Handout 10: Likelihood-Ratio Test

In addition to the nice point-estimator properties of MLE estimators, they also come along with a built-in general procedure for building hypothesis tests. This process is called the **likelihood-ratio test**; while it only generates asymptotically correct tests in the limit of large data, it often performs very well even for small datasets. It will be helpful for us to a start with a slightly more theoretical description of the test than we needed for the introductory methods we coved in the first weeks.

Assume that we are working with a distribution that has k unknown parameters. We are going to define a null-hypothesis as a subset of all possible configurations of the unknown parameters, which is in turn a subset of $\mathbb{R}^k$. Symbolically, we have $\Theta_0 \subset \Theta \subseteq \mathbb{R}^k$. We can define the G-score as the following:

$$G = -2 \cdot \log \left[\frac{\sup_{\theta \in \Theta_0} \mathcal{L}(\theta)}{\sup_{\theta \in \Theta} \mathcal{L}(\theta)} \right].$$

The quantity inside of the brackets must be between zero and one, and therefore G will always be positive. If we define θ_0 as the argmax of the numerator and $\hat{\theta}$ as the argmax of the numerator, this can be simplified in terms of the log-likelihood $l(\cdot)$:

$$G = 2 \cdot \left[l(\hat{\theta}) - l(\theta_0) \right].$$

There is an important (but not easy to prove) result called **Wilks' Theorem** that establishes that G will be approximately distributed as a χ^2. The degrees of freedom will be the difference in dimensionality between Θ and Θ_0.

A common case that the null hypothesis Θ_0 is just a single, fixed value. In this case, the distribution of G will be a chi-squared with degrees of freedom equal to the number of unknown parameters. The case of a larger null-hypothesis is often needed. We will see an example of this on today's worksheet.