Worksheet 01 (Solutions)

1. Assume we have a random sample of size $n = 5$ with the following data: $x_1 = 2$, $x_1 = 6$, $x_1 = 1$, $x_1 = 0$, $x_1 = 6$. What is the observered sample mean $\bar{x}$? [1]

[1] I am using the standard convention that we replace upper-case random variable names with lower-case variables when we have specific observations of them.

Solution: We have:

$$\bar{x} = \frac{2+6+1+0+6}{5} = \frac{15}{5} = 3.$$

2. Let $X_1, \ldots, X_n \overset{\text{i.i.d.}}{\sim} \mathcal{G}$ be a random sample from a distribution with mean μ_X and variance σ_X^2. What is the expected value of the sample mean $\bar{X}$?[2] Does this imply that $\bar{X}$ is an unbiased estimator of μ_X?

[2] I gave the answer on the handout. Make sure that you can justify the result.

Solution: NA

3. Using the same set-up as the previous question, what is $Var(\bar{X})$?

Solution: NA

4. Let Y be a random variable with mean m and variance v. Chebyshev's Inequality tells us that if for any $a > 0$,

$$\mathbb{P}[|Y - m| \geq a] \leq \frac{v}{a^2}.$$

Use this result to show that $\bar{X}$ is a consistent estimator of μ_X.

Solution: NA

5. Assume that $\mathcal{G}$ has a normal distribution. Define the following:

$$Z = \frac{\mu_X - \bar{X}}{\sqrt{\sigma_X^2/n}}$$

What is the distribution of Z?

Solution: NA