Worksheet 14

1. Consider a simple linear regression where we know that $b_0 = 0$. You can write $b_1 \to b$ to simplify the notation. Write down the likelihood function for the sample. Do not yet simplify.

2. Now, (a) compute the log-likelihood function and simplify. (b) Without doing any calculus (that is, just looking at the function), maximizing the log-likelihood with respect to b is equivalent to minimizing what quantity in terms of y_i, x_i, and b? (c) Why might it make sense to minimize this quantity? Note: Ask me about the correct solution before proceeding.

3. Take the derivative of the quantity that you had in part (b) from the previous question with respect to the parameter b. Set this equal to zero to get the MLE.

4. What obsevations will have the most influence on the estimate of the slope? Does this make sense?

5. What is the distribution of the MLE of b? Is the estimator unbiased? Under what conditions will it be consistent? Note: This will take several steps.

6. Go back to the full log-likelihood function. Take the derivative with respect to σ^2 (remember, this is a single parameter, not the square of a parameter). Set this to zero and solve to get the MLE of σ^2. Does this equation make sense to you?

7. The MLE estimator for σ^2 is biased, but we can fix this by dividing by $n - 1$ instead of n, just as we did with the one-sample mean. This unbiased version is independent of $\widehat{b}$. If we take this unbiased estimator and divide by σ^2, we will have a chi-squared distribution with $n - 1$ degrees of freedom. Using this, create a pivot statistic that depends only on b and not σ^2.