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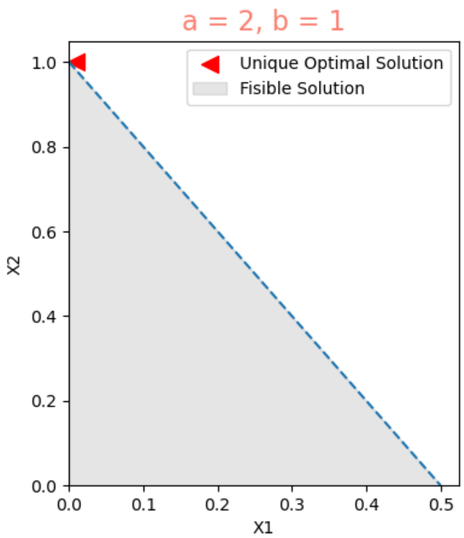
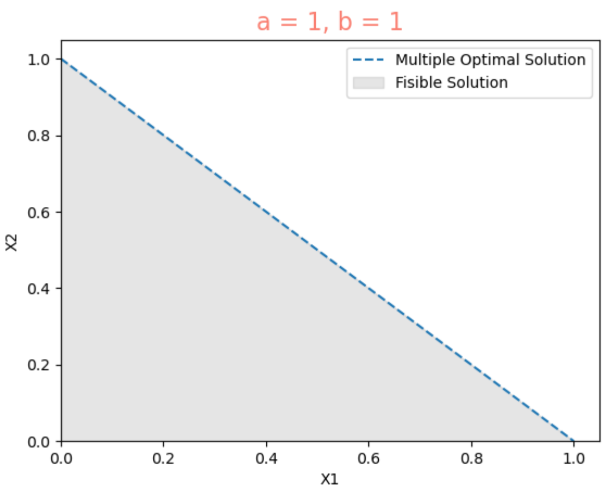
Advanced Algorithms - Homework 4

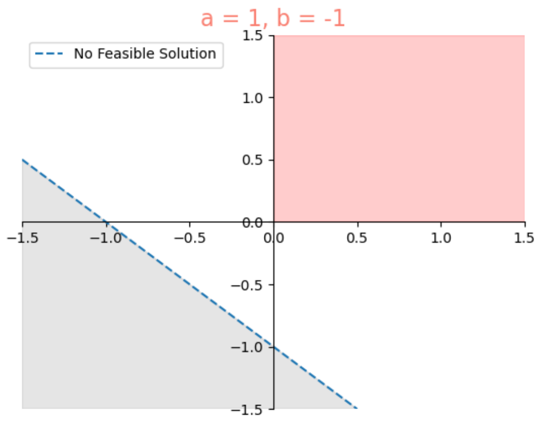
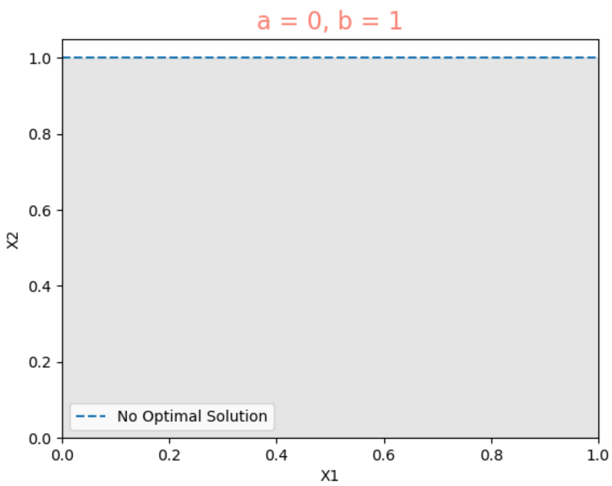
Question 1

A chef knows how to bake two cakes, cheese, and chocolate. The profit of each cake is the same and assume that every cake that the chef baked is sold immediately.
Cheesecake requires a eggs and chocolate cake require only 1 egg.
In the refrigerator there are only b eggs, the chef needs to maximize his profit, how many cheese and chocolate cakes he needs to bake?

Dual LP

Minimize t, subject to
$a \cdot t \geq 1$
$t \geq 1$
$t \geq 0$

Case	b.1	b.2
1		Minimize t, subject to $2 \cdot t \geq 1$ $t \geq 1$ The dual problem has also a unique optimal solution.
2		Minimize t, subject to $t \geq 1$ The dual problem has a unique optimal solution.

3		Minimize $-t$, subject to $t \geq 1$ The dual problem has no solution (as the primal problem)
4		Minimize t, subject to $0 \cdot t \geq 1$ $t \geq 1$ The dual problem is not feasible.

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Question 2

Given an undirected graph $G = (V, E)$, a cut is a bipartition $(S, V - S)$.

We know that x_v determine in which partition the vertex is in, and y_e determine whether y_e is in the cut or not. This is working because if $x_u \neq x_v$ then $y_{uv} - 1 \leq 0$, $y_{uv} + 1 \leq 2$ → $y_{uv} = 1$, Otherwise, assume that $x_u = x_v$ then $y_{uv} \leq 2$, $y_{uv} \leq 0$ → impossible. ($y_{uv} = 1$)

There is no such algorithm, to get the LP relaxation, just let $1 \geq x_v \geq 0$ and $1 \geq y_{uv} \geq 0$.

Assume for every $v \in V$ $x_v = 0.5$ → $y_{uv} - \frac{1}{2} - \frac{1}{2} \leq 0$, $y_{uv} + \frac{1}{2} + \frac{1}{2} \leq 2$ → $y_{uv} \leq 1$, $y_{uv} \leq 1$ → $y_{uv} = 1$.

We have 2 options to do rounding, round all vertices to 0 or to 1 (because all vertices are equals to 0.5). No matter which option we choose we'll have that $y_{uv} = 0$ → for every graph we get that the sum is 0, which is not a 2-approximation.

Question 3

Let x_a be a variable that $x_a = 1$ if a is chosen into X and $x_a = 0$ if a not chosen into X
Let w_a be the weight of element a

$$\text{Min} \sum_{a \in S} w_a x_a$$

s.t

$$\forall C_j \subseteq S \sum_{a \in C_j} x_a \geq 1$$

$$\forall a \in S, x_a \in \{0,1\}$$

Primal problem - there is a **variable** for each element in S and there is **constraint** for each set (C_j)

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = B, C = \begin{pmatrix} 4 & 4 & 4 & 4 & 4 \end{pmatrix}$$

$$\text{Max} \sum_{j=1}^{m} y_j$$

s.t

$$\forall a \in S \sum_{i: a \in C_j} y_i \leq w_a$$

$$y_i \geq 0 \; \forall i \in \{1, \cdots, n\}$$

Dual problem - there is a **variable** for each set (C_j) and there is **constraint** for each element in S

K-approximation algorithm

1. Solve the LP-Relaxation.
2. Do smart rounding where all $x_i \geq \frac{1}{k}$ round to 1, else round to 0. The worst case is that we increased x_i by a factor k.
3. Our output is a hitting set.

Analysis

The solution is feasible: for each element either $\text{element} \geq \frac{1}{k}$ or $\text{element} \leq \frac{1}{k}$, in our solution we've increased x_i by a factor of at most k, Hence the cost of the solution is at most $K \cdot OPT_{LP}$. Since $OPT_{LP} \leq OPT_{HS}$, the cost of the solution is $\leq K \cdot OPT_{HS}$

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Question 4

The integer programming problem is:

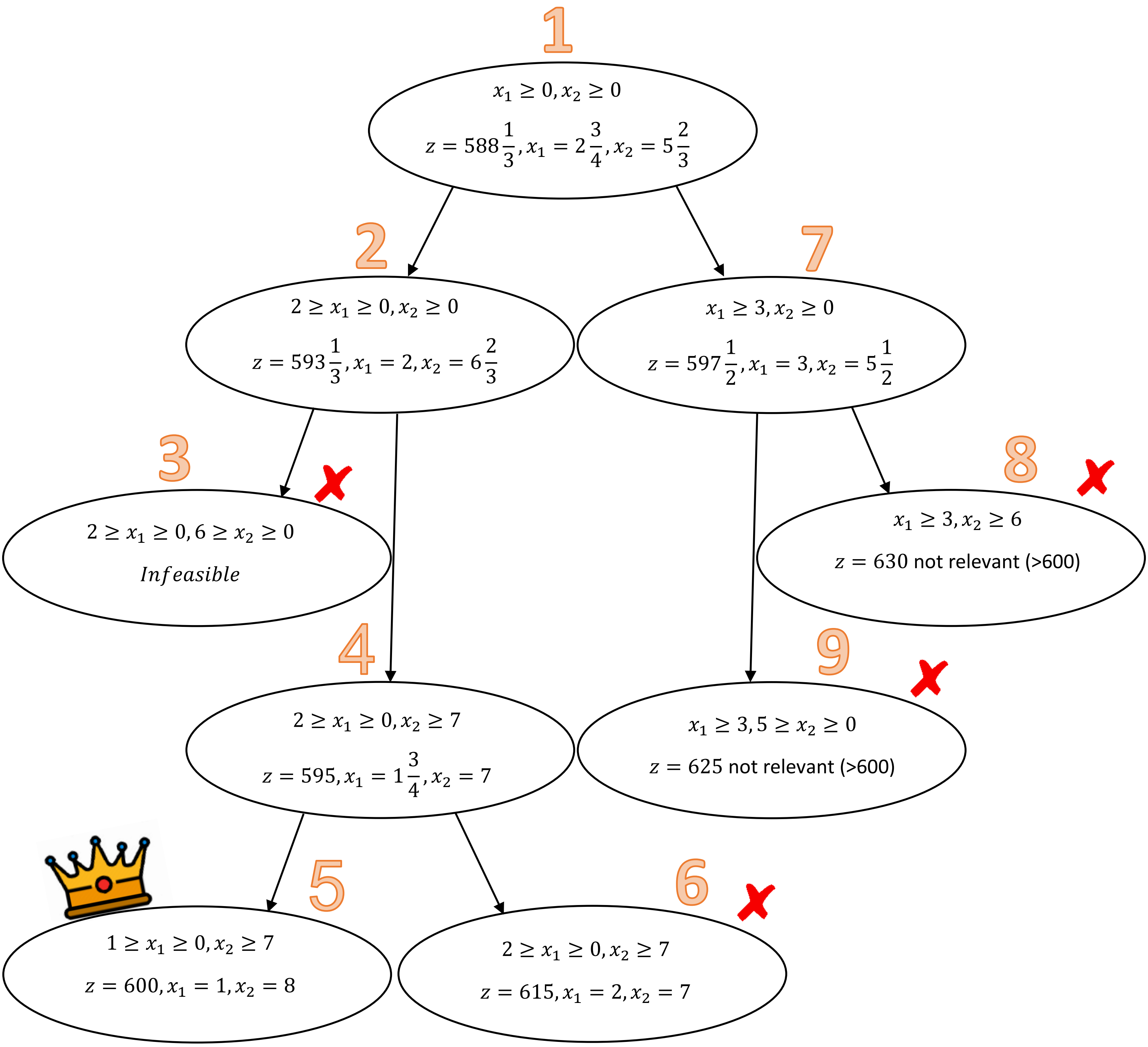
Min $z = 80x_1 + 65x_2$

s.t

$4x_1 + 6x_2 \geq 45$

$4x_1 + 3x_2 \geq 28$

$x_1 \geq 0,\ x_2 \geq 0$ and integers.



Explanation

The root of the tree (1) corresponds to the linear programming problem as described above. branch on x_1. The left subproblem is smaller (=2, 2), and the right higher (=3, 7). On the left side we are branching once more on x_2, the left side (3) is infeasible and the right (4) requires another branching on x_1 because x_2 is an integer. The left subproblem is better than the right problem (600<615), at this moment (5) is the best candidate, $z = 600$. Going back to subproblem 7, branching x_2 resulting having (9, 625 and 8, 630) worse results than 5, therefore the winner is subproblem 5, $x_1 = 1, x_2 = 8, z = 600$