Worksheet 17

1. Consider a prior distribution $p \sim Beta(\alpha, \beta)$ for some fixed α and β for a likelihood given by $X|p \sim Bin(n, p)$. Derive the posterior distribution $p|X$.

2. For reasons that we will explore in more next time, the Bayesian point estimator (the best single-number estimator) is the expected value of the posterior distribution. Under the set up from the previous question, what the Bayesian point estimator $\widehat{p}$ in terms of X, α, and β?

3. Consider observing $X \sim Bin(n, p)$. We know that the MLE estimator of p is given by $\hat{p}_{MLE} = X/n$. The Binomial comes from doing n Bernoulli trials and adding the number of 1s. Consider creating a new Y in which we artificially augment the data X by adding (in effect) an extra 0 and an extra 1. In other words, we create a $Y = X + 1$ with the assumption that $Y \sim Bin(n + 2, p)$. What is the MLE of p using the data from the augmented data Y? Where have you seen this before?

4. Consider your solution to the previous set of questions. If α and β are non-negative integers, how could you describe the Bayesian estimator based on adding data to X?

5. The standard uniform distribution is equivalent to $Beta(1, 1)$. In the notes I started by implicitly assuming that this was a fairly neutral starting assumption for indicating that we do not have any strong prior knowledge of p. Based on your results above, what would actually seem to be the best netural position if we do not want the prior to have a strong influence on the posterior mean?