Intro to AI and ML

Matrix Project

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Geometry Question-

A circle passes through the points $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$. If its centre lies on the line

$$\begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} + 3 = 0 \tag{1}$$

find its radius.

Solution-

Assume centre of required circle is given by $\mathbf{c}$ and radius r. Then equation of circle will be:

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} = r^2 - \mathbf{c}^T\mathbf{c} \tag{2}$$

Let $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ be the given points. Substituting the two points $\mathbf{A}$ and $\mathbf{B}$ in the equation of circle:

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2\mathbf{c}^T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = r^2 - \mathbf{c}^T\mathbf{c} \tag{3}$$

and

$$\begin{pmatrix} 4 & 5 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \end{pmatrix} - 2\mathbf{c}^T\begin{pmatrix} 4 \\ 5 \end{pmatrix} = r^2 - \mathbf{c}^T\mathbf{c} \tag{4}$$

Solution(contd)-

On taking difference of above two equations ((4)-(3)) we get:

$$41 - 13 - 2\mathbf{c}^T \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 0 \tag{5}$$

or

$$2\mathbf{c}^T \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 28 \tag{6}$$

or

$$\mathbf{c}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 7 \tag{7}$$

on taking transpose both sides

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{c} = 7 \tag{8}$$

Solution(contd)-

It is also given that centre lies on line:

$$\begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} = -3 \tag{9}$$

Using equations (8) and (9) we have the following linear eqn in two variables:

$$\begin{pmatrix} -1 & 4 \\ 1 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -3 \\ 7 \end{pmatrix} \tag{10}$$

or

$$\mathbf{c} = \begin{pmatrix} -1 & 4 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ 7 \end{pmatrix} \tag{11}$$

or

$$\mathbf{c} = \frac{1}{5} \begin{pmatrix} -1 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 7 \end{pmatrix} \tag{12}$$

Solution(contd)-

or

$$\mathbf{c} = \frac{1}{5}\begin{pmatrix} 31 \\ 4 \end{pmatrix} \tag{13}$$

or

$$\mathbf{c} = \begin{pmatrix} \frac{31}{5} \\ \frac{4}{5} \end{pmatrix} \tag{14}$$

Now radius can be found by calculating distance between **C** and **A**. Therefore,

$$radius = (\mathbf{c} - \mathbf{A})^T(\mathbf{c} - \mathbf{A}) \tag{15}$$

Substituting values of **c** and **A** we have:

$$radius = \begin{pmatrix} \frac{21}{5} & \frac{-11}{5} \end{pmatrix} \begin{pmatrix} \frac{21}{5} \\ \frac{-11}{5} \end{pmatrix} \tag{16}$$

Solution(contd)-

or

$$radius^2 = \frac{441}{25} + \frac{121}{25} \tag{17}$$

or

$$radius^2 = \frac{562}{25} \tag{18}$$

or

$$radius = 4.74 \tag{19}$$

Solution Figure-