

## Worksheet 18 (Solutions)

1. Consider a prior distribution where  $\lambda \sim \text{Gamma}(\alpha, \beta)$  and a likelihood  $X|\lambda \sim \text{Poisson}(\lambda)$ . Find (a) the prior distribution and (b) the Bayesian point estimator.

*Solution:* TODO

2. Consider a prior distribution where  $\lambda \sim \text{Gamma}(\alpha, \beta)$  and a likelihood  $X|\lambda \sim \text{Exp}(\lambda)$ . Find (a) the prior distribution and (b) the Bayesian point estimator.

*Solution:* TODO

3. Consider a prior distribution where  $\beta \sim \text{Gamma}(\alpha_0, \beta_0)$  and a likelihood  $X|\beta \sim \text{Gamma}(1, \beta)$ . Find (a) the prior distribution and (b) the Bayesian point estimator.

*Solution:* TODO

4. Consider a prior distribution where  $\mu \sim N(0, 1)$  and a likelihood  $X|\mu \sim N(\mu, 1)$ . Find (a) the prior distribution and (b) the Bayesian point estimator.

*Solution:* TODO

5. The Pareto distribution is a distribution with two parameters:  $x_m > 0$  and  $\alpha > 0$ . It has a pdf equal to zero for values of  $x < x_m$  and a pdf equal to  $\frac{\alpha x_m^\alpha}{x^{\alpha+1}}$  for  $x > x_m$ . If  $\alpha > 1$ , the expected value of the Pareto is given by  $\frac{\alpha x_m}{\alpha - 1}$ . Consider a prior distribution where  $\theta$  has a Pareto distribution with parameters  $x_m = 0$  and  $\alpha > 0$  with a likelihood having a density  $X|\theta \sim U(0, \theta)$ . Find (a) the prior distribution and (b) the Bayesian point estimator for  $\theta$ .

*Solution:* TODO