1) $$L(p|n,x) = \left(\frac{n!}{x!(n-x)!}\right) p^x (1-p)^{n-x}$$

$$\ln\left(L(p|n,x)\right) = \ln\left(\left(\frac{n!}{x!(n-x)!}\right) p^x (1-p)^{n-x}\right)$$

$$\ln\left(L(p|n,x)\right) = \ln\left(\frac{n!}{x!(n-x)!}\right) + \ln\left(p^x\right) + \ln\left((1-p)^{n-x}\right)$$

$$\ln\left(L(p|n,x)\right) = \ln\left(\frac{n!}{x!(n-x)!}\right) + x\ln(p) + (n-x)\ln(1-p)$$

$$\frac{\partial \ln\left(L(p|n,x)\right)}{\partial p} = 0 + x\cdot\frac{1}{p} + x\cdot\frac{1}{1-p} - n\cdot\frac{1}{1-p}$$

$$\frac{x}{p} + \frac{x}{1-p} - \frac{n}{1-p} = 0 \qquad x(1-p) + xp - np = 0$$

$$x - np = 0 \qquad x = np \qquad p = \frac{x}{n}$$

**Note -** We can add ln to the function because we derive it and trying to find maxima (which is the same point with or without ln)

2) P - pass $\bar{P}$ - failed

S - studied $\bar{S}$ - not studied

$P(P)=0.6 \quad P(\bar{P})=0.4$

$P(S|P)=0.95 \quad P(S|\bar{P})=0.6$

$P(S)=P(S|P)\cdot P(P)+P(S|\bar{P})\cdot P(\bar{P})$

$=0.95\cdot 0.6+0.6\cdot 0.4=0.81 \qquad P(\bar{S})=0.19$

$P(S|P)=\frac{P(S\cap P)}{P(P)} \qquad 0.95=\frac{P(S\cap P)}{0.6} \qquad P(S\cap P)=0.57$

a) $P(P|S)=\frac{P(P\cap S)}{P(S)}=\frac{0.57}{0.81}=\frac{19}{27}=0.703$

$P(P)=P(P|S)\cdot P(S)+P(P|\bar{S})\cdot P(\bar{S})$

$0.6 \quad \frac{19}{27} \quad 0.81 \qquad 0.19$

b) $P(P|\bar{S})=\frac{3}{19}=0.157$

3)

| X | Y | C |
|---|---|---|
| 6 | 6 | 0 |
| 6 | 6 | 0 |
| 6 | 6 | 0 |
| 6 | 6 | 0 |
| 6 | 6 | 0 |
| 3 | 6 | 0 |
| 3 | 6 | 1 |
| 6 | 3 | 1 |
| 3 | 3 | 1 |
| 6 | 3 | 1 |
| 6 | 3 | 1 |
| 6 | 3 | 1 |
| 6 | 3 | 1 |
| 3 | 3 | 1 |
| 3 | 6 | 1 |
| 6 | 6 | 1 |
| 6 | 6 | 1 |
| 6 | 6 | 1 |
| 6 | 6 | 1 |
| 3 | 6 | 1 |
| 3 | 3 | 1 |
| 3 | 3 | 1 |

a)

$P(X=3, Y=3 \mid C=0) = 0$ ✓

$\overset{?}{=} P(X=3 \mid C=0) \cdot P(Y=3 \mid C=0) = 0$ (0)

$P(X=3, Y=3 \mid C=1) = \frac{4}{14} = \frac{2}{7}$ ✓

$\overset{?}{=} P(X=3 \mid C=1) \cdot P(Y=3 \mid C=1) = \frac{2}{7}$ (7/14, 8/14)

$P(X=3, Y=6 \mid C=0) = \frac{1}{6}$ ✓

$\overset{?}{=} P(X=3 \mid C=0) \cdot P(Y=6 \mid C=0) = \frac{1}{6}$ (1/6, 1)

$P(X=3, Y=6 \mid C=1) = \frac{3}{14}$ ✓

$\overset{?}{=} P(X=3 \mid C=1) \cdot P(Y=6 \mid C=1) = \frac{3}{14}$ (1/2, 3/7)

$P(X=6, Y=3 \mid C=0) = 0$ ✓

$\overset{?}{=} P(X=6 \mid C=0) \cdot P(Y=3 \mid C=0) = 0$ (0)

$P(X=6, Y=3 \mid C=1) = \frac{4}{14}$ ✓

$\overset{?}{=} P(X=6 \mid C=1) \cdot P(Y= \mid C=1) = \frac{4}{14}$ (1/2, 4/7)

$P(X=6, Y=6 \mid C=0) = \frac{5}{6}$ ✓

$\overset{?}{=} P(X=6 \mid C=0) \cdot P(Y=6 \mid C=0) = \frac{5}{6}$ (5/6, 1)

$P(X=6, Y=6 \mid C=1) = \frac{3}{14}$ ✓

$\overset{?}{=} P(X=6 \mid C=1) \cdot P(Y=6 \mid C=1) = \frac{3}{14}$ (7/14, 6/14)

$X \not\perp Y \mid C$

b) $\frac{3}{5} = \frac{12}{20} = P(X=6) \overset{?}{\neq} P(X=6 \mid Y=3) = \frac{P(X=6 \wedge Y=3)}{P(Y=3)} = \frac{\frac{4}{20}}{\frac{8}{20}} = \frac{4}{8} = \frac{1}{2}$

$\frac{3}{5} \neq \frac{1}{2}$ ✓

X and Y are not independent

c) ✓ d) ✓

Relevant values: (6)

$P(X=3, Y=3, C=1) = \frac{1}{5}$ $P(X=3, Y=6, C=0) = \frac{1}{20}$ $P(X=3, Y=6, C=1) = \frac{3}{20}$

$P(X=6, Y=3, C=1) = \frac{1}{5}$ $P(X=6, Y=6, C=0) = \frac{1}{4}$ $P(X=6, Y=6, C=1) = \frac{3}{20}$

4) This question is a binomial distribution

$B(7, 0.75)$

a) $P(x=2) = \binom{7}{2} \cdot 0.75^{2} \cdot 0.25^{5} = \frac{189}{16384} = 0.01153$

b) $P(x \geq 4) = P(x=4) + P(x=5) + P(x=6) + P(x=7)$

$= \frac{2835}{16384} \quad \frac{5103}{16384} \quad \frac{5103}{16384} \quad \frac{2187}{16384}$

$= \frac{3807}{4096} = 0.9294$

c) We expecting the average be the mean of binomial distribution

Which is $n \cdot p$ $\quad 7 \cdot 0.75 = 5.25$