Standard Statistical Tests

One Unknown Mean (One-Sample T-Test)

Consider a random sample $X_1, \dots, X_n \overset{iid}{\sim} G$ where we want to estimate the expected value μ_X of the distribution G. The sample mean $\bar{X}$ is an unbiased and consistent estimator of μ_X with variance $\frac{\sigma_X^2}{n}$. Under the assumption that G is normal (or asymptotically in the limit of large n), we have the following confidence interval:

$$\bar{X} \pm t_{\alpha/2}(n-1) \times \sqrt{S_X^2/n}$$

For a null hypothesis $H_0 : \mu_X = \mu_0$ for some fixed μ_0, the corresponding T test statistic is:

$$T = \frac{\bar{X} - \mu_0}{\sqrt{S_X^2/n}}$$

With a rejection region of signficance α given by:

$$R = \{T < t_{1-\alpha/2}(n-1)\} \cup \{T > t_{\alpha/2}(n-1)\}.$$

One Unknown Variance (Chi-squared Test for Variance)

Consider a random sample $X_1, \dots, X_n \overset{iid}{\sim} G$ where we want to estimate the variance σ_X^2 of the distribution G. The sample variance S_X^2 is an unbiased and consistent estimator of the variance. Under the assumption that G is normal, we have the following upper and lower bounds on the confidence interval:

$$L = \frac{(n-1)S_X^2}{\chi^2_{\alpha/2}(n-1)}$$
$$U = \frac{(n-1)S_X^2}{\chi^2_{1-\alpha/2}(n-1)}$$

For a null hypothesis $H_0 : \sigma_X^2 = \sigma_0^2$ for some fixed σ_0^2, the corresponding chi-squared test statistic C is:

$$C = \frac{(n-1)S_X^2}{\sigma_0^2 \cdot \chi_{\alpha/2}(n-1)}$$

With a rejection region of signficance α given by:

$$R = \{C < \chi^2_{1-\alpha/2}(n-1)\} \cup \{T > \chi^2_{\alpha/2}(n-1)\}.$$

Difference of Means (Two-Sample T-Test)

Consider a random sample $X_1, \ldots, X_n \overset{iid}{\sim} G_X$ with expected value μ_X and an independent second sample $Y_1, \ldots, Y_n \overset{iid}{\sim} G_Y$ with expected value μ_Y. We want to estimate the difference $\mu_X - \mu_Y$. The estimator $\bar{X} - \bar{Y}$ is an unbiased and consistent estimator of the mean difference. Under the assumption that G is normal (or asymptotically in the limit of large n and m) and $\sigma_X^2 = \sigma_Y^2$, we have the following confidence interval:

$$(\bar{X} - \bar{Y}) \pm t_{\alpha/2}(n + m - 2) \times \sqrt{S_p^2 \cdot \left[\frac{1}{n} + \frac{1}{m}\right]}$$

For a null hypothesis $H_0 : \mu_X - \mu_Y = d_0$ for some fixed d_0 the corresponding T test statistic is:

$$T = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{S_X^2 / n}}$$

With a rejection region of signficance α given by:

$$R = \{T < t_{1-\alpha/2}(n + m - 2)\} \cup \{T > t_{\alpha/2}(n + m - 2)\}.$$

Difference in Variances (F-Test)