

## Worksheet 17

1. Consider a prior distribution  $P \sim \text{Beta}(\alpha, \beta)$  for some fixed  $\alpha$  and  $\beta$  for a likelihood function  $X|P \sim \text{Bin}(n, P)$ . Derive the (a) prior distribution and (b) the Bayesian point estimator.

2. Consider observing  $X \sim \text{Bin}(n, p)$ . We know that the MLE estimator of  $p$  is given by  $\hat{p}_{MLE} = X/n$ . The Binomial comes from doing  $n$  Bernoulli trials and adding the number of 1s. Consider creating a new  $Y$  in which we artificially augment the data  $X$  by adding (in effect) an extra 0 and an extra 1. In other words, we create a  $Y = X + 1$  with the assumption that  $Y \sim \text{Bin}(n + 2, p)$ . What is the MLE of  $p$  using the data from the augmented data  $Y$ ? Where have you seen this before?

3. Consider your solution to the previous two questions. If  $\alpha$  and  $\beta$  are non-negative integers, how could you describe the Bayesian estimator based on adding data to  $X$ ?

4. The standard uniform distribution is equivalent to  $\text{Beta}(1, 1)$ . In the notes I started by implicitly assuming that this was a fairly neutral starting assumption for indicating that we do not have any strong prior knowledge of  $P$ . Based on your results above, what would actually seem to be the best neutral position if we do not want the prior to have a strong influence on the posterior mean?