where $\sigma_n^2 = Var(\sum_{j=1}^n X_j)$. We need to show that:

We will prove both directions of the statement.
First, assume that $\frac{1}{n}\sum_{j=1}^n(X_j - E[X_j])[]p0$. We will show that $\frac{\sigma_n}{n}[]p0$.
By Chebyshev's inequality, for any $\epsilon > 0$:

Since $\frac{\sum_{j=1}^n(X_j - E[X_j])}{\sigma_n}[]dN(0,1)$, we know that:

where $Z \sim N(0,1)$. By hypothesis, we have that $\frac{1}{n}\sum_{j=1}^n(X_j - E[X_j])[]p0$. This means that for any $\epsilon > 0$, we have:

Using Markov's inequality, we have:

Since $X_1, X_2, \ldots$ are independent, we have:

where $\sigma_j^2 = Var(X_j)$. Combining the above results, we have: