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Letter from the Editor

It was exactly two Hilaries ago that I was at my desk in Jericho perusing the feedback for an essay I had written. It was about successful language revitalisation programmes and I had spent the holidays reading extensively about language policies, language loss, and linguistic diversity. From the cold expanses of Siberia to the dense rainforests of the Amazon, I had gone on intellectual journeys that led me to reflect not only about the topic at hand, but also about wider issues such as globalisation, climate change, and human rights. I spent days constructing arguments in favour of language revitalisation movements and nights commiserating with the Evenki, the Kayapó and other indigenous peoples fighting to keep their languages alive. As cliché as it might sound, I poured my heart and soul into this essay and, in the end, it was simply reduced to a number. Steeping in the incipient anticlimax, I wondered about other similar works at Catz that, with the exception of the markers and the author, would never see the light of day. That's when the idea to start a student-run, inter-disciplinary journal took root in my head. A day later, I sent out an email to the MCR outlining my ideas and calling for volunteers. It took all of two minutes for the first response, and by the end of that day, my blown-up inbox was a sight to behold! Now, two years later, I am incredibly proud to present to you the inaugural volume of *St Catherine's Academic Review (SCAR)*.

As a large Oxford college with a diverse student body, everyday life at Catz is replete with intellectual conversations, innovative ideas, and critical opinions. I envisioned SCAR as a window into this world preserved in print—the abstract reflected in the concrete. The abstract is encompassed in the work of our authors and editors. The articles and book reviews cover a broad range of topics from a number of disciplines. From nearest-neighbour regressions and cognitive impairment to refugee education in Lebanon and colonial photography, the academic diversity and intellectual curiosity at Catz are on prominent display in this volume. The concrete is embodied in our design, which draws inspiration from the balance of the old and the new that our young Oxford college has strived to achieve. Our logo, designed by Sonja Witte, is a minimalist representation of the Catherine wheel inspired by the college crest. The cover image depicting the undergraduate accommodation as viewed from the Old Quad was taken by Maximilian Langefeld. Titled *Adenine (Old Quad)*, it pays homage to Arne Jacobsen's iconic architecture as one of the bases of our collective college DNA. In an attempt to blend form and function while maintaining our commitment to the synthesis of the traditional and the modern, Karl Welzel and I picked out two humanist, open-type fonts. For the body text we use *EB Garamond*, a modern revival of a renaissance letterform. It makes for easy reading and beautifully typeset math, while also reflecting the 'low, horizontal expression and the serial repetition of identical modules' of Arne Jacobsen's architecture. For titles and headings, we use Alegreya Sans, a modern take on the traditional calligraphic letter that lends a dynamic contrast to the body text. Overall, I was humbled as I watched the abstract turn concrete, and I hope that you find this volume to be a faithful vignette of the intellectual environment cultivated at College.

Between conception and fruition came two years of hard work by a number of people without whom SCAR would not have succeeded. Owing to time and space constraints (and strict orders from Production!) I mostly list names here, but know that I am acutely aware of and immensely grateful for every individual contribution made to SCAR. That being said, I would like to start by thanking our founding members who are no longer with SCAR—Emma Hedley, Parth Patel, Sonja Witte, Anthony Rajecki-Doyle, Sarah Marshall, Ananya Sharma, Lucy Bartel, Noah Wescombe, Ella Liang, Rafael Hunt-Stokes, Roxanna Abhari, Yuwei Zheng, Louis Finegan, Xiaofan Wu, Katie Prosser, Romana Meereis, and the JCR crew (Eira Murphy, Alexander Pollard, Rachael Rajah & Irene Zhang). Your contributions in the early days laid the groundwork and helped shape the future of SCAR as an institution. I am grateful for the support given to us by Ashok Handa (Tutor for Graduates), Cressida Chappell (Academic Registrar), and the members of our SCR advisory board—Orestis Adamidis, Joanna Bullivant, Tommaso Pizzari, Mackenzie Graham, Kia Nobre, Gaia Scerif, and Kersti Börjars. Knowing that senior members of Catz encouraged and supported our endeavour was instrumental in boosting morale, especially when times were tough. There are also people outwith Catz that I must thank—Malika Ihle from Reproducible Research

Oxford, Cathy Scutt from the Bodleian Libraries, Hamish Chalmers (my MSc supervisor!) from the Department of Education, Alexandra Diouk from St Antony's International Review, Ye-Ye Xu from St Anne's Academic Review, and Nathalie Gontier from the Faculty of Science, University of Lisbon. Thank you for sharing your knowledge, expertise, and advice on navigating the world of academic publishing.

Turning my attention to everyone that helped get this show on the road, I'd like to thank our Executive Committee—John Kainer, Benjamin Wagenvoort, Jake Reeve, Felice Wallner, and Michael Hinz. I'm pleased that the MCR has such dynamic, motivated, and amazing people. John, thank you for keeping the ball rolling and the wheels turning. The world would burn without you. I am immensely grateful and honoured to have such an outstanding editorial board. Francesco Feriozzi, Abelardo De Anda Casas, Valentina Semenova, Alexandra Hertlein, Samantha Morito, Annika Theodoulou, Arunima Cheruvathoor, and everyone who worked tirelessly and meticulously editing, finding peer reviewers, copy-editing, and proofreading, I am indebted to you for your effort, endurance, and enthusiasm. Here, I must italicize, bold, and underline the gargantuan effort put in by Samantha Morito to find peer-reviewers. Your emails made all the difference! You are kind, intelligent, and sincere, which shines through in everything you do. Thank you for going above and beyond what was expected and I'm sure everyone on the editorial board will agree with me when I say that your effort carried this volume through to production.

That brings me to my production team—Karl Welzel and Maximilian Langefeld, this volume would not be the same without your painstaking attention to detail. Max, your work on the cover and design is nothing less than stellar. Thank you for making sense of my vague, adjective-filled ramblings on the overall vision for the design and then bringing them to life. You have a keen eye and a good ear. Karl, words cannot express my gratitude for your absolutely incredible work on the layout and template, and for creating an entire LaTeX article class for SCAR! I was left utterly speechless when you unveiled the template and all of its functionalities. I don't think I've ever met anyone as *klar*, *deutlich*, and *ordentlich* as you.

On behalf of the board, I would like to extend a special thank you to the MCR for their continued support and generosity, our peer reviewers who have dedicated countless hours to carefully evaluating and providing feedback on each submission, and our authors for entrusting us to share their work and ideas. We are pleased to have received submissions which explore complex issues, challenge assumptions, and promote interdisciplinary dialogue. We hope that their contributions will inspire and engage you, our readers, as they did us.

Finally, I would be remiss if I didn't thank my friends—Abelardo, Francesco, Karl, and Sam. Thank you for embarking on this adventure with me! Words cannot express how grateful I am to have had friends by my side on this long, arduous, and rewarding journey. To those whom I have forgotten to thank, my sincerest apologies. As is evident from these acknowledgements, it takes a village to start a journal. I have had the good fortune of being part of such a great network of kind, insightful, and intelligent people who have helped bring this first volume to life.

Sydelle de Souza
Founding Editor-in-Chief
Edinburgh, February 2023

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Part I.

Social Sciences

Acknowledgements

Being part of the first issue of SCAR has been a great privilege. Finalising this issue was also a great challenge—a challenge that took an even greater team to overcome. I want to sincerely thank our editor-in-chief, Sydelle, for trusting me with the responsibility of bringing her vision of the journal into the Social Sciences section. Syd: starting a new project with you is always an adventure—thank you for letting me be part of this one. What started as a dream is now a reality. Thank you for your passion, your unwavering enthusiasm, and your leadership.

Thank you to Noah Wescombe for his work as the previous Social Sciences Section Chief. Thank you to the Social Sciences team: Jake, Michael, Yuwei, Eilia; and a very special thank you to Samantha Morito, whose efforts were fundamental in keeping the section afloat. Sam: publishing the issue would not have been possible without your work. Thank you. One amazing thing about SCAR is that when all hands on deck were needed, help was offered in spades. Thank you to Francesco, Emma, Karl, Valentina, Jake, Maximilian, John, Alexandra, and the rest of the team. Thank you to all the past members of SCAR who helped diligently right when we were just starting the project.

Thank you as well to our reviewers for their engagement, keen interest, and expertise. Finally, and at the centre of it all, I want to thank our authors for trusting us with their work and ideas. SCAR was created precisely for people like you. Going forward, I would like SCAR to continue being a platform to foster the discussion of novel and insightful research from young academics.

I am proud of the work we are presenting in the Social Sciences section. These articles are characterised by the urgency of the topics they cover and the rigour of the methods employed in analysing them. Diversity, interdisciplinarity, and academic excellence are values that are fundamental to St. Catherine's College—a college that I am proud to be an alumnus of. These values are also represented in the articles comprising this section. I hope our readers enjoy them.

Abelardo De Anda Casas
Social Sciences Section Chief

Extended Abstract of a Realist Review that Seeks to Understand the Role of the Paramedic in Primary Care

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Background: Recent recommendations to improve UK NHS workforce capacities have led to a major push to increase the number of paramedics recruited into primary care. However, gaps exist in the evidence base regarding how and why these changes would work, for whom, in what context and to what extent. **Methods:** A realist approach was used to search electronic databases. Included documents were from the UK, Australia, Canada and the Americas—countries within which the paramedic role within primary care is well established. **Results:** 205 documents were included in the review, from which data were extracted to produce a programme theory. The results outline that paramedics are more likely to be effective in contributing to primary care workforces when they are supported to expand their existing role through formal education and clinical supervision. We also found that unless paramedics were fully integrated into primary care services, they did not experience the socialisation needed to build trusting relationships with patients or physicians. **Conclusions:** This review is the first to offer insight into understanding the impact paramedics may have on the international primary care workforce and shaping how they might be optimally deployed.

Keywords: Extended roles, allied health personnel, paramedic, primary health care, realist review, urgent care.

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1 Background

Building workforce capacity in health systems is a priority for many countries, and access to primary care has been a persistent problem for decades in the National Health Service (NHS). Current policy to address this issue has focused on increasing the number and range of clinicians working in primary care. As a result, more and more paramedics have been employed to work in primary care, a transition also mirrored within Australia, Canada and the United States of America (USA).

Paramedics within the United Kingdom (UK) are traditionally associated with the provision of emergency care within an Emergency Medical Service (EMS), responding to life-threatening emergencies through the 999-call system. However, a combination of a reduced amount of emergency calls (8% of 999 calls are for life-threatening illnesses or injuries (NHS England, 2014) and a sociocultural dependence on EMS (Wankhade, 2010) has led to the evolution of the paramedic role. As well as advanced life support, paramedics now need to be skilled in managing long-term conditions, acute presentations of mental ill-health, social-care assessments, and a range of urgent care presentations (National Institute for Health and Care Excellence, 2018a, 2018b). For the UK, this expanded role for paramedics to focus on urgent care has coincided with a move to degree-level pre-registration programmes (Health and Care Professions Council, 2018), and a career framework for paramedics to progress

in specialist practice in urgent or critical care, before moving onto more generalist advanced roles through postgraduate study (College of Paramedics, 2015).

Recent recommendations to improve UK NHS workforce capacities have led to a major push to increase the number of paramedics recruited into primary care. We previously undertook a scoping review of evidence published since 2005 (Eaton et al., 2020), which outlined that paramedics can safely apply their extended skills to assess and treat patients in primary care, but there were conflicts in relation to job titles, roles and responsibilities. We also found a lack of standardisation and complexity of the role of paramedics in primary care and that paramedics working in primary care are most helpfully conceptualised as a complex intervention.

Understanding complex interventions requires a clear theoretical model outlining the contributing components and how these work together to produce outcomes (Craig et al., 2008), which are context-sensitive. The factors that underpin how paramedics work well (or not) in primary care are unclear and likely to depend on a range of different contexts. To understand the ways in which paramedics impact (or not) the primary care workforce, we conducted a realist review.

2 Methods

This realist review builds on the aforementioned scoping review (Eaton et al., 2020) to offer an in-depth understanding of how paramedics might work in practice, for whom, in what circumstance and how to optimize the contribution of paramedics to primary care. A realist approach aims to provide causal explanations through the generation and articulation of contexts, mechanisms, and outcomes. Realist reviews also seek to relate substantive theory to the findings of the review, in order to make sense of the complex intervention.

Our search of electronic databases (Cochrane Database of Systematic Reviews [29/01/2021], MEDLINE (OvidSP) [2002–29/01/2021], PsycINFO (OvidSP) [2002–29/01/2021], Embase (OvidSP) [2002–29/01/2021], CINAHL (EBSCOHost) [2002–29/01/2021], NHS EED and DARE via CRDWeb (https://www.crd.york.ac.uk/CRDWeb/) [01/01/2002–29/01/2021], ERIC (Pro-Quest), Joanna Briggs Institute (https://jbi.global/), EBP (https://jbi.global/ebp) and OpenGrey (http://www.opengrey.eu/)) was supplemented with Google and citation checking to locate grey literature including news items and workforce reports. Included documents were from the UK, Australia, Canada and the Americas—countries within which the paramedic role within primary care is well established. Our review is reported following the RAMSES publication standards for realist synthesis (Wong et al., 2013).

3 Results

Our searches resulted in 205 pieces of literature from Australia, Canada, the USA and the UK. From these documents, data were extracted to produce context-mechanism-outcome configurations (CMOCs) within a final programme theory. Our engagement and incorporation of substantive theory to develop our CMOCs followed an abductive process to elaborate on the proposed mechanisms and continue the process of refinement until the programme theory became more nuanced. We drew on theories of professional role boundaries (Lamont & Molnár, 2002), professional identity (Freidson, 2001), and liminal states (Meyer & Land, 2003) to develop our final programme theory.

Our results outline that paramedics are more likely to be effective in contributing to primary care workforces when they are supported to expand their existing role through formal education and clinical supervision. We also found that unless paramedics were fully integrated into primary care services, they did not experience the socialisation needed to build trusting relationships with patients or physicians. Indeed, for patients to accept paramedics in primary care, their role and its implications for patients' care should be outlined by a trusted source.

We have provided a narrative overview of three key abstract categories that were developed from these documents, and combined with substantive theories, to produce a programme theory about how paramedics work in primary care roles, outlined below:

3.1 Expectations of Paramedics Working in Primary Care

Understanding the expectations of how paramedics may contribute and work within primary care was viewed through different perspectives:

Patient Perspectives Patients may view the role of the paramedic in primary care favourably after being informed of it by a trusted source. Uncertainty exists when the role is not made clear to patients or their expectation is not met if they attend an appointment with a paramedic when they believed they were seeing their usual GP.

GP Perspectives Whilst there was much positivity when considering the paramedic in primary care, in some reviewed literature, GPs saw paramedics as offering assessment-only roles (i.e. to make a diagnosis but not treat the patient). Deployment of paramedics in such a way was unlikely to free up GP time and often led to unintended consequences, such as patient frustration in the unnecessary duplication of consultations.

Paramedic Perspectives Paramedics perceive themselves as generalist clinicians who, by virtue of their work within emergency medical services, need to respond to all types of patients, across all ages, with any presenting complaint. Due to their generalist nature, paramedics would seek opportunities to work in primary care, believing their capabilities would fit well within this workforce.

Contribution to Primary Care Teams The idea that paramedics were pluripotential (i.e. able to do a range of tasks) was considered a useful addition for primary care teams. However, where the skills and competencies of the paramedic duplicated existing services (such as when urgent assessment clinics were already being run by another discipline, such as Nurses), paramedics were not considered to be a useful addition to the team.

3.2 Transition from EMS into Primary Care Roles

Our research found that paramedics are more likely to be effective in contributing to primary care workforces when they have significant experience as a paramedic and are supported to expand their existing role through formal education and clinical supervision.

Education The clinical gaps in paramedic knowledge that need to be filled for a successful transition to primary care centred around biochemistry (for the understanding and interpretation of blood tests), pharmacotherapy (to support independent prescribing for long-term conditions or complex patient groups), and some technical skills such as wound care, urinalysis, and imaging.

Supervision Clinical supervision enabled paramedics to feel supported as they adjusted their skill set to a new clinical setting and gave them confidence and satisfaction in their new role. Supervision also enabled GPs to build trusting relationships with the paramedics. Where clinical supervision was not provided or where there were difficulties in the supervisory relationship, paramedics reported feelings of isolation and lower satisfaction with the work in their role, opting to return to EMS employment.

Experience Paramedics, employers and policymakers emphasised the need for paramedics to have had significant experience within the ambulance service prior to working in primary care. This role consolidation was considered crucial for the successful transition and development of clinical capabilities.

3.3 Role and Responsibilities

Unless paramedics are fully integrated into primary care services, they do not experience the socialisation needed to build trusting relationships within the team to work to the best of their capabilities.

Working in a Team Integration into the primary care team is crucial to avoid role duplication. These are less likely to occur when the professional role boundaries of the paramedic in primary care do not overlap with existing healthcare professionals. However, where role boundaries became blurred or where the paramedic was viewed as a jack-of-all-trades, resistance could occur from other healthcare professionals due to a lack of confidence in the capabilities of the paramedic or feelings of threat in terms of their own job security.

Interpersonal Skills The ability of paramedics to build rapport and trusting relationships in a short amount of time (as required during emergencies) was considered an important component for replication in primary care. Patients were more satisfied when attended by paramedics with strong interpersonal skills and enthusiasm, citing their ability to connect to these healthcare professionals as a key marker of the success of their work in primary care.

This is the first published systematic synthesis of the literature using a realist lens to explore how this role can be implemented optimally. Based on this realist review, the employment and integration of paramedics into primary care should consider the framework to support implementation outlined in Figure 3.1 below.

4 Conclusions

Our final programme theory shows that paramedics are more likely to be effective in contributing to primary care workforces when supported to develop their knowledge through formal education (such as a postgraduate degree) combined with clinical supervision within the primary care setting. This also builds trust between the paramedic-GP and helps the paramedic to find their role within the workforce without threatening the contributions of other professions. Paramedics who are trusted to practice at their full potential are more satisfied working in primary care, and this may contribute to the enthusiasm perceived by patients in their role. Paramedics with strong interpersonal skills are highly rated by patients, and the development of a trusting relationship between patient and paramedic is paramount in meeting patient expectations, but also acceptance of the role. For patients to accept paramedics in primary care, the role and its implications for their care should be outlined by a trusted source, such as the primary care clinic or surgery. When this is done, it engenders support for these new roles.

Understanding of the deployment of paramedics into primary care roles was also gained from the literature. Paramedics were able to integrate well within primary care and EMS when they worked in a rotational role. This was attractive from a personal, professional identification point of view, as well as by EMS, who otherwise would risk losing their most experienced and highly educated staff. Such a peripatetic nature may not enable paramedics working in this way to be embedded or socialised enough in primary care or socialised enough to build trusting relationships with patients or GPs. However, paramedics employed by EMS providing primary care services in remote settings were able to address healthcare access gaps and were embedded within local communities accessing these services.

Our realist review highlights the complexity surrounding the introduction of paramedics into primary care roles. As well as offering an insight into understanding the paramedic professional identity, we also discuss the

Figure 3.1: A framework to support the implementation of paramedics in primary care (Eaton et al., 2021).

range of expectations this professional group will face in the transition to primary care. These expectations come from patients, General Practitioners (Family Physicians) and paramedics themselves. This review is the first to offer insight into understanding the impact paramedics may have on the international primary care workforce and shaping how they might be optimally deployed. Our realist review has been published open access and is available at:

> Eaton, G., Wong, G., Tierney, S., Roberts, N., Williams, V., & Mahtani, K. R. (2021). Understanding the role of the paramedic in primary care: a realist review. *BMC Medicine*, *19*(1), 145. https://doi.org/10.1186/s12916-021-02019-z

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Generations in Crisis: Unravelling Syrian Refugee Education in Lebanon

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With four distinct refugee groups and a crumbling economic system, Lebanon is under extreme pressure. As of 2019, 58% of refugee children in Lebanon were out of school, and 48% did not have access to learning opportunities. Despite the work of the Lebanese government, multilateral agencies, and NGOs to aid the crisis, obstacles that prevent refugee children from accessing educational opportunities persist daily. This paper utilises a systems analysis to understand Lebanon's refugee education crisis at the individual, non-state, and governmental levels in order to identify potential solutions. Its methodology includes an extensive literature review and eighteen engagements with academics, the UNHCR, NGO heads, and psychologists: all working across realms within the sphere of refugee education in Lebanon. Moving forward, three key levers of change are identified as potential solutions to avoid a 'lost generation' of children without any education: language development, the need for accreditation, and formally expanding employment opportunities. Although the focus is on Syrian refugees in Lebanon, refugee children's lack of access to education is a global reality. The points of intervention identified through this research will enable the international community to take steps, across contexts, in resolving an issue for all of humanity.

Keywords: Syrian refugees, education access, iceberg analysis, 5R framework, stakeholder map, tech-based education, theory of change.

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1 Introduction

1.1 Displacement, Education & The Issue at Hand

Every day, children around the world face myriad obstacles that constrain their access to education. In Lebanon, the combined impediments of transportation, costs of school fees and books, discrimination from classmates and teachers, and pressure to financially provide for their families bar access to learning for approximately half of the 660,000 student-age Syrian refugees currently living there. Despite reform efforts, 58% of all refugee children in Lebanon are not attending school, and 48% do not have access to informal learning. Without urgent action, half a generation will move into the workforce undereducated, likely forcing them into a life of poverty and reliance on government aid (UNHCR, 2019b).

Missed school time in their home countries, challenges to school enrolment when displaced or resettled, and negative psychosocial impacts of trauma combine to create significant—and often persistent—academic, social, and emotional barriers which these children must surmount. Further exacerbating the education gap is the trauma of displacement, which can lead to adverse educational, physical, and mental health outcomes. Fleeing

war and violence, refugees often experience prolonged tensions, which can give rise to a toxic stress response in developing children that leads to a strong activation of their body's stress management system. This can interfere with their brain and other organ development and may cause lifelong cognitive impairment (Center on the Developing Child, 2016; Pritzker & Redford, 2016). To prevent such devastating outcomes, policymakers need to consider the role trauma and toxic stress play in educating refugees. In this context, access to quality learning opportunities has the power to prevent a generational learning gap and alleviate the stresses of displacement.

Given the deeper dynamics driving the systemic challenges, it is critical to accept that solution efforts will require long-term investment. However, this must be weighed against the psychosocial harm and lifelong opportunity constraints endured by each successive generation of children as a result of the current system. To identify intervention points and levers for change, this research focuses on these children and their urgent needs, seeking mechanisms to bring immediate change at the level of the child while recognising the potential long-term impact of improved education on the integration, stability, and security needed to address the root causes of the systemic challenges.

1.2 Conceptual Bases, Methodology & Analytical Frameworks

The value of systems thinking as a conceptual foundation to this research must be outlined. Systems thinking is a growing tool used by both researchers and practitioners across diverse disciplines (Hossain et al., 2020). This is also true for the authors of this paper, who come from a number of disciplines and focus their research on education—an interdisciplinary field in itself (Medeiros, 2015). This is further bolstered by the range of disciplines represented by the literature scoped in this research.

Across the literature, systems thinking is the 'primary conceptual framework as a school of thought, as a holistic approach to problem-solving and analysis'—a framework centred on studying patterns, interactions, and correlations, seeking to 'accommodate the management of complexity' in relation to a particular context (Junpho & Rosenkranz, 2018). In line with these goals, this paper has sought to further two pillars of systems thinking in its methodology. The first of these pillars is 'Recognising Interconnections', or 'the ability to identify key connections between parts of a system' (Arnold & Wade, 2015). The second pillar, 'Understanding System Structure', builds on the first in delineating greater structure around interconnections identified and, as will be elaborated across visual representations in this paper, requires a level of understanding of the feedback loops that exist as part of these interconnections—impacting systems behaviour as a whole (Arnold & Wade, 2015). Herein, these interconnections are seen to be, at a minimum, equally important to the components of the system itself (Monat & Gannon, 2015). Additionally, this is embedded in the belief that driving social change as part of systems thinking requires understanding and engaging stakeholders embedded into the dynamics of the system (Caulfield & Maj, 2001).

Throughout this research, several frameworks were leveraged to perform a robust analysis of the system in which our primary stakeholder is embedded. These models provide reference points when conducting an exhaustive investigation into the factors that influence educational outcomes for Syrian refugee children in Lebanon, directly and indirectly. These frameworks, as well as their utility and applicability to this research, are summarised in Table 1.1.

To further understand these interconnections, the authors spoke with both academics and practitioners, well-versed and active in the context studied, to understand their perceptions. This allowed us to consolidate, both within and external to the authors, the policy, practitioner, and research lenses applied to this work, especially given its vulnerable context. The recruitment of these individuals was through individual and institutional networks that eventually snowballed to further recruitment. These conversations were not recorded except in the form of notes during personal communication; all quotations in this text have been sought permission for and consented to. Although these individuals do not exhaustively represent all levels of the system, this was the best possible effort, given travel limitations of the pandemic and accessibility across networks. The intertwining of

Table 1.1: Summary of Analytical Frameworks.

Framework	Figure	Rationale	Application	Source
Stakeholder Map	Figure 2.1: Education ecosystem of the Syrian refugee child	Enables the identification of all stakeholders that have a direct or indirect impact on the primary stakeholder and establishes the nature and extent of the relationships between each. This ensures no strong or significant influence is left out while understanding the factors that perpetuate the crisis.	Stakeholders were identified under three categories: State actors (i.e. governmental institutions), Non-State Actors (i.e. private sector organisations), and the immediate ecosystem surrounding children, including parents, teachers and local communities.	Hester and Adams, 2013
Iceberg Analysis	Figure 4.1: Mapping the gaps within existing solution efforts	Enables an understanding of the problem beyond what is visible and easily perceived, as it pushes an engagement with underlying patterns, structures and mental models that cause, aggravate or exacerbate the problem at hand. Without such, solutions could merely address the symptoms without investigating root causes.	Analysing structures and mental models that could be causing the status quo to persist allowed the identification of deeply entrenched issues like lack of certainty around the return of refugees to their native land, the historical conflict between locals and refugees and the lack of political will to finance education for refugees given economic conditions in Lebanon.	The Iceberg Model by Goodman, 2002
Theory of Change	Figure 5.1: Proposed Theory of Change	Creates a better understanding of the mechanisms and resources involved for a proposed solution to have a real impact. This helps to ensure a feasible and sustainable intervention while challenging implicit assumptions.	This framework maps out the inputs, activities, outcomes, and impact of the proposed solutions. This helps show the impact of these three potential solutions and creates a preliminary implementation guide.	Valters, 2014
5R Framework	Figure 5.2: 5Rs framework for identifying and monitoring system interventions	Identifies the five dimensions of a given system: Results, Roles, Relationships, Rules and Resources. Assessing and monitoring the 5Rs provides a model for tracking interventions and strengthening them for improved effectiveness. Traditional monitoring systems may otherwise not account for tracking of relationships, rules and roles.	This framework helped identify stakeholder roles, relationships and rules in the system that would require monitoring. Shifts under the 5Rs would be necessary for the solutions proposed to work.	USAID, 2016

these inputs, along with an extensive literature review, concretises the reliability and rigour of the work presented below.

2 The Educational Challenge

2.1 Regional Crises and Lebanon's Burden

Lebanon has been 'the most generous country in [the] region' in its acceptance of refugees (A. Sultan-Khan, personal communication, 10 March 2020). However, because it is not a signatory to the 1951 UN Convention or 1967 Protocol relating to the Status of Refugees, refugees are not granted asylum, have no legal status, and face changing requirements to enter and stay in the country (O'Donnell & Newland, 2008). The contextual realities of the refugee experience are further complicated by the coexistence of the many religious and political affiliations that have sought asylum in the state and the Lebanese government's sectarian makeup.[1] With refugee inflows, the Lebanese government has established policies to protect Lebanese interests against refugee interests,—including greater securitisation and implementation of curfews—which acutely affect male refugees threatened by arrest and deportation. Refugees are also barred from most professions due to strict government policies, discussed in detail later in this text (O'Donnell & Newland, 2008).

Syrians are also required to either register as refugees with the UNHCR or pay for a work visa; the former forbids Syrians from working but allows benefit from international aid, while the latter is often cost prohibitive (Hamadeh, 2019).[2] The complexity of the situation 'coexists with politically polarised perceptions of the effects of the refugees on Lebanon', as is further reflected in instances of fighting and violence within the country (Chamas et al., 2020; Hamadeh, 2019). The ongoing uprising initially sparked in response to a proposed fuel tax represents the nation's response to the current political, social, and economic crises and illustrates the tensions herein. Activists are protesting corruption in government, lack of basic service provision, and the sectarianism that defines the Lebanese state (UNICEF, 2019). This policy environment has clear repercussions for refugees' capacity to integrate into Lebanese society economically and socially, and therefore, a significant impact on refugee children's access to educational opportunities. Drawing from her own experience working on the ground to support an intervention involving refugee education, Dr. Yasmine El Masri observes that current pressures have 'laid bare all the problems in the system'—a system that 'is collapsing' and one the Lebanese population is now revolting against (personal communication, 13 March 2020).

2.2 Refugee Children's Disrupted Education

Within this context, 'education has the power to make a real and lasting difference to young lives who have suffered through Syria's conflict' (UNHCR, 2016; UNICEF, 2013). Attempts to accommodate the influx of migrants have seen Lebanon open public schools to refugee children in 2012 and 2014, with registration fees covered through international donors' assistance. However, to enrol all school-aged refugees into the Lebanese system would require 'tripling existing capacity', the feasibility of which remains in question (Hamadeh, 2019).

This systemic shortfall necessitates a greater focus on understanding and improving non-formal learning environments (NFE), often provided by NGOs and multilateral organisations (Karam et al., 2017). Dr. Athar Sultan-Khan, political advisor to UNHCR, refers to this as 'burden sharing' amongst the international community (personal communication, 10 March 2020). However, the accreditation of non-formal programmes remains a key issue, often resulting in students without academic credentials even if they complete a programme (Ahmadzadeh et al., 2014).

[1] Lebanon is no stranger to acting as a refuge for displaced peoples, as can be traced with Armenian settlement in the state from 1915, the arrival of Palestinian refugees between 1948 and 1967, and Iraqi refugees thereafter.

[2] Moreover, failing to register with the government could risk detainment or prison time.

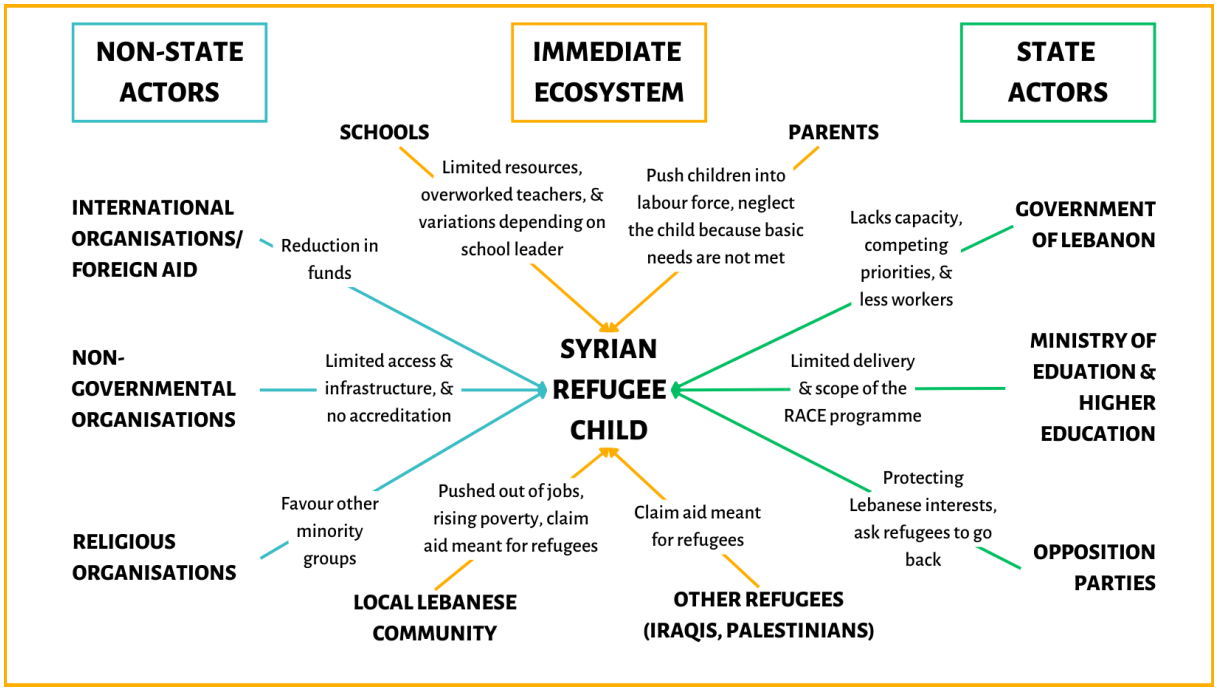

Figure 2.1: Education ecosystem of the Syrian refugee child.

The stakeholder map below places the Syrian refugee child at the centre of these multiple interconnected systems. An examination of the values, motivations, and needs of pertinent actors captures the nuanced impacts of state and non-state ecosystems on education, revealing competing expectations between refugee groups and Lebanese nationals, trade-offs between economic and political decisions taken by the government, and a severely resource-constrained environment in which front-line providers like teachers operate. Hence, it is useful to examine the various problems that sustain the current status quo through a systemic, rather than one-dimensional, lens.

2.3 COVID-19 Related School Closures Further Exacerbate Learning Losses

The impact of coronavirus has been near universal in the shutting down of educational institutions across the world. In the first year of the pandemic, 190+ countries implemented country-wide school closures, which affected more than 70% of the world's student population (UNESCO, 2020b). Even though the exact impact of these school closures is unknown, prolonged school shutdowns have kept children from disadvantaged families away from remote learning altogether. Additionally, unmet nutritional needs and an increase in child abuse have adversely affected children globally. As a result, governments around the world are rolling-out education response plans to mitigate the negative consequences of school closures (UNESCO, 2020b).

As part of the government's response, 1.2 million children in Lebanon have been out of school since the beginning of COVID-19, when all educational institutions shut down. Since the beginning of the pandemic until April 2021, the children in school have received a maximum of 11 weeks of education, with an even lower number for Syrian children (Save the Children, 2021). The impact of COVID-19-related school closures and other interruptions in other vital services have been disproportionately high on refugee children. For instance, the Lebanese Ministry of Education and Higher Education (MEHE) launched a distance learning program that relies on three transmission models: media, online platforms, and non-ICT methods (UNICEF, 2020).

However, refugee families have limited resources to support the infrastructure needs of online learning. Hence, lack of internet access, high cost of data, and in some cases, lack of devices limit Syrian children's online access to learning.

Additionally, the shutting down of nine unlicensed private schools by the Education Ministry during the summer of 2020 has left a further 5,000 Syrian students out of school (Human Rights Watch, 2021a). This is compounded by the disastrous August 4th explosion in Beirut in 2020, which has destroyed at least 163 public and private schools—disrupting learning for a minimum of 85,000 learners (UNESCO, 2020a)—comprising of Lebanese and refugee learners, the latter of whom remain most disenfranchised from the education system and physical access to school.

The international funding and response to combat such losses have been limited, with only 8.4% funded of the total USD $342 million required for the education sector, as per the COVID-19 Global Humanitarian Response Plan (UN OCHA FTS, 2020). Against the backdrop of such funding deficits, private foundations and non-state actors are developing programs to bring remote learning to refugee children in Lebanon. The Abdul Aziz Al Ghurair Refugee Education Fund, for instance, has partnered with Discovery Education to collaborate with local organisations in Lebanon to ensure that high-quality digital learning resources reach students in Grades 8 to 12 (Discovery Education, 2020).

2.4 Contextual Factors: Lebanon's Political Architecture, Crumbling Economy and Donor Funding

Obstacles to educating Syrian refugee children in Lebanon are rooted in complex political, social, cultural, and historical dynamics, extending beyond mere resource shortfalls. Systemically, the Lebanese government divides its political power amongst stakeholders from the predominant religious groups (Barnett, 2020).[3] Drawing from her extensive ethnographic work in the region, Dr. Dawn Chatty sums this up by enunciating that Lebanon 'is about its multiple ethno-religious communities and solidarity at the local level'—a system drawn from the French example of sectarianism during their imperial rule in Lebanon (personal communication, 15 March 2020).

It is globally acknowledged that countries hosting Syrian refugee children cannot provide for their education without significant external support. Lebanon has seen a disproportionate rise in the number of Syrian refugees enrolling in Lebanese schools, further stretching an already strained public education system. Between 2011-12 and 2017-18, there was a 7,000% increase in the number of non-Lebanese students enrolled in public schools. In 2017, more non-Lebanese students (213,358) were enrolled in Lebanese public schools than Lebanese students (209,409).

At the 'Supporting Syria and the Region Conference' in 2016, participants showed unanimous commitment to reducing the pressure on countries hosting Syrian refugees, especially focusing on access to education. Funding of at least USD $1.4 billion a year was promised to prevent a 'lost generation' of children (Supporting Syria and the Region Conference, 2016). At the subsequent conference in Brussels in 2017, the international community reaffirmed their commitment to support both the first- and second-shift enrolment costs as part of its new education plan—'Reaching All Children with Education (RACE) II'—at an estimated cost of €350 million (USD $410 million) per year (Council of the European Union, 2017). The Government of Lebanon, in its partnership paper at the 2018 conference, promised to work towards strengthening the existing governance system on the basis of sound performance measurement, cost efficiency, and transparency of financial and delivery data (Council of the European Union, 2018).

[3] Through unofficial convention, the prime minister is always a Sunni Muslim, the president is a Maronite Christian, and the speaker of parliament is a Shia Muslim. The Lebanese parliament contains 68 seats for the government and 60 for the opposition. Hezbollah is a Shiite Muslim political party and militant group founded during the Lebanese Civil War. Although deemed a terrorist organisation by the US and EU, the party holds thirteen seats in Lebanon's parliament.

Through RACE II, MEHE's goal was to enrol 250,000 Syrian children in 2018-2019—an increase of almost 40,000 students. However, in 2018-19 the number of non-Lebanese students instead dropped to 206,061. Disaggregated data reveals that, of these students, 153,286 were enrolled in exclusive second-shifts as part of RACE, while the remaining 52,775 were enrolled in first-shifts, against regulation (Lebanese Ministry of Education and Higher Education, 2019). The funds required to meet enrolment goals were estimated at €149 million (USD $174 million; European Union, 2018).[4] As of November, MEHE reported receiving only €100 million (USD $117 million) in donor funding, with the present funding gap estimated at USD $30,055,613 (Human Rights Watch, 2018).

Compounding these shortfalls, Lebanon's economy is in the midst of total collapse. Financial policy over-reliant on debt (supplied by central bank investments, 'which analysts have likened to a state-sponsored Ponzi scheme'), and a currency pegged to the US dollar, have made Lebanon 'the third most indebted state in the world' (Hubbard, 2020). The underlying troubles surfaced in late 2019 as protests erupted 'against decades of corruption and mismanagement by political elites who bled state coffers dry' (Noueihed & Khraiche, 2020). Since then, protests have intensified as the currency devalued by 60% and, for the first time, Lebanon defaulted on its foreign debt in March 2020. The combined impact of economic collapse, 'currency chaos', and the COVID-19 'pandemic lockdown [which] shuttered businesses and produced mass layoffs' has been calamitous for Lebanon's middle class, exacerbating tensions between Lebanese citizens and refugee groups in a context when almost all are in need of increased government support and international aid (Vohra, 2020). The above context contains and has a significant part in creating lived experiences of refugee children, distinct across refugee groups (i.e., Palestinian, Kurdish,[5] Iraqi,[6] Armenian,[7] etc.) living in Lebanon. With varied realities of inclusion and access to resources in education, further research necessarily needs to detail and delineate the distinct and nuanced layers of disenfranchisement that exist for these groups in Lebanon. For Syrian and Iraqi refugee students, scattered living arrangements undermine education access as transport, which most refugee families are unable to afford, is often required to reach schools (O'Donnell & Newland, 2008). Many refugees, particularly in women-headed households, are in severe poverty, earning less than $1 per day, a reality likely to drive child involvement in illegal labour (CARE International, 2018). For registered Syrian and Iraqi refugees, who, unlike Palestinians, are not recognised as refugees by Lebanon, it can take up to two years to register with UNHCR and start receiving aid. In addition, these displaced persons face discrimination and are subject to manipulation and abuse from employers. Dr. Chatty highlights a general sentiment that Syrians have 'outstayed their welcome' (personal communication, 15 March 2020).

2.5 Conceptualising Purpose: Inclusion, Assimilation, Integration, and Return

Current realities in Lebanon have clear repercussions for refugees' capacity to integrate into Lebanese society economically and socially. This complexity boils down to educating the vast numbers of refugee children in Lebanon—not only to enable national integration but also in the hope of working against the promulgation of a

[4] The requirement for funds is calculated on the previously agreed 'unit costs' of USD $600 per Syrian child enrolled in second shift classes, and USD $363 per Syrian child enrolled in first shift classes.

[5] Some Kurds were granted under-study identification cards which eased travel restrictions and allowed them to enrol in public schools, but they could not vote or be employed in the public sector, contributing to citizens' perceptions of their community.

[6] To work in Lebanon, Iraqi refugees must pay USD $2,000/year for an employment visa and have a Lebanese sponsor who takes legal responsibility for them. If arrested, they can face jail sentences of one to three months unless they choose to move back to Iraq; even when they have served their time, many are forcibly deported (Karam et al., 2017). These instances of refugee securitization are particularly high in impoverished areas and exacerbate poverty within refugee groups (D. Chatty, personal communication, 15 March 2020).

[7] Starting in 1930, Armenians set up their own private schools in Lebanon, which teach in Armenian and by 1948, Lebanon was home to over 50 Armenian schools. They can also attend colleges in Lebanon, and the top two universities both have a chair of Armenian studies and an Armenology major (Hamadeh, 2019).

'lost generation'—now applicable to the 'millions of Syrian children currently fighting for the basic human right of education' (Dryden-Peterson et al., 2019). This is in line with UNHCR's mandate that sees the provision of education to refugees as central to its mission and is echoed by several international organisations who believe 'education has the power to make a real and lasting difference to young lives who have suffered through Syria's conflict' (Dryden-Peterson et al., 2019).

Global pragmatism suggests that a 'future most relevant for refugees would be integration into a country of exile', with quality education presumed to flow from inclusion (Dryden-Peterson et al., 2019). Meanwhile, national models tend to centre between those that work towards no inclusion whatsoever, those that see inclusion as a pragmatic and temporary response to the situation at hand, and those that see inclusion as 'creating futures' for 'refugees in long-term exile' (Dryden-Peterson et al., 2019). As can be inferred by labour market policies that limit refugee employment and hence, do not seek to fully integrate Syrian refugees into the Lebanese workforce, the Lebanese context works on the basis of inclusion as a pragmatic response to the influx of Syrian refugees.[8]

2.6 Toxic Stress and Psycho-Emotional Well-Being

According to the Adverse Childhood Experiences study, there are ten major adverse childhood experiences (ACEs); the higher the number of ACEs experienced, the higher the likelihood of developing adverse health outcomes; a score of four or more correlates to a thirty-two-fold increase in behavioural problems, and a score of six or more is associated with a twenty-year decrease in life expectancy (Pritzker & Redford, 2016). All refugees are affected by the trauma of displacement, which gives them an ACE score of one. Forced displacement likely leads to economic hardship, poor mental health outcomes, and parental psychological distress, all reinforcing negative parenting techniques such as physical punishment (Cluver et al., 2018). These traumas are also associated with toxic stress, which affects the structure and function of the brain and can negatively impact students' academic performance (Hamadeh, 2019). This is exacerbated by the constant existential threat of deportation. Although 'resignation syndrome' has been found only amongst refugees in Sweden, the psychological impact of potential deportation remains detrimental, both physically and psychologically (Aviv, 2017).

Dr. Solfrid Raknes, who develops and implements psychosocial programmes across Lebanon, also notes the acute impact of the 'trauma of poverty', wherein many children live below the poverty line and are likely to have lost family members and, therefore, a support structure (S. Raknes, personal communication, 12 June 2020). The increasing levels of trauma these children face daily 'really impact their health and development'. Amidst the COVID-19 lockdown, this is exacerbated as families now feel 'imprisoned in their tents' (M. MacDonald, personal communication, 11 June 2020). Consequently, refugee children are unlikely to be learning in the way stakeholders assume. To make matters worse, there is a dearth of understanding of how this plays out for refugee populations in Lebanon. Dr. Ellie Ott, in her review of mental health and psychosocial interventions for refugee and asylum-seeking populations, found an insufficient understanding of how to support these children on a global scale, let alone in national contexts. Dr. Ott suggests this academic gap is a particular loss, given that her experience in the field has shown the incredible 'educational resilience' these children desire to persist with (E. Ott, personal communication, 11 March 2020); this sentiment is echoed at Jusoor Lebanon (M. MacDonald, personal communication, 11 June 2020).

Schools and educational environments are key to providing refugee children 'opportunities for connection' and hope. Dr. Chatty asserts that schools do 'more than providing education', particularly so for refugee children (personal communication, 15 March 2020; Ahmadzadeh et al., 2014). Skilling and education afford refugees a sense of belief 'in themselves, and this gives them stewardship', empowering them to continue to carve their own learning path. Furthermore, the structure of a school-like day provides security to children in an already uncertain

[8] For example, in 2014, the Ministry of Labour published a list of protected jobs for Lebanese citizens that could not be given to an individual of other nationalities unless no qualified Lebanese worker was available.

environment. The importance of 'creating a positive school culture' is also underlined in practice. Jusoor was the first organisation in Lebanon to hire a counsellor, Ms. MacDonald, whose strategy in building psychosocial well-being has centred on empowering teachers. Teacher training can 'help the teachers to help themselves' and so reduces dependence on external and potentially inconsistent modes of assistance (M. MacDonald, personal communication, 11 June 2020).[9] Empowering teachers with psychosocial knowledge is also prominent in Dr. Raknes's work, a lever she believes is key for 'systems changes' to enable impact on children's lives at scale.[10]

3 Understanding Existing Solutions

Within the complex education and policy system described above, solution efforts span both state and non-state actors. Although the literature suggests the latter is likely to provide quality learning to refugee students, most fail to formally enrol children in public schools, leaving students without qualifications to account for it (Ahmadzadeh et al., 2014).

3.1 Second Shift Schools

The recent implementation of the Reaching All Children with Education in Lebanon (RACE) strategy, an initiative from the Lebanese government stemming from UNICEF's Regional Response Plans (RRP), was aimed at benefiting an average of 413,000 vulnerable children in Lebanon. To work towards this goal, there has been an introduction of the second shift in Lebanese public schools dedicated to refugee children, lowering barriers to access but providing a separationist solution (N. Weaver, personal communication, 9 April 2020). Tensions remain as global actors continue to fund education reforms, and teachers on the ground continue to experience many challenges with implementation, with questions surrounding the contextualisation of curricula, for example. Another government-backed solution to this education crisis is introducing night-shift classes. These could benefit students in places where public schools do not have enough room to enrol refugees and enable children to work during the day while attending classes at night. However, this option must be sensitive to the psychosomatic needs of children to ensure their well-being, as the reality of their context often means having to simultaneously balance the needs of work, learning, self, and family.

Despite these well-intentioned reforms, Lebanon spends only 2% of their GDP on education, about a quarter of the OECD nations' average (Loo, 2017). It is worth noting that this has been part of creating a reality where over 60% of Lebanese children used to attend private schools, and since 2019, up to 120,000 of these children can no longer afford such schooling—putting added strain on a system already severely under-resourced (Human Rights Watch, 2021b). The consequent reality has been that schools are overcrowded, and teachers often do not show up; additionally, there are safety concerns as children must walk to and from school at night, often through dangerous areas, since their families cannot cover the cost of taking the bus. Furthermore, no policies account for refugee children beyond grade nine, which is particularly concerning given that only 2% of Syrian refugee children aged 14-18 are enrolled in school. According to Dr. al Zoubi, an Oxford Senior Departmental Lecturer who researches refugee livelihoods, this is worsened by the Education Ministry's policy requiring Syrians to register for national exams when most refugees cannot afford the financial burden nor brave the dangerous conditions necessary to return to Syria to obtain their school documents (S. Al Zoubi, personal communication, 20 March 2020). This is exacerbated by the policy requirement for rights to residency when

[9] Through regular class observations, a constant feedback loop is enabled to consistently adapt and cater to children's needs, with additional personalised support possible should need require it. Furthermore, activities to support psychosocial well-being are inculcated as integral to academic activity at Jusoor's three centres.

[10] In working to support refugees digitally, especially critical amidst the current pandemic, Dr. Raknes has also recently launched 'The Happy Helping Hand App'; the content of which is available in Arabic and amidst utilisation in class, its capacity to enable engagement with psychosocial realms has already been noted.

fewer than 16% of Syrian refugees have residency rights in Lebanon (Human Rights Watch, 2021c). Brian Lally, an Education Specialist with MultiAid Programs (MAPS) Lebanon, notes that in his experience, it has been 'difficult to move the conversation' as there isn't 'one aspect of this issue that is not (...) heavily politicised' (B. Lally, personal communication, 17 April 2020). This leaves NGOs who work in this space to buffer learning significantly in the dark with respect to the levels of meaningful learning occurring for refugee children in public school classrooms. Therefore, reform to the national education system is crucial to ensure all students can access accredited educational opportunities.

3.2 Instructional Language: A Fundamental Barrier

Because of a language's connection to culture, it is 'an essential component in enhancing the resilience of individuals, communities, and institutions' (British Council, 2016). Students educated in their home language are better equipped for academic success at all levels, and proficiency in specialised language is a prerequisite for entering middle- and upper-class professions such as medicine or business. Given the ongoing years of displacement, the question of language's role in refugee integration should also be of note. Facilitating a multilingual educational environment benefits both Syrian refugee and Lebanese students as being bilingual is associated with a better understanding of complex maths concepts, stronger cognitive thinking skills, use of logic, superior memory and decision-making skills (US Department of Education, n.d.).

The relationship between Syrian refugees and the Lebanese system presents an interesting challenge for accommodating instructional languages. Many Syrians see Lebanon as a temporary home and plan to go back to Syria when its political turmoil has settled (Chulov, 2018). Therefore, many students may not understand why they need to learn in English and French, the languages of instruction in Lebanon, and not in Arabic, the language used in Syrian schools (Loo, 2017). Although elementary schools in Lebanon teach in Arabic, several fundamental subjects—including maths, physics, and chemistry—must be taught in either English or French (Loo, 2017).[11] While speaking either language may help students access higher education internationally and is therefore important for social mobility, almost 60 per cent of school-age Syrian refugees are not even enrolled in school. Therefore, for impact at scale, it is arguably important to first prioritise refugee access to primary and secondary school education over the linguistic ability of the few able to study at the post-secondary level (Human Rights Watch, 2021c). Teaching only in English and French excludes Syrian students, affecting their ability to learn and create relationships with their fellow students. Furthermore, teaching in two languages adds more stress to already overworked teachers (US Department of Education, n.d.). The complexity that then exists herein and that the public education system needs to accommodate for is that English fluency, in particular, is often key for access to higher education globally.

Coordination among Non-Formal learning Environments (NFE) is essential to ensure students can access educational resources and teaching in their own language, as well as further opportunities for English-language acquisition. A key complexity remains in the decentralised decision-making structure in this sphere; issues like the language of instruction in NFEs are made by individual NGOs, which makes it more difficult to transition students from NFEs to public schools, continuing to leave students with no educational credentials (Karam et al., 2017).

3.3 Civil Society and Non-Formal Learning

The importance of the NGO network in providing learning opportunities to refugee children has been key; there has been an 'amazing response of solidarity at the civil society level' (D. Chatty, personal communication, 15 March 2020). For example, several NGOs work with the aim of creating full curriculums or acting as 'mobile

[11] Syria and Lebanon both use the Levantine dialect of Arabic. However, there are some regional differences in accent, vocabulary, and usage (Nassra, n.d.).

educators', moving through a number of targeted areas in an attempt to cover any 'educational gaps' for students (Hamadeh, 2019). As is true for a number of refugee populations globally, the UNHCR also provides 'access to basic education (...) by funding implementing partners (INGOs and NGOs)' (Karam et al., 2017). Projects carried out by international agencies include those by the International Rescue Committee, which 'teaches life skills, literacy, language courses, and business skills development' (Ahmadzadeh et al., 2014).

Multi-Aid Programs (MAPS) is one of the few actors within this space that was originally set up by a community of Syrian refugees. MAPS' educational arm includes nine educational centres as well as a vocational training program located in parts of the Bekaa Valley in Lebanon. The aforementioned confusions around curriculum are seen by MAPS to be key opportunities—since the centres are 'not shackled by the curriculum for certification', teachers are far freer to pursue elements of the 21st-century curriculum, providing both students and service providers herein the space to innovate in developing creativity and problem-solving skills across learning, albeit within a context of limited resources (B. Lally, personal communication, 17 April 2020).

Significantly, technology is an increasingly utilised tool for expanding and attempting to equalise access: innovative organisations deliver offline content, adapt multilingual digital curricula, and provide critical digital skilling.[12] Dr. Nayla Fahed, founder of Lebanese Alternative Learning, having noted that a key obstacle is 'getting the refugee into the formal system', has sought to create digital and multilingual versions of Lebanese curricula that are accessible to thousands of refugees across partners in the NGO network for free (N. Fahed, personal communication, 16 March 2020). Edutek, founded by Mariam Haidar, and DOT Lebanon, whose programs are managed by Walid Abu Saifan, are examples of organisations that have sought to curate unique solutions to accredit their programs, particularly in digital skilling, enabled by their work with organisations like Microsoft and Cisco. Nevertheless, resource and temporal constraints remain an obstacle to scaling this further into the system (W. A. Saifan, personal communication, 31 March 2020; M. Haidar, personal communication, 1 April 2020). As enunciated, the sector is really 'struggling trying to find a way with the resources we have available' (M. Haidar, personal communication, 1 April 2020).

Furthermore, given that most non-state efforts focus on primary education, a 'secondary school bottleneck' persists—a gap true for refugees globally (N. Weaver, personal communication, 9 April 2020). The World Bank notes that only 10% of Syrian refugees in Lebanon are in secondary school, while UNHCR reports that it is 5% (Karasapan & Shah, 2018)—evidently, actual numbers are uncertain and worryingly low. This limits not only refugee access to education, but also universities' ability to provide opportunities for higher education globally, as has been noted by Nina Weaver, who is the Director of Research & Partnerships for the University of Southern New Hampshire's refugee higher education programs. Amala Education, co-founded by Polly Akhurst, is working to bridge this with a competency-based high school curriculum, made available through a blended learning model launched in Jordan, as a step in expanding across the region and continuing work in Lebanon (P. Akhurst, personal communication, 21 April 2020). Their amalgamation of synchronous and asynchronous learning models developed has also meant smooth adaptation to a COVID learning environment, wherein remote learning is possible. Several NGOs interviewed noted that COVID has accelerated developments for their future visions—be it in the presentation, delivery or usability of content and digital programs offered to students.

In tandem with accessing secondary and high school learning, Mosaik works with students who want to access higher education by working with facilitators on the ground in Lebanon to train and develop students' digital literacy and English language capacity, which is key to enabling access to universities. Ben Webster, the founder of Mosaik, notes the recent goal announced by UNHCR to have 15% of refugees entering higher education by 2030 is a greater than five-fold increase from the current rate. The potential for efficacy here is increased by utilising open-source resources and involving refugees in the design of their programs (B. Webster, personal

[12] Other NGOs in this space include DOT Lebanon, the Lebanese arm of Canadian-based Digital Opportunity Trust, Edutek and Salam, which has been funded by Syrian expatriates since 2013 (Karam et al., 2017).

communication, 19 May 2020; UNHCR, 2019a).

Opportunities accessible to refugees are significantly impacted by intra-national contexts. Malaak works with Syrian refugees in a further deprived part of Lebanon on the Northern border with Syria where, during the school year, informal classes centre on supporting children through their lack of understanding of the national curriculum in schools. Most recently, six children of the total 500 classes that attend the Malaak centre passed the national secondary school exams and, therefore, are now fully integrated into the public school system (A. Rasamny, personal communication, 7 May 2020). Although such efforts are extremely commendable within this context, far more needs to be done to support the integration of a greater proportion of refugee children into the Lebanese school system.

3.4 Accreditation Amiss: The Missing Piece

Currently, tensions have resulted in many non-formal learning opportunities unable to ascertain nationally accredited opportunities for qualification. The state-sanctioned school curriculum should work to give refugee children qualifications for future careers. As Dr. Chatty states, the 'Lebanese school system should be extended (...) instead of providing a shortcut education.' Currently, the modified Lebanese diploma for refugee children does not translate into substantial returns in Lebanon or Syria. An alternative solution to a Lebanese diploma could be to offer the Arab League's alternative form of the Baccalaureate; this would enable a far greater proportion of refugee children (compared to the current 2%) to earn a high school degree and later pursue educational and professional opportunities across the Arab League (D. Chatty, personal communication, 15 March 2020). In the future, Amala hopes to acquire formal accreditation for their programs and, if achieved, would likely be an enabler for flexible and long-term refugee education globally. Nevertheless, the current reality remains wherein 'very few are able to make a transition to higher education, or even to attain a final certificate' (B. Lally, personal communication, 17 April 2020).

In addition to reforming the formal learning space, there needs to be accreditation of quality NFE programs, which can enable refugees to gain relevant job skills, enhance their psychosocial well-being, and give them the opportunity to contribute to the Lebanese economy now and the Syrian economy in the future. These programs should be prioritised in areas such as Bekaa Valley, which hosts the highest number of Syrian refugees and have poor education for all students, even Lebanese citizens (S. Al Zoubi, personal communication, 20 March 2020). Furthermore, it is noted that living conditions in Northern Lebanon, which borders Syria, are relatively deprived. Overpopulation and starvation are rampant there, with open sewages and plastic pollution openly evident (UNICEF, 2019). However, education provides great hope; as Asma Rasamny, the founder of Malaak NGO notes, 'if there is a classroom in every informal settlement, there is hope' (A. Rasamny, personal communication, 7 May 2020).

Incorporating technology can further assist in scaling up the reach and empowerment enabled by these programs. Rudayna Abdo, the founder of Thaki, emphasises the benefits of technology-enabling educational resources her organisation puts together to reach rural areas and also the teaching of digital literacy to allow students to become lifelong learners within the international economy (R. Abdo, personal communication, 18 March 2020). Post-secondary vocational courses run by the likes of DOT Lebanon and Edutek, which centre on digital skilling (e.g., robotics and coding), also reflect the potential for online and international means of accreditation.

In practice, leaders in organisations focused on refugee education—like Suha Tutunji, Director of Refugee Education in Jusoor Lebanon—remain concerned over the reality that Lebanon is globally ranked the fourth worst country in terms of internet infrastructure (Johnson, 2011). Simply put, internet access for citizens in Lebanon, let alone refugees, remains expensive (S. Tutunji, personal communication, 22 April 2020). Therefore, attention must also be directed to developing infrastructure so that technology can be incorporated into classrooms throughout the country more seamlessly (W. A. Saifan, personal communication, 31 March 2020).

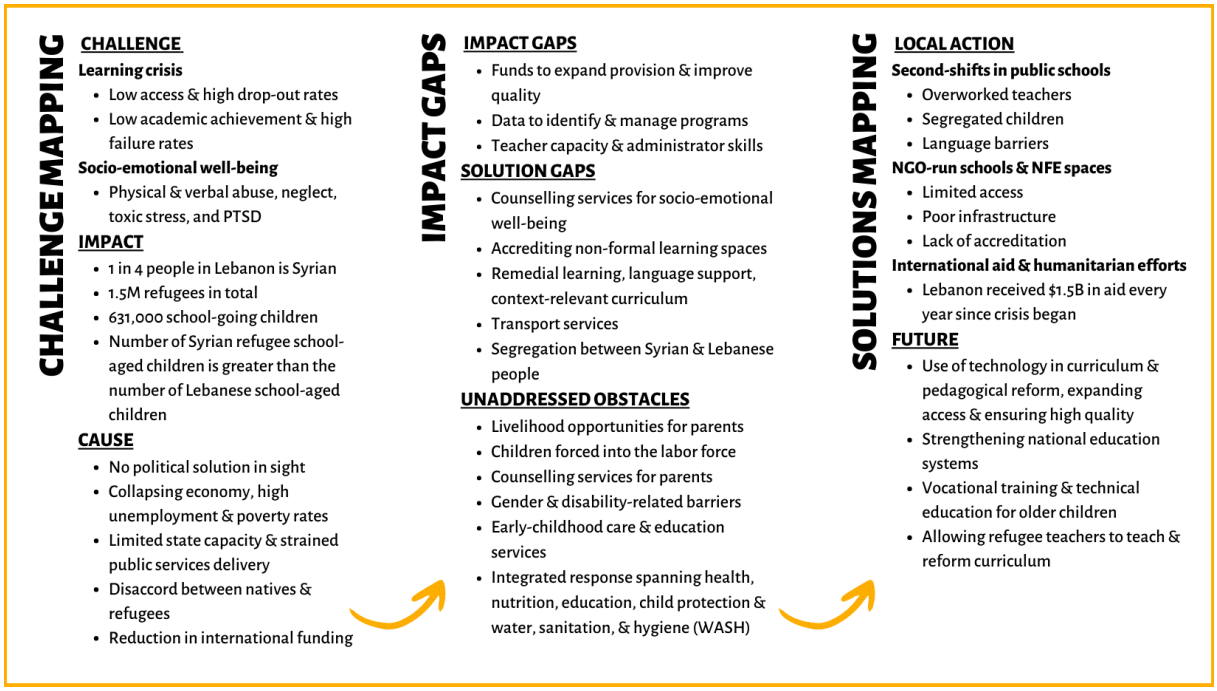

Figure 4.1: Mapping the gaps within existing solution efforts.

4 Isolating Levers of Systemic Change

As elaborated below, *language development*, *the need for accreditation*, and *formally expanding employment opportunities* are key levers of opportunity where the lives of Syrian refugee children may be enriched through targeted intervention efforts. These three intervention areas share commonalities in their need to incorporate considerations of language support, social integration, and support services, as well as the crucial need for government buy-in for policy shifts to enable coordinated implementation at scale. Moreover, it is fundamentally important that they are integrated across the entire system and can work as complements of one another (A. Sultan-Khan, personal communication, 10 March 2020). The impact gaps canvas below illustrates how the analysis thus far helps to visualise potential intervention points.

Going beyond surface-level system mapping, the iceberg analysis[13] reveals the deeply entrenched mental models that sustain the status-quo of the Lebanese educational ecosystem. From the insurmountable uncertainty around the end of the Syrian civil war and the duration of this crisis to the political, religious and cultural divide between the Lebanese citizens and the stigma attached to Syrian refugees, there are many complexities that have contributed to forming the current system.

Several potential intervention points have been identified below. Although this will be a promising start, this is not an exhaustive list. Rather, this ever-evolving field must react to changes over time. Therefore, these recommendations should be the start of a conversation that continues to grow. Fundamentally, policymakers and practitioners must work towards implementing these intervention points together, as it is key that they are integrated across the system (A. Sultan-Khan, personal communication, 10 March 2020).

The levers of intervention identified seek to balance the need for both long-term and short-term strategies to tackle this education crisis in a holistic manner while acknowledging and appreciating the diversity in potential

[13]The iceberg model uses an iceberg as an analogy to represent the underlying structures generating perceived events and issues (Rogers, n.d.).

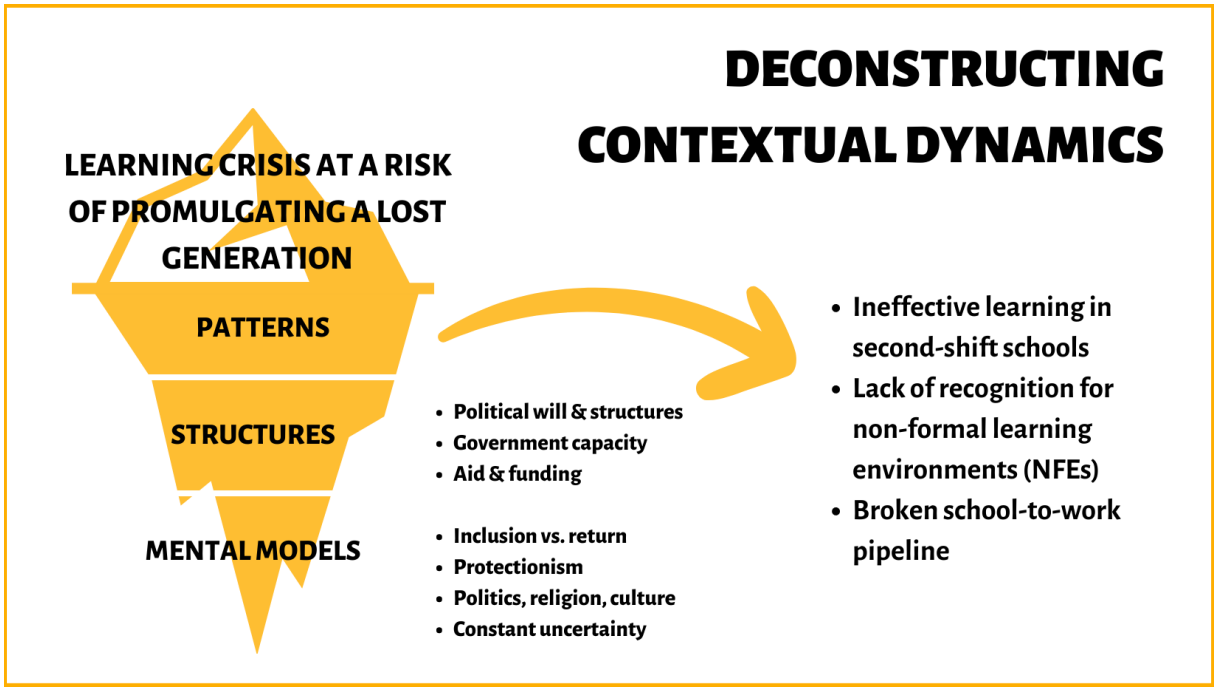

Figure 4.2: Visualising the iceberg analysis.

solutions to bridge the gaps identified above. The authors' goal is to identify gaps that will be the most feasible to fill right now and, at the same time, identify those that are the most impactful for the stakeholder at the centre: the Syrian refugee child.

4.1 Critical Levers of Change

The three points of intervention can be summed up as language development, the need for accreditation, and formally expanding employment opportunities. These together are aimed at reforming the education sector in Lebanon and bringing back the focus on the most important stakeholder—the child. Hence, the approach is to ensure that the children have a meaningful learning experience at school and are ready to join the workforce with the skills and qualifications they need. Collectively, these will transform the system so that it recognises and prioritises the needs of the child.

Language development is particularly key to empowering refugees to manoeuvre Lebanon's linguistic context; with Arabic spoken colloquially and both English and French forming the language of instruction in schools, students in the public school system are expected to be, at the very least, bilingual. Although most civil society organisations describe English language curricula as one of their key programs, English fluency remains a key obstacle to refugee education in Lebanon. To overcome this, more comprehensive efforts must be made to build English language capacity for refugee students. Focus on English is suggested so that a greater range of higher education institutions are likely made available to a growing refugee child as they upskill and work to contribute to the international economy.

The insufficient capacity of the Lebanese public school system and the prevention of formal accreditation in non-formal learning environments for refugees leaves refugees disenfranchised from an educational system. This is greatly exacerbated by an outcome wherein a lack of opportunity for certification means reduced opportunity for refugees to acquire meaningful employment thereafter. In bridging this gap, formalised efforts need to be

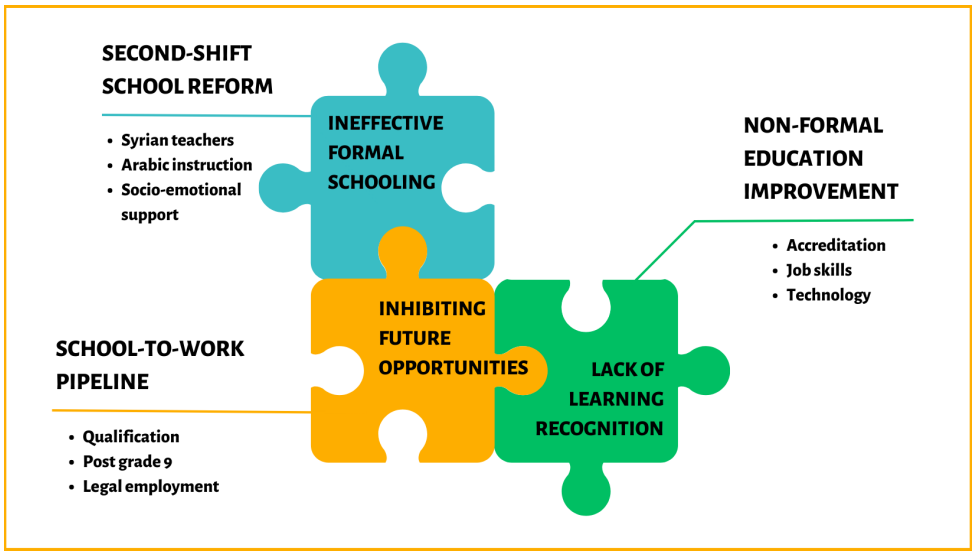

Figure 4.3: Critical levers of change.

made to ascertain accreditation across learning environments. In cognisance of the protracted nature of the crisis, future versions of RACE must also better account for refugee children's education beyond Grade 9. In addition, ensuring all student-age children are within a reasonable distance of a school will alleviate high transport costs and dangerous walking conditions that currently act as barriers to parents enrolling their students. Parents reported their children getting injured by vehicles along busy roads, experiencing harassment and bullying, and being targets of kidnapping attempts on their walk home from school (Human Rights Watch, 2016).

Finally, legal blockades in Lebanon prevent refugees from acquiring meaningful employment beyond limited blue-collar occupations, even if their skills and qualifications could enable far more. In overcoming this, organisations like Human Rights Watch must continue to advocate for policy change; this would be accompanied by appeals to the Lebanese government to enable greater scope of employment for refugees. This will be far more conducive to rebuilding what continues to be a crumbling economy in the state of Lebanon. However, building the political will to enable such is a long-term vision. Therefore, efforts to increase the capacity for digital skilling of refugees in the non-formal learning space will be crucial. In doing so, the potential for online employment, not constrained by state boundaries, opens far greater scope for refugees to nurture financial independence in the short term. This will ensure the empowerment of refugee families to better lift themselves out of their own poverty and feed into Lebanon's deteriorating economy.

It is necessary to create a conducive educational ecosystem for the successful implementation of the aforementioned levers. Reform efforts at the state level, i.e. for second-shift schools, must include Syrian teachers, improve geographical proximity, and increase funding for NGOs. With Lebanese teachers overworked, hiring Syrian teachers not only ensures sufficient capacity and quality but also helps fill the language gap refugee children face in school (Y. El Masri, personal communication, 13 March 2020). Their participation will enable greater power and agency to change the status quo, acting as a key lever to ensure refugee voices are heard and responded to while their skills are tapped into to empower their own community and uplift a crumbling Lebanese economy. This enables a key stakeholder, previously a passive agent, to become an instrumental and transformative participant in the change-making process.

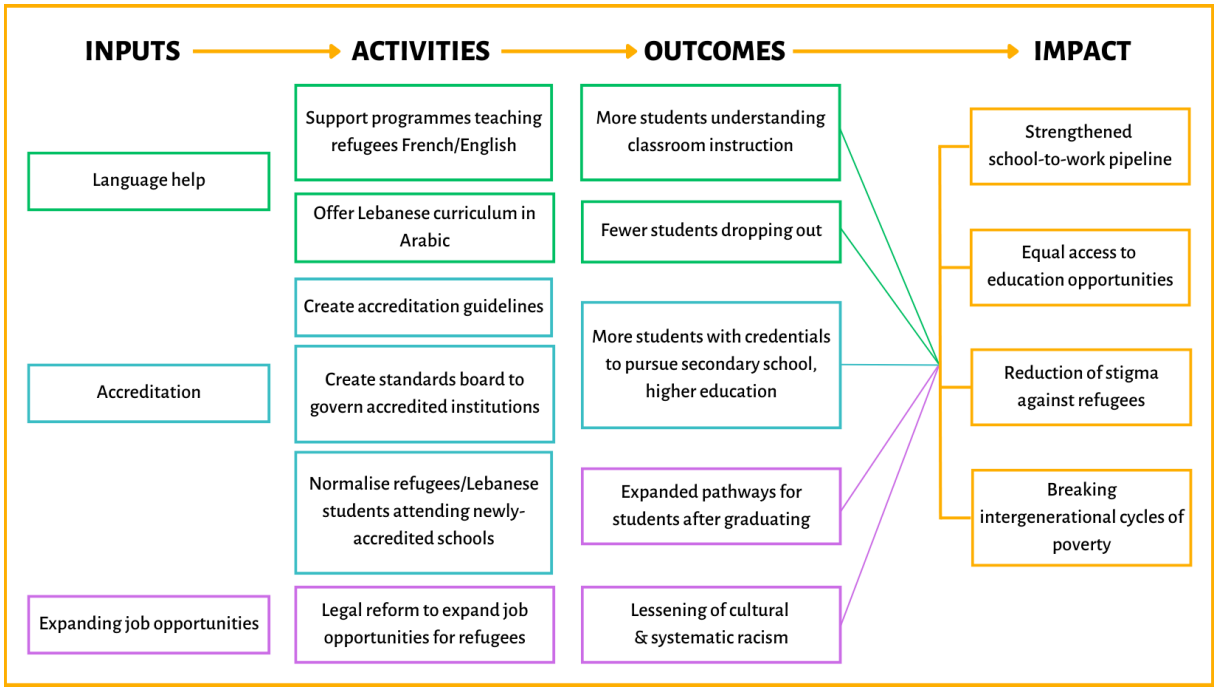

Figure 5.1: Proposed Theory of Change.

5 Effecting Systemic Change

5.1 Logic Model: Visualising Proposed Solutions, Challenges, and Assumptions

Taking into account the existing system and its stakeholders, this theory of change (see Figure 5.1) lays out how three proposed solutions—language help, accreditation opportunities, and expanding job opportunities—combine to strengthen the school-to-work pipeline for refugees, while also ensuring equal access to educational opportunities for all students in Lebanon and providing opportunities to the most disadvantaged groups living in Lebanon to boost their contributions to the economy, reducing stigma and breaking intergenerational cycles of poverty.

5.2 Implementation: Leveraging Reality for Future Efficacy

Implementation refers to how a planned intervention works on the ground, relying on measures of acceptability, fidelity, cost, population reach, and sustainability to ensure the intervention is effective at a large scale (Gardner, 2019). Effectively implementing the proposed solutions must rely on five major steps: engaging stakeholders, refining the strategy, engaging local partners, pilot testing, and scaling. To begin, further stakeholder research must involve a broader range of expert voices—particularly teachers and government education officials—as well as engaging students, parents, and the rest of the education community to better understand the context and determine the best solutions going forward. These conversations will inform the strategy, refining these proposed solutions so they can effect sustainable, positive change.

Partnering with local governments, NGOs, and international donors already entrenched in the local community will increase resources, boost credibility, and maximise sustainability. Local counterpart cooperation will be indispensable in promoting language courses, creating accreditation guidelines, and lobbying the Lebanese government to expand job options for refugees. Pilot testing must take place in close collaboration with local

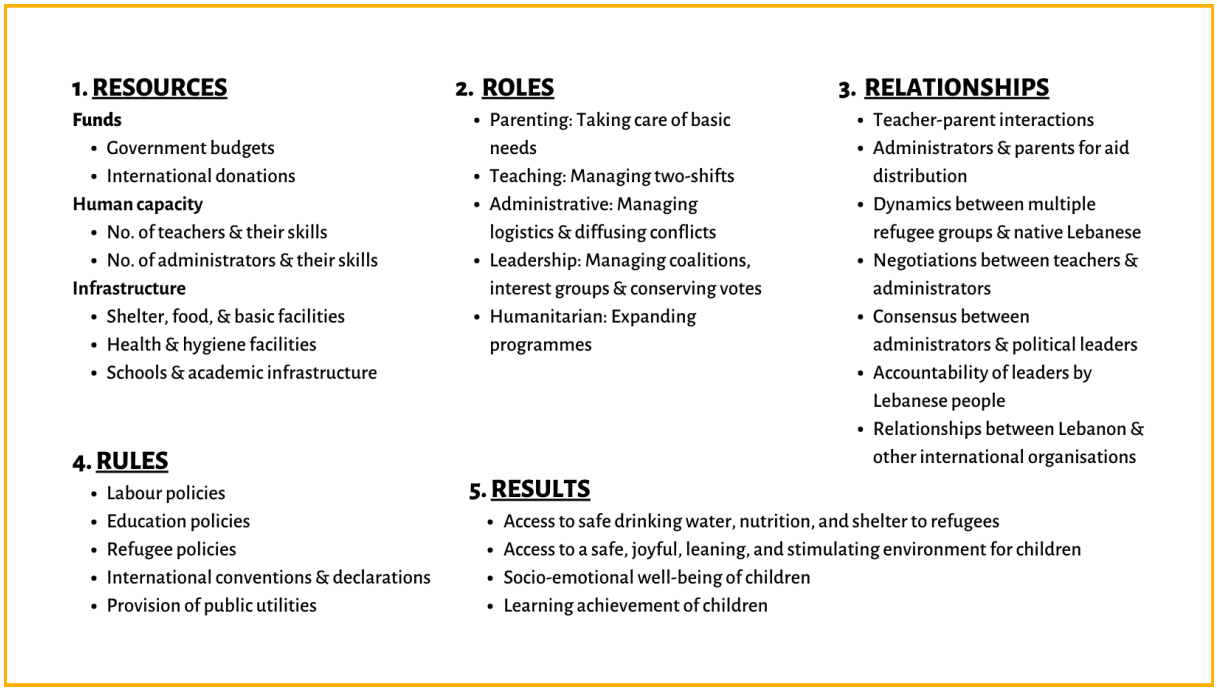

Figure 5.2: 5Rs framework for identifying and monitoring system interventions.

stakeholders—and in compliance with government regulations and ethics requirements. Evaluation strategies must include both an outcome and process evaluation to understand which mechanisms lead to intended outcomes and see whether there are any potential harms.

5.3 Evaluation: Ensuring Ongoing Understanding and Impact

For each of these proposed solutions, communication, transparency, ongoing monitoring, and evaluation will be crucial to success, such that all students, regardless of background, can attain quality education that will prepare them for future careers. Figure 5.2 illustrates a potential framework for achieving and measuring systemic change in education access for Syrian refugees, adapted from USAID's 5Rs Framework (USAID, 2016). By building on the existing structures and stakeholders identified through the systems analysis, this framework presents a pathway for sustainable change.

Partnerships with stakeholders will allow for comprehensive data collection in measuring the impact of interventions. This data collection strategy must ensure evaluation of overall outcomes, such as student achievement, percentage of students going to higher education and a process evaluation to look at which mechanisms are most effective in boosting overall outcomes, with a subgroup analysis to see which groups benefited most.

6 Lessons for the Future

6.1 Learning from this Challenge: Context-Bound Change

The research presented here scopes a significant breadth and depth of research while undertaking several layers of robust analysis to answer directly to its research objectives. In doing so, it reveals just how many stakeholder groups are involved with refugee education in Lebanon and the complexity therefore embedded in the system. Getting children into school does not necessarily solve the problem, as there remain issues around the language

of instruction, curriculum content, the impact of toxic stress and trauma, and bullying from classmates. Given the lack of progress made with government interventions, it is apparent that the reality of education is largely supported by non-formal educational provisions. The available literature reviewed for this report suggests that organisations attempting to tackle aspects of this education crisis are not employing a similar depth of thinking and understanding of the stakeholders at play across multiple layers in context. Therefore, our systems thinking framework presents a novel approach to addressing this challenge.

6.2 Informing Impact Across Global Systems

The importance of prioritising refugee education is a growing and pressing issue of global concern. This research uncovers this in relation to displacement caused by political conflict; while this is ongoing, it is also estimated that up to 140 million people could be climate refugees by the year 2050 (World Bank, 2018). Many of these people, including children and adults, will need to access education so that we can avoid lost generations of children who are never educated. If, as suggested, Lebanon can reform their education system to include the millions of refugees already in Lebanon, the country can be an example of how other countries with increasing numbers of refugees can handle the issue and potentially thrive despite additional pressures on their education systems.

These proposed solutions are adaptable to different contexts regarding educating refugees. The power in the levers of change presented here lies not only in a national context but also has significant repercussions for advances that global systems need and should make in the context of greater uncertainty and scale of global displacement. The proposal of accreditation of non-formal learning environments, for example, should come with a call for greater innovation in accreditation systems globally—not just in regard to the types of qualifications required but considerations where digital skilling and technology are far better accounted for. This is true not only for refugee education but for all children—especially given technological shifts in learning with COVID-19. Moving forward within a pandemic-induced financial recession requires the international community to support empowerment from the ground up, bleeding change across levels for all stakeholders.

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Part II.

Humanities

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Francesco Feriozzi
Humanities Section Chief

Narratives of Image: Views of Ceylon, Veins of Influence

Interdisciplinary Discovery for Curatorial Investigation

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The use of photography as a scientific and commercial tool was fairly well established in Ceylon by the end of the 19th century. The camera was used in archaeology, astronomy, anthropology, agriculture, engineering and industry between 1860 and 1880, as well as in commercial studio practice. Scholarship and recent exhibitions of colonial photographs from Ceylon to date (of which there is a dearth), however, have focussed on image classification, the relevant photographers and their individual practices. Departing from that didacticism, I outline the dynamics in collection and collecting, with a broader consideration of how the collection might have interacted with 'oscillating potentialities' beyond the collection materials themselves. Notably, most surviving collections and images are located outside Ceylon (e.g., UK, Europe and USA) and assuredly exist today as a result of being thus protected from the devastating effects of tropical weather on this medium. This essay covers the finding of an unstudied collection of colonial-era photographs and related materials on Ceylon in an album titled Views of Ceylon (the 'Collection') held by the Pitt Rivers Museum, Oxford University. This collection, largely unstudied, serendipitously provides a case study for tracing narratives of influence that operate from image to person and person to image. I recreate networks of individuals and their relationship to the various images, providing an example of the ways in which imperial networks can be traced through university collections.

Keywords: Ceylon, Sri Lanka, colonial, photography, Joseph Lawton, Jermyn, Pitt Rivers Museum.

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1 Introduction

During the course of research of early colonial photography of Ceylon (now Sri Lanka), discoveries relating to the photograph itself and extraneous factors not reflected in the image led to my development of the concept of '*veins of influence*.' This idea invites inter-disciplinary discourse to consider the dynamics of the photographic image, including those reflected in the image and through the image to viewers at any given time, varying with time. Though such interdisciplinary investigation is regularly practised by curators and researchers, this term, as coined during the course of this research, more poetically reflects the pulsating dynamics, whether they be imagined, possible, or proven.

In the context of early photographic images by colonial photographers of Ceylon, in particular, contemporaneous or operating influential writings on Ceylon, policy edits, political propaganda, commercial objectives and artistic endeavour are some examples of what might have operated on and through any given image or images.

Further to this research, I was invited to review, as a Visiting Fellow of the Pitt Rivers Museum ('PRM'), Oxford University, a previously unstudied collection of colonial-era photography on Ceylon, in an album, titled,

Views of Ceylon (the 'Collection'), now part of the PRM collection. Though, at the time, my image focus had been early portrait photography from Ceylon and its influence on national and native identity perspectives, this Collection has serendipitously provided a case study for tracing and activating the study of *veins of influence* in the context of these images. Considering collection dynamics, collection materials, collectors, owners and creators: a curatorial narrative has unfolded to reveal the surprising significance of these materials, inviting further study and inter-disciplinary engagement. These *veins of influence* develop a broader narrative with and around these collection materials, also revealing exciting possibilities for curatorial presentation whilst simultaneously identifying and elaborating on these dynamics.

In Ceylon, the use of photography as a scientific and commercial tool was fairly well established by the end of the 19th century. The camera was used by commercial studios to capture images of archaeology, anthropology, agriculture, engineering and industry between 1860 and 1880,[1] as well as in studio practice. However, photographs of colonial Ceylon were and remained much fewer than those documenting its much favoured colonial neighbour, the 'Jewel in the Crown', India. In India, photography had been officially and extensively employed (including important documentation of the varied races and castes of the country), occasionally supported with funding, since the early 1860s. In contrast, there is little evidence that such scientific and ethnographic application of the medium was widely employed in Ceylon.[2]

The rarity of photographs of Ceylon during the days of early photography would have given any such images an assumed status of value and often insight into the unfamiliar. Most viewers could not have known better than to see the few as a representational and authentic pictorial record of the contents shown.[3]

Most scholarship and exhibitions of colonial photographs from Ceylon to date have focussed on gathering images, indexing and categorising,[4] with bio-data on photographers and their individual practices. Departing from this approach, this review emphasises the dynamics in the collection and of collecting through an organic analogy of life-giving veins.[5] Guiding markers for review thus include: the dynamics of collecting, namely why and how the collection started, developed and travelled; what the images and their selection might offer; and implicit or stated perspectives of the creators, collectors and viewers; with the broader consideration of how the collection might have nurtured 'oscillating potentialities,'[6] namely the varied and changing possibilities of influence, operating outside, within and from the collection itself.

The Collection's route to the PRM in 2010 remains unconfirmed, though the materials refer to five names: *L.A.S. Jermyn*, '*C.F.G.C.*' (shown to be Constance Francis Gordon Cumming), *C. Symons* and, in unclear script, *Revd Cliff Godwin* and *J Lawton*. The starting point for the provenance trail was L.A.S. Jermyn because of the album notation 'Presented by L.A.S. Jermyn.'

The collection under consideration consists of:

- 1 vignette print of vine/flower fixed on the inside of the first page, signed *C Symons 1873*;
- 44 albumen prints (each approx. 28cm x 21.5cm), 37 fixed on consecutive individual pages, followed by 7 fixed in different arrangement (front and back of each page);
- 2 photographs of coffee plantation scenery, fixed on the recto;

[1] Ismeth Raheem and Percy Colin-Thomé, *Images of British Ceylon: Nineteenth Century Photography of Sri Lanka* (Times Editions, 2000).

[2] John Falconer, *Regeneration, A Reappraisal of Photography in Ceylon 1850–1900* (British Council, 2000) 17.

[3] H. D. Gower, *The Camera as Historian (1916)* (FA Stokes & Co., 1916) 3.

[4] There was a typical stock list of the 19th Century commercial photographer, which included: Middle-Class Mansions; The Working Class situation in Colombo; Clubs and other Institutions; Streets and Urban Landscape; Landscape; Urban Outposts; Portraiture; Ethnographic Studies; Agricultural and Plantation Views; Archaeological Views and Buried Cities; and Botanical Gardens and Tropical Plants.

[5] This review intentionally bypasses post-colonial theories, such as Orientalism, to bring a fresh focus to primary source materials, namely the images and contemporaneous sources that connect to inform on the collecting, creation and context of these images.

[6] Christopher Pinney, 'Foreword,' *Visual Histories of South Asia*, edited by Annamaria Motrescu-Mayes and Marcus Banks (Primus Books, 2018) xii.

- 4 photographs of artworks, affixed on the recto;
- Loose, handwritten 'Liste of Photographs'—one page with notations on both sides in two different scripts;[7]
- Loose, 1 studio photograph of a turbaned man ('Portrait');
- Loose, sepia-toned drawing, handwritten title and the notation '*C.F.G.C.*' ('Drawing');
- Leather bound album with gold embossed title *Views of Ceylon*, photographs pasted on pages, with numerous unused pages.

In what follows, these materials are collectively referred to as the 'collection' or 'Jermyn Folio'. Materials pasted in the *Views of Ceylon* album are referred to as the 'Album.' All materials are unsigned except where otherwise indicated.

2 The Collection

2.1 Provenance, Owners and Collectors

L.A.S. Jermyn (1880–1962), Donor Lancelot Ambrose Scudamore Jermyn ('L.A.S.') was born in Lucknow, India, where his father served as a priest. After studying at Keble College, Oxford University, he became a teacher, spending time in Malaya[8] in the late 1930s through to the 1940s, the relevance of which we see later. There is, however, to date, no record of any visit by him to Ceylon. L.A.S.'s only child and son predeceased him, being a war casualty in his early 20s, and this absence of descent may have prompted L.A.S.'s eventual gift of the Collection outside the family.

In Malaya, he taught and headed private English-medium schools, including serving as Head Master of the prestigious Malay College, Kuala Kangsar (1919–23) and Malacca Highschool (1934–1941).[9] During the Japanese invasion of Malaya in 1941, L.A.S. was taken prisoner and spent the rest of the war in Changi Prison, Singapore, during which time he translated into English Virgil's Georgics, furthering his personal interests in poetry and classical literature. His translation of the Georgics became part of his work *The Singing Farmer* that included notes on his personal experiences in South East Asia and, interestingly, applied Virgil's guidelines on agriculture to his own observations of soils and plants in Malaya.[10] L.A.S.'s indicated affinity for nature and agriculture offers another reason why this Collection might have appealed to him and stayed with him.

Despite the notation 'Presented by L.A.S. Jermyn', we still do not know the immediate recipient of that presentation. The PRM accession note states that the Album came from the School of Geography, but there is no record of when that School itself received the Album.[11] It is more probable that L.A.S. gifted the Collection to his Oxford College, Keble, before his death in 1962. Keble College—with strong ties to the School of Geography—could have passed the materials there, for greater access and appreciation.[12]

[7] An interesting linguistic point is the writing of '*Liste*' a French word, rather than the English 'List', and what such notation indicates about the writer of this list.

[8] Throughout this article, the word 'Malaya' indicates the area of the Malay peninsula and of the island of Singapore, which were under the British colonial power.

[9] Ramli Hj. Khamis, *The First Hundred Years : Malay College Kuala Kangsar* (Word Wizards, 2005); Bok Chye Chua, *Our Story: Malacca High School, 1826–2006* (MHS Anniversary, 2006).

[10] Laura Sayre, '"How / to make fields fertile": Ecocritical Lessons from the History of Virgil's Georgics in Translation,' *Ecocriticism, Ecology, and the Cultures of Antiquity*, edited by Christopher Schliephake (Lexington Books, 2017) 191; L.A.S. Jermyn, *The Singing Farmer. A Translation of Vergil's 'Georgics'* (Blackwell, 1947).

[11] Discussion with Susan Squibb (a.k.a. Sue Bird), Retired Geography Subject Librarian, Bodleian Libraries, Oxford University, 23 March 2020.

[12] Discussion with Susan Squibb (a.k.a. Sue Bird).

Rt. Rev. Hugh Jermyn (1820-1903), Collector & Original Owner The intersection between L.A.S. Jermyn, the Collection and Ceylon is certainly his family inheritance and his paternal grandfather, Bishop Hugh Willoughby Jermyn, the Bishop of Colombo from 1871–1875. Bishop Jermyn's stewardship of the Church of Ceylon was exemplary and garnered much praise, as noted in church records. He 'crowded into less than four brief years an amount of work that would have done credit to twice that length of service,'[13] and built up the Church reserves, increased chaplaincies, particularly in the coffee districts, and promoted missionary work through financial and physical support. Described vividly as 'striking' in stature, with 'herculean mould' and 'flowing beard' giving a 'patriarchal appearance,'[14] he actively toured outstation parishes, travelling the country and worked with 'zeal and energy that boded no good for his health.'[15] Sadly, he was forced into early resignation due to ill health.

Based on recorded dates of his residence in Ceylon and the noted dates for most of these collection materials, it is likely that Bishop Jermyn was the primary collector and original owner of the Views of Ceylon Album, though not necessarily of all materials in the Collection, as hinted by its different constructs of presentation. The Album's image presentations vary: showing quality large format photographs, one per album page, at the start of the Collection; then clusters of faded photographs, similar and varied in subject matter, in other sections; and other images either pasted or in loose format, indicating that the Collection was completed over time and by different owners.

3 Veins of Influence – Collecting and Creation

3.1 The Albumen Prints

The majority of the albumen prints in the Album capture the picturesque, showing rolling hills, exotic rock formations and tall waterfalls. We also see a botanical focus, with lush greens, ferns and palms – landscapes of a tropical paradise. Included also are distinctive architectural images, most with religious connections: the Buddhist Temple of the Tooth (Kandy), the Hindu Temple (Colombo) and the Christian Christ Church Cathedral (Colombo). Notably missing is an image of a mosque, even though the Muslim faith was practised by a minority in Ceylon then (and continues to be so today).[16] Though it is noted that it is very typical of travel albums of Ceylon of this period to not have included images of mosques, this exclusion remains curious as this collection is more than a mere travel album due to the extended and interactive residency of the Jermyns in Ceylon, which is later discussed in this article.

These first 44 albumen prints can confidently be credited to Joseph Lawton, a renowned British studio photographer active in Ceylon in the late 1860s and early 1870s. The dates of his commercial studio activity and stay in Ceylon overlap with Bishop Jermyn's four years there.

These views (the 'Lawton photographs') bear a striking resemblance to photographs in The Princeton Art Museum Collection specifically titled *Lawton, Views of Ceylon* (1872–73), containing 25 albumen prints (titled the *Princeton Album*) and six matching images including the Mahavali Ganga (shown below).[17] The photographs from both collections show similar photographic perspectives with centralised focus, a balance of dense and open spaces, and the absence of man.

Notwithstanding Lawton's importance as a colonial photographer, there is frustratingly little biographical information on him, including no confirmed portraits, birth year, or death date. His memory is notably only preserved and celebrated through his imaging of Ceylon.

[13] F. Lorenz Beven and J. A. Martensz, *A History of the Diocese of Colombo: A Centenary Volume* (Times of Ceylon Co, 1946) 90.
[14] Beven and Martensz 91.
[15] Beven and Martensz 92.
[16] Survey of 1891 shows a 7.9% Muslim segment of the population.
[17] Joseph Lawton, 'Lawton's Views of Ceylon (1872-73),' *Princeton University Art Museum Collection*,

Mahavali River 2010.75.43.4.

Kandy Lake 2010.75.43.4.

Lawton came to Ceylon working for the trading company H.C Byrde and Co, and around 1866, opened a photography studio in Kandy.[18] He was famously commissioned by the Ceylon Archaeological Committee to photograph ruins at Anuradhapura, Sigiriya, Mihintale and Polonnaruwa. Those 227 highly skilled and valued photographs can be accessed through UK public collections, including in the National Archives, Kew (previously in the Colonial Office Collections) and the Victoria & Albert Museum Collection, London (V&A Collection).[19] He stopped photographing in early 1872, and is believed to have returned to England due to his ill health and died by November 1872, as indicated in an announcement at the Ceylon Branch of the Royal Asiatic Society at that time.[20] His wife continued to manage the studio and sell his photographs in Ceylon until at least 1876.[21]

The Jermyn Folio has no visible markings or signings by Lawton, unlike the images in the Princeton Album and V&A Collection, which prints show the name 'Lawton'.[22] Perhaps for the collector of these images, the photographer's identification was not material. Rather, his focus may have been on the images which reflected personal experience, aesthetic preference and memory.

The titling of this album *Views of Ceylon*, would have likely been done through Lawton's studio and indicates not only familiar association but also standard title classification, as studios often offered leather-bound albums that would then be printed on order with a selected title.[23]

These images, as *Views of Ceylon*, would have been intended to introduce to the uninformed viewer impressions of Ceylon, though the extent to which they did, in fact, do so is pure conjecture.[24] The image content was in line with colonial photographic efforts to capture picturesque views much desired by British buyers,[25] and perhaps

[18] Raheem and Colin-Thomé 22-23.

[19] Falconer 22.

[20] 'Proceedings, 1872,' *The Journal of the Ceylon Branch of the Royal Asiatic Society of Great Britain & Ireland*, 1872, xix-xx.

[21] Joachim K. Bautze, 'The Photographic Studios in Landscapes of Sri Lanka – Early Photography in Ceylon / Die Foto-Ateliers in Landscapes of Sri Lanka – Frühe Fotografie in Ceylon,' *Landscapes of Sri Lanka. Early Photography in Ceylon / Frühe Fotografie in Ceylon*, edited by Raffael Dedo Gadebusch (Museum für Asiatische Kunst, Staatliche Museen zu Berlin, 2013) 23.

[22] Princeton Album has embossed in gold lettering on the cover, 'Lawton, Views of Ceylon.' Compare also V&A Lawton Collection, photographs from the special commission by the Ceylon Archeological Committee. Those photographs show 'LAWTON' scraped in the right corner of each image.

[23] *Views of 'place'* was a common title.

[24] Commercial Studio categories included Portraiture; Ethnographic Studies; Agricultural and Plantation Views; Railway Views; Royal Visits; Archaeological Views, and Buried Cities.

[25] James R. Ryan, *Picturing Empire, Photography and the Visualization of the British Empire* (The U of Chicago P, 1997) 46.

Temple of the Tooth, Kandy 2010.75.43.35.

Hindu Temple 2010.75.43.26.

also to entice travel to, mission work, and stay in such an idyllic colony.[26] Such images, with their relative period novelty, would have been powerful influencers of the perception of this otherwise unfamiliar isle.[27] Furthermore, we can assume that Bishop Jermyn's likely status as an 'influencer' (derived from his social and religious positions, lineage and access to decision-makers) would have empowered these images further as definitive depictions of Ceylon, following the narratives that he may have shared to entice further missionary work, travel, investment and commitment to the Empire in Ceylon.

The selection—and if not by him, then the maintenance—of these particular images by Bishop Jermyn not only likely reflects his personal journey to and memory of the sites but arguably also a larger commitment to the Empire's power over land and religion.[28] Further still, the idealised images of lush landscapes with their Edenic sensibility[29] soothe and belie the challenges of climate and illness that many a colonial succumbed to, including, ironically, both collector Bishop Jermyn and creator Joseph Lawton.[30] Nowhere here do we find Lawton's iconic archaeological images, perhaps indicating this collector's preference for natural landscapes and living institutions rather than impressions of 'perpetual antiquity' often associated with the imaging of Victorian Ceylon.[31]

If this Collection started with the first 26 photographs as the *Liste* suggests, then it is plausible that additions were made in groupings and stages by Bishop Jermyn during his four-year sojourn in Ceylon (during which time Lawton's studio would have been in operation). As the photographs are not unique, they are likely from the studio's commercial inventory rather than by special commission. It is in the later group of images that we find the Christ Church Cathedral, the Bishop's headquarters in Ceylon. Notably, there are also four perspectives of the main Buddhist temple (the Kandyan Temple of the Tooth), as compared to the single image of the Hindu temple, and the absence of mosques altogether, arguably indicating a subtle emphasis on the Buddhist and Hindu populace as more amenable to Christian conversion and/or architectural admiration for these particular structures.

Captured with an upward angle, accentuating the height and stature of the cathedral tower with its thick

[26] C. F. Gordon Cumming, *Two Happy Years in Ceylon* (Chatto & Windus, 1901) Foreword.
[27] Pinney, 'Foreword' x.
[28] Falconer 11.
[29] Vindhiya Bhuthpitiya, ''Paradise' in Missing Pictures: A Brief and Incomplete History of Sri Lankan Photography,' *History Workshop*, 15 July 2019, 2.
[30] Lawton around 1872, and Jermyn in 1875. Beven and Martensz 42.
[31] Annamaria Motrescu-Mayes, 'Perpetual Antiquity in Early Photographs of Ceylon,' *Visual Histories of South Asia*, edited by Annamaria Motrescu-Mayes and Marcus Banks (Primus Books, 2018) 93-123.

Christ Church Cathedral 2010.75.43.33.

Waterfalls 2010.75.43.30.

boundary walls and solid foundations, this image reflects stability, the desired message of Christianity that the Bishop might have wanted to share through this image inclusion. The cathedral, surrounded by tropical foliage, shows architectural resemblance to many a town church in England of that time (and now), hinting at anglicised civilisation and communion.

The last page of albumen photographs in the Jermyn Album taper into mediocrity, both in image quality and presentation, showing in two images what appear to be coffee plantation clearings. Given this variance in quality, size, content and presentation, these prints are unlikely to be by Lawton, appearing in the Collection as a later addition, showing cleared and broken tree areas, notably at odds with the picturesque displayed earlier. The inclusion of these images makes sense, given the concentration of chaplaincies that Bishop Jermyn established in coffee districts.

A vignette of the tropical flower *Stephanotis floribunda*[32] signed faintly in pencil, 'C. Symons 1873,' is pasted at the front of the photographic collection. The image loosely connects to a theme of tropical flora. Dated 1873, we might initially assume that C. Symons was a guest of the Jermyns in Ceylon, but records show that amateur photographer, Charles Edward Hood Symms, was a Royal Artillery Officer (as per the British Army List). He was stationed in Trincomalee in the 1860s.[33] Thus, though he may not have visited the Jermyns in Ceylon, he may well have been personally known to the Jermyns, as indicated through the inclusion of this image in such prominent placement.

3.2 The Drawing and Photographs of Illustrations

The Collection features a loose, original drawing signed 'C.F.G.C.' (Constance Frederica Gordon Cumming), which has been confirmed to be the original for one illustration from her publication, *Two Happy Years in Ceylon.* Also pasted on one album page are four poor-quality photographs, identified now as four illustrations from the same publication.[34] We can only speculate how the original came to be included, whether gifted by Gordon Cumming, selected by Jermyn or entering the Collection in another way.

Gordon Cumming was an unconventional period persona. An ardent traveller, recognised artist and writer, she visited Ceylon from 1872–1873 at the invitation of the Jermyns. This trip marked her first journey out of

[32] Also known as Madagascar Jasmine, found in tropical climates of South Asia and South East Asia.
[33] Discussion with John Falconer, 21 January 2022.
[34] C. F. Gordon Cumming, *Two Happy Years in Ceylon*, vol. 1 (W. Blackwood / Sons, 1892); C. F. Gordon Cumming, *Two Happy Years in Ceylon*, vol. 2 (Charles Scribner's Sons, 1892); *Two Happy Years in Ceylon [1901]*.

Avenue of Indian Rubber Trees, Ficus Elastica at Peradeniya, Ceylon.
C.F.G.C. 2010.75.43.53 (mounted)-O [loose painting].[39]

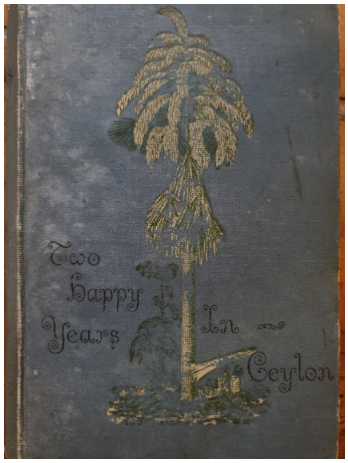
Scotland and started with stays in India, then Malta, and then Ceylon.[35]

She published *Two Happy Years in Ceylon* documenting her extensive travels with the Bishop and his daughter around Ceylon. Gordon Cumming aimed to share remote parts of the world for the benefit of people who were never likely to see such places for themselves. 'In this, she offered reassurance as well as instruction and diversion,' using the picturesque as propaganda to entice the reader to turn traveller.[36] The fact that three editions were published between 1892–1901 indicates a strong market for the publication, with rave reviews declaring it to be 'by far the most valuable account of Ceylon.'[37] The 1901 edition combines the contents of the earlier two volumes with additional data points on commerce.[38]

This artwork exemplifies 'clear intentionality of purpose,'[40] through her painter's hand by which she emphasises the botanical significance and exotic appearance, recreating this 'magnificent avenue of the old india-rubber trees.'[41] The image serves to show more than 'prized beauty,'[42] but informs the reader, through her writing, of the commercial viability of the rubbery sap of the *Ficus elastica* and related genus, the Pará rubber tree, because the 'ever-increasing demand … would certainly be satisfactory if their cultivation in a British colony can be made to pay.'[43]

She omits an explanation of the relationship between the *Ficus elastica* and its grand relative *Ficus religiosa* (also known as the 'Bodhi tree'), which has seminal religious significance for Buddhists and Hindus as the tree of enlightenment, under the shelter of which the Buddha is believed to have attained enlightenment. In Gordon Cumming's publication and in the Collection, interest at first reading and glance is only in the aesthetics of the species. Extended cultural context or local meaning is offered only through implication through inclusion.

[35] Hugh Laracy, *Watriama and Co: Further Pacific Islands Portraits* (ANU P, 2013) 69-91.
[36] Laracy 85.
[37] Gordon Cumming, *Two Happy Years in Ceylon [1901]*.
[38] Gordon Cumming, *Two Happy Years in Ceylon [1901]*.
[39] Gordon Cumming, *Two Happy Years in Ceylon [vol. 1, 1892]*.
[40] Christopher Pinney, *The Coming of Photography in India* (The British Library, 2008) 2-3.
[41] Gordon Cumming, *Two Happy Years in Ceylon [1901]* 187.
[42] Gordon Cumming, *Two Happy Years in Ceylon [1901]* 187.
[43] Gordon Cumming, *Two Happy Years in Ceylon [1901]* 188.



Cover of *Two Happy Years in Ceylon*.
Talipot Palm in Blossom (Author's Collection).

Talipot Palm, 2010.75.43.37.

Interestingly, Gordon Cumming did not use photography to illustrate any of the editions, most likely because these publications served to promote her own artistry. The photographs of her illustrations in the Jermyn Folio are amateurish in capture and casual in presentation, but they serve as a record of shared memory of a favoured four from a total of 28 illustrations in that publication.

The initial anonymity of the illustration photographs, lacking reference to a source, indicates that their inclusion and relevance were for personal appreciation rather than for any official record. It would appear that these images were also meant to be explained through personal encounters, otherwise offering little guidance on their relevance. We can imagine Bishop Jermyn or his daughter leafing through the album during a quiet afternoon, reminiscing about their experiences in Ceylon, with such memories accessible to others through their retelling, and now with that source lost, with the helpful vein of Gordon Cumming's publication.

Given the placement of the illustrations in this collection, it is only natural to ask: what might the points of intersection have been between the Lawton photographs and Gordon Cumming's eye? What veins of influence from his images might have touched her views? How does her work intersect with his photographs? Could she have participated in their selection? Did she peruse the Album contents to find inspiration for her own painterly capture? Though these ideas are speculation, there certainly are visual similarities between her illustrations and Lawton's photographs, all being of nature or structures, and most notably of the towering gold embossed talipot palm that decorates the cover of *Two Happy Years in Ceylon*.

Lawton's photographs, too, like Gordon Cumming's illustrations, convey impressions of a fertile, lush colony, with extended horizons and tropical breezes, imagining glorious dawns and sunsets over the misted mountain ranges shown. These sensibilities are aptly reflected in the excerpt from the poem quoted in her publication, which is an excerpt from *The Voyage Of Maeldune* by Alfred Lord Tennyson, unapologetically appropriated by her and retitled *Ceylon*:

And we came to the isle of Flowers;
Their breath met us out in the seas,
For the Spring and the Middle Summer
Sat each on the lap of the breeze.'[44]

[44]Gordon Cumming, *Two Happy Years in Ceylon [vol. 1, 1892]*.

Portrait. Man wearing a turban, carte. Loose.

3.2.1 Views of Ceylon—The Anomaly

At first glance, reviewing the album pages, a viewer would be justified in concluding that Ceylon is a beautiful but abandoned isle devoid of human habitation, with religious sites left as trace evidence of civilization. Nowhere in the album do we meet living souls (save in nondescript miniature to indicate scale or by error).

Had the collector so wanted, he could have inserted local identities easily onto the album's empty pages, selecting from commercial studio stocks of '*Ethnographic Studies*' or '*Native Types*', or specific studio portraits.[45] The absence of personal images offers another perspective on this collecting exercise. Perhaps these images, free of people, simply showed the distance between man and environment and spoke more easily to God's land. Perhaps he just wanted to maintain the content within the literal classification of '*Views of Ceylon*'. The absence of people might also make it easier for viewers to imagine themselves in the image.

Hence the exaggerated and misleading single image of a man placed loosely between the Album pages—the anomaly. Who was this man dressed in foreign garb? Was he a friend from Ceylon? Was he a typical Ceylonese of that period? It turns out to be neither. This most 'human' image is, in fact, an impersonal commercial carte, printed in multiple, with the same carte in the collection of the Rijksmuseum, Netherlands. Museum records describe it as a 'studio portrait of a man wearing a turban, carte' from Singapore, circa late 19th/early 20th century.[46]

The significance of this carte then lies in the fact that it is from Malaya (of which Singapore was a part prior to 1965). Through this object, we find the only trace of L.A.S. Jermyn's contribution to the Collection, his only imprint other than his signing. We can only speculate as to why this loose addition was made. Perhaps the image spoke to him of Malaya? Perhaps the portrait reminded him of a friend? Whatever the reason, this lone, well-preserved carte, provides a distinctive presence with a transfixed gaze of encounter and disrupts an otherwise faceless engagement with the Collection. Here for the first and only time, we see someone looking back at us—someone not from Ceylon.

[45] Raheem and Colin-Thomé 44.

[46] Unknown, 'August Sachtler Collection. Studio Portrait of a Man Wearing a Turban, Singapore circa 1870–1900. Carte de Visite. Object No: RP-F-00-5225 LL/76427,' Sachtler in fact operated in the 1860s and thus, it is unlikely that this image was from L.A.S. Jermyn's lifetime or that the subject was personally known to him.

4 Conclusion

The best exhibitions and collections tell strong human stories that we might not otherwise know. This Collection reveals numerous, magically interwoven human stories to reflect a variety of *veins of influence.* The Collection, when seen as a collection also of choices, reveals individual and overlapping influences, and demands to be shared and celebrated.

What can the photographs themselves tell us? As a superimposition of reality and of the past, it is true that the camera cannot censor what it captures.[47] In this sense, the photographs are an objective representation of what is shown within the frames. However, the collector's act of selection and editing introduces a critical and subjective curation of what appears as the '*Views of Ceylon.*' Furthermore, the images informed by the writings of Gordon Cummings provide fresh perspectives and narratives. The travel writing brings added information, in the absence of express guidance by the long-gone owners.

At the time of its development in the late 19th century, the Collection could have arguably been a quality 'influencer' primarily because of Bishop Hugh Jermyn's status, the relative novelty of images of Ceylon and unfamiliarity with the colony. Thus, the Collection could have provided a significant *vein of influence* for the imaging of Ceylon to a wider gaze.

Of key significance is that the collecting exercise can be confidently attributed to a collector who had personal engagement with the images shown. It is reasonable to say that the Collection was very personal, not only because it was passed through the family for generations (rather than being given earlier to the public domain, such as church archives) but also based on the addition and arrangement of newer content.

Nearly 150 years have passed since Bishop Hugh Jermyn's stay in Ceylon. The Lawton photographs would be at least that old and remain well preserved away from the harsh, tropical environments that their origins would have offered. With unfolding veins leading to important attributions to key church figure Bishop Hugh Jermyn, seminal photographer Joseph Lawton, and renowned Victorian artist and writer Constance Frederica Gordon Cumming, the Collection provides a rich resource for continued inter-disciplinary research. A collection that appears to have been private, kept in the family until its serendipitous giving to Oxford University, can now reveal and fully realise its own continuing *vein of influence*.

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5 Images

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Part III.

Mathematical, Physical and Life Sciences

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Convergence Rates for Nearest Neighbour Regression in Infinite Dimensions

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Nearest neighbour methods are one of the most widely used regression techniques due to the simplicity of their implementation and their wide applicability. They are one of the few approaches viable for regression on non-Euclidean manifolds and infinite dimensional function spaces, which increasingly come up in applications in engineering, data science, and other fields. However, currently, proofs of consistency and rates of convergence are only available for Euclidean domains. In this paper, we prove that nearest neighbour regression on general metric spaces—which includes general manifolds and function spaces—is, under minimal assumptions, universally uniformly L^2-consistent, and present convergence rates in terms of small ball probabilities of the regressor. We use our general framework to derive explicit convergence rates for cases where the regressor is finite-dimensional (in the Hausdorff sense) or a Gaussian random function.

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1 Introduction

Consider a task where a random quantity Y is to be predicted from a related quantity X, based on a set of training samples $(x_1, y_1), \dots, (x_n, y_n)$. One approach called the k-nearest neighbour technique takes the average over k of the training samples which are closest to a given observation $X = x$. As an example, consider a streaming platform that collects user ratings for films they have watched in order to predict which other films they are likely to enjoy. The platform needs a way of predicting the rating (in our notation, Y) that a user would give to a new film X based on previous ratings $(x_1, y_1), \dots, (x_n, y_n)$. The k-nearest neighbour technique in this setting would take the average of the ratings that the user assigned to the k previously watched films that are closest to the new one based on genre, age rating, year of production, etc. In reality, the user's rating will also depend on external factors such as current mood, whether they watch the film together with a group, or the quality of available snacks. The easiest way to model this is as random fluctuations. Hence Y is assumed to be random rather than a fixed function of X. A good estimation procedure would then be expected to predict a user's rating of a film averaged over external factors. If this prediction becomes more accurate with an increasing amount of training data, the estimator is called *consistent*. There are various notions that make this idea of consistency mathematically precise—see [2] for a comprehensive list.

A vital part of theoretical research on nearest neighbour regression is proving that estimation is guaranteed to be consistent under certain assumptions. Preferably, such a result also makes assertions about the speed at which the estimator converges to the true function, called the *rate of convergence*. There are many results available in this direction, but they are all concerned with the case where the domain of X is a subset of the

Euclidean space $\mathbb{R}^d$ [6, 9, 8, 17, 18], or where a more general state space is projected on $\mathbb{R}^d$ by extraction of a finite number of features [13]. Many classical applications can be modelled in such a way, including our earlier example wherein each film could be represented by a vector of features (genre, age rating, production year, etc.). However, many recent applications do not fit this assumption. Fuchs et al. [11] study applications of nearest neighbour techniques on functional data in speech recognition and sensor technologies, in which case the domain of X is an infinite-dimensional space of functions. Lang et al. [14] use regression in the context of 6D object tracking. They use quaternions to describe rotations, which reside on a so-called *manifold*—a space that "locally looks Euclidean" but generally cannot be identified with a subset of $\mathbb{R}^d$. The authors use Gaussian process regression, but nearest neighbour techniques could be applied in the same context.

The most general setting in which nearest neighbour techniques can be applied would be the case where the domain of X is a set with a notion of distance between objects called a *metric space*. More formally, a metric on a set E is a symmetric function $\rho\colon E \times E \to [0, \infty)$, such that $\rho(x, y) = 0$ if and only if $x = y$, and $\rho(x, y) \leq \rho(x, z) + \rho(z, y)$ for all $x, y, z \in E$. This includes $\mathbb{R}^d$ with the Euclidean distance and general manifolds. Function spaces can also be equipped with metrics. Several have been suggested and studied by Fuchs et al. [11] in the context of nearest neighbour regression.[1]

Nearest neighbour *classification* in this setting, that is, the case where Y takes values in $\{0, 1\}$, has been studied in [4, 5, 7]. In this paper, we study *regression*, which is the more general case where Y is real-valued, and establish consistency of nearest neighbour estimation under minimal assumptions. We give a general result that applies to arbitrary metric spaces and show how these results can be used to obtain explicit convergence rates in the case where X is finite-dimensional and in the infinite-dimensional case where X is a Gaussian random function. To our knowledge, this makes the present paper the first to formally prove consistency of nearest neighbour regression in the most general setting of arbitrary metric spaces.

We give a more precise summary of the type of consistency we show. To put our result in context with existing literature, we introduce some important distinctions with regard to the sense in which the estimator converges to the correct function—convergence can hold *pointwise*, i.e. individually on every point of the domain, versus *uniformly* on certain subsets of it; and the convergence can be *weak* (convergence in probability), *strong* (almost sure convergence), or in the L^p-sense, which means that the difference of the estimator and the target function vanishes in the probabilistic L^p-norm. Both strong and L^p-consistency imply weak consistency, but neither implies the other. In the case of Euclidean domains, results on pointwise L^2-consistency were first presented in [17], uniform L^p-consistency is given in [18] and [8]. Uniform strong consistency was established in [9] and [6]. In our setting, where X takes values in a general metric space, we prove uniform L^2-consistency, with convergence rates in terms of small ball probabilities of X, under mild continuity assumptions and sufficient integrability of Y, with no assumptions on the metric domain space or the distribution of X.

In Section 2, we introduce the general framework of the paper, recall some basic probabilistic notions, and define the estimators. In Section 3, we present our main results, which we apply in Section 4 to derive convergence rates in the cases where X is finite-dimensional or a Gaussian random function. The proofs for these results can be found in Section 5.

[1] They also consider *semi-metrics*, that is functions $\rho\colon X \times X \to [0, \infty)$ which are metrics except for the fact that there may be $x_1, x_2 \in X, x_1 \neq x_2$ with $\rho(x_1, x_2) = 0$. Note that in such cases any nearest neighbour estimator will, in general, not be consistent: If $\rho(x_1, x_2) = 0$, then the estimator cannot distinguish between observations at x_1 and x_2, hence will always estimate $\widehat{f}(x_1) = \widehat{f}(x_2)$, which is wrong if the estimated function satisfies $f(x_1) \neq f(x_2)$. Semi-metrics generally perform worse than true metrics for this reason but are often more computationally efficient. To trade off performance and efficiency, one often tries to find a semi-metric which is as simple as possible while still being expected to satisfy $f(x_1) \approx f(x_2)$ whenever $\rho(x_1, x_2) = 0$.

2 Preliminaries and Setup

We first introduce some notation. If $f, g\colon \mathbb{N} \to [0,\infty)$ are two functions, we write $f = O(g)$ if there exists $K > 0$ with $f \leq Kg$, and we write $f = o(g)$ or $f \ll g$ if there is $\alpha_n \downarrow 0$ with $f(n) = \alpha_n g(n)$ for all $n \in \mathbb{N}$. We write $f \sim g$ if there is $\alpha_n \to 1$ with $f(n) = \alpha_n g(n)$ for all $n \in \mathbb{N}$.

Let (X, Y) be a pair of random variables such that Y is real-valued, and X takes values in a metric space (E, ρ). Denote the Borel σ-algebras on E and $\mathbb{R}$ by $\mathcal{E}$ and $\mathcal{B}$ respectively, and the underlying probability space by $(\Omega, \mathcal{A}, \mathbb{P})$. Assume that $\|Y\|_{L^2} < \infty$, where $\|\cdot\|_{L^p} \coloneqq \mathbb{E}\left[|\cdot|^p\right]^{1/p}$ for $p \geq 1$.

2.1 Bayes Estimator

We recall some basic probabilistic notions, details on all of which can be found in [1]. If $Z\colon \Omega \to \mathbb{R}$ is an integrable random variable, then the conditional expectation $\mathbb{E}[Z \mid X]$ of Z given X is the unique (up to modification on a $\mathbb{P}$-null set) integrable, $X^{-1}(\mathcal{E})$-measurable random variable for which $\mathbb{E}\left[\mathbb{E}[Z \mid X]\, \mathbb{1}_{\{X\in A\}}\right] = \mathbb{E}\left[Z\mathbb{1}_{\{X\in A\}}\right]$ for all $A \in \mathcal{E}$. Let $(\mathbb{P}^{Y|X=x})_{x\in E}$ be a conditional distribution of Y given X. That is, $\mathbb{P}^{Y|X=x}(\cdot)\colon \mathcal{B} \to [0,1]$ is a probability measure on $\mathbb{R}$ for every $x \in E$, and $x \mapsto \mathbb{P}^{Y|X=x}(B)$ is measurable for every $B \in \mathcal{B}$, and

$$\mathbb{P}(X \in A, Y \in B) = \int_A \mathbb{P}^{Y|X=x}(B)\mathbb{P}^X(\mathrm{d}x),$$

for any $A \in \mathcal{E}$ and $B \in \mathcal{B}$, where $\mathbb{P}^X \coloneqq \mathbb{P}(X^{-1}(\cdot))\colon \mathcal{E} \to [0,1]$ denotes the law of X. Then, for any measurable $g\colon \mathbb{R} \to \mathbb{R}$, let

$$\mathbb{E}[g(Y) \mid X = x] \coloneqq \int_{\mathbb{R}} g(y)\mathbb{P}^{Y|X=x}(\mathrm{d}y).$$

Denoting the left-hand side above by $f(x)$, it is elementary to confirm that $f(X) = \mathbb{E}[g(Y) \mid X]$. Other characteristics of Y, such as its variance, can also be expressed conditional on $X = x$ by defining them in terms of $\mathbb{P}^{Y|X=x}$ instead of $\mathbb{P}^Y$.

For $x \in E$ and $k \in \mathbb{N}$, if $\|Y\|_{L^k} < \infty$, define

$$m_k(x) \coloneqq \mathbb{E}\left[Y^k \,\middle|\, X = x\right], \tag{1}$$

and put $m \coloneqq m_1$. Then, $m(X) = \mathbb{E}[Y \mid X]$, and $m\colon E \to \mathbb{R}$ is the so-called Bayes estimator of Y given X, which minimises the mean squared error in the sense that, for any measurable $\widetilde{m}\colon E \to \mathbb{R}$,

$$\mathbb{E}\left[|m(x) - Y|^2 \,\middle|\, X = x\right] \leq \mathbb{E}\left[|\widetilde{m}(x) - Y|^2 \,\middle|\, X = x\right]$$

holds for $\mathbb{P}^X$-almost all $x \in E$. Even though the Bayes estimator is optimal in the above sense, it is by no means perfect. If we put $\varepsilon \coloneqq Y - m(X)$, then

$$Y = m(X) + \varepsilon,$$

where ε is a noise with $\mathbb{E}[\varepsilon \mid X] = 0$ that describes the fluctuations of Y around $m(X)$. A measure for the strength of these fluctuations is the variance of the noise,

$$v(x) \coloneqq \mathbb{V}(\varepsilon \mid X = x) = \mathbb{V}(Y \mid X = x) = m_2(x) - m(x)^2.$$

By estimating v alongside m, one obtains a sense of the uncertainty associated with the estimation.

2.2 Nearest Neighbour Estimation

Suppose that a sequence (X_i, Y_i), $i \in \mathbb{N}$, of independent copies of (X, Y) is defined on $(\Omega, \mathcal{A}, \mathbb{P})$. Let $k_n \in \{1, \dots, n\}$, $n \in \mathbb{N}$, be a non-decreasing sequence with $k_n \to \infty$ and $k_n/n \to 0$. We now define the k_n-nearest neighbour estimators. For fixed $n \in \mathbb{N}$ and $x \in E$, let $(\sigma_1, \dots, \sigma_n)$ be the (random) permutation of $(1, \dots, n)$ such that

$$\rho(X_{\sigma_1}, x) \leq \rho(X_{\sigma_2}, x) \leq \dots \leq \rho(X_{\sigma_n}, x),$$

where, if several X_i have the same distance from x, they are ordered uniformly at random, independently of all other quantities and choices. This way, $(\sigma_1, \dots, \sigma_n)$ is any of the $n!$ permutations with equal probability. Denote the inverse permutation by $(\Sigma_1, \dots, \Sigma_n)$, that is, $\Sigma_i = \sum_{j=1}^n j \mathbb{1}_{\{\sigma_j = i\}}$ is the position of X_i in the ordered tuple $(X_{\sigma_1}, \dots, X_{\sigma_n})$. Then, in particular,

$$\mathbb{P}(\Sigma_i \leq k) = \frac{k}{n}, \qquad \mathbb{P}(\Sigma_i \vee \Sigma_j \leq k) = \frac{k(k-1)}{n(n-1)}, \tag{2}$$

whenever $i, j, k \in \{1, \dots, n\}$, $i \neq j$, where $x \wedge y \coloneqq \min(x, y)$ and $x \vee y \coloneqq \max(x, y)$ for $x, y \in \mathbb{R}$. Note that σ_i and Σ_i depend on n and x, so strictly speaking we should specify $\sigma_{(i,n)}(x)$ and $\Sigma_{(i,n)}(x)$, but we omit either or both if they are clear from the context. For $n \in \mathbb{N}$ and $k \in \mathbb{N}$, the k_n-nearest neighbour estimators of m_k and v based on the first n observations are defined by

$$\widehat{m}_k^{(n)}(x) \coloneqq \frac{1}{k_n} \sum_{i=1}^{k_n} Y^k_{\sigma_{(i,n)}(x)} = \frac{1}{k_n} \sum_{i=1}^{n} Y_i^k \mathbb{1}_{\{\Sigma_{(i,n)}(x) \leq k_n\}}, \tag{3}$$

$$\widehat{v}^{(n)}(x) \coloneqq \frac{1}{k_n} \sum_{i=1}^{k_n} \left(Y_{\sigma_i} - \widehat{m}^{(n)}(x)\right)^2 = \widehat{m}_2^{(n)}(x) - \widehat{m}^{(n)}(x)^2, \tag{4}$$

for $x \in E$, where $\widehat{m}^{(n)} \coloneqq \widehat{m}_1^{(n)}$.

2.3 Small Ball Probabilities

We introduce some notation and elementary facts regarding small ball probabilities of X. For $x \in E$, $\varepsilon, \delta > 0$, define

$$p_x(\delta) \coloneqq \mathbb{P}(X \in \overline{B}(x, \delta)), \tag{5}$$

$$p_x^{-1}(\varepsilon) \coloneqq \inf\{\delta \geq 0 \colon p_x(\delta) \geq \varepsilon\}, \tag{6}$$

where $\overline{B}(x, \delta) = \{y \in E \colon \rho(x, y) \leq \delta\}$. The support of X is the closed set

$$\mathrm{S}(X) \coloneqq \{x \in E \colon \forall \delta > 0 \colon p_x(\delta) > 0\}.$$

Lemma 2.1. *Let $x \in E$.*

(i) $p_x \colon [0, \infty) \to [0, 1]$ *and* $p_x^{-1} \colon [0, 1] \to [0, \infty)$ *are increasing and respectively right- and left-continuous.*

(ii) $p_x(0) = \mathbb{P}(X = x)$, $p_x^{-1}(0) = 0$.

(iii) $p_x(p_x^{-1}(\varepsilon)) \geq \varepsilon$, *for all* $\varepsilon > 0$.

(iv) *If* $x \in \mathrm{S}(X)$, *then* $p_x^{-1}(\varepsilon) \downarrow 0$ *as* $\varepsilon \downarrow 0$.

Proof. (i) Monotonicity is obvious and right-continuity of p_x is just continuity from above of $\mathbb{P}^X$. Let $0 < \varepsilon_n \uparrow \varepsilon_0$, and put $\delta_n := p_x^{-1}(\varepsilon_n)$, $n \in \mathbb{N}_0$. Then, $0 \leq \delta_n \uparrow \delta$ for some $0 \leq \delta \leq \delta_0$ (because $\delta_n \leq \delta_0$ for all $n \in \mathbb{N}$). Hence it suffices to show that $\delta \geq \delta_0$, that is, that $p_x(\delta) \geq \varepsilon_0$. But this is clear because $p_x(\delta) \geq p_x(\delta_n) \geq \varepsilon_n \uparrow \varepsilon_0$.

(ii) Clear from the definitions.

(iii) Follows from Eq. (6) and right-continuity of p_x.

(iv) If $x \in \mathrm{S}(X)$, then $p_x(\delta) > 0$ for all $\delta > 0$. Hence, for every $\delta > 0$ there is $\varepsilon > 0$ with $p_x(\delta) \geq \varepsilon$ and thus $p_x^{-1}(\varepsilon) \leq \delta$. □

3 Main Results

The following two theorems are our main results. Recall the definitions of the moments m_k ($k \in \mathbb{N}$) of Y given X, and of the nearest neighbour estimator $\widehat{m}_k^{(n)}$ of m_k based on the first n observations (see Eqs. (1) and (3)). Also, recall that we assume a sequence $(k_n)_{n\in\mathbb{N}}$, $k_n \in \{1, \ldots, n\}$ with $k_n \to \infty$ and $k_n/n \to 0$ to be given. We call a function $h\colon E \to \mathbb{R}$ γ-Hölder continuous at $x \in E$ for $\gamma \in (0, 1]$ if there are $c, \delta > 0$ such that $|h(y) - h(z)| \leq c\rho(y, z)^\gamma$ for all $y, z \in B(x, \delta)$.

Theorem 3.1. *Let* $k \in \mathbb{N}$. *Suppose there exists* $p > 2k$ *such that* $\|Y\|_{L^p} < \infty$, *that* m_k *is continuous, and that* m_{2k} *is locally bounded. In this case, the following statements hold:*

(i) *If* $C \subset \mathrm{S}(X)$ *is compact, then* $\widehat{m}_k^{(n)}(\cdot) \xrightarrow{L^2} m_k(\cdot)$ *uniformly on* C. *That is,*

$$\sup_{x\in C} \left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2} \to 0, \quad n \to \infty.$$

(ii) *If* $x \in \mathrm{S}(X)$ *and* m_k *is* γ*-Hölder continuous at* x *for some* $\gamma \in (0, 1]$, *then, for any sequence* $0 < \alpha_n = o(n/k_n)$, *there is a* $c > 0$ *such that*

$$\left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2}^2 \leq c \left(\frac{1}{k_n} + p_x^{-1}\left((1 + \alpha_n)\frac{k_n}{n} \right)^\gamma + n^2 \exp\left(-\frac{k_n}{2q} \frac{\alpha_n^2}{1 + \alpha_n} \right) \right), \tag{7}$$

where $q = \frac{p}{p-k} \in (1, 2)$. *In particular,*

$$\left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2}^2 \leq c \left(\frac{1}{k_n} + p_x^{-1}\left(\frac{2k_n}{n} \right)^\gamma + n^2 \mathrm{e}^{-k_n/8} \right).$$

Theorem 3.2. *Suppose there is* $p > 4$ *such that* $\|Y\|_{L^p} < \infty$, *and that* m, m_2 *are continuous and* m_4 *is locally bounded. Then,* $\widehat{v}^{(n)}(\cdot) \xrightarrow{L^1} v(\cdot)$ *as* $n \to \infty$, *uniformly on compact subsets of* $\mathrm{S}(X)$.

4 Examples

Before moving on to the proofs, we demonstrate how Theorem 3.1 can be used to derive explicit convergence rates in the cases where X is finite-dimensional (in the following sense), or a Gaussian random function.

Definition 4.1. Let $s > 0$. We say that X is *at most s-dimensional at $x \in E$* if there exists $c > 0$ and $\delta_0 \in (0, 1)$ such that

$$p_x(\delta) \geq c\delta^s, \quad \delta \in (0, \delta_0). \tag{8}$$

We say that X is *at most s-dimensional* if X is at most s-dimensional at every $x \in \mathrm{S}(X)$.

Remark 4.2. (i) The reason we call X 'at most' s-dimensional in Definition 4.1 is that if Eq. (8) holds for some $s > 0$, then it also holds for all $s' > s$. One might thus be tempted to define the dimension s_X of X as the infimum over all s for which Eq. (8) holds, but in this case Eq. (8) does not in general also hold for s_X.

(ii) Suppose that E has Hausdorff dimension $s > 0$, and that there are measurable sets $E_n \uparrow E$ such that the Hausdorff measure $\mathcal{H}^s \colon \mathcal{B}(E) \to [0, \infty]$ satisfies $\mathcal{H}^s(E_n) \in (0, \infty)$ for all $n \in \mathbb{N}$. If X has a positive, continuous density with respect to $\mathcal{H}^s$ (which in this case is a sort of uniform measure on E) then X is at most s-dimensional in the sense of Definition 4.1. This includes the case of absolutely continuous random variables X on $\mathbb{R}^d$. See [10] for details.

Theorem 4.3. *Let $k \in \mathbb{N}$, $x \in \mathrm{S}(X)$, suppose that m_k is γ-Hölder continuous at x for some $\gamma \in (0, 1]$, and that X is at most s-dimensional at x. Then, if $k_n \gg \log n$,*

$$\left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2}^2 = O\left(\frac{1}{k_n} + \left(\frac{k_n}{n} \right)^{\gamma/s} \right). \tag{9}$$

If k_n, $n \in \mathbb{N}$, optimises this bound, then

$$k_n \sim c n^{\frac{1}{1+s/\gamma}},$$

where $c = \left(\frac{s}{\gamma} \right)^{s/(s+\gamma)}$. In that case, and in fact whenever $k_n \sim c' n^{\frac{1}{1+s/\gamma}}$ for some $c' > 0$, then

$$\left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2} = O\left(n^{-\frac{1}{2+2s/\gamma}} \right). \tag{10}$$

Note that we only considered $k_n \gg \log n$ above, but we know that $k_n = O(\log n)$ cannot transform Eq. (7) into a better bound than Eq. (10), as it would be no smaller than $\frac{1}{k_n} \geq \frac{1}{C \log n} \gg n^{-1/(2+2s/\gamma)}$.

Remark 4.4. (i) The assumptions of Theorem 4.3 include as domains for X manifolds that cannot be embedded into $\mathbb{R}^d$, as well as more general fractal spaces. For example, this includes applications that regress on quaternions, as Lang et al. [14] have previously done in the context of 6D object tracking.

(ii) Theorem 4.3 includes the case where $s = d \in \mathbb{N}$ and $E = \mathbb{R}^d$. In this setting, known results assert a slightly faster convergence rate of $O(n^{-1/(2+s/\gamma)})$, albeit under stronger regularity conditions such as sufficient differentiability of m [2], or with a weaker notion of convergence [12]. Still, it is reasonable to assume that a refinement of the arguments presented here yields the aforementioned stronger rate of convergence also in this more general setting.

We now turn to the case where X is a *centred Gaussian process* on $[0, 1]$. That is, we assume that $X = (X_t)_{t \in [0,1]}$ takes values in $C([0, 1], \mathbb{R})$—which is a complete and separable normed space when equipped with the supremum norm—and that for any $n \in \mathbb{N}$ and $t_1, \ldots, t_n \in [0, 1]$, $(X_{t_1}, \ldots, X_{t_n})$ has an n-dimensional centred Gaussian distribution. In this case, X is fully determined by its *covariance function* or *kernel*

$$K(t, s) \coloneqq \mathbb{E}\left[X_t X_s \right], \quad t, s \in [0, 1]. \tag{11}$$

We suppose that there is $\beta > 0$ such that for every $t, s \in [0,1]$,

$$|K(t,s) - K(t,t)| \leq c\,|t-s|^{2\beta}\,, \tag{12}$$

for some constant $c > 0$. For example, if K is Lipschitz continuous, then Eq. (12) is satisfied with $\beta = 1/2$. One can think of β as a measure of the regularity of X.

The *reproducing kernel Hilbert space (RKHS)* or *Cameron–Martin space* $\mathcal{H}$ of X is a dense linear subspace of $\mathrm{S}(X)$ with a scalar product that turns $\mathcal{H}$ into a Hilbert space, see [3] for details.

Remark 4.5. Similar to the universality of the Gaussian distribution in the context of naturally occurring, real-valued data, Gaussian processes are commonly used to model functional data. The following are two of the most commonly used kernels:

(i) The *exponential kernel* is defined by

$$K(t,s) = \mathrm{e}^{-|t-s|/\sigma}, \quad t, s \in [0,1],$$

for some $\sigma > 0$. In this case $\beta = 1/2$ and $\mathcal{H} = C^1([0,1])$.

(ii) The *squared exponential kernel* is defined by

$$K(t,s) = \mathrm{e}^{-(t-s)^2/(2\sigma^2)}, \quad t, s \in [0,1],$$

where $\sigma > 0$. In this case $\beta = 1$, and $\mathcal{H}$ contains $C^\infty([0,1])$.

In both cases, σ determines the length scale of X. The squared exponential kernel is often used if the data is expected to be smooth, while the exponential kernel tends to work well for more ragged functions. A discussion of a variety of kernels in the context of Gaussian process regression can be found in section 4.2.1 of [16].

Theorem 4.6. *Let $k \in \mathbb{N}$, $x \in \mathcal{H}$, and suppose that m_k is γ-Hölder continuous at x for some $\gamma \in (0,1]$. Then,*

$$\left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2}^2 = O\left(\frac{1}{k_n} + \log\left(\frac{n}{k_n} \right)^{-\gamma\beta} \right). \tag{13}$$

If $k_n \in \mathbb{N}$ optimises this bound, then

$$k_n \sim \tfrac{1}{\gamma\beta}\,(\log n)^{1+\gamma\beta}\,.$$

In that case, and in fact whenever $(\log n)^{\gamma\beta} \ll k_n = O(n^a)$ for some $a \in (0,1)$, then

$$\left\| \widehat{m}_k^{(n)}(x) - m_k(x) \right\|_{L^2} = O\left((\log n)^{-\gamma\beta/2} \right). \tag{14}$$

Remark 4.7. (i) The convergence rate obtained in Theorem 4.6 seems rather slow at first glance, but considering that the optimal rate for nearest neighbour estimation in $\mathbb{R}^r$ is $O(n^{-1/(2+r/\gamma)})$, and that function spaces such as $C([0,1],\mathbb{R})$ are infinite-dimensional, it is not surprising that the convergence rate is slower than n^{-c} for any $c > 0$. It is also noteworthy that although the bound in Eq. (13) is optimised by a logarithmic k_n, its asymptotic form Eq. (14) remains unchanged through a wide range of values for k_n, including $k_n \sim n^a$ for any $a \in (0,1)$. A possible direction for future work could aim for lower bounds on convergence rates in the functional setting which may lead to further insights regarding the optimal choice for k_n.

(ii) These arguments can be generalised to Gaussian processes in $C([0,1]^d, \mathbb{R})$ for $d \in \mathbb{N}$. In that case, the statements of Lemma 5.9 and Theorem 4.6 remain true with β replaced by β/d. In particular, the convergence rates are improved by higher regularity of X and m_k (i.e. large β and γ), and small dimension d.

(iii) The above can readily be extended to non-centred Gaussian processes X by considering the centred process $X - m$, where $m(\cdot) = \mathbb{E}[X(\cdot)]$ is the mean function of X.

5 Proofs

Recall Section 2.2, in particular the fact that we assume a sequence $k_n \in \{1, \dots, n\}$, $n \in \mathbb{N}$, to be given that satisfies $k_n \to \infty$ and $k_n/n \to 0$. Write $\rho_i := \rho_i(x) := \rho(X_i, x)$ for $x \in E$ and $i \in \mathbb{N}$, where x is omitted if clear from context. For $n \in \mathbb{N}$ and $F \subset E$

$$N_n(F) := |\{i \in [n] \colon X_i \in F\}|, \quad N_n(x, \delta) := N_n(B(x, \delta)),$$

where $[n] = \{1, \dots, n\}$. The following two lemmas show that, for fixed $\delta > 0$ and $x \in E$, it is exponentially likely that all of the k_n closest X_i's lie inside $B(x, \delta)$.

Lemma 5.1. *Let $n \in \mathbb{N}$, and $F \subset E$. If $p_F := \mathbb{P}(X \in F) > k_n/n$, then*

$$\mathbb{P}(N_n(F) < k_n) \le \exp\left(-\frac{n}{2p_F}\left(p_F - \frac{k_n}{n}\right)^2\right).$$

If $\delta > 0$ is fixed, and either $A = B(x_0, \delta/2)$ with $x_0 \in \mathrm{S}(X)$, or $A \subset \mathrm{S}(X)$ is compact, then there exists $c > 0$ such that

$$\mathbb{P}(\exists x \in A \colon N_n(x, \delta) < k_n) = O\left(\mathrm{e}^{-cn}\right).$$

Proof. Each X_i falls into F independently with probability p_F, so $N_n(F) \sim \mathrm{Bin}(n, p_F)$. A standard Chernoff bound yields, since $np_F > k_n$,

$$\begin{aligned}
\mathbb{P}(N_n(F) < k_n) &= \mathbb{P}\left(\mathrm{Bin}(n, p_F) < np_F\left(1 - \left(1 - \frac{k_n}{np_F}\right)\right)\right) \\
&\le \exp\left(-\frac{np_F}{2}\left(1 - \frac{k_n}{np_F}\right)^2\right) \\
&= \exp\left(-\frac{n}{2p_F}\left(p_F - \frac{k_n}{n}\right)^2\right).
\end{aligned}$$

If $A = B(x_0, \delta/2)$ for some $x_0 \in \mathrm{S}(X)$, then $B(x_0, \delta/2) \subset B(x, \delta)$ for all $x \in A$, and $p' := \mathbb{P}(X \in B(x_0, \delta/2)) > 0$, so by what we have already shown,

$$\begin{aligned}
\mathbb{P}(\exists x \in A \colon N_n(x, \delta) < k_n) &\le \mathbb{P}\left(N_n(x_0, \delta/2) < k_n\right) \\
&\le \exp\left(-\frac{n}{2p'}\left(p' - \frac{k_n}{n}\right)^2\right) \\
&= O\left(\mathrm{e}^{-np'/8}\right),
\end{aligned}$$

where we used in the last step that $\frac{k_n}{n} \leq \frac{p'}{2}$ for sufficiently large $n \in \mathbb{N}$. The claim for compact $A \subset \mathrm{S}(X)$ follows because A can be covered in finitely many balls of the form $B(x, \delta/2)$ with $x \in A$. □

Lemma 5.2. *If $n \in \mathbb{N}$, $x \in E$, $\delta > 0$, and $I \subset [n]$, $|I| \leq k_n$, then*

$$\mathbb{P}\left(\exists i \in I \colon \rho_i \geq \delta \,\middle|\, \bigvee_{i \in I} \Sigma_i \leq k_n\right) \leq \mathbb{P}(N_n(x, \delta) < k_n).$$

Proof. We assume $I = \{1\}$, the general proof merely requires more notation. Denote by S_n the set of permutations of $[n]$, put $A_\pi \coloneqq \{(\sigma_1, \ldots, \sigma_n) = \pi\}$ for $\pi \in S_n$, and abbreviate $B \coloneqq \{N_n(x, \delta) < k_n\}$. Then,

$$\begin{aligned}
\mathbb{P}(B \cap A_\pi) &= \mathbb{P}\left(\text{Less than } k_n \text{ of the } (X_i)_{i=1}^n \text{ lie in } B(x, \delta), A_\pi\right) \\
&= \mathbb{P}\left(\text{Less than } k_n \text{ of the } (X_{\pi(i)})_{i=1}^n \text{ lie in } B(x, \delta), A_\pi\right) \\
&= \mathbb{P}\left(\text{Less than } k_n \text{ of the } (X_i)_{i=1}^n \text{ lie in } B(x, \delta), A_{\mathrm{id}}\right) \\
&= \mathbb{P}(B \cap A_{\mathrm{id}}),
\end{aligned}$$

where $\mathrm{id} \coloneqq (1, \ldots, n) \in S_n$, and we used in the third step that $(X_1, \ldots, X_n)$ and $(X_{\pi(1)}, \ldots, X_{\pi(n)})$ are equal in distribution. Hence, for any $\pi \in S_n$,

$$\mathbb{P}(B) = \sum_{\tau \in S_n} \mathbb{P}(B \cap A_\tau) = \sum_{\tau \in S_n} \mathbb{P}(B \cap A_\pi) = n! \mathbb{P}(B \cap A_\pi).$$

Now observe that $\{\Sigma_1 \leq k_n\} = \bigcup_{\substack{\pi \in S_n \\ \pi(1) \leq k_n}} A_\pi$, so

$$\begin{aligned}
\mathbb{P}\left(B \cap \{\Sigma_1 \leq k_n\}\right) &= \sum_{\substack{\pi \in S_n \\ \pi(1) \leq k_n}} \mathbb{P}(B \cap A_\pi) = k_n (n-1)! \cdot \frac{1}{n!} \mathbb{P}(B) = \frac{k_n}{n} \mathbb{P}(B) \\
&= \mathbb{P}(\Sigma_1 \leq k_n) \mathbb{P}(B),
\end{aligned}$$

where we used Eq. (2) in the last step. Finally, note that $\{\rho_1 \geq \delta\} \cap \{\Sigma_1 \leq k_n\} \subset B \cap \{\Sigma_1 \leq k_n\}$, so

$$\mathbb{P}(B) \mathbb{P}(\Sigma_1 \leq k_n) = \mathbb{P}(B, \Sigma_1 \leq k_n) \geq \mathbb{P}(\rho_1 \geq \delta, \Sigma_1 \leq k_n).$$

Dividing by $\mathbb{P}(\Sigma_1 \leq k_n)$ finishes the proof. □

Theorem 3.2 will turn out to be an immediate corollary from Theorem 3.1. The idea behind the proof of the latter is to show that $\mathbb{E}\left[\widehat{m}_k^{(n)}(\cdot)\right]$ goes to $m_k(\cdot)$ and $\mathbb{V}\left(\widehat{m}^{(n)}(\cdot)\right)$ vanishes as $n \to \infty$. We begin by proving some lemmas.

Lemma 5.3. *If $k \in \mathbb{N}$ and $\|Y\|_{L^k} < \infty$, then*

$$\mathbb{E}\left[\widehat{m}_k^{(n)}(x)\right] = \mathbb{E}\left[m_k(X_1) \,|\, \Sigma_1(x) \leq k_n\right], \quad n \in \mathbb{N}, x \in E.$$

Proof. We may assume $k = 1$ (otherwise consider $\widetilde{Y} := Y^k$). Let $n \in \mathbb{N}$ and $x \in E$. Then, by Eq. (3),

$$\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] = \frac{1}{k_n}\sum_{j=1}^{n}\mathbb{E}\left[Y_j \mathbb{1}_{\{\Sigma_j(x)\le k_n\}}\right] = \frac{n}{k_n}\mathbb{E}\left[Y_1 \mathbb{1}_{\{\Sigma_1(x)\le k_n\}}\right].$$

Using the independence of all the (X_i, Y_i) and basic properties of conditional expectation,

$$\begin{aligned}\mathbb{E}\left[Y_1 \mathbb{1}_{\{\Sigma_1(x)\le k_n\}}\right] &= \mathbb{E}\left[\mathbb{E}\left[Y_1 \mathbb{1}_{\{\Sigma_1(x)\le k_n\}} \,\middle|\, X_1, \dots, X_n\right]\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[Y_1 \,|\, X_1, \dots, X_n\right] \mathbb{1}_{\{\Sigma_1(x)\le k_n\}}\right] \\ &= \mathbb{E}\left[m(X_1)\mathbb{1}_{\{\Sigma_1(x)\le k_n\}}\right] \\ &= \frac{k_n}{n}\mathbb{E}\left[m(X_1) \,|\, \Sigma_1(x) \le k_n\right],\end{aligned}$$

where we used Eq. (2) in the last step. □

If $h\colon E \to \mathbb{R}$ and $A \subset E$, we write $\|h\|_A := \sup_{x\in A}|h(x)|$.

Lemma 5.4. *Let $k \in \mathbb{N}$. Suppose that there exists $p > k$ with $\|Y\|_{L^p} < \infty$, and put $q := \frac{p}{p-k}$. Let $\delta > 0$.*

(i) For $x \in E$ and $n \in \mathbb{N}$,

$$\mathbb{E}\left[|m_k(X_1) - m_k(x)|\,\mathbb{1}_{\{\rho_1\ge\delta\}} \,\middle|\, \Sigma_1 \le k_n\right] \le \left(n\,\|Y\|_{L^p} + |m_k(x)|\right)\mathbb{P}\left(N_n(x,\delta) < k_n\right)^{1/q},$$

(ii) For $x \in E$ and $n \in \mathbb{N}$,

$$\begin{aligned}\mathbb{E}\left[\left|m_k(X_1)m_k(X_2) - m_k(x)^2\right|\mathbb{1}_{\{\rho_1\vee\rho_2\ge\delta\}} \,\middle|\, \Sigma_1 \vee \Sigma_2 \le k_n\right] \\ \le \left(n^2\,\|Y\|_{L^p}^2 + |m_k(x)|^2\right)\mathbb{P}\left(N_n(x,\delta) < k_n\right)^{1/q}.\end{aligned}$$

For fixed $\delta > 0$, if $A \subset E$ satisfies $\|m_k\|_A < \infty$ and is either a ball of radius $\delta/2$ with centre in $\mathrm{S}(X)$ or $A \subset \mathrm{S}(X)$ is compact, then both bounds vanish uniformly on A as $n \to \infty$.

Proof. We may assume that $k = 1$ (otherwise consider $\widetilde{Y} := Y^k$ and $\widetilde{p} := p/k > 1$). Fix $\delta > 0$, and let $x \in E$ and $n \in \mathbb{N}$.

(i) Abbreviate $A := \{\Sigma_1(x) \le k_n\}$, and observe that

$$\begin{aligned}\mathbb{E}\left[|m(X_1) - m(x)|\,\mathbb{1}_{\{\rho(X_1,x)\ge\delta\}} \,\middle|\, A\right] \\ \le \mathbb{E}\left[|m(X_1)|\,\mathbb{1}_{\{\rho(X_1,x)\ge\delta\}} \,\middle|\, A\right] + |m(x)|\underbrace{\mathbb{P}\left(\rho(X_1,x)\ge\delta \,|\, A\right)}_{\le\mathbb{P}(\rho(X_1,x)\ge\delta\,|\,A)^{1/q}}.\end{aligned}$$

Since $\frac{1}{q} + \frac{1}{p} = 1$, Hölder's inequality gives

$$\begin{aligned}
\mathbb{E}\left[|m(X_1)|\,\mathbb{1}_{\{\rho(X_1,x)\geq\delta\}} \,\middle|\, A\right] &= \frac{\mathbb{E}\left[|m(X_1)|\,\mathbb{1}_{\{\rho(X_1,x)\geq\delta\}\cap A}\right]}{\mathbb{P}(A)} \\
&\leq \mathbb{E}\left[|m(X_1)|^p\right]^{1/p} \frac{\mathbb{P}(\rho(X_1,x)\geq\delta,\, A)^{1/q}}{\mathbb{P}(A)} \\
&= \mathbb{E}\left[|\mathbb{E}\left[Y_1 \mid X_1\right]|^p\right]^{1/p} \mathbb{P}(A)^{1/q-1} \mathbb{P}\left(\rho(X_1,x)\geq\delta \mid A\right)^{1/q} \\
&\leq \mathbb{E}\left[\mathbb{E}\left[|Y_1|^p \mid X_1\right]\right]^{1/p} \left(\frac{n}{k_n}\right)^{1-1/q} \mathbb{P}\left(\rho(X_1,x)\geq\delta \mid A\right)^{1/q} \\
&\leq n\,\|Y_1\|_{L^p}\,\mathbb{P}\left(\rho(X_1,x)\geq\delta \mid A\right)^{1/q},
\end{aligned}$$

where we used Jensen's inequality and Eq. (2) in the penultimate step. Finally, $\|Y_1\|_{L^p} = \|Y\|_{L^p}$, and $\mathbb{P}\left(\rho(X_1,x)\geq\delta \mid A\right) \leq \mathbb{P}(N_n(x,\delta) < k_n)$ by Lemma 5.2.

(ii) Abbreviate $A \coloneqq \{\Sigma_1(x) \vee \Sigma_2(x) \leq k_n\}$. Then,

$$\begin{aligned}
&\mathbb{E}\left[\left|m(X_1)m(X_2) - m(x)^2\right| \mathbb{1}_{\{\rho_1\vee\rho_2\geq\delta\}} \,\middle|\, A\right] \\
&\qquad\leq \underbrace{\mathbb{E}\left[|m(X_1)m(X_2)|\,\mathbb{1}_{\{\rho_1\vee\rho_2\geq\delta\}} \,\middle|\, A\right]}_{(*)} + |m(x)|^2 \underbrace{\mathbb{P}\left(\rho_1\vee\rho_2\geq\delta \mid A\right)}_{\leq\mathbb{P}(\rho_1\vee\rho_2\geq\delta \mid A)^{1/q}}.
\end{aligned}$$

Hölder's inequality gives

$$\begin{aligned}
(*) &= \frac{1}{\mathbb{P}(A)}\mathbb{E}\left[|m(X_1)m(X_2)|\,\mathbb{1}_{\{\rho_1\vee\rho_2\geq\delta\}\cap A}\right] \\
&\leq \mathbb{E}\left[|m(X_1)m(X_2)|^p\right]^{1/p} \frac{\mathbb{P}\left(\rho_1\vee\rho_2\geq\delta, A\right)^{1/q}}{\mathbb{P}(A)} \\
&= \mathbb{E}\left[|m(X)|^p\right]^{2/p} \mathbb{P}(A)^{1/q-1} \mathbb{P}\left(\rho_1\vee\rho_2\geq\delta \mid A\right)^{1/q} \\
&\leq \|Y\|_{L^p}^2 \left(\frac{n(n-1)}{k_n(k_n-1)}\right)^{1-1/q} \mathbb{P}\left(\rho_1\vee\rho_2\geq\delta \mid A\right)^{1/q} \\
&\leq n^2\,\|Y\|_{L^p}^2\,\mathbb{P}\left(\rho_1\vee\rho_2\geq\delta \mid A\right)^{1/q},
\end{aligned}$$

where we used Eq. (2) and that X_1 and X_2 are independent and equal in distribution to X. Lemma 5.2 finishes the argument.

For fixed $\delta > 0$, if $A \subset E$ satisfies $\|m\|_A < \infty$ and is either a ball of radius $\delta/2$ with centre in $\mathrm{S}(X)$ or $A \subset \mathrm{S}(X)$ is compact, then Lemma 5.1 implies that both bounds vanish uniformly on A as $n \to \infty$. □

Lemma 5.5. *Let $k \in \mathbb{N}$, and suppose there is $p > k$ with $\|Y\|_{L^p} < \infty$. Then, for any $x_0 \in E$ and $\delta > 0$,*

$$\overline{\lim_{n\to\infty}} \left(\sup_{x\in B(x_0,\delta)} \left|\mathbb{E}\left[\widehat{m}_k^{(n)}(x)\right]\right|\right) \leq \|m_k\|_{B(x_0,3\delta)}.$$

In particular, if m_k is locally bounded, then so is $\sup_{n\in\mathbb{N}} \left|\mathbb{E}\left[\widehat{m}_k^{(n)}(\cdot)\right]\right|$.

Proof. We may assume $k = 1$. Let $x_0 \in E$, $\delta > 0$, and abbreviate $A_n(\cdot) \coloneqq \{\Sigma_{(1,n)}(\cdot) \leq k_n\}$. If $\|m\|_{B(x_0,3\delta)} = \infty$, there is nothing to show, hence, we assume otherwise. Then, by Lemma 5.3,

$$\begin{aligned}\sup_{x\in B(x_0,\delta)} \left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right]\right| &= \sup_{x\in B(x_0,\delta)} \left|\mathbb{E}\left[m(X_1)(\mathbb{1}_{\{\rho_1<2\delta\}} + \mathbb{1}_{\{\rho_1\geq 2\delta\}}) \,\middle|\, A_n(x)\right]\right| \\ &\leq \|m\|_{B(x_0,3\delta)} + \sup_{x\in B(x_0,\delta)} \mathbb{E}\left[|m(X_1)|\,\mathbb{1}_{\{\rho_1\geq 2\delta\}} \,\middle|\, A_n(x)\right].\end{aligned}$$

By Lemmas 5.1 and 5.4, the latter summand vanishes as $n \to \infty$. □

Proposition 5.6. *Let $k \in \mathbb{N}$, suppose there exists $p > k$ with $\|Y\|_{L^p} < \infty$, and put $q \coloneqq \frac{p}{p-k}$.*

(i) If m_k is continuous, then $\mathbb{E}\left[\widehat{m}_k^{(n)}(\cdot)\right] \to m_k(\cdot)$ uniformly on compact subsets of $\mathrm{S}(X)$.

(ii) If $x \in \mathrm{S}(X)$ and m_k is γ-Hölder continuous at x for some $\gamma \in (0,1]$, then, for any sequence $0 < \alpha_n = o(n/k_n)$,

$$\left|\mathbb{E}\left[\widehat{m}_k^{(n)}(x)\right] - m_k(x)\right| = O\left(p_x^{-1}\left((1+\alpha_n)\frac{k_n}{n}\right)^{\gamma} + n\exp\left(-\frac{\alpha_n^2}{1+\alpha_n}\frac{k_n}{2q}\right)\right).$$

Proof. Assume $k = 1$ and $\mathrm{S}(X) = E$ (otherwise consider $\widetilde{Y} \coloneqq Y^k$, $\widetilde{p} \coloneqq p/k > 1$, and $\widetilde{E} \coloneqq \mathrm{S}(X)$). Abbreviate $A \coloneqq \{\Sigma_1(x) \leq k_n\}$ (n and x will be clear from context). If $x \in E$, and $n \in \mathbb{N}$, then Lemma 5.3 implies that, for any $\delta > 0$,

$$\begin{aligned}\left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right| &= \left|\mathbb{E}\left[m(X_1) - m(x) \mid A\right]\right| \\ &\leq \mathbb{E}\left[|m(X_1) - m(x)|\,\mathbb{1}_{\{\rho(X_1,x)<\delta\}} \mid A\right] \\ &\quad + \mathbb{E}\left[|m(X_1) - m(x)|\,\mathbb{1}_{\{\rho(X_1,x)\geq\delta\}} \mid A\right].\end{aligned} \tag{15}$$

(i) If $C \subset E$ is compact, then, by continuity of m, for any $\varepsilon > 0$ there exists some $\delta > 0$ such that $|m(x) - m(x')| < \varepsilon$ whenever $x \in C$ and $x' \in E$ with $\rho(x,x') < \delta$, so that, for such δ, the former summand on the right-hand side (RHS) of Eq. (15) is at most ε for any $x \in C$. The latter summand vanishes uniformly on C as $n \to \infty$ by Lemmas 5.1 and 5.4 and since $\|m\|_C < \infty$ by continuity of m. Hence,

$$\overline{\lim_{n\to\infty}}\left(\sup_{x\in C}\left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right|\right) \leq \varepsilon,$$

for any $\varepsilon > 0$.

(ii) Suppose that $x \in E$ and that there exist $\gamma \in (0,1]$ and $c_1, \delta_0 > 0$ such that $|m(y) - m(z)| \leq c_1\rho(y,z)^\gamma$ for all $y, z \in B(x,\delta_0)$. Then, for any $n \in \mathbb{N}$ and $0 < \delta < \delta_0$ with $p_x(\delta) > \frac{k_n}{n}$, Eq. (15) and Lemmas 5.1 and 5.4 imply

$$\begin{aligned}\left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right| &\leq c_1\delta^\gamma + (|m(x)| + n\|Y\|_{L^p})\,\mathbb{P}\left(N_n(x,\delta) < k_n\right)^{1/q} \\ &\leq c_1\delta^\gamma + (|m(x)| + n\|Y\|_{L^p})\exp\left(-\frac{n}{2qp_x(\delta)}\left(p_x(\delta) - \frac{k_n}{n}\right)^2\right),\end{aligned}$$

where $q = \frac{p}{p-1} \in (1,\infty)$. Let $(\alpha_n) \in (0,\infty)^{\mathbb{N}}$ be such that $\alpha_n = o(n/k_n)$, so that $0 < \varepsilon_n := (1+\alpha_n)\frac{k_n}{n} \to 0$. Then, by Lemma 2.1, $0 < \delta_n := p_x^{-1}(\varepsilon_n) < \delta_0$ for large enough $n \in \mathbb{N}$, and $p_x(\delta_n) \geq \varepsilon_n > k_n/n$. Thus, for large enough $n \in \mathbb{N}$,

$$\begin{aligned}\left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right| &\leq c_1\delta_n^\gamma + 2n\left\|Y\right\|_{L^p}\exp\left(-\frac{n}{2q\varepsilon_n}\left(\varepsilon_n - \frac{k_n}{n}\right)^2\right)\\ &= c_1 p_x^{-1}\left((1+\alpha_n)\frac{k_n}{n}\right)^\gamma + 2n\left\|Y\right\|_{L^p}\exp\left(-\frac{k_n}{2q}\frac{\alpha_n^2}{1+\alpha_n}\right),\end{aligned}$$

which implies (ii). □

Proposition 5.7. *Let $k \in \mathbb{N}$, and suppose there exists $p > 2k$ with $\|Y\|_{L^p} < \infty$, and that m_k is continuous and m_{2k} is locally bounded. Put $q := \frac{p}{p-k}$.*

(i) $\mathbb{V}\left(\widehat{m}_k^{(n)}(\cdot)\right) \to 0$ *uniformly on compact subsets of* $\mathrm{S}(X)$,

(ii) If $x \in \mathrm{S}(X)$ and m_k is γ-Hölder continuous at x for some $\gamma \in (0,1]$, then, for any sequence $0 < \alpha_n = o(n/k_n)$,

$$\mathbb{V}\left(\widehat{m}_k^{(n)}(x)\right) = O\left(\frac{1}{k_n} + p_x^{-1}\left((1+\alpha_n)\frac{k_n}{n}\right)^\gamma + n^2\exp\left(-\frac{k_n}{2q}\frac{\alpha_n^2}{1+\alpha_n}\right)\right).$$

Proof. We may again assume $k = 1$ and $\mathrm{S}(X) = E$. Let $x \in E$ and $n \in \mathbb{N}$. Then,

$$\begin{aligned}\mathbb{E}\left[\widehat{m}^{(n)}(x)^2\right] &= \frac{1}{k_n^2}\sum_{i,j=1}^n \mathbb{E}\left[Y_iY_j\mathbb{1}_{\{\Sigma_i(x)\leq k_n,\Sigma_j(x)\leq k_n\}}\right]\\ &= \frac{1}{k_n^2}\Big(\sum_{i=1}^n \mathbb{E}\left[Y_i^2\mathbb{1}_{\{\Sigma_i(x)\leq k_n\}}\right] + n(n-1)\mathbb{E}\left[Y_1Y_2\mathbb{1}_{\{\Sigma_1(x)\leq k_n,\Sigma_2(x)\leq k_n\}}\right]\Big) \qquad (16)\\ &= \frac{1}{k_n}\mathbb{E}\left[\widehat{m}_2^{(n)}(x)\right] + \frac{n(n-1)}{k_n^2}\mathbb{E}\left[Y_1Y_2\mathbb{1}_{\{\Sigma_1(x)\leq k_n,\Sigma_2(x)\leq k_n\}}\right].\end{aligned}$$

Conditioning on X_1, X_2 in the second summand, and using Eq. (2) yields

$$\begin{aligned}\mathbb{E}\left[Y_1Y_2\mathbb{1}_{\{\Sigma_1(x)\vee\Sigma_2(x)\leq k_n\}}\right] &= \mathbb{E}\left[m(X_1)m(X_2)\mathbb{1}_{\{\Sigma_1(x)\vee\Sigma_2(x)\leq k_n\}}\right]\\ &= \frac{k_n(k_n-1)}{n(n-1)}\mathbb{E}\left[m(X_1)m(X_2)\,|\,\Sigma_1(x)\vee\Sigma_2(x)\leq k_n\right],\end{aligned}$$

which we combine with Eq. (16) to obtain, abbreviating $A := A_n(x) := \{\Sigma_1(x)\vee\Sigma_2(x)\leq k_n\}$,

$$\mathbb{E}\left[\widehat{m}^{(n)}(x)^2\right] = \frac{1}{k_n}\mathbb{E}\left[\widehat{m}_2^{(n)}(x)\right] + \left(1-\frac{1}{k_n}\right)\mathbb{E}\left[m(X_1)m(X_2)\,|\,A\right].$$

We conclude that

$$\mathbb{V}\left(\widehat{m}^{(n)}(x)\right) = \mathbb{E}\left[\widehat{m}^{(n)}(x)^2\right] - \mathbb{E}\left[\widehat{m}^{(n)}(x)\right]^2 \leq \overbrace{\left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right]^2 - m(x)^2\right|}^{(*_1)} + \underbrace{\left|\mathbb{E}\left[m(X_1)m(X_2) \mid A\right] - m(x)^2\right|}_{(*_2)} + \underbrace{\frac{1}{k_n}\left(\left|\mathbb{E}\left[\widehat{m}_2^{(n)}(x)\right]\right| + \left|\mathbb{E}\left[m(X_1)m(X_2) \mid A\right]\right|\right)}_{(*_3)}, \tag{17}$$

where we introduced $m(x)^2$ using the triangle inequality. Put $\omega(m, A) := \sup_{y,z\in A} |m(y) - m(z)|$ for $A \subset E$. Then, whenever $\delta > 0$ and $x_1, x_2 \in B(x, \delta)$,

$$\begin{aligned} \left|m(x_1)m(x_2) - m(x)^2\right| &= \left|m(x_1)m(x_2) - m(x_1)m(x) + m(x_1)m(x) - m(x)^2\right| \\ &\leq |m(x_1)|\,|m(x_2) - m(x)| + |m(x)|\,|m(x_1) - m(x)| \\ &\leq 2\,\|m\|_{B(x,\delta)}\,\omega(m, B(x, \delta)). \end{aligned} \tag{18}$$

Hence, and by Lemma 5.4, for any $\delta > 0$,

$$\begin{aligned} (*_2) &\leq \mathbb{E}\left[\left|m(X_1)m(X_2) - m(x)^2\right|\left(\mathbb{1}_{\{\rho_1\vee\rho_2\geq\delta\}} + \mathbb{1}_{\{\rho_1\vee\rho_2<\delta\}}\right) \mid A\right] \\ &\leq \left(n^2\,\|Y\|_{L^p}^2 + |m(x)|^2\right)\mathbb{P}\left(N_n(x, \delta) < k_n\right)^{1/q} \\ &\quad + 2\,\|m\|_{B(x,\delta)}\,\omega(m, B(x, \delta)), \end{aligned} \tag{19}$$

where $q = \frac{p}{p-1} \in (1, 2)$.

(i) Fix a compact set $C \subset \mathrm{S}(X)$, and let $\varepsilon > 0$. Since C is compact, there is $\delta > 0$ such that $\|m\|_{B(x,\delta)} \leq \|m\|_C + 1$ and $\omega(m, B(x, \delta)) \leq \varepsilon$ for all $x \in C$. Together with Eq. (19) and Lemma 5.1, this implies

$$\overline{\lim_{n\to\infty}}\left(\sup_{x\in C}(*_2)\right) \leq 2(\|m\|_C + 1)\varepsilon$$

for all $\varepsilon > 0$, so $(*_2)$ vanishes uniformly on C. In particular,

$$\sup_{n\in\mathbb{N},x\in C} |\mathbb{E}\left[m(X_1)m(X_2) \mid A_n(x)\right]| < \infty,$$

which together with Lemma 5.5 implies that $(*_3)$ also vanishes uniformly on C, and $(*_1)$ does the same by Proposition 5.6.

(ii) Suppose that $x \in E$ and $\gamma \in (0, 1]$, $\delta_0, c_1 > 0$ are such that $\omega(m, B(x, \delta)) \leq c_1\delta^\gamma$ for all $\delta \in (0, \delta_0)$, so by Eq. (19) and Lemma 5.1, for any $\delta \in (0, \delta_0)$,

$$(*_2) \leq 2c_1\,\|m\|_{B(x,\delta)}\,\delta^\gamma + \left(n^2\,\|Y\|_{L^p}^2 + |m(x)|^2\right)\exp\left(-\frac{n}{2qp_x(\delta)}\left(p_x(\delta) - \frac{k_n}{n}\right)^2\right).$$

Now let $(\alpha_n) \in (0, \infty)^{\mathbb{N}}$ be such that $\alpha_n = o(n/k_n)$, so that $0 < \varepsilon_n := (1 + \alpha_n)\frac{k_n}{n} \to 0$. Then, by Lemma 2.1, $0 < \delta_n := p_x^{-1}(\varepsilon_n) < \delta_0$ for large enough $n \in \mathbb{N}$, and $p_x(\delta_n) \geq \varepsilon_n > k_n/n$. Thus, for

large enough $n \in \mathbb{N}$,

$$(*_2) \leq 2c_1 \|m\|_{B(x,\delta_n)} \delta_n^\gamma + \left(n^2 \|Y\|_{L^p}^2 + |m(x)|^2\right) \exp\left(-\frac{n}{2q\varepsilon_n}\left(\varepsilon_n - \frac{k_n}{n}\right)^2\right)$$
$$= O\left(p_x^{-1}\left((1+\alpha_n)\frac{k_n}{n}\right)^\gamma + n^2 \exp\left(-\frac{k_n}{2q}\frac{\alpha_n^2}{1+\alpha_n}\right)\right).$$

We have shown in (i) that $\mathbb{E}\left[m(X_1)m(X_2) \mid A_n(x)\right] \to m(x)^2$ as $n \to \infty$, and, by Lemma 5.5, $\overline{\lim}_{n\to\infty} \left|\mathbb{E}\left[\widehat{m}_2^{(n)}(x)\right]\right| \leq m_2(x)$, so

$$(*_3) \leq (1+o(1))\frac{m_2(x) + m(x)^2}{k_n} = O\left(\frac{1}{k_n}\right).$$

Finally, Proposition 5.6 and Lemma 5.5 imply

$$(*_1) = \left|\left(\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] + m(x)\right)\left(\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right)\right|$$
$$= O\left(p_x^{-1}\left((1+\alpha_n)\frac{k_n}{n}\right)^\gamma + n \exp\left(-\frac{k_n}{2q}\frac{\alpha_n^2}{1+\alpha_n}\right)\right). \qquad \square$$

Theorem 3.1 is now an easy consequence of Propositions 5.6 and 5.7.

Proof of Theorem 3.1. As always we assume $k = 1$ and $E = \mathrm{S}(X)$. If $x \in E$, then

$$\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2} \leq \left\|\widehat{m}^{(n)}(x) - \mathbb{E}\left[\widehat{m}^{(n)}(x)\right]\right\|_{L^2} + \left\|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right\|_{L^2}$$
$$= \sqrt{\mathbb{E}\left[\left(\widehat{m}^{(n)}(x) - \mathbb{E}\left[\widehat{m}^{(n)}(x)\right]\right)^2\right]} + \left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right|$$
$$= \sqrt{\mathbb{V}\left(\widehat{m}^{(n)}(x)\right)} + \left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right|.$$

This bound vanishes uniformly on compact sets by Propositions 5.6 and 5.7, which proves (i). Now suppose $x \in E$ and $\gamma \in (0,1]$, $c_1, \delta_0 > 0$, such that $|m(y) - m(z)| \leq c_1 \rho(y,z)^\gamma$ for all $y, z \in B(x,\delta)$, and fix a sequence $0 < \alpha_n = o(n/k_n)$. Then, Proposition 5.7(ii) and Proposition 5.6(ii) respectively give asymptotic bounds $\mathbb{V}(\widehat{m}^{(n)}(x)) = O((\mathrm{v}))$ and $\left|\mathbb{E}\left[\widehat{m}^{(n)}(x)\right] - m(x)\right| = O((\mathrm{e}))$. A comparison yields $(\mathrm{e}) = O((\mathrm{v}))$, and since both go to zero as $n \to \infty$, $(\mathrm{e}) = o(\sqrt{(\mathrm{v})})$. Thus,

$$\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2} = O\left(\sqrt{(\mathrm{v})} + (\mathrm{e})\right) = O(\sqrt{(\mathrm{v})}).$$

The final assertion follows by choosing $\alpha_n = 1$, $n \in \mathbb{N}$, and because $q < 2$ (since $\frac{1}{q} + \frac{1}{p} = 1$ and $p > 2$). $\square$

Proof of Theorem 3.2. Let $C \subset \mathrm{S}(X)$ be compact. Recall from Eq. (4) that $\widehat{v}^{(n)}(\cdot) = \widehat{m}_2^{(n)}(\cdot) - \widehat{m}^{(n)}(\cdot)^2$, so, for $x \in E$,

$$\left\|\widehat{v}^{(n)}(x) - v(x)\right\|_{L^1} \leq \left\|\widehat{m}_2^{(n)}(x) - m_2(x)\right\|_{L^1} + \left\|\widehat{m}^{(n)}(x)^2 - m(x)^2\right\|_{L^1}$$
$$\leq \left\|\widehat{m}_2^{(n)}(x) - m_2(x)\right\|_{L^2} + \left\|\widehat{m}^{(n)}(x) + m(x)\right\|_{L^2}\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2}.$$

where we used the Hölder inequality on both summands in the second step. Applying Theorem 3.1 with $k = 1$ and $k = 2$ gives $\widehat{m}^{(n)}(\cdot) \xrightarrow{L^2} m(\cdot)$ and $\widehat{m}_2^{(n)}(\cdot) \xrightarrow{L^2} m_2(\cdot)$ uniformly on C. In particular, $\sup_{n\in\mathbb{N},x\in C} \left\|\widehat{m}^{(n)}(x)\right\|_{L^2} < \infty$, so the bound above vanishes uniformly on C. □

We close with the proofs of Theorems 4.3 and 4.6.

Proof of Theorem 4.3. Assume $k = 1$ and $\mathrm{S}(X) = E$. By assumption, there are $c_1, \delta_0 > 0$ such that $p_x(\delta) \geq c_1\delta^s$ for all $\delta \in (0, \delta_0)$, and thus $p_x^{-1}(\varepsilon) \leq \left(\frac{\varepsilon}{c_1}\right)^{1/s}$ for $\varepsilon \in (0, c_1\delta_0^s)$. Hence, and by Theorem 3.1, for any $0 < \alpha_n = o(n/k_n)$,

$$\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2}^2 = O\left(\frac{1}{k_n} + \left((1+\alpha_n)\frac{k_n}{n}\right)^{\gamma/s} + n^2 \exp\left(-\frac{k_n}{4}\frac{\alpha_n^2}{1+\alpha_n}\right)\right). \tag{20}$$

Choose $\alpha_n > 0$ such that $\frac{k_n}{4}\frac{\alpha_n^2}{1+\alpha_n} = (2+\gamma/s)\log n$. That is,

$$\alpha_n = 2(2+\gamma/s)\frac{\log n}{k_n}\left(1 + \sqrt{1 + \frac{1}{2+\gamma/s}\frac{k_n}{\log n}}\right) \sim 2\sqrt{(2+\gamma/s)\frac{\log n}{k_n}},$$

since $\log n/k_n \to 0$. Then, $\alpha_n = o(1) = o(n/k_n)$, so Eq. (20) is valid and turns into

$$\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2}^2 = O\left(\frac{1}{k_n} + \left((1+\alpha_n)\frac{k_n}{n}\right)^{\gamma/s} + n^{-\gamma/s}\right) = O\left(\frac{1}{k_n} + \left(\frac{k_n}{n}\right)^{\gamma/s}\right),$$

where we used that $1 + \alpha_n = O(1)$ and $n^{-\gamma/s} = o(\left(\frac{k_n}{n}\right)^{\gamma/s})$ since $k_n \to \infty$.

Claim. *If $a, b, \eta > 0$, then $f\colon (0,\infty) \to (0,\infty); x \mapsto \frac{a}{x} + bx^\eta$ has a unique minimum at $x_0 = \left(\frac{a}{\eta b}\right)^{1/(\eta+1)}$. Furthermore, if $k \in \mathbb{N}$ minimises f on $\mathbb{N}$, then $k \in \{\lfloor x_0 \rfloor, \lceil x_0 \rceil\}$.*

Proof of Claim. Clearly $f \in C^1((0,\infty))$, and if $\bowtie \in \{<, >, =\}$, then

$$f'(x) = -\frac{a}{x^2} + \eta b x^{\eta-1} \bowtie 0 \iff x \bowtie x_0 := \left(\frac{a}{\eta b}\right)^{1/(\eta+1)}.$$

Hence f has a unique minimum at x_0, is decreasing on $(0, x_0)$, and increasing on (x_0, ∞). In particular, if $k \in \mathbb{N}$ minimises f on $\mathbb{N}$, then $k \in \{\lfloor x_0 \rfloor, \lceil x_0 \rceil\}$. ◇

Applying this with $a = 1, b = n^{-\gamma/s}, \eta = \gamma/s$ yields that the bound in Eq. (9) is optimised over $(0,\infty)$ by $\kappa_n := cn^{1/(1+s/\gamma)}, n \in \mathbb{N}$, where $c = \left(\frac{s}{\gamma}\right)^{s/(s+\gamma)}$, and that an optimal $k_n \in \mathbb{N}$ satisfies $k_n \in \{\lfloor \kappa_n \rfloor, \lceil \kappa_n \rceil\}$. Now suppose that $k_n \sim c' n^{1/(1+s/\gamma)}$ for some $c' > 0$. Then, $\log n \ll k_n$, and $k_n = o(n)$, so Eq. (9) is valid and turns into

$$\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2}^2 = O\left(\left(1/c' + c'^{\gamma/s}\right) n^{-\frac{1}{1+s/\gamma}}\right) = O\left(n^{-\frac{1}{1+s/\gamma}}\right),$$

which implies Eq. (10). □

Remark 5.8. The choice for α_n we made in the proof above was optimal in the sense that the achieved asymptotic bound Eq. (9) is a strict lower bound on the RHS of Eq. (20) for any choice of $\alpha_n > 0$.

We turn to the proof of Theorem 4.6. Recall that we assume X to be a centred Gaussian process taking values in $E = C([0,1])$, which satisfies Eq. (12). We again denote by $\mathcal{H} \subset E$ the reproducing kernel Hilbert space of X.

Lemma 5.9. *There is a $c > 0$ such that, for any $x \in \mathcal{H}$,*

$$p_x(\delta) \geq \mathrm{e}^{-\|x\|_{\mathcal{H}}^2/2} \exp\left(-c\delta^{-1/\beta}\right), \quad \delta > 0,$$

where $\|\cdot\|_{\mathcal{H}}$ denotes the norm induced by the scalar product on $\mathcal{H}$.

Proof. For $t, s \in [0,1]$, by Eq. (12),

$$\begin{aligned}\mathbb{E}\left[|X_t - X_s|^2\right] = K(t,t) - 2K(t,s) + K(s,s) &\leq |K(t,t) - K(t,s)| + |K(s,t) - K(s,s)| \\ &\leq 2c\,|t-s|^{2\beta}.\end{aligned}$$

Now the claim follows from Theorems 3.2 and 5.2 in [15]. □

Proof of Theorem 4.6. Assume $k = 1$ and $\mathrm{S}(X) = E$. By Lemma 5.9, there are $c_1, c_2 > 0$ such that $p_x(\delta) \geq c_1 \exp\left(-c_2\delta^{-1/\beta}\right)$ for all $\delta > 0$, and thus $p_x^{-1}(\varepsilon) \leq \left(\frac{1}{c_2}\log\frac{c_1}{\varepsilon}\right)^{-\beta}$ for $\varepsilon \in (0,1)$. Hence, and by Theorem 3.1, for any $0 < \alpha_n = o(n/k_n)$,

$$\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2}^2 = O\left(\frac{1}{k_n} + \log\left(\frac{n}{(1+\alpha_n)k_n}\right)^{-\gamma\beta} + n^2\exp\left(-\frac{k_n}{4}\frac{\alpha_n^2}{1+\alpha_n}\right)\right). \quad (21)$$

Choose $\alpha_n > 0$ such that $\frac{k_n}{4}\frac{\alpha_n^2}{1+\alpha_n} = 3\log n$, that is,

$$\begin{aligned}\alpha_n &= 6\frac{\log n}{k_n}\left(1 + \sqrt{1 + \frac{3k_n}{\log n}}\right) \leq 12\frac{\log n}{k_n}\sqrt{1 + \frac{3k_n}{\log n}} \\ &\leq 12\frac{\log n}{k_n}\begin{cases}2, & k_n \leq \log n \\ \sqrt{\frac{4k_n}{\log n}}, & k_n > \log n\end{cases} \\ &\leq 24\begin{cases}\frac{\log n}{k_n}, & k_n \leq \log n \\ 1, & k_n > \log n.\end{cases}\end{aligned}$$

In either case, $\alpha_n = o(n/k_n)$, so Eq. (21) is valid. If $k_n > \log n$, then

$$\log\left(\frac{n}{(1+\alpha_n)k_n}\right) \geq \log\left(\frac{n}{25k_n}\right) = \log\frac{n}{k_n} - \log 25 \sim \log\frac{n}{k_n}.$$

If $k_n \leq \log n$, then

$$\begin{aligned}\log\left(\frac{n}{(1+\alpha_n)k_n}\right) &\geq \log\left(\frac{n}{k_n + 24\log n}\right) \geq \log\left(\frac{n}{(k_n+24)\log n}\right)\\ &= \log n - \log(k_n+24) - \log\log n \sim \log n - \log k_n\\ &= \log\frac{n}{k_n}.\end{aligned}$$

Hence, for any $n \in \mathbb{N}$, Eq. (21) turns into

$$\begin{aligned}\left\|\widehat{m}^{(n)}(x) - m(x)\right\|_{L^2}^2 &= O\left(\frac{1}{k_n} + \log\left(\frac{n}{(1+\alpha_n)k_n}\right)^{-\gamma\beta} + \frac{1}{n}\right)\\ &= O\left(\frac{1}{k_n} + \log\left(\frac{n}{k_n}\right)^{-\gamma\beta}\right),\end{aligned}$$

where we used that $\frac{1}{n} = o(\frac{1}{k_n})$ since $k_n = o(n)$.

Claim. *If $\eta > 0$, $n \in \mathbb{N}$, then $f\colon (0,n) \to (0,\infty) : x \mapsto \frac{1}{x} + \left(\log\frac{n}{x}\right)^{-\eta}$ has a unique minimum at $x_0 = x_0(n) \in (0,n)$ with*

$$x_0 \sim \frac{1}{\eta}\left(\log n\right)^{\eta+1}.$$

If $k \in \mathbb{N} \cap (0,n)$ minimises f, then $k \in \{\lfloor x_0\rfloor, \lceil x_0\rceil\}$.

Proof of Claim. Clearly, $f \in C^1((0,n))$, and $\lim_{x\downarrow 0} f(x) = \lim_{x\uparrow n} f(x) = \infty$, so f has at least one local minimum, all of which must be zeros of f'. For $x \in (0,n)$,

$$\begin{aligned}f'(x) = 0 &\iff -\frac{1}{x^2} + \frac{\eta}{x}\left(\log\frac{n}{x}\right)^{-(\eta+1)} = 0\\ &\iff (\eta x)^{1/(\eta+1)} = \log\frac{n}{x}\\ &\iff n = x\exp\left((\eta x)^{1/(\eta+1)}\right)\\ &\iff x_0 = aW\left((n/a)^{1/(1+\eta)}\right)^{1+\eta},\end{aligned}$$

where $a = \frac{1}{\eta}(1+\eta)^{(1+\eta)}$, and W denotes the Lambert-W function. For $z > 0$, $w = W(z)$ is the unique positive number for which $we^w = z$. It is well-known and easy to show that $W(z) \sim \log z$, so

$$x_0 \sim a\left(\log\left((n/a)^{1/(1+\eta)}\right)\right)^{1+\eta} = \frac{1}{\eta}\left(\log n - \log a\right)^{1+\eta} \sim \frac{1}{\eta}(\log n)^{1+\eta}.$$

Now suppose that $k \in \mathbb{N}\cap(0,n)$ minimises f, and assume for contradiction that $k \notin \{\lfloor x_0\rfloor, \lceil x_0\rceil\}$, say $k \in (0, \lfloor x_0\rfloor)$. Then $f(k) < f(\lfloor x_0\rfloor)$, but since $\lim_{x\downarrow 0} f(x) = \infty$, this would imply that f has a local minimum in $(0, \lfloor x_0\rfloor)$, which contradicts the fact that x_0 is the only local minimum. ◇

Applying this with $\eta = \gamma\beta$ yields that the bound in Eq. (13) is minimised over $(0,n)$ by a unique κ_n that satisfies $\kappa_n \sim \frac{1}{\gamma\beta}(\log n)^{1+\gamma\beta}$, and that an optimal $k_n \in \mathbb{N}$ satisfies $k_n \in \{\lfloor\kappa_n\rfloor, \lceil\kappa_n\rceil\}$.

Finally, suppose that there is $a \in (0,1)$ and $K > 0$ with $(\log n)^{\gamma\beta} \ll k_n \leq Kn^a$. Then, $k_n = o(n)$ and $\log \frac{n}{k_n} = \log n - \log k_n \geq \log n - a \log n - \log K \sim (1-a) \log n$, so Eq. (13) turns into

$$\left\| \widehat{m}^{(n)}(x) - m(x) \right\|_{L^2}^2 = O\left(\frac{1}{k_n} + (\log n)^{-\gamma\beta} \right) = O\left((\log n)^{-\gamma\beta} \right). \qquad \square$$

Again, note that the choice for α_n we made in the proof above was optimal in the sense that the achieved asymptotic bound Eq. (13) is a lower bound on the RHS of Eq. (21) for any choice of $\alpha_n > 0$.

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White Matter Hyperintensities: Red Flags for Cognitive Impairment?

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Background: White matter hyperintensities (WMHs) on T2-weighted imaging are considered classic signs of cerebral small vessel disease (SVD). Although WMHs are prevalent in healthy ageing, they are also associated with cognitive decline and dementia. Potential mechanisms underlying this interaction have been proposed, but it remains unclear to what extent these mechanisms hold true in the context of different concomitant pathologies, such as the plaques and tangles found in Alzheimer's disease (AD). **Aims:** This article first evaluates the common approaches to assess WMH severity before addressing what is currently understood about the variable aetiology of WMHs. It then explores the interaction between WMHs and cognitive decline in the contexts of healthy ageing, mild cognitive impairment, and AD, culminating in a schematic representation of the current hypotheses and uncertainties regarding this complex interaction. In light of these interactions, the clinical potential of WMHs is appraised as clinical biomarkers. Finally, this article considers the outstanding limitations and future directions to improve the way we investigate and ultimately understand both WMHs and cognitive impairment in general. **Significance:** By examining the association between WMHs and cognition, this body of work supports the relevance of WMHs in the broader context of cognitive impairment and dementia. This, in turn, highlights the wider importance of WM structural integrity and cerebrovascular health for sustained cognitive performance throughout ageing.

Keywords: neuroimaging, dementia, biomarkers, magnetic resonance imaging, cerebrovascular disease.

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1 Introduction

White matter hyperintensities (WMHs), appearing as bright areas on T2-weighted magnetic resonance imaging (MRI), are traditional markers of cerebral small vessel disease (SVD). In addition to WMHs, other SVD markers include lacunes, cerebral microbleeds, and enlarged perivascular spaces [63], examples of which can be found in Shi and Wardlaw [62] and Lee et al. [38] (Figure 1.1). This article will focus on WMHs, which are prevalent in ageing and have been linked to dementia and cognitive decline. The terms leukoaraiosis and white matter lesion have been used in other studies, but for consistency, this article will refer to these MRI signs as WMHs when equivalence can be determined. Although they are best seen with T2-Fluid-Attenuated Inversion Recovery (T2-FLAIR) imaging, WMHs also appear hyperintense with T2-weighted and proton-density imaging. They most commonly develop around the lateral ventricles (i.e., periventricular) and in the deep, or subcortical, WM. Depending on their severity and location, WMHs may appear as punctate foci, thin periventricular lines, or extensive, confluent lesions.

WMHs and other signs of SVD are hallmark features of vascular cognitive impairment and vascular dementia. However, Alzheimer's disease (AD), a dementia characterised by beta-amyloid (Aβ) plaques and tau tangles, also involves SVD signs, including WMHs [33]. This indicates possible shared risk factors across dementia subtypes

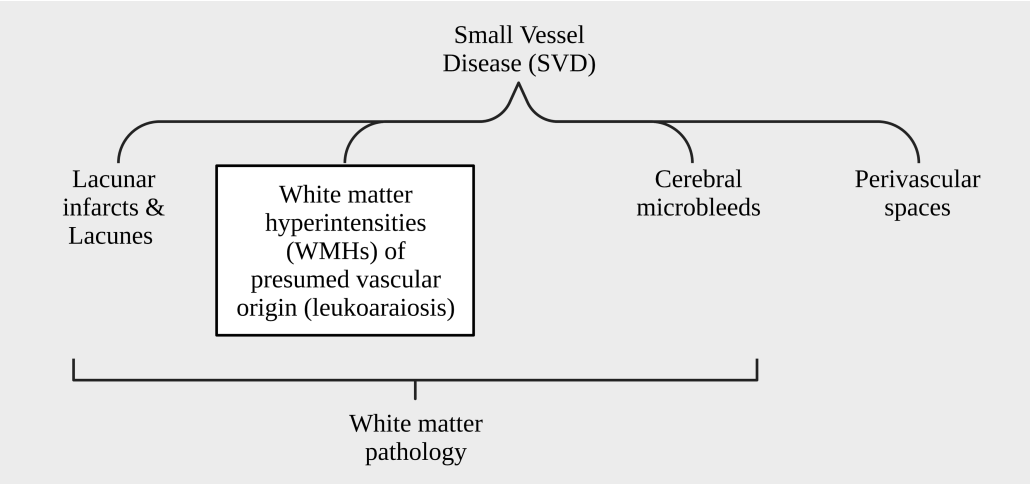

Figure 1.1: Types of small vessel disease (SVD). The term white matter pathology includes white matter hyperintensities (WMHs) of presumed vascular origin (leukoaraiosis), but it can also describe lacunes and cerebral microbleeds. Created with BioRender.com.

and points to a growing category of mixed dementia [32].

Considering the prevalence of cerebrovascular changes in cognitive impairment, I aim to explore the wider association between WMHs and cognitive decline. Given the recent developments in WMH quantification and our evolving understanding of WMH aetiology, an updated review that spans this nuanced interaction between WMHs and cognition is pertinent. I will first compare popular approaches for assessing WMHs. I will then consider the underlying pathology of WMHs before turning to evaluate their connection with cognitive decline in three contexts: cognitively-normal ageing, mild cognitive impairment (MCI), and AD. I will conclude by discussing the potential of WMHs as biomarkers and in the therapeutic context, as well as the current limitations and future directions for WMH investigations.

2 Assessing WM Pathology

There are various qualitative and quantitative approaches to measure WMH severity (Figure 2.1). At one end of this spectrum are the original visual rating scale, the Fazekas scale [16], and more recent scales, including the Schelten's scale [61] and the age-related white matter changes (ARWMC) scale [74]. These visual ratings are relatively quick to perform, are less affected by lower-quality scans (e.g., lower field strengths), and do not require advanced computing resources. Moreover, these scales may categorise WMHs in clinically-relevant ways (e.g., by proximity to the lateral ventricles or by brain lobe). Although qualitative ratings are subjective, less precise, and dependent on an expert radiologist's opinion, they are still commonly used in clinical and research settings.

By contrast, fully automated approaches like BIANCA (Brain Intensity AbNormality Classification Algorithm) yield probabilistic lesion maps, which can be used to calculate WMH volumes [23]. Volumetric measures correlate well with subjective ratings [18] and are more sensitive to subtle, longitudinal differences. BIANCA tends to be less biased than subjective WMH ratings, although its accuracy is subject to that of its training dataset of manually-labelled lesions [23]. This automated approach is higher-throughput and supports multimodal analyses for improved accuracy. BIANCA has been widely adopted in the research domain [2], although it has not yet been extensively adopted in clinical settings. Other sophisticated approaches, including deep learning algorithms for WMH classification [57], are also accurate but currently too memory-intensive

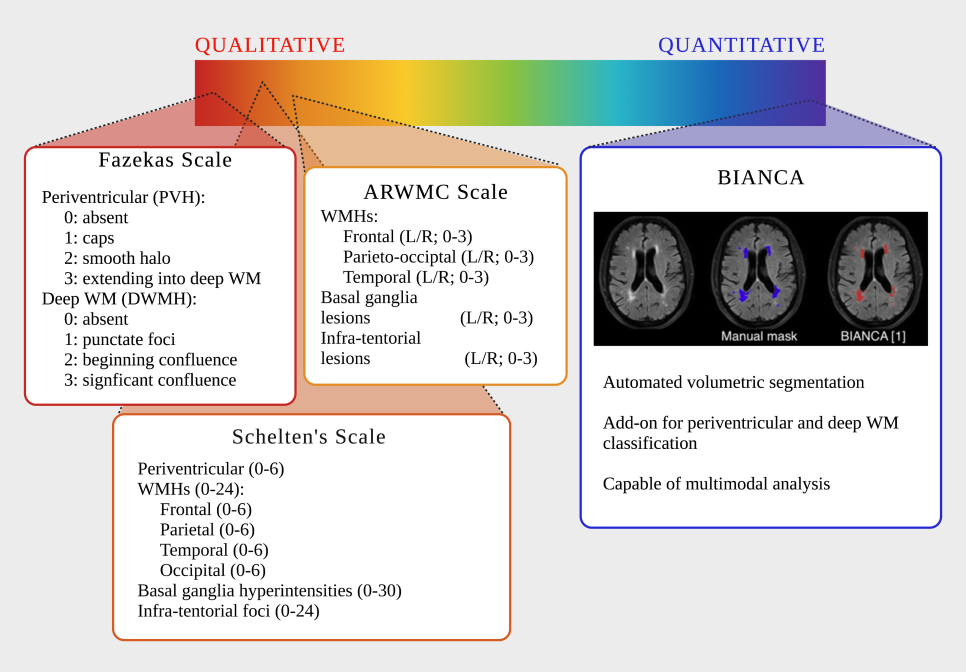

Figure 2.1: White matter hyperintensity (WMH) rating scales: spectrum from qualitative to quantitative. Three qualitative rating scales are included: the Fazekas scale, Schelten's scale, and ARWMC scale. A popular quantitative approach, BIANCA, is also shown along with some of its features. ARWMC, age-related white matter changes; L/R, left/right lateralised ratings; BIANCA, Brain Intensity AbNormality Classification Algorithm. BIANCA example image reproduced from Griffanti et al. [23]. Created with BioRender.com.

for widespread clinical use. Therefore, BIANCA is a more promising tool for clinical applications since it is relatively less computationally demanding.

3 WMH Aetiology Is Heterogeneous

The aetiology of WMHs is heterogeneous, even within those of presumed vascular origin (Figure 3.1). Microvascular disruption may include ischaemia, hypoperfusion, or blood-brain barrier disruption [21]. Despite their similar hyperintense appearance on T2-FLAIR imaging, post-mortem histological analyses have revealed diverse underlying structural changes in WMHs, ranging from minor changes in the extracellular matrix to substantial demyelination and axonal loss [21]. Concomitant pathologies, such as amyloid angiopathy, also promote the appearance of WMHs [21]. In controls, WMHs were associated with demyelination, likely reflecting an ischaemic origin, but WMHs in AD were also associated with axonal loss, likely reflecting Wallerian degeneration [47]. Pathological origins may also be region-dependent, with periventricular WMHs more commonly linked to ischaemia [42]. Multimodal T1-weighted [48] and diffusion tensor [49] imaging may distinguish WMH types, but nevertheless, this mixed aetiology of WMHs likely contributes to the poor correlation between clinical symptomatology and radiological phenotype [21].

Additionally, WMHs may represent the most extreme presentation on a continuum of WM pathology [53]. Other MRI-detectable WM changes (e.g., cerebral microbleeds [19]) are associated with WMHs. Altered microstructural integrity, detected as changes in fractional anisotropy and mean diffusivity on diffusion-weighted

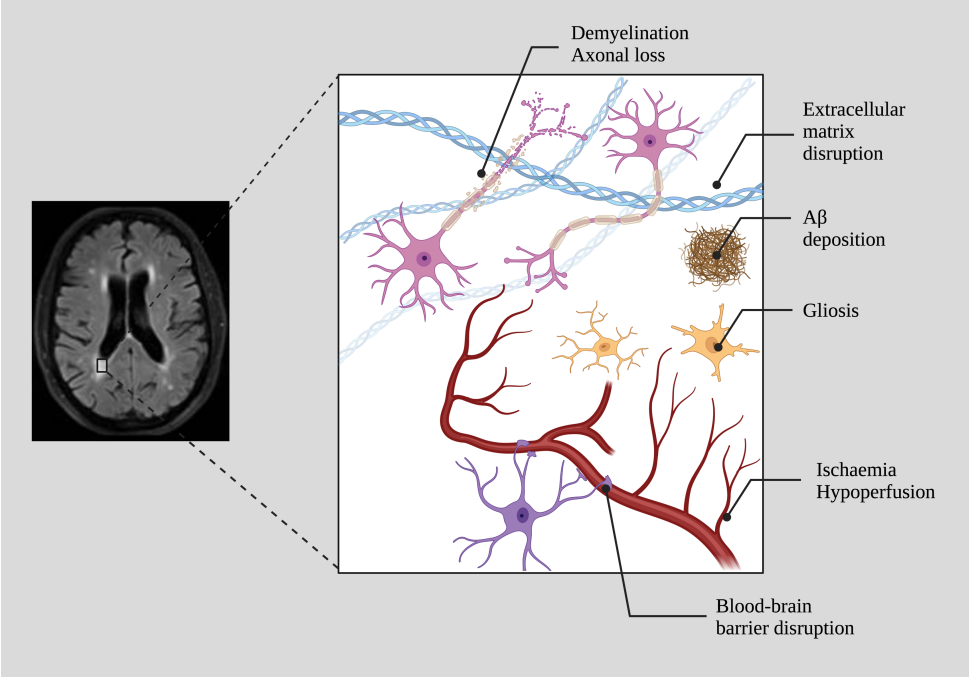

Figure 3.1: Pathological changes that may contribute to the manifestation of white matter hyperintensities (WMHs). Depending on the region and disease context, a mixture of these changes may contribute to the appearance of WMHs on MRI. Aβ, beta-amyloid. T2-FLAIR image reproduced from Griffanti et al. [23]. Created with BioRender.com.

imaging, often precede WMH development [26, 41]. Although Zeng et al. [78] reported that these microstructural changes are only apparent with severe WMHs, subtle WM changes may long precede WMH development and thus are not significantly different between controls and mild cases. Widespread integrity changes throughout the WM [67] may further contribute to the poor clinico-radiological correlation observed.

4 WMHs in Cognitively-Normal Ageing

WMHs are some of the most commonly-detected changes in cognitively-normal elderly populations [77]. In adults over 65, WMH prevalence has been estimated around 96% [45]. Age consistently correlates with automated [24, 23] and qualitative [78] measures of WMH burden. Despite this prevalence, it remains unclear how WMHs in healthy ageing differ from those observed in cognitive decline and dementia.

Aside from age, cardiovascular risk factors are also associated with WMH incidence in the general population. Leeuw et al. [40] found that the presence and severity of midlife aortic atherosclerosis was linearly associated with the presence of periventricular WMHs 20 years later. They did not, however, find any associations with late-life atherosclerosis or subcortical WMHs. Hypertension, particularly if uncontrolled during midlife, is also consistently reported as predictive of WMH severity [24] and progression [39, 60, 72]. Overall, these studies reinforce the significance of midlife cardiovascular health and earlier interventions regarding subsequent WMH appearance.

Table 5.1: Cross-sectional studies on white matter hyperintensities (WMHs) in relation to cognitive impairment. Relevant studies were selected using PubMed. MCI, mild cognitive impairment; AD, Alzheimer's disease; ARWMC, age-related white matter changes; MoCA, Montreal Cognitive Assessment.

Reference	Sample Size	Methods	Key Findings	Strengths	Limitations
Kaskikallio et al. [35]	86 [42 older controls, 44 MCI/AD; 79 of which were R-handed]	Multistage WMH segmentation algorithm; measures of semantic and phonological fluency	• Excluding L-handed individuals, frontal, parieto-occipital, and right temporal WMH volumes correlated with semantic fluency performance • In the whole sample, after multiple comparison correction, no associations were found between WMHs and semantic fluency • No group-specific associations were significant for controls or MCI/AD	• Ran analyses including and excluding L-handed subjects • Used the Benjamini-Hochsberg procedure to control for Type I errors • Controlled for age and education • Region-wise analysis	• The verbal fluency tasks used may not be very sensitive to the word-finding difficulties associated with cognitive decline • Sample size limited their statistical power
Kaskikallio et al. [34]	148 [56 older controls, 40 MCI, 52 AD]	Visually-rated MRI data (ARWMC Scale)	• Frontal and parieto-occipital WMHs affected processing speed (generally and in the AD group specifically) • Left frontal WMHs affected visual memory • No effects of WMHs were observed for verbal-logical memory or verbal functions	• Controlled for age and education • Region-wise analysis (including hemispheric analysis) • 9 neuropsychological assessments of 4 cognitive domains	• Did not control for cortical atrophy (although this may be advantageous if the effects of WMHs are indeed mediated through atrophy) • Did not examine executive function • Groups differed in age and education (possible confounding effects) • Did not correct for multiple comparisons

Reference	Sample Size	Methods	Key Findings	Strengths	Limitations
Wei et al. [76]	161 [113 with WMHs (35 cognitively normal), 48 controls]	Fazekas scale to assess WMH presence	• Individuals with WMHs (combined normal cognition and vascular cognitive impairment) had lower cognitive performance scores (assessed with MoCA and an executive function battery) than controls	• No significant difference in age, sex, or cardiovascular risk factors across groups • Fully automated processing	• Excluded participants with non-vascular dementia • Hospital-based (possible selection bias) • No correlational analyses with WMH load
Zeng et al. [78]	321 [all cognitively normal]	Fazekas scale and quantitative assessment of WMHs; 6 cognitive domains measured	• Negative association between WMH severity and working and episodic memory (starting at Fazekas grades 3 and 4) • Periventricular WMHs appeared first (in milder cases) • Only Fazekas grades 3 and 4 had significant changes in FA or significant cognitive differences • Fazekas grade 3 may be predictive for future cognitive decline	• Sampled community-dwelling individuals • Adjusted for age (Model 2) and additional factors (Model 3)	• Cognitive assessments may have lacked sensitivity to subtle changes • Strong assumption that disease progression is represented in this cross-sectional data • Excluded participants with MCI or AD
Defrancesco et al. [12]	60 [all MCI]	Modified Fazekas scale and Schelten's scale	• Baseline periventricular WMHs were negatively associated with psychomotor speed, executive function, attention, and cognitive flexibility • Baseline subcortical WMHs were negatively associated with visual memory	• Avoids selection biases (recruitment based on patient initiative) • Detailed neuropsychological evaluation	• Small sample size • Recruitment style may not represent wider population

Reference	Sample Size	Methods	Key Findings	Strengths	Limitations
Kramer et al. [36]	39 [27 controls, 12 cognitively normal with subcortical lacunes]	Group comparison (lacunes vs controls); WMH volume	• Subjects with lacunes had significantly greater WMH volume and subtle impairments in executive functioning and visual memory • 2 out of 3 executive function measures correlated with WMH extent	• Bonferroni correction to correct for multiple comparisons across executive function tests • No significant difference between patients' and controls' age or education level	• Very small sample size • Subcortical lacunes were the focus for group assignment (but used interchangeably with WMHs, defined as hyperintense on proton-density imaging) • Did not correct for multiple comparisons across other cognitive domains • Lacked the power to investigate the effects of lacune location or number
Mungas et al. [51]	157 [90 controls, 37 MCI, 30 dementia; with and without WMHs]	Quantitative assessment of subcortical lacunes and WMH volumes	• Subcortical lacunes did not independently relate to cognitive measures • WMHs independently predicted performance on timed tasks, indicating a possible effect on cognitive speed • Cortical grey matter and hippocampal volumes predicted cognitive performance	• Volumetric approach with automated segmentation • Subclassified lacunes by subcortical region • Measured cortical grey matter and hippocampal volumes to control for these factors • Age and education were included as covariates	• Groups (controls, MCI, dementia) significantly differed in age and education level • Did not examine effects of WMH location • Correlational, limiting ability to infer causality

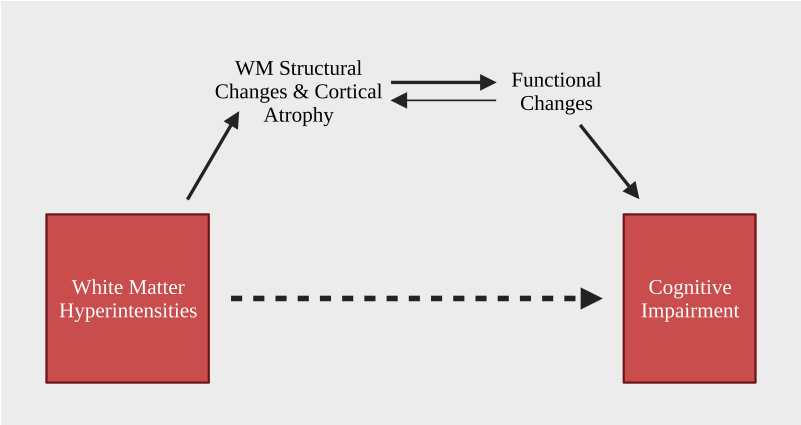

Figure 5.1: Possible interaction between WMHs and cognitive impairment. The nature of the interactions contributing to the co-occurrence of white matter hyperintensities and cognitive decline remains unclear. This interaction may be mediated by structural WM changes and cortical atrophy, which in turn contribute to functional network changes and cognitive decline. Further work is necessary to shed light on any additional mediators or direct interactions (dashed arrow). Created with BioRender.com.

5 WMHs and Cognitive Impairment

In the general population, WMHs are associated with subjective and objective measures of cognitive impairment [25]. Numerous cross-sectional studies support this association (Table 5.1).

However, the cognitive domains affected by WMHs remain controversial. Impairments in information processing speed and executive functioning are most commonly cited [12, 50]. In a large prospective study, the presence and progression of periventricular WMHs were associated with a baseline reduction and accelerated decline in mental processing speed [28]. A recent meta-analysis similarly found the greatest WMH effect sizes for the following frontal lobe functions: attention, executive function, and processing speed [4]. While some have proposed that memory is not associated with WMHs, others have reported impaired episodic and working memory in association with severe WMHs [78] or even mild WMHs [65]. Despite these domain-specific findings, Overdorp et al. [52] reported only an independent effect of WMHs on global cognition. However, this study regressed out medial temporal atrophy, which may be an over-adjustment if neurodegeneration is a mediator for the effects of WMHs, as proposed by Rizvi et al. [58].

Beyond simply the presence of WMHs, growing attention has been paid to their location. WMHs are non-uniformly distributed, with periventricular and frontal WMHs developing before subcortical and dorsal ones [27, 78]. Periventricular WMHs tend to associate more strongly with cognitive impairments [14], and WMH location influences which cognitive domains are affected [12]. While frontal WMHs were associated with executive dysfunction, parieto-temporal WMHs were linked with memory impairments in one study [37]. This likely contributes to the aforementioned variability between studies. The absence of region-wise analysis by Overdorp et al. [52] may, therefore, also contribute to the insensitivity to domain-specific changes.

Since most studies are correlational, causality is difficult to assess, especially in the presence of concomitant and multifactorial pathologies. Although supported by animal models of cerebrovascular disease [6], the causality of WMH-related changes in cognitive impairment is difficult to determine in clinical settings. In practice, expert opinion considers severe WMHs preceding cognitive impairment suggestive of vascular dementia [55]. However, this assumption does not indicate mechanisms, mediators, or the general involvement of WMHs in cognitive decline. Considering that WMHs are associated with accelerated cortical atrophy, Rizvi et al. [58] propose that

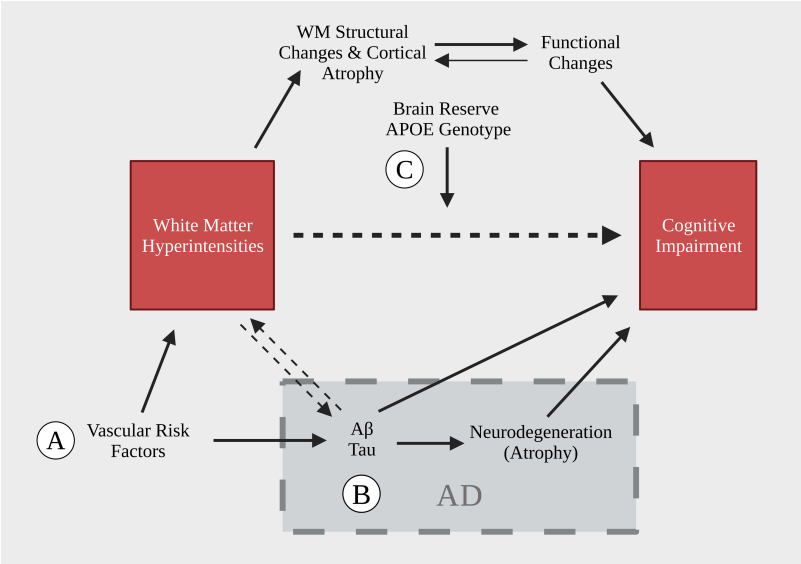

Figure 6.1: WMHs and cognitive decline in the context of Alzheimer's disease (AD). A) Vascular risk factors contribute to both the development of WMHs and AD hallmark pathologies like Aβ deposition. B) The classical AD aggregates of Aβ and hyperphosphorylated tau may bi-directionally interact with WMH development, in addition to contributing to cognitive impairment in other ways. C) The extent of brain reserve and the AD risk gene APOE may moderate the strength of the association between WMHs and cognitive impairment. Aβ, beta-amyloid. Created with BioRender.com.

WMH-related changes may drive neurodegeneration with global and localised changes in cortical thickness mediating the relationship between WMHs and cognitive decline. Jokinen et al. [30], on the other hand, found that WMHs and atrophy have independent but synergistic effects on cognition. Functional connectivity changes linked to WMHs suggest another potential mechanism of executive dysfunction [65, 66]. Clarifying the links between WMHs and cognition is crucial (Figure 5.1).

6 Interactions Between WMHs and AD

In the context of Alzheimer's Disease (AD), both vascular factors and disease-specific mechanisms may contribute to the formation of WMHs. Cardiovascular risk factors, which directly promote WMH development, also increase dementia risk (Figure 6.1A; [44]). These vascular factors influence the amyloid pathway, with blood-brain barrier disruption increasing the risk of Aβ deposition [3, 29]. Aβ may also contribute to WMH development in addition to its traditional involvement in AD (Figure 6.1B; [47, 73]). Although a recent study showed no association between global amyloid burden and WMHs [20], periventricular WMHs have been linked with PET measures of Aβ [22]. Overall, this provides preliminary support for a bidirectional interaction between Aβ and WMH pathology in AD.

WMHs may also relate to hyperphosphorylated tau, the other trademark aggregate in AD, which parallels cognitive decline more closely. In a small post-mortem study on varying degrees of AD pathology, only hyperphosphorylated tau immunoreactivity independently predicted WMH severity [46]. Although this suggests an association of tau with WMHs, a much larger study on non-demented individuals reported no association between regions with elevated tau burden on PET and WMH burden [22]. This does not refute the possibility of disease-specific WMH-tau interactions, but larger-scale investigations are necessary to determine the extent of

any such interactions.

WMHs and AD-specific features (i.e., Aβ, tau, and neurodegeneration) may have additive or synergistic effects contributing to cognitive decline. The heightened prevalence of signs of cerebrovascular disease, including WMHs, in AD compared to other neurodegenerative disorders supports an interaction [69]. Even among patients with AD, those with severe frontal WMHs had slower information processing speed than those with milder WMHs [34]. Other AD-related factors, such as APOE genotype, may modify the relationship between WMHs and cognitive decline (Figure 6.1C). Specifically, the cognitive effects of the APOE4 allele may weaken or overpower any interaction with WMHs [55]. Reserve, including brain reserve and cognitive reserve, can also influence the strength of the interaction between changes like WMHs and cognitive decline [64]. Therefore, beyond contributing to the development of WMHs, these disease-specific mechanisms may mediate or overshadow any direct interactions between WMHs and cognitive impairment.

7 WMHs as Biomarkers

Longitudinal studies suggest that both WMH baseline severity and progression may be linked to risk of future cognitive decline (Table 7.1). This supports their prognostic value as MRI markers to identify individuals likely to develop cognitive impairment. Although group-level differences may be present, however, the high degree of within-group variability currently precludes the use of WMHs as a selective biomarker at the individual level. Furthermore, the ability of WMHs to predict conversion to specific subtypes of dementia is also unclear. While Verdelho et al. [71] found that WMH severity generally predicted cognitive decline to dementia, subtype analysis revealed that AD was only predicted by medial temporal atrophy, whilst vascular dementia was also predicted by WM changes. More promisingly, Brickman et al. [7] found that both baseline parietal WMH severity and parietal WMH progression independently predicted conversion to AD in a study of 300 non-demented individuals.

Given their association with both cognitive decline and dementia, WMHs could theoretically also improve diagnostic accuracy. Although considered in the diagnosis of vascular dementia [43], including WMHs in the diagnostic criteria for other dementia subtypes currently offers little benefit because they tend to be more closely related to disease progression as opposed to disease status. Rather than a signal of impending diagnosis, WMHs may more accurately be an indication of additional risk factors for disease development, including vascular status. Although the variable aetiology of WMHs complicates this interpretation, the possibility of disease-specific mechanisms highlights their unique value as relatively comprehensive biomarkers of neurological state.

Moreover, there is an ever-present drive to develop better objective measures of therapeutic efficacy in clinical trials. WMHs hold potential in this context as a covariate or a surrogate marker of cerebrovascular disease progression [1, 53]. For this application, however, the possibility of additional disease-specific mechanisms obscures direct interpretations in the context of neurodegenerative disease.

8 Therapeutic Value of WMHs

Given the association between WMHs and cognitive impairment, it is worth examining their therapeutic implications. Detection of WMHs may inform strategies for both primary (i.e., prior to overt cognitive impairment) and secondary (i.e., after the presentation of mild cognitive impairment) prevention. Thus far, WMH-related therapies have centred on targeting cardiovascular risk factors to slow WMH progression and the associated cognitive outcomes. The other disease-specific origins of WMH development, however, remain more difficult to target.

Epidemiological evidence supports the influence of good cardiovascular health and effective treatment on WMH severity. A large prospective cohort study demonstrated that, while individuals with successfully-treated hypertension had moderately increased risks of WMHs compared to normotensive individuals, those with

Table 7.1: Longitudinal studies on the predictive value of white matter hyperintensities (WMHs) for disease progression and cognitive decline. Relevant studies were selected using PubMed. AD, Alzheimer's disease; MCI, mild cognitive impairment; CERAD, Consortium to Establish a Registry for Alzheimer's Disease; MMSE, Mini-Mental State Examination; ADAS-Cog, Alzheimer's Disease Assessment Scale-Cognitive Subscale; SIVD, subcortical ischaemia vascular disease.

Reference	Sample Size	Follow-Up (Mean)	Methods	Key Findings	Strengths	Limitations
Bilello et al. [5]	158 [57 AD, 66 MCI, 35 controls]	1–1.5 years	Automatic WMH segmentation and voxel-wise atrophy measures at baseline	• In AD patients, WMHs in the corpus collosum and fornices correlated with cognitive decline (CERAD score) • No significant correlations between WMHs and cognitive decline in the MCI or control groups	• Combines WMH and grey matter volumes in sophisticated voxel-wise analysis • Groups did not significantly differ in age or education	• Did not investigate individual cognitive domains • Did not address patient conversion
Defrancesco et al. [12]	60 [all MCI; 31 → AD]	1.5 years	Retrospective analysis; modified Fazekas scale and Scheltens scale	• Converters had higher Fazekas scores and more periventricular WMHs • WMH location impacts which cognitive domains are affected • WMH severity was not associated with memory decline • Vascular changes were not predictive for conversion from MCI to AD (i.e., risk factor but not early symptom of conversion)	• Avoids selection biases (recruitment based on patient initiative) • Single-centre, therefore reduced inter-investigator variability • Age did not affect results of regression analysis	• Retrospective analysis introduces possible bias (higher conversion rate) • Small sample size • Significant age difference between converters and non-converters

Reference	Sample Size	Follow-Up (Mean)	Methods	Key Findings	Strengths	Limitations
Carmichael [9]	804 [cognitively normal, MCI, or AD]	1 year	WMH volume (3 timepoints)	• Higher baseline WMH volume associated with worse baseline performance and greater change in both MMSE and ADAS-Cog	• Large study • Model included many covariates (age, ApoE genotype, cardiovascular risk score, atrophy, etc.) • WMH progression included • Similar parameters to a clinical trial	• Convenience sample • Linear models were fitted to non-linear trajectories for cognitively-normal individuals • MMSE is not very sensitive to subtle changes (as may be expected after 1 year) • Short time period for detectable changes
Jokinen et al. [31]	639 [age 65-84]	3 years	Subcortical ischaemic vascular disease (SIVD), based on WMHs and lacunar infarcts	• SIVD patients had steeper cognitive decline (executive function, psychomotor speed, and global cognition) • SIVD patients had a greater risk of developing dementia (3-fold)	• Large study • Included a battery of neuropsychological assessments (examining different domains) • Controlled for age, education, and medial temporal atrophy • Included 2 WM pathologies	• Did not stratify by severity of WM pathology

Reference	Sample Size	Follow-Up (Mean)	Methods	Key Findings	Strengths	Limitations
Dijk et al. [14]	668 [age 60-90]	3.4 years	Semi-quantitative 9-point scale for periventricular WMHs; volume estimates for subcortical WMHs	• Progression of periventricular WMHs associated with decline in information processing speed, global cognitive function, and MMSE score • Subcortical WMH progression was not associated with cognitive decline	• Large study • Repeated MRI scanning allowed tracking of WMH progression • 5 neuropsychological tests used to create compound scores • Prospective study design	• Those who followed up tended to be younger and healthier with better cognitive function (possible selection bias) • Semiquantitative approach less objective than volumetric measures
Pantoni et al. [54]	639 [age 65-84]	3 years	Fazekas scale for WMHs; extent of disability measured with the Disability Assessment for Dementia	• Association between baseline WMH severity and functional impairment • Impairment in executive function correlated with increasing WMH severity • WMH severity associated with higher autonomy → disability transition rates	• Large sample • Multi-centre and multinational	• Risk of inter-rater variability • Non-specific disability measurement • Single MRI scan (no analysis of WMH progression)

Reference	Sample Size	Follow-Up (Mean)	Methods	Key Findings	Strengths	Limitations
Prins et al. [56]	832 [non-demented at baseline]	5.2 years	Baseline periventricular WMHs (9-point scale) and subcortical WMHs (volume estimates); neuropsychological evaluation (3 timepoints)	• Periventricular WMH severity, infarcts, and generalised atrophy each predicted steeper decline in information processing speed and executive function • No associations were observed with rate of memory decline • With stroke cases excluded, many associations no longer significant (i.e., stroke involved in the interaction between SVD and cognitive decline)	• Large study • Used a global index of cognitive function, which they propose to be more robust	• A higher percentage of those who dropped out had memory decline • Excluding participants with incident stroke is an over-adjustment
De Groot et al. [11]	563 [non-demented elderly]	7.3 years	Semi-qualitative scale for periventricular and subcortical WMHs; MMSE for cognitive assessment	• Subjects with severe periventricular WMHs declined three times faster than average • Subcortical WMHs not independently associated with cognitive decline • No association between baseline MMSE score and WMH severity	• Long follow-up period • Sampled from general population • Controlled for age, education, atrophy, and infarcts • Reported results with and without correcting for baseline MMSE	• MMSE relatively insensitive to subtle cognitive changes • Those with less severe WMHs tended to participate longer (possible bias) • Single MRI scan (no analysis of WMH progression)

poorly-controlled hypertension had substantially higher relative risks [39]. A more recent longitudinal study showed that treated individuals, even if their hypertension was not successfully controlled, had less WMH progression compared to untreated patients [72]. This supports the importance of hypertension treatment and overall good cardiovascular health to slow WMH progression. Despite these findings, randomised controlled trials of cardiovascular interventions for slowing WMH progression are limited but ongoing [75].

In vascular cognitive impairment, the cognitive benefits of slowing WMH progression are evident [17], but the benefits in other forms of cognitive impairment are less clear. WMH pathology may have a late effect on cognitive decline [78], which opens up a therapeutic window to slow WMH progression prior to cognitive changes. A six-year randomised control trial demonstrated no differences in dementia incidence with a cardiovascular intervention program compared to usual treatment, but this study also failed to show any differences in cardiovascular disease incidence, likely due to the high standard of care in the usual treatment arm [10]. Another recent trial did not observe a significant dementia risk reduction with intensive hypertension treatment, but they did report an associated reduction in mild cognitive impairment as a non-primary outcome [68]. Although these reports do not rule out the value of WMH-directed cardiovascular interventions, they are far from conclusive. Considering the importance of midlife factors on later dementia onset, however, these treatments may also be targeted too late in disease progression.

9 Limitations and Future Directions

Despite the consistent association between WMHs and cognitive impairment, the correlational nature of our analyses limits our ability to infer mechanisms or causality. Currently, cortical atrophy [58] and WM microstructural changes [76] have been proposed as mediators, while factors like brain reserve are modifiers [8]. Other controversy surrounds the scale of WM changes (i.e., global or tract-specific) that influence cognitive decline [67, 78]. Further work is necessary to clarify the nature of any interaction between WMHs and cognitive decline, including how AD-specific pathologies contribute to or modify this interaction. Bearing in mind that WMHs are simply visible signs of underlying pathologies, the upstream pathological mechanisms (e.g., cardiovascular factors) must be targeted to have any meaningful impact. Nevertheless, this does not negate the suggested value of WMHs as biomarkers.

Objective and sensitive measures of cognitive impairment are difficult to consistently achieve. To standardise our working definition of cognitive impairment, the DSM-5 outlines six neurocognitive domains with subdomains (Table 9.1). This framework, however, does not include objective assessments for each domain, and thus assessments vary between studies. The standard choices (Table 9.1) often lack sensitivity for subtle cognitive decline. Meanwhile, other cognitive assessments, including the Mini-Mental State Examination (MMSE) and the Montreal Cognitive Assessment (MoCA), remain popular choices to easily assess cognitive function, but they, too, lack sensitivity and do not directly correspond to the cognitive domains.

Greater standardisation across the field is necessary to enable better comparison across studies and effective meta-analyses. Terms to describe WM pathology are often vaguely defined and interchanged. Standardisation of rating scales is also necessary, which can be achieved by a greater shift towards quantitative approaches. Even volumetric measures currently lack complete standardisation (e.g., in the definition of periventricular [24]). As our computing resources improve, so should our optimisation of standardised, automated pipelines and machine-learning approaches to segment and stratify WMHs with greater accuracy and consistency. Optimising these approaches, in conjunction with multimodal neuroimaging, may also allow clinically-relevant subcategorization of WMHs, such as based on T1 intensity [48].

Our current categorisation of dementia subtypes, in particular the delineation of vascular dementia, often leads to misdiagnosis. The widespread contribution of cerebrovascular disease across dementia subtypes calls into question our current distinction between vascular dementia and other dementia subtypes like AD. Instead,

Table 9.1: DSM-5 neurocognitive domains and subdomains. Some commonly-used objective assessments are listed for each. Italicised domains are particularly relevant in the context of WMH-related cognitive decline. See Sachdev et al. [59] and Eramudugolla et al. [15].

DSM-5 Neurocognitive Domain	Example Subdomains	Example Assessments
Complex Attention	• Processing speed • Divided attention • Selective attention	• Trail Making Test A • Simple Reaction Time • Choice Reaction Time
Executive Function	• Planning • Decision-making • Cognitive flexibility	• Colour-word inference (Stroop test) • Trail Making Test B • Digit Span Backwards
Learning and Memory	• Free recall • Recognition memory • Implicit learning	• California Verbal Learning Test
Language	• Object naming • Word finding • Fluency	• Letter Fluency • Boston Naming Test
Perceptual-Motor Function	• Visual perception • Perceptual-motor coordination	• Purdue Pegboard Test • Benton Visual Retention Copy
Social Cognition	• Recognition of emotions • Theory of mind • Insight	

common risk factors and pathomechanisms may point to a growing category of mixed dementia [32]. By looking at WMHs in the context of other WM pathologies (e.g., microbleeds and infarctions), we may be able to (i) unpick the case-by-case contribution of cerebrovascular factors to WMH formation and (ii) separate this mechanism from other disease-specific processes. This, in turn, may better inform our understanding of the wider contribution of cerebrovascular changes in cognitive decline [13, 70]. Clarifying the extent to which vascular mechanisms are shared across dementia subtypes may further shape our diagnostic practices, clinical trial recruitment, and therapeutic strategies.

10 Conclusions

The general association between WMHs and cognitive impairment is evident, and more recent studies indicate possible region-specific effects on certain cognitive domains. Given the correlational nature of current research and the lack of standardisation in the field, however, any inference of specific pathological mechanisms is obscured. The latest evidence highlights the varied origins of WMHs in different contexts. While the vascular origin of WMHs is widely supported, WMH-related studies in the context of AD also point to distinct disease-specific mechanisms. Nevertheless, these findings highlight the wider relevance of WM pathology in cognitive impairment

and dementia, even across dementia subtypes. By further elucidating the details of this interaction, including the role of disease-specific mediators and moderators, we may better understand the potential of WMHs as biomarkers and any therapeutic implications.

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Part IV.

Book Reviews

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Alexandra Hertlein
Book Reviews Section Chief

Book Reviews

Beauchamps, Marie (2018). *Governing Affective Citizenship: Denaturalization, Belonging and Repression*. Frontiers of the Political. New York: Rowman & Littlefield Publishers. 186 pp. Hardcover $138.00; Paperback $40.00; eBook $38.00. ISBN: 9781538158678.

Marie Beauchamps' *Governing Affective Citizenship* is a timely book, as the publication of this critical study on denaturalisation comes at a time when states around the world have begun weaponising the practice in their 'war against terrorism'. More crucially, her archival work shows that denaturalisation cannot be understood solely as a contemporary phenomenon. By turning to pivotal moments in French national history (i.e., the French Revolution, World War I, World War II, and the War on Terror), Beauchamps argues that denaturalisation was and remains caught up in affective governmental discourses that seek to differentiate between those who ought to belong in the nation-state and those subject to repression.

The book derives from Beauchamps' 2015 doctoral dissertation at the Amsterdam School for Cultural Analysis and is composed of chapters that draw from her previous journal article publications. Here, I will only address the contents as presented in the book. The introduction begins by framing denaturalisation in the French context of *déchéance*, which 'expresses a demotion, a moral downgrading' as well as 'the deprivation of rights'. Beauchamps emphasises that while denaturalisation is often framed as a means to ensure security against threats, 'its effect is not security as such'. Rather, she argues, 'denaturalisation turns nationality and citizenship into affective technologies of government, turning questions of inclusion and exclusion into matters of belonging and repression'.

Part I looks at the French Revolution, and chapter one begins by addressing the rise of the citizen as a universal subject with the Declaration of Rights of Man and of the Citizen (1789). Citing debates by political theorists, she points out that despite the Declaration, the creation of a new political and juridical community led to distinctions between the citizen and the foreigner, effectively undermining the idea of the universal subject endowed with rights. In this context, denaturalisation serves as an indicator of who ought to be included and excluded. In chapter two, Beauchamps introduces Sara Ahmed's idea of 'affective economies' and the 'metonymic slide'[1] as a lens to help identify how categories such as foreigner and threat, as well as immigration and criminality, have come to accompany each other via emotions. Beauchamps brings up the case of Jean Mathieu Scholler to demonstrate how foreignness was associated with one's birthplace, and the trial of Olympe de Gouges in the next chapter to demonstrate how foreignness also depended on a juridical interpretation of who ought to (not) belong. Chapter four extends the conversation to the 1848 abolition of slavery and Furcy Madeleine's case to probe into the relationship between the law and political subjectivity.

Part II focuses on the World Wars and the interwar period, and the fifth chapter seeks to demonstrate how 'denaturalisation contributed to model a performative image of a national self' as well as how denaturalisation centred around discourses of safety, security, and emergency. Beauchamps demonstrates how the first denaturalisation law in 1915—which was invoked as a temporary measure for the duration of WWI—came to be normalised through a discourse that especially targeted new nationals as compared to native-born ones. Chapter six looks at WWII and how a 1939 bill, along with a law enacted by Vichy France in 1940, sought to legitimise denaturalisation as a means to preserve the national community. The next chapter looks at the Commission for the Review of Naturalization at their denaturalisation of Jews and other human beings, as well as a case where an individual was charged with *degradation nationale*, to demonstrate how governments sought to impose ideas about nationality from above.

Part III looks at terrorism since the 1970s and its effect on denaturalisation. Chapter eight brings up Article 25 of the civil code law (the 1996 version) and how it allowed the denaturalisation of those who engaged in

[1] Sara Ahmed (2004). 'Affective Economies'. In: *Social Text* 22.2 (79), pp. 117–139. ISSN: 0164-2472. DOI: 10.1215/01642472-22-2_79-117

acts of terrorism. Highlighting the fluidity and indefiniteness of the term 'act of terrorism' and using Djamel Beghal's case as an indicative example, Beauchamps argues that terrorism is 'not necessarily about an act, but about the affective economy surrounding those acts'. She points out how the suspect of terrorism is 'ensnared in an economy of fear and suspicion' and how the state can 'make foreign those who are prosecuted in the name of the nation's security'. Chapter nine discusses President Hollande's attempt to subject all French nationals (not just foreign-born ones) who engage in terrorist activities to denaturalisation and the then-Minister of Justice Christiane Taubira's resignation and protest against it. Seeing that rhetoric about denaturalisation has not ceased even under President Macron's France, Beauchamps ends the book by calling for the abolition of denaturalisation.

Beauchamps' book is relatively short but rich with theoretical insights, drawing from a variety of sources from continental philosophy, political theory, citizenship studies, and security studies. Notwithstanding the interdisciplinary character of her book, she was able to keep a consistent theme throughout, which revolves around the question of how affective considerations shape governmental discourses and practices of denaturalisation. Her insight challenges public perceptions about governmental policies as deriving from rational and objective considerations about threat and security. It invites readers to adopt a critical view towards the practice of denaturalisation and pushes us to think about whether democratic states ought to continue with it, which is an especially important question to ask as states today are increasingly adopting denaturalisation measures in a post-9/11 world.

Beauchamps' dive into the French National Archives enables her to present discourses of government officials in their considerations of who counts as a citizen and who does not. What is missing in her state-centric analysis (which could be an avenue for future study) is a consideration of public discourse that may have contributed to the shift of boundaries between the citizen and the foreigner. The power to denaturalise lies with the state, but the criteria of (non-)belonging may not have been the sole product of governmental discourse. Indeed, Beauchamps points out that multiple discourses exist and that the one she presents is not the only one. Hence, conducting a bottom-up analysis is important if one were to resist denaturalisation practices since the ones perpetuating the practice may not only be government officials but also parts of the citizenry.

Furthermore, Beauchamps tends to focus on governmental discourses that actively seek to denaturalise individuals or groups of people. Hence, people who conduct acts that lead to their denaturalisation that are not necessarily criminal (e.g., marrying a foreign national, (in)voluntary renunciations of citizenship, etc.) are left out from Beauchamps' analysis of law and normalisation. Paying attention to the micro-occurrences of denaturalisation (as opposed to ones exclusively sanctioned by the state) may introduce other affective considerations among the denaturalised themselves. This may be difficult to achieve with archival research only. Hence, it presents a venue for contemporary research that involves interviews of denaturalised subjects.

A final, key concern of mine comes from Beauchamps' framing of denaturalisation through the lens of security and her subsumption of all forms of citizenship revocation under the single term 'denaturalisation'. While I believe framing denaturalisation within government discourses on security does indeed reveal a lot about the genealogy of denaturalisation practices (especially in a post-9/11 world), a wider perspective of denaturalisation requires us to shed light on other forms of it—especially those that seek to denaturalise 'native-born' citizens. Beauchamps' archival analysis showed that in France, there is a recurring discourse that tends to shield native-born nationals from denaturalisation, whereas foreign-born ones were continuously exposed to that risk. However, her analysis does not substantially deal with discourses about the denaturalisation of native-born nationals outside of concerns about terrorism. On pages 18 and 77, Beauchamps does bring up the loss of nationality of 'native-born' citizens prior to 1939, but they are understood as an example of how citizenship criteria revolved around territorial considerations. She points out how new nationals came to be conceptually linked with 'foreigners' in the affective-juridical discourse of denaturalisation, but what is missing is an analysis of the affective discourse of how 'native-born' citizens were capable of being 'foreign' and denaturalised. While territorial

considerations are more than likely to be relevant, looking at the denaturalisation of 'natives' through affective and demographic discourses pertaining to, for example, international marriage, miscegenation, conscription (especially in the context of the Napoleonic Wars), and military service may add crucial insights to the genealogy of denaturalisation.

Beauchamps' archival work in France has opened ways of looking at denaturalisation through affective discourses of (non-)belonging and security, but a number of other avenues need to be investigated to have a broader historical and contemporary understanding of denaturalisation as practised around the world. Hence, my review is less of a critique of Beauchamps and more of a call to keep pointing our flashlights against the unintended and inevitable shadows cast by our predecessors.

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Felski, Rita (2020). *Hooked: Art and Attachment*. Chicago: University of Chicago Press. 200 pp. Paperback $22.50; Cloth $95.00; eBook $21.99. ISBN: 9780226729466.

Notions of a kind of countercurrent in literary studies, avowedly opposed to what Eve Sedgwick termed 'paranoid' hermeneutic practices, are hardly new. In Rita Felski, this surface reading has always had one of its staunchest proponents. Her latest work, *Hooked: Art and Attachment* seems in many ways a maturation of this trend, combining questions of aesthetic value, the sustainability of literary study, and the confluences between what she calls 'lay' readers and the academy, explored through the central theme of our various forms of attachment to works of art.

Having demarcated this line of enquiry in a brief introduction, the remainder of the book is given to four chapters, each with a particular facet of this attachment as its central theme, with her notions of actor-network theory (ANT), heavily influenced by Bruno Latour, acting as a kind of conceptual framework, on which each definition is, to different degrees, comfortably hung. The first of these is a development of the introduction, engaging with questions of what artistic attachment constitutes as a means of expositing the relational structures of ANT. Feslki then moves on to what she calls *attunement*, defined as our response to the presences of different pieces of art, and the social evaluative structures that determine our preferences for one form of art over another. Having examined the sociality of artistic preferences, she then embarks on a 'defence' of the practice of identifying in some form or other with an artwork, variously affectively or ironically. Finally, she concludes by offering a sense of how the relational systems she has expounded on might be made a part of the academy, posing the foundational question of an alternative and more affective approach to literature.

It would be tempting, from a brief description like the one above, to dismiss this work as that of a Susan Sontag–esque public intellectual and this book as offering nothing more revolutionary than a new 'erotics of art' or similar. In the tradition of many of the most readable theorists, however, Felski's style, in a sense, enacts her conclusions, offering a prose that speaks simultaneously of the profound affective engagement she espouses and a wealth of scholarly erudition. What we are given, then, is a relatively brief but spirited and engaging polemic that resists straying into simplistic impressionism. Those readers familiar with her earlier writing will find less that is new here, with much of *The Limit of Critique*'s work on ANT and suspicious hermeneutics acting as a kind of foundation to this work. Much as, ironically, it can pay to be suspicious when a critic raises the possibility of 'an overhaul of humanistic methods', Felski's contribution to this supposed paradigm shift is nevertheless a novel and clearly delimited one. In many ways, one is compelled to feel, her return to the basic structures of artistic attachment says more about most of our experiences of reading than much of the critical writing that has emerged since the advent of close reading as an academic discipline.

The work's basic premise is the exploration of the means by which 'artworks compel our attention or solicit our devotion'. Literature, it seems fair to suggest, survives in the main not through formalist dissections in the academy but by its treatment by the average reader as affective aesthetic experience. Perhaps this overstates the motivations of a tourist at an airport searching for a means to fill time, for instance, but the affective seems nevertheless closer to said 'ordinary' reader's approach than the disciplined analysis imposed by the traditional academy. The solution for Felski, it seems, is collapsing these two poles into a universalising network of textual circulation. In drilling down to the basic motivations for reading, perceptively noting that 'affective ties are often stronger in academia than elsewhere, because more is at stake', she offers a refreshing, almost Cartesian, approach to literary studies, returning to first principles so simple and unspoken as to often be erased. The defence of identification is a particularly strong example of this, offering one of the clearer examples of Felski's decentring and relational tendencies. In the broad claim that 'identifying involves ideas and values as well as persons; may confound or remake a sense of self as well as confirming it; and is practised by sceptical scholars as well as wide-eyed enthusiasts', she seems to attempt to offer to criticism what Sullivan and Klein offered to psychoanalysis. As we see, the monumental and the individual mentioned here are made fluid, organic, and responsive through a movement beyond straightforward notions of art's self-containment as a realm beyond ordinary experience.

In refusing to accept the simplistic dismissal of identification as interwoven with questions of empathy and identity, the affective foundations of literary studies are laid bare beyond that of our approach to the text. In the shrewd observation that 'the readings offered by literary critics … are not outside identification, but premised upon it', the drive to level the disparity between reader and academy is given an aesthetic theoretical backing through what she goes on to describe as 'a sense of estrangement and dissociation [that] is the connecting tissue that binds character and reader'. Through this continual premising of commonality, we see the beginnings of her attempts to redress the 'missed connections between humanities scholarship and lay audiences' with which she closes the work. In so doing, there is an attempt at creating a kind of universal network of specialist and non-specialist readerships alike, with neither's approach given privilege over the other. Her premise that such a levelling will revitalise discourse around the discipline of literary studies is an optimistic one, but not one beyond the realms of possibility explored by this book.

If there is one flaw in Felski's innovative interpretive methods, it is a tendency to sublimate or otherwise dissolve too quickly the issues raised by her theoretical frameworks. Her early suggestion, for instance, that ANT 'allows us to circumvent a series of surprisingly stubborn dichotomies: art versus society, text versus context, sophisticated versus naïve response' in order to 'walk around them and arrive somewhere else' seems to raise more questions than the rest of the text can answer. Her dismissal of these fundamental concerns as 'surprisingly stubborn', if not in itself surprising, reveals something of the limitations of this approach, as fundamental concerns over textual stability and permeability are dismissed in favour of 'social meanings'. The issue here is that Felski seems, at least initially, to be far clearer on what her approach is not than what it is. Perhaps inevitably, when setting out, as she claims, on a 'rethinking of the fundaments of aesthetic experience', there is a strong sense early in the text that more interpretive frameworks are being dismissed or 'circumvented' than are being refreshed or reconstructed. Part of this problem emerges from the vagueness by which she initially defines her terms, with statements such as 'I avoid overpsychologizing or oversociologizing the word by forcing it into the exclusive ambit of particular disciplines' almost guaranteed to raise hermeneutic eyebrows. The insistence on avoiding specificity, as we see here, can give an uncomfortable sense of vacuity to her arguments as alternatives to the familiar concepts she does away with are not always forthcoming. In so keenly pointing out the absences in her work, we might suggest, Felski seems to do much of her critics' work for them.

Equally, for an exploratory work of theory, one could fairly suggest this kind of oppositional logic is to be expected. Demarcation, after all, is essential to the establishment of any new interpretive avenue, a process that is clearly and effectively undertaken here. So too, once this is done, does Felski present a cogent and convincing

polemic, unsettling traditional category boundaries without offering straightforwardly comforting replacements. To a certain extent, one wonders how strategically sound this argument might be. In a society increasingly reticent to recognise the value of creative and humanistic disciplines, this move to strip away the primacy of detailed, professional analysis might be called reckless. As the drive to challenge notions of canonicity grows, on the other hand, perhaps the approach posed here of exploring the relational structures generated by attachments to text is precisely the kind of decentring that will prove corrective. Whether an attempt to homogenise the responses of reader and academy will help or harm the discipline, it is a notion comprehensively and fascinatingly explored in Felski's work, of value to any student of aesthetics.

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Lindley, David (2020). *The Dream Universe: How Fundamental Physics Lost Its Way*. First edition. New York: Doubleday. 240 pp. Hardcover $26.95; eBook $13.99. ISBN: 9780385543859.

From Plato's attempts to understand the celestial sphere via pure reason alone, to Galileo's invention of an experimentally driven approach to scientific understanding, and on to the furthest horizons of theoretical physics, David Lindley weaves a gripping yarn concerning the development of modern science and humanity's quest to understand the universe. *The Dream Universe* is accessible to the non-specialist and experienced scientist alike: general audiences will be held captive by the historical narrative and rich character portraits Lindley paints of the various revolutionary figures; more technical audiences benefit from a unique take on the underlying philosophical framework guiding current research in fundamental physics, which should rightly garner greater attention. *The Dream Universe* is impressive in its scope whilst not shying away from complexities. This is all the more masterly considering the book's relatively light page count: a testament to Lindley's illuminating and concise writing style.

The book is split into four parts, each describing a broad chapter in the development of science. First, Galileo's struggles against the Christian orthodoxy of Renaissance Europe are recounted, culminating in the birth of the scientific method, where detailed observations and mathematics displaced Aristotelian physics. Here, Lindley engages in an analysis of the origins of Christian orthodoxy, and along the way, the reader is exposed to the teachings of Plato, Eudoxes and the cult of Pythagoras. These fragments build up an explanation as to why, as Lindley puts it, a 'highly refracted interpretation of Aristotle's principles and opinions' was given such importance by the Catholic Church. This opening may seem like a general introduction to the history of science with little bearing on modern physics, but many of the ideas expressed here are skilfully woven into the final chapter's arguments, which gives the book a pleasingly circular structure.

Second, the golden age of classical physics, with its 'universe as machine' worldview, is presented. Here, Lindley tells how Galileo's scientific method, with mathematics serving as a tool for practising physicists, was rigorously applied to a wide range of phenomena, with great strides being made in thermodynamics, mechanics and electromagnetism. The overriding message is thus: the language of mathematics provides a description of the underlying laws of nature; it is not in and of itself their source. While many parts of this chapter will be familiar to those with a scientific background, Lindley peppers the usual narrative with interesting anecdotes, bringing the characters of Newton, Halley, and Maxwell to life. Lindley's account of Michael Faraday, a humble blacksmith's son turned physicist, and his groundbreaking insight into the relationship between electric and magnetic phenomena is the highlight of the chapter. The author's enthusiasm for Faraday is clear, and it's infectious: that such a deep insight into the nature of reality can be obtained from an intuitive, visual picture

and that such a conceptual image may be transcribed into a quantitative mathematical theory (with the help of the brilliant James Clerk Maxwell), is deservedly couched as one of the crowning achievements of classical physics. This exemplifies how physics should be carried out according to Lindley: intuition and imagination, motivated by precise experimentation, build a picture of the relevant physical system. Mathematics is then employed afterwards to provide a quantitative description.

Third, Lindley dives into the quantum revolution of the twentieth century. This chapter's purpose is to reveal the first divergence in scientific thought from Galilean philosophy: as physicists began to delve into the microscopic world, they encountered phenomena further and further away from their everyday experience. Intuitive physical pictures described by Newtonian mechanics were no longer sufficient to explain the properties of fundamental particles and their interactions. This led to more abstract areas of mathematics being brought to light and incorporated into the physicist's toolbox, such as the (re-)invention of matrix mechanics by Heisenberg and Schrodinger's description of particles in terms of his 'wavefunction'. As theorists grappled with these less intuitive mathematical tools, a gulf began to grow between the physical reality we can measure and the mathematical objects populating quantum mechanics. This is in stark contrast to classical physics, where mathematical variables were directly linked to physically measurable quantities. Lindley best describes the twentieth century's departure from Galilean physics with his account of Paul Dirac's 'discovery' of antimatter, the idea of which originated from the allowed solutions of Dirac's equation alone. Dirac inverted the paradigm of classical physics: rather than starting with self-evident phenomena and using mathematics as a descriptive tool, mathematics was now used to predict the existence of new physics, and it was down to experiment to confirm or falsify these predictions.

The final chapter is a startling one where the reader is exposed to many strange ideas which pervade modern theoretical physics. This section will delight even seasoned practitioners of science, who, by its conclusion, will surely share Lindley's discomfort over the path taken by fundamental physics. The reader encounters the eleven dimensions of string theory, ideas of supersymmetry, the multiverse, electroweak symmetry breaking and the Higgs boson, inflationary cosmology, and the universe modelled as an incomprehensibly large quantum computer. Lindley hammers home his point that many of these theories cannot be falsified. Either they make no physical predictions or require experiments operating at prohibitively high energies. Thus, theorists judge their theories based on vague notions of aestheticism and beauty, determined by the theoretical community themselves rather than being driven by experimental results. Lindley has by now built up to his controversial final statement: 'research in this area, no matter its intellectual pedigree and exacting demands, is better thought of not as science but as philosophy.' Lindley aptly describes modern theoretical physics as an exercise in a slightly revamped Platonism; mathematics and pure logic are once again seen as the source of physical laws and deeper insights into the universe's nature. A slight difference is, as Lindley puts it, 'Plato never supposed that by explaining the heavens he would also explain the earth, but that's exactly what fundamental physics proposes'. This chapter pleasingly comes full circle, returning us to the ancient philosophies detailed in the opening section.

The Dream Universe is an excellent read, providing a satisfying elucidation on the origins of modern physics and its apparent regression to platonist ideologue. Throughout, Lindley sprinkles his narrative with gripping anecdotes, serving to humanise the various characters and thus maintain the reader's interest. While the book has depth, it would be impossible for such a short volume to contain the entire story. Lindley acknowledges this throughout and signposts the reader to many resources which deal with specific topics in more detail. This is deftly done, and Lindley instils his own passion for the history and philosophy of science in the reader. Thus, *The Dream Universe* serves as a self-contained tale of the development of modern science, accessible to experienced scientists and general audiences alike, but may also be seen as an entryway to wider study.

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