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Exercise 2

We perform CTC prefix beam search with beam size equal to 2.
For each prefix l, we track:

- p_b(l): probability of paths ending in the blank symbol "-"
- p_nb(l): probability of paths ending in a non-blank symbol
- Total probability:
  p(l) = p_b(l) + p_nb(l)

Given model output probabilities

Time	-	a	b
1	0.3	0.6	0.1
2	0.2	0.4	0.4
3	0.5	0.2	0.3

CTC prefix update rules

For each prefix l:

1. Blank extension (using -)

p_b′(l) += p(-) · (p_b(l) + p_nb(l))

2. Character extension with character c

- If c is the last character of l:
  - Repeat without blank:
    p_nb′(l) += p(c) · p_nb(l)
  - Add the same character only if the previous path ended with blank (-):
    p_nb′(l + c) += p(c) · p_b(l)
- If c is different from the last character of l:

p_nb′(l + c) += p(c) · (p_b(l) + p_nb(l))

Frame 1

Initialization:
p_b("") = 1, p_nb("") = 0

After time step t1:

- "" : p_b = 0.3, p_nb = 0 → p = 0.3
- "a": p_b = 0, p_nb = 0.6 → p = 0.6
- "b": p_b = 0, p_nb = 0.1 → p = 0.1

Keep top 2 prefixes:
"a" (0.6), "" (0.3)

Frame 2 (expanding "a" and "")

From "" (total probability 0.3):

- "" (blank -): p_b = 0.3 × 0.2 = 0.06
- "a": p_nb = 0.3 × 0.4 = 0.12
- "b": p_nb = 0.3 × 0.4 = 0.12

From "a" with p_b = 0, p_nb = 0.6:

- "a" (blank -): p_b += 0.6 × 0.2 = 0.12
- "a" (repeat a): p_nb += 0.6 × 0.4 = 0.24
- "ab": p_nb += 0.6 × 0.4 = 0.24

Totals after Frame 2:

- "a": p_b = 0.12, p_nb = 0.36 → p = 0.48
- "ab": p = 0.24
- "b": p = 0.12
- "": p = 0.06

Keep top 2 prefixes:
"a" (0.48), "ab" (0.24)

Frame 3 (expanding "a" and "ab")

From "a" with p_b = 0.12, p_nb = 0.36:

- "a" (blank -): p_b += 0.48 × 0.5 = 0.24
- "a" (repeat a): p_nb += 0.36 × 0.2 = 0.072
- "aa": p_nb += 0.12 × 0.2 = 0.024
- "ab": p_nb += 0.48 × 0.3 = 0.144

From "ab" with p_b = 0, p_nb = 0.24:

- "ab" (blank -): p_b += 0.24 × 0.5 = 0.12
- "ab" (repeat b): p_nb += 0.24 × 0.3 = 0.072
- "aba": p_nb += 0.24 × 0.2 = 0.048

Totals after Frame 3:

- "ab": p_b = 0.12, p_nb = 0.216 → p = 0.336
- "a": p_b = 0.24, p_nb = 0.072 → p = 0.312
- "aba": p = 0.048
- "aa": p = 0.024

Keep top 2 prefixes:
"ab" (0.336), "a" (0.312)

The most probable decoding hypothesis after three time frames is: "ab", with probability 0.336