

Worksheet 01

1. Assume we have a random sample of size $n = 5$ with the following data: $x_1 = 2$, $x_2 = 6$, $x_3 = 1$, $x_4 = 0$, $x_5 = 6$. What is the observed sample mean \bar{x} ?¹

2. Let $X_1, \dots, X_n \stackrel{iid}{\sim} \mathcal{G}$ be a random sample from a distribution with mean μ_X and variance σ_X^2 . What is the expected value of the sample mean \bar{X} ?² Does this imply that \bar{X} is an unbiased estimator of μ_X ?

3. Using the same set-up as the previous question, what is $\text{Var}(\bar{X})$?

4. Let Y be a random variable with mean m and variance v . Chebyshev's Inequality tells us that if for any $a > 0$,

$$\mathbb{P}[|Y - m| \geq a] \leq \frac{v}{a^2}.$$

Use this result to show that \bar{X} is a consistent estimator of μ_X .

5. Assume that \mathcal{G} has a normal distribution. Define the following:

$$Z = \frac{\mu_X - \bar{X}}{\sqrt{\sigma_X^2/n}}$$

What is the distribution of Z ?

¹ I am using the standard convention that we replace upper-case random variable names with lower-case variables when we have specific observations of them.

² I gave the answer on the handout. Make sure that you can justify the result.