Advanced Algorithms - Homework 4

Question 1

A chef knows how to bake two cakes, cheese, and chocolate. The profit of each cake is the same and assume that every cake that the chef baked is sold immediately.

Cheesecake requires eggs and chocolate cake require only 1 egg.

In the refrigerator there are only eggs, the chef needs to maximize his profit, how many cheese and chocolate cakes he needs to bake?

Dual LP

Minimize , subject to

|  |  |  |
| --- | --- | --- |
| **Case** | **b.1** | **b.2** |
| **1** |  | Minimize , subject to  The dual problem has also a unique optimal solution. |
| **2** |  | Minimize , subject to  The dual problem has a unique optimal solution. |
| **3** |  | Minimize , subject to  The dual problem has no solution (as the primal problem) |
| **4** |  | Minimize , subject to  The dual problem is not feasible. |

Question 2

Given an undirected graph , a cut is a bipartition .

We know that determine in which partition the vertex is in, and determine whether is in the cut or not. This is working because if then , 🡪 , Otherwise, assume that then , 🡪 impossible. ()

There is no such algorithm, to get the LP relaxation, just let and .

Assume for every 🡪 , 🡪  
, 🡪 .

We have 2 options to do rounding, round all vertices to 0 or to 1 (because all vertices are equals to 0.5). No matter which option we choose we'll have that 🡪 for every graph we get that the sum is 0, which is not a 2-approximation.

Question 3

Let be a variable that if is chosen into and if not chosen into

Let be the weight of element

s.t

Primal problem - there is a **variable** for each element in and there is **constraint** for each set ()

,

s.t

Dual problem - there is a **variable** for each set () and there is **constraint** for each element in

K-approximation algorithm

1. Solve the LP-Relaxation.
2. Do smart rounding where all round to 1, else round to 0. The worst case is that we increased by a factor .
3. Our output is a hitting set.

Analysis

The solution is feasible: for each element either or , in our solution we've increased by a factor of at most , Hence the cost of the solution is at most . Since , the cost of the solution is

Question 4

The integer programming problem is:

s.t

and integers.

**1**



**6**

**4**

**3**

**2**

**8**

not relevant (>600)

**9**

**7**

not relevant (>600)

**5**





Explanation

The root of the tree (1) corresponds to the linear programming problem as described above. branch on . The left subproblem is smaller (=2, 2), and the right higher (=3, 7). On the left side we are branching once more on , the left side (3) is infeasible and the right (4) requires another branching on because is an integer. The left subproblem is better than the right problem (600<615), at this moment (5) is the best candidate, . Going back to subproblem 7, branching resulting having (9, 625 and 8, 630) worse results than 5, therefore the winner is subproblem 5,