Statistical Tests

The following table provides a summary of the key results from the five tests we have to estimate the mean or variance of one, two, or many samples. All of the results assume normality. The two-sample and multiple-sample tests for the mean assume that all groups have the same variance. The distribution column provides the distribution of the pivot (in general) and the distribution of the test statistic (conditional on H_0). See the handouts for the details of the notation.

Type	**Parameter**	**P. Estimator**	**Pivot**	**Confidence Interval**	**H$_0$**	**Test Statistic**	**Distribution**
One-Sample	μ_X	$\hat{\mu} = \bar{X}$	$\frac{\bar{X}-\mu_X}{\sqrt{S_X^2/n}}$	$\bar{X} \pm t_{\alpha/2} \times \sqrt{S_X^2/n}$	$\mu_X = \mu_0$	$\frac{\bar{X}-\mu_0}{\sqrt{S_X^2/n}}$	$T \sim t(n-1)$
	σ_X^2	$\hat{\sigma}^2 = S_X^2$	$\frac{(n-1)S_X^2}{\sigma_X^2}$	$\left[\frac{(n-1)S_X^2}{\chi^2_{1-\alpha/2}}, \quad \frac{(n-1)S_X^2}{\chi^2_{\alpha/2}}\right]$	$\sigma_X^2 = \sigma_0^2$	$\frac{(n-1)S_X^2}{\sigma_0^2}$	$C \sim \chi^2(n-1)$
Two-Sample	$\delta = \mu_X - \mu_Y$	$\hat{\delta} = \bar{X} - \bar{Y}$	$\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{S_p^2 \cdot \left[\frac{1}{n}+\frac{1}{m}\right]}}$	$(\bar{X}-\bar{Y}) \pm t_{\alpha/2} \times \sqrt{S_p^2 \cdot \left[\frac{1}{n}+\frac{1}{m}\right]}$	$\delta = \delta_0$	$\frac{(\bar{X}-\bar{Y})-(\delta_0)}{\sqrt{S_p^2 \cdot \left[\frac{1}{n}+\frac{1}{m}\right]}}$	$T \sim t(n+m-2)$
	$\Delta = \frac{\sigma_Y^2}{\sigma_X^2}$	$\hat{\Delta} = \frac{S_Y^2}{S_X^2}$	$\frac{S_X^2}{S_Y^2} \times \frac{\sigma_Y^2}{\sigma_X^2}$	$\left[\frac{S_Y^2}{S_X^2} \cdot f_{1-\alpha/2}, \frac{S_Y^2}{S_X^2} \cdot f_{\alpha/2}\right]$	$\Delta = \Delta_0$	$\frac{S_X^2}{S_Y^2} \times \Delta_0$	$F \sim F(n-1, m-1)$
Many-Sample	·	·	·	·	$\forall k, \mu_k = \mu_0$	$\frac{\frac{1}{K-1}\sum_{j=1}^{K} n_j \cdot \left[\bar{X}_j - \bar{X}\right]^2}{\frac{1}{N-K}\sum_{j=1}^{k}(n_j-1)S_j^2}$	$F \sim F(K-1, N-K)$