Handout 07: CLT and T-Test

Let k_X be a constant defined, for any i, by:

$$k_X = \text{Var}\left[\frac{X_i - \mu_X}{\sigma_X}\right]^2.$$

If $K_X < \infty$, then, we have, in the limit of large n, the following chain of relationships:

$$\frac{S_X^2}{\sigma_X^2} = \frac{1}{n-1}\sum_i \left[\frac{X_i - \bar{X}}{\sigma_X}\right]^2 \rightarrow \frac{1}{n}\sum_i \left[\frac{X_i - \mu_X}{\sigma_X}\right]^2 \rightarrow N(1, \frac{X}{n}) \rightarrow_P 1.$$

Plugging this into the formula for the T statistic, we have:

$$T = \frac{\frac{\mu_X - \bar{X}}{\sqrt{\sigma_X^2/n}}}{\sqrt{\frac{(n-1)S_X^2}{\sigma_X^2 \cdot (n-1)}}} = \frac{\frac{\mu_X - \bar{X}}{\sqrt{\sigma_X^2/n}}}{\sqrt{\frac{S_X^2}{\sigma_X^2}}} \rightarrow \frac{N(0,1)}{1} = N(0,1).$$

The numerator is just a straightforward application of the central limit theorem to $\bar{X}$; the denominator comes from the chain of relationships above. We can combine them in the "natural" way that you might assume due to a result called Slutsky's Theorem.