

Worksheet 16

1. Consider a prior distribution $P \sim \text{Beta}(\alpha, \beta)$ for some fixed α and β for a likelihood function $X|P \sim \text{Bin}(n, P)$. Derive the (a) prior distribution and (b) the Bayesian point estimator.

2. Consider observing $X \sim \text{Bin}(n, p)$. We know that the MLE estimator of p is given by $\hat{p}_{MLE} = X/n$. The Binomial comes from doing n Bernoulli trials and adding the number of 1s. Consider creating a new Y in which we artificially augment the data X by adding (in effect) an extra 0 and an extra 1. In other words, we create a $Y = X + 1$ with the assumption that $Y \sim \text{Bin}(n + 2, p)$. What is the MLE of p using the data from the augmented data Y ? Where have you seen this before?

3. Consider your solution to the previous two questions. If α and β are non-negative integers, how could you describe the Bayesian estimator based on adding data to X ?

4. The standard uniform distribution is equivalent to $\text{Beta}(1, 1)$. In the notes I started by implicitly assuming that this was a fairly neutral starting assumption for indicating that we do not have any strong prior knowledge of P . Based on your results above, what would actually seem to be the best neutral position if we do not want the prior to have a strong influence on the posterior mean?