Worksheet 18

1. Consider a prior distribution where $\lambda \sim Gamma(\alpha, \beta)$ and a likelihood $X_j|\lambda \sim Poisson(\lambda)$ for an i.i.d. sample of size n. Find (a) the prior distribution, (b) the Bayesian point estimator, and (c) the limit of the point estimator when the data dominates the prior.

2. Assume you have a sample of size $n = 10$ from a Poisson distribution. The average of the data is $\bar{x} = 3$. What are the (a) MLE estimator of λ, (b) the Bayesian estimator of λ with a $Gamma(1, 1)$, and (c) the Bayesian estimator of λ with a $Gamma(10, 1)$?[1]

[1] Take a moment to compare the results and see how the relate the means of the two priors.

3. Consider a prior distribution where $\lambda \sim Gamma(\alpha, \beta)$ and a likelihood $X|\lambda \sim Exp(\lambda)$ for an i.i.d. sample of size n.. Find (a) the prior distribution, (b) the Bayesian point estimator, and (c) the limit of the point estimator when the data dominates the prior.

4. Assume you have a sample of size $n = 30$ from an Exponential. The average of the data is $\bar{x} = 0.5$. What are the (a) MLE estimator of λ, (b) the Bayesian estimator of λ with a $Gamma(1, 1)$, and (c) the Bayesian estimator of λ with a $Gamma(1, 4)$?

5. Consider a prior distribution where $p \sim Beta(a, b)$ and a likelihood $X|p \sim Geometric(1, \beta)$for an i.i.d. sample of size n. Find (a) the prior distribution, (b) the Bayesian point estimator, and (c) the limit of the point estimator when the data dominates the prior.

6. Assume you have a sample of size $n = 12$ from a Geometric distribution. The average of the data is $\bar{x} = 1$. What are the (a) MLE estimator of p, (b) the Bayesian estimator of λ with a $Beta(1, 10)$, and (c) the Bayesian estimator of λ with a $Beta(10, 1)$? What are the means of the two priors?