

## Worksheet 01

1. Assume we have a random sample of size  $n = 5$  with the following data:  $x_1 = 2$ ,  $x_2 = 6$ ,  $x_3 = 1$ ,  $x_4 = 0$ ,  $x_5 = 6$ . What is the observed sample mean  $\bar{x}$ ?<sup>1</sup>

2. Let  $X_1, \dots, X_n \stackrel{iid}{\sim} \mathcal{G}$  be a random sample from a distribution with mean  $\mu_X$  and variance  $\sigma_X^2$ . What is the expected value of the sample mean  $\bar{X}$ ?<sup>2</sup> Does this imply that  $\bar{X}$  is an unbiased estimator of  $\mu_X$ ?

3. Using the same set-up as the previous question, what is  $\text{Var}(\bar{X})$ ?

4. Let  $Y$  be a random variable with mean  $m$  and variance  $v$ . Chebyshev's Inequality tells us that if for any  $a > 0$ ,

$$\mathbb{P}[|Y - m| \geq a] \leq \frac{v}{a^2}.$$

Use this result to show that  $\bar{X}$  is a consistent estimator of  $\mu_X$ .

5. Assume that  $\mathcal{G}$  has a normal distribution. Define the following:

$$Z = \frac{\mu_X - \bar{X}}{\sqrt{\sigma_X^2/n}}$$

What is the distribution of  $Z$ ?

<sup>1</sup> I am using the standard convention that we replace upper-case random variable names with lower-case variables when we have specific observations of them.

<sup>2</sup> I gave the answer on the handout. Make sure that you can justify the result.