Worksheet 10

1. **(Ratio Test)** Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. What is the test statistic G for the corresponding likelihood ratio test for the null hypothesis $H_0 : \lambda = 1$.

2. **(Ratio Test)** Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. What is the test statistic G for the corresponding likelihood ratio test for the null hypothesis $H_0 : \lambda = 1$.

3. **(Regression I)** Consider a completely different task, where we start with a set of fixed real values $x_1, \ldots, x_n$. We observe a random sample of independent observations $Y_1, \ldots, Y_n$, where $Y_i \sim N(b \cdot x_i, \sigma^2)$.[1] So, the observations are independent but not identically distributed (they have different means). What is the log-likelihood of the sample?

[1] In these questions, y_i will be play the role that we have previously been calling x_i. This is the unquestioned standard notation for regression, so I wanted to use it even though it requires a bit of translation work.

4. **(Regression II)** Continuing from the previous question, what are $\hat{sigma}^2_{MLE}$ and $\hat{a}_{MLE}$ for the regression problem?

5. **(Regression III)** Consider an hypothesis test with $H_0 : a = 0$. Using the results we established the first few weeks (that is, without using the log-likelyhood test), find a pivot statistic to test this hypothesis.

6. **(MLE)** Let $X_1, \ldots, X_n \overset{iid}{\sim} Uniform(0, a)$. Find the MLE estimator for a. Note: You cannot do this using the derivative. Just think about it!