

# Simulation results

General model:

$$\mu(t|Z) = f(t, \beta_0^\top Z) g(\gamma_0^\top Z),$$

where for fixed  $x \in \mathbb{R}$ ,  $f(\cdot, x)$  is an unspecified density function on  $[0, \tau]$ , and  $g(x)$  is unknown but monotone in  $x$ . We assume  $\|\beta_0\| = \|\gamma_0\| = 1$ .

## Simulation settings used in the paper:

- $Z$  is generated from a multivariate truncated normal distribution satisfying  $Z \sim N_2(0, I_2)$  and  $\|Z\| \leq 1$ .
- Censoring time is an exponential distribution with mean  $10 \cdot (1 + |z_1|)$ .
- Recurrent event times are generated from Poisson process with rate functions:

**M1:**  $\mu(t|Z) = \mu_0(t) \exp(\gamma_0^\top Z)$

- $\mu_0(t) = \frac{2}{1+t}$ .
- $\beta_0 = (\beta_1, \beta_2) = (0, 0)$ ,  $\gamma_0 = (\gamma_1, \gamma_2) = (0.28, 0.96)$ .

**M2:**  $\mu(t|Z) = \mu_0(t) + \alpha_0^\top Z$

- $\mu_0(t) = e^{0.1t}$ .
- $\beta_0 = \gamma_0 = \alpha_0 = (0.6, 0.8)$ .

**M3:**  $\mu(t|Z) = \mu_0\{t \exp(\alpha_0^\top Z)\}$

- $\mu_0(t) = e^{-t}$ .
- $\beta_0 = \gamma_0 = \alpha_0 = (0.6, 0.8)$ .

**M4:**  $\mu(t|Z) = \mu_0\{t, \exp(\beta_0^\top Z)\} \exp(\gamma_0^\top Z)$

- $\mu_0(t, x) = \frac{t \cdot (1-t)^{1+x}}{B(2, 1+x)}$
- $\beta_0 = (0.6, 0.8)$ ,  $\gamma_0 = (0.28, 0.96)$

- Set  $\tau = 10$  for **M1**, **M2**, **M3** and  $\tau = 1$  for **M4**.

- **M1-ind** solves  $\gamma_0$  under shape-independence.

Table 1: Point estimator (PE) and empirical standard error (ESE) for **M1** to **M4**. Sample size is  $n = 200$ , with 1000 replications. **M1-ind** assumes  $\beta_0 = (0, 0)$  and shape-independence.

	<b>M1</b>		<b>M1-ind</b>		<b>M2</b>		<b>M3</b>		<b>M4</b>	
	PE	ESE	PE	ESE	PE	ESE	PE	ESE	PE	ESE
$\beta_1$	0.187	0.655			-0.583	0.208	-0.584	0.170	-0.578	0.274
$\beta_2$	0.215	0.701			-0.788	0.206	-0.791	0.189	-0.774	0.267
$\gamma_1$	0.278	0.057	0.277	0.051	0.592	0.062	-0.595	0.119	0.276	0.090
$\gamma_2$	0.959	0.016	0.959	0.014	0.801	0.055	-0.787	0.114	0.957	0.025