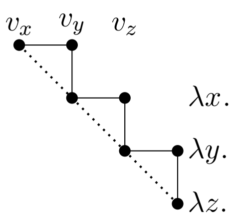
$$C_1[A_1[\lambda x.C_2[E[x]]]\,A_2[v]] \quad \boldsymbol{\beta}_{\mathbf{need}} \quad C_1[A_1[A_2[C_2[E[x]]\{x := v\}]]]$$
$$C_1[C_2] \in A$$

- C_1 = around everything
- A_1 = around the $\lambda x.$, but not the $A_2[v]$
- C_2 = under the $\lambda x.$, around the $E[x]$

Example 1 (AF style)

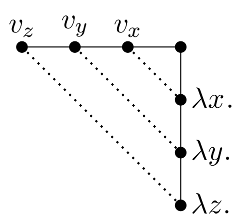


1. $(\underline{\lambda x.}(\lambda y.(\lambda z.x)\,v_z)\,v_y)\,\underline{v_x}$
   $A_2[v] = v_x$
   $C_1 = A_1 = [\,]$
   $C_2[E] = (\lambda y.(\lambda z.[\,])\,e_z)\,e_y$

2. $(\lambda x.(\underline{\lambda y.}(\lambda z.y)\,v_z)\,\underline{v_y})\,v_x$
   $A_2[v] = v_y$
   $C_1 = (\lambda x.[\,])\,v_x$
   $A_1 = [\,]$
   $C_2[E] = (\lambda z.[\,])\,e_z$

3. $(\lambda x.(\lambda y.(\underline{\lambda z.}z)\,\underline{v_z})\,v_y)\,v_x$
   $A_2[v] = v_z$
   $C_1 = (\lambda x.(\lambda y.[\,])\,e_y)\,e_x$
   $A_1 = [\,]$
   $C_2 = E = [\,]$

Example 2 (anti-AF style)

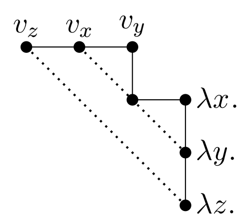


1. $(\underline{\lambda x.}\lambda y.\lambda z.x)\,\underline{v_x}\,v_y\,v_z$
   $A_2[v] = v_x$
   $C_1 = [\,]\,v_y\,v_z$
   $A_1 = [\,]$
   $C_2 = \lambda y.\lambda z.[\,]$
   $E = [\,]$

2. $(\lambda x.\underline{\lambda y.}\lambda z.y)\,v_x\,\underline{v_y}\,v_z$
   $A_2[v] = v_y$
   $C_1 = [\,]\,v_z$
   $A_1 = \lambda x.[\,]\,v_x$
   $C_2 = \lambda z.[\,]$
   $E = [\,]$

3. $(\lambda x.\lambda y.\underline{\lambda z.}z)\,v_x\,v_y\,\underline{v_z}$
   $A_2[v] = v_z$
   $C_1 = [\,]$
   $A_1 = (\lambda x.\lambda y.[\,])\,v_x\,v_y$
   $C_2 = E = [\,]$

Example 3 (via Casey)

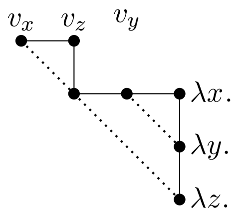


1. $(\underline{\lambda x.}(\lambda y.\lambda z.x)\,v_y)\,\underline{v_x}\,v_z$
   $A_2[v] = v_x$
   $C_1 = [\,]\,v_z$
   $A_1 = [\,]$
   $C_2 = (\lambda y.\lambda z.[\,])\,v_y$
   $E = [\,]$

2. $(\lambda x.(\underline{\lambda y.}\lambda z.y)\,\underline{v_y})\,v_x\,v_z$
   $A_2[v] = v_y$
   $C_1 = (\lambda x.[\,])\,v_x\,v_z$
   $A_1 = [\,]$
   $C_2 = \lambda z.[\,]$
   $E = [\,]$

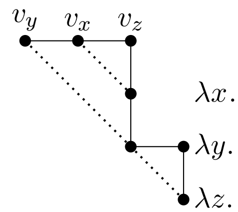
3. $(\lambda x.(\lambda y.\underline{\lambda z.}z)\,v_y)\,v_x\,\underline{v_z}$
   $A_2[v] = v_z$
   $C_1 = [\,]$
   $A_1 = (\lambda x.(\lambda y.[\,])\,v_y)\,v_x$
   $C_2 = E = [\,]$

Example 4 (compose 1 + 2)



1. $(\underline{\lambda x.}(\lambda y.\lambda z.x)\,v_y\,v_z)\,\underline{v_x}$
   $A_2[v] = v_x$
   $C_1 = [\,]$
   $A_1 = [\,]$
   $C_2[E] = (\lambda y.\lambda z.[\,])\,v_y\,v_z$
2. $(\lambda x.(\underline{\lambda y.}\lambda z.y)\,\underline{v_y}\,v_z)\,v_x$
   $A_2[v] = v_y$
   $C_1 = (\lambda x.[\,]\,v_z)\,v_x$
   $A_1 = [\,]$
   $C_2 = \lambda z.[\,]$
   $E = [\,]$
3. $(\lambda x.(\lambda y.\underline{\lambda z.}z)\,v_y\,\underline{v_z})\,v_x$
   $A_2[v] = v_z$
   $C_1 = (\lambda x.[\,])\,v_x$
   $A_1 = (\lambda y.[\,])\,v_y$
   $C_2 = E = [\,]$

Example 5 (compose 2 + 1)



1. $(\underline{\lambda x.}\lambda y.(\lambda z.x)\,v_z)\,\underline{v_x}\,v_y$
   $A_2[v] = v_x$
   $C_1 = [\,]\,v_y$
   $A_1 = [\,]$
   $C_2[E] = \lambda y.(\lambda z.[\,])\,v_z$
2. $(\lambda x.\underline{\lambda y.}(\lambda z.y)\,v_z)\,v_x\,\underline{v_y}$
   $A_2[v] = v_y$
   $C_1 = [\,]$
   $A_1 = (\lambda x.[\,])\,v_x$
   $C_2[E] = (\lambda z.[\,])\,v_z$
3. $(\lambda x.\lambda y.(\underline{\lambda z.}z)\,\underline{v_z})\,v_x\,v_y$
   $A_2[v] = v_z$
   $C_1 = (\lambda x.\lambda y.[\,])\,v_x\,v_y$
   $A_1 = [\,]$
   $C_2 = E = [\,]$