Worksheet 18

1. Consider a prior distribution where $\lambda \sim Gamma(\alpha, \beta)$ and a likelihood $X|\lambda \sim Poisson(\lambda)$. Find (a) the prior distribution and (b) the Bayesian point estimator.

2. Consider a prior distribution where $\lambda \sim Gamma(\alpha, \beta)$ and a likelihood $X|\lambda \sim Exp(\lambda)$. Find (a) the prior distribution and (b) the Bayesian point estimator.

3. Consider a prior distribution where $\beta \sim Gamma(\alpha_0, \beta_0)$ and a likelihood $X|\beta \sim Gamma(1, \beta)$. Find (a) the prior distribution and (b) the Bayesian point estimator.

4. Consider a prior distribution where $\mu \sim N(0, 1)$ and a likelihood $X|\mu \sim N(\mu, 1)$. Find (a) the prior distribution and (b) the Bayesian point estimator.

5. The Pareto distribution is a distribution with two parameters: $x_m > 0$ and $\alpha > 0$. It has a pdf equal to zero for values of $x < x_m$ and a pdf equal to $\frac{\alpha x_m^\alpha}{x^{\alpha+1}}$ for $x > x_m$. If $\alpha > 1$, the expected value of the Pareto is given by $\frac{\alpha x_m}{\alpha - 1}$. Consider a prior distribution where θ has a Pareto distribution with parameters $x_m = 0$ and $\alpha > 0$ with a likelihood having a density $X|\theta \sim U(0, \theta)$. Find (a) the prior distribution and (b) the Bayesian point estimator for θ.