - X_{t-1}$$

The next step in the identification process is to find the initial values for the orders of seasonal and non-seasonal parameters, p, q. They could be obtained by looking for significant autocorrelation and partial autocorrelation coefficients. Say, if second order auto correlation coefficient is significant, then an AR (2), or MA (2) or ARMA (2) model could be tried to start with. This is not a hard and fast rule, as sample autocorrelation coefficients are poor estimates of population autocorrelation coefficients. Still they can be used as initial values while the final models are achieved after going through the stages repeatedly. Note that usually up to order 2 for p, d, or q are sufficient for developing a good model in practice.

(ii) Estimation

At the identification stage one or more models are tentatively chosen that seem to provide statistically adequate representations of the available data. Then we attempt to obtained precise estimates of parameters of the model by least squares as advocated by Box and Jenkins.

(iii) Diagnostics

Different models can be obtained for various combinations of AR and MA individually and collectively. The best model is obtained with following diagnostics.

(a) **Low Akaike Information Criteria (AIC**

AIC is given by

AIC = (-2 log L + 2 m)

where m=p+ q+ P+ Q and L is the likelihood function.

(b) **Plot of residual ACF**

Once the appropriate ARIMA model has been fitted, one can examine the goodness of fit by means of plotting the ACF of residuals of the fitted model. If most of the sample autocorrelation coefficients of the residuals are within the limits $\pm 1.96 / \sqrt{N}$ where N is the number of observations upon which the model is based then the residuals are white noise indicating that the model is a good fit.

(c) **Non-significance of auto correlations of residuals via Portmonteau tests (Q-tests based on Chisquare statistics)-Box-Pierce or Ljung-Box texts**

After tentative model has been fitted to the data, it is important to perform diagnostic checks to test the adequacy of the model and, if need be, to suggest potential improvements. One way to accomplish this is through the analysis of residuals. It has been found that it is effective to measure the overall adequacy of the chosen model by examining a quantity Q known as Box-Pierce statistic (a function of autocorrelations of residuals) whose approximate distribution is chi-square and is computed as follows:

$$\mathbf{Q = n \Sigma r^2 (j)}$$

where summation extends from 1 to k with k as the maximum lag considered, n is the number of observations in the series, r (j) is the estimated autocorrelation at lag j; k can be any positive integer and is usually around 20. Q follows Chi-square with (k-m_1) degrees of freedom where m_1 is the number of parameters estimated in the model. A modified Q statistic is the Ljung-box statistic which is given by

$$\mathbf{Q = n(n+2) \Sigma r^2 (j) / (n-j)}$$

The Q Statistic is compared to critical values from chi-square distribution. If model is correctly specified, residuals should be uncorrelated and Q should be small (the probability value should be large). A significant value indicates that the chosen model does not fit well.

All these stages require considerable care and work and they themselves are not exhaustive.

The SAS syntax is

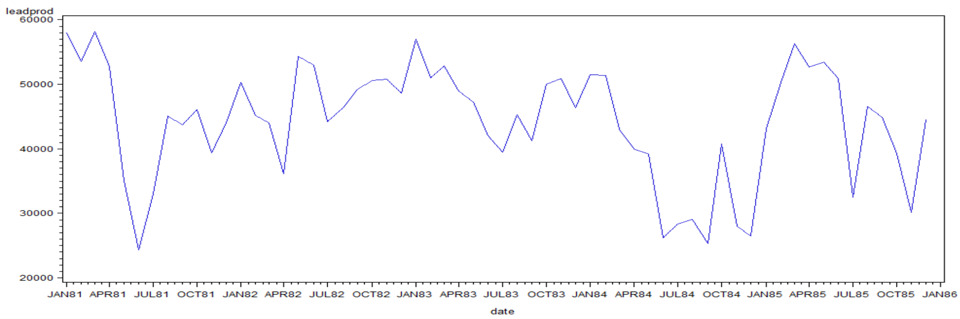
```
data lead;
input date:monyy. leadprod @@;
format date monyy.;
cards;
```

Jan81	57984
Feb81	53569
Mar81	58170
Apr81	52741
May81	35038
Jun81	24378
Jul81	33027
Aug81	45097
Sep81	43704
Oct81	46082
Nov81	39412
Dec81	44036
Jan82	50257
Feb82	45193
Mar82	43994
Apr82	36111
May82	54323
Jun82	52974
Jul82	44224
Aug82	46362
Sep82	49201
Oct82	50574
Nov82	50816
Dec82	48585
Jan83	57029
Feb83	50943
Mar83	52825
Apr83	48938
May83	47230
Jun83	42045
Jul83	39525
Aug83	45283
Sep83	41226
Oct83	49972
Nov83	50886
Dec83	46381
Jan84	51500
Feb84	51400
Mar84	43000
Apr84	40000
May84	39200
Jun84	26200
Jul84	28400
Aug84	29100

```
Sep84    25300
Oct84    40800
Nov84    28000
Dec84    26500
Jan85    43100
Feb85    50300
Mar85    56300
Apr85    52700
May85    53400
Jun85    50900
Jul85    32500
Aug85    46600
Sep85    44900
Oct85    39300
Nov85    30100
Dec85    44500
;
proc print data=lead(obs=20);
run;

proc gplot data=lead;
plot leadprod*date;
symbol1 i=join;
run;

proc arima data=lead;
identify var=leadprod;
run;
proc arima data=lead;

i var=leadprod noprint;
e p=1;
forecast lead=10;
run;
```

The output of the above SAS codes is obtained as follows. To start with, the plot obtained using proc gplot is displayed.

Autocorrelations



```
Lag    Covariance    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0      77907043        1.00000    |                   |********************|
  1      46262812        0.59382    |              .    |************        |
  2      24820147        0.31859    |            .      |******.             |
  3      13652130        0.17524    |            .      |****  .             |
  4       2570698        0.03300    |            .      |*     .             |
  5      -4490918        -.05764    |            .     *|      .             |
  6     -12385840        -.15898    |            .   ***|      .             |
  7     -12692244        -.16292    |            .   ***|      .             |
  8     -14068258        -.18058    |            .  ****|      .             |
  9     -11112953        -.14264    |           .    ***|       .            |
 10      -3817838        -.04901    |           .      *|       .            |
 11      -1165090        -.01495    |           .       |       .            |
 12       2399981        0.03081    |           .       |*      .            |
 13       2340982        0.03005    |           .       |*      .            |
 14       7761548        0.09963    |           .       |**     .            |
 15       4965254        0.06373    |           .       |*      .            |

                            "." marks two standard errors
```

Interpretation of the above output: The ACF tails off exponentially in an oscillating fashion. This is an indication that the process has an autoregressive component.

```
                         Inverse Autocorrelations

              Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                1       -0.51539    |         **********|    .               |
                2        0.14446    |              .    |*** .               |
                3       -0.12357    |              .  **|    .               |
                4        0.09154    |              .    |**  .               |
                5       -0.07940    |              .  **|    .               |
                6        0.08223    |              .    |**  .               |
                7       -0.02848    |              .   *|    .               |
                8        0.00475    |              .    |    .               |
                9        0.08168    |              .    |**  .               |
               10       -0.10324    |              .  **|    .               |
               11        0.08693    |              .    |**  .               |
               12       -0.10803    |              .  **|    .               |
               13        0.15068    |              .    |*** .               |
               14       -0.14794    |              . ***|    .               |
               15        0.06106    |              .    |*   .               |

                         Partial Autocorrelations

              Lag    Correlation    -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

                1        0.59382    |              .    |************        |
                2       -0.05258    |              .   *|    .               |
                3        0.01135    |              .    |    .               |
                4       -0.09972    |              .  **|    .               |
                5       -0.04867    |              .   *|    .               |
                6       -0.12939    |              . ***|    .               |
                7        0.01352    |              .    |    .               |
                8       -0.08617    |              .  **|    .               |
                9        0.02841    |              .    |*   .               |
               10        0.05807    |              .    |*   .               |
               11       -0.01958    |              .    |    .               |
               12        0.02489    |              .    |    .               |
               13       -0.03937    |              .   *|    .               |
               14        0.10776    |              .    |**  .               |
               15       -0.09285    |              .  **|    .               |
```

Interpretation of the above output: The PACF have spikes at lag 1 and cuts off to zero. This tells that the process is autoregressive with order 1.

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	32.79	6	<.0001	0.594	0.319	0.175	0.033	-0.058	-0.159
12	38.73	12	0.0001	-0.163	-0.181	-0.143	-0.049	-0.015	0.031

Interpretation of the above output: While testing the null hypothesis that "The time series observations are uncorrelated (white noise)", it has been rejected (probability values highly significant), hence it can be concluded that the given time series is not simply white noise and there is meaning to exploit the dependency between observations and TS models can tried.

Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	44997.1	2236.5	20.12	<.0001	0
AR1,1	0.59925	0.10550	5.68	<.0001	1

Interpretation of the above output: The fitted model is $y_t = 44997.1 + 0.59925\ y_{t-1}$.

Constant Estimate	18032.71
Variance Estimate	52061990
Std Error Estimate	7215.4
AIC	1238.315
SBC	1242.504
Number of Residuals	60

* AIC and SBC do not include log determinant.

Correlations of Parameter Estimates

Parameter	MU	AR1,1
MU	1.000	0.083
AR1,1	0.083	1.000

Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	1.57	5	0.9046	0.031	-0.042	0.046	-0.034	0.001	-0.131
12	3.22	11	0.9875	-0.029	-0.095	-0.086	0.047	-0.012	0.054
18	7.24	17	0.9803	-0.055	0.123	0.001	0.043	0.076	-0.146
24	10.72	23	0.9859	-0.007	-0.101	-0.089	-0.036	-0.069	0.106

Interpretation of the above output: While testing the null hypothesis that "The residuals of the fitted model are uncorrelated (white noise)", it has been found not rejected (probability values not significant), hence it can be concluded that the fitted time series model is a good fit.

```
                    Model for variable leadprod

                    Estimated Mean    44997.05


                      Autoregressive Factors

                   Factor 1:  1 - 0.59925 B**(1)


                  Forecasts for variable leadprod

 Obs       Forecast    Std Error       95% Confidence Limits

  61     44699.1950    7215.3995     30557.2717     58841.1182
  62     44818.5619    8411.7349     28331.8645     61305.2593
     ...
  69     44992.0987    9012.4443     27328.0323     62656.1650
  70     44994.0835    9012.7311     27329.4551     62658.7119
```

References

Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1994). *Time Series Analysis: Forecasting and Control*, Pearson Education, Delhi.

Croxton, F.E., Cowden, D.J. and Klein, S.(1979). *Applied General Statistics.* Prentice Hall of India Pvt. Ltd., New Delhi.

Makridakis, S., Wheelwright, S.C. and Hyndman, R.J. (1998). *Forecasting Methods and Applications*, 3rd Edition, John Wiley, New York.

Pankratz, A. (1983). Forecasting with univariate Box – Jenkins models: concepts and cases, John Wiley, New York.