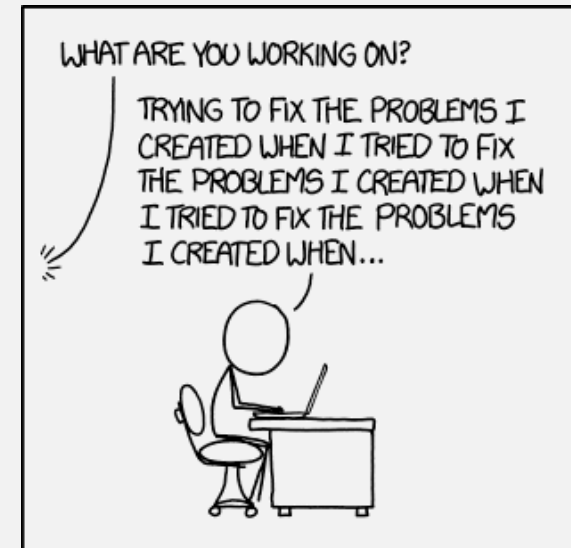


UMass Boston Computer Science
CS450 High Level Languages (section 2)
Recursive Data Definitions

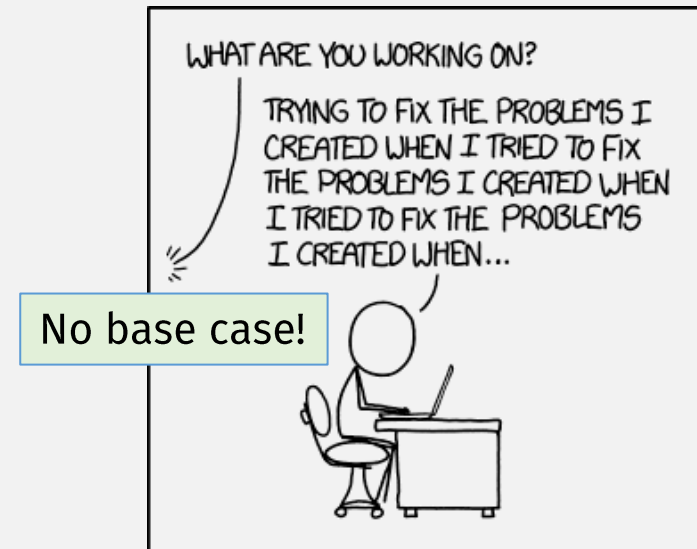
Monday, October 2, 2023



Logistics

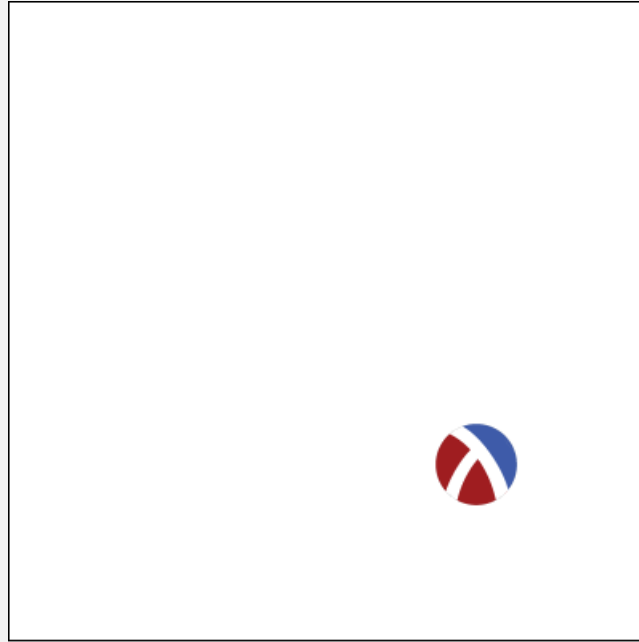
- HW 2 in
 - ~~due: Sun 10/1 11:59 pm EST~~
- HW 3 delayed
 - out: tomorrow
 - due: Sun 10/15 11:59 pm EST
 - (2 weeks)
- No class: next Monday 10/9
 - Indigenous Peoples Day

(What's wrong with this recursion?)



*Last
Time*

Bouncing Ball



*Last
Time*

Multi-ball Animation

Design a **big-bang** animation that:

- Start: a single ball, moving with random x and y velocity
- If a ball “hits” an edge:
 - for vertical edge, flip x velocity direction
 - for horizontal edge, flip y velocity direction

Randomness

[bracketed args] = optional

```
(random k [rand-gen]) → exact-nonnegative-integer?
```

```
k : (integer-in 1 4294967087)
```

```
rand-gen : pseudo-random-generator?
```

```
= (current-pseudo-random-generator)
```

When called with an integer argument *k*, returns a random exact integer in the range 0 to *k*-1.

Optional arg Default value

```
(random min max [rand-gen]) → exact-integer?
```

```
min : exact-integer?
```

```
max : (integer-in (+ 1 min) (+ 4294967087 min))
```

```
rand-gen : pseudo-random-generator?
```

```
= (current-pseudo-random-generator)
```

When called with two integer arguments *min* and *max*, returns a random exact integer in the range *min* to *max*-1.

What is “random”???

Not secure!
e.g., for generating
passwords

A pseudorandom number generator (PRNG), also known as a **deterministic random bit generator (DRBG)**,^[1] is an **algorithm** for generating a sequence of numbers whose properties approximate the properties of sequences of **random numbers**. The PRNG-generated sequence is **not truly random**, because it is completely determined by an initial value, called the PRNG's **seed**

VS

A **cryptographically secure** pseudorandom number generator (CSPRNG) or **cryptographic pseudorandom number generator (CPRNG)** is a **pseudorandom number generator** (PRNG) with properties that make it suitable for use in **cryptology**.

Designing Random Functions: Same Recipe!

```
;; A Velocity is a non-negative integer
;; Interp: reresents pixels/tick change in a ball coordinate
(define MAX-VELOCITY 10)
```

```
;; random-velocity : -> Velocity
;; returns a random velocity between 0 and MAX-VELOCITY
(define (random-velocity)
  (random MAX-VELOCITY))
```

Functions can
have zero args

```
(check-true (< (random-velocity) MAX-VELOCITY))
(check-true (>= (random-velocity) 0))
(check-true (integer? (random-velocity)))
(check-pred (λ (v) (and (integer? v)
                        (< v MAX-VELOCITY)
                        (>= v 0))))
(random-velocity))
```

```
;; random-x      : -> ???
;; random-y      : -> ???
;; random-ball   : -> ???
```

Can still **test!**
Just less precise

Last
Time

Multi-ball Animation

Design a **big-bang** animation that:

- Start: a single ball, moving with random x and y velocity
- On a click: add a ball at random location with random velocity
- If a ball “hits” an edge:
 - for vertical edge, flip x velocity direction
 - for horizontal edge, flip y velocity direction

;; A **WorldState** is ... an unknown number of balls!

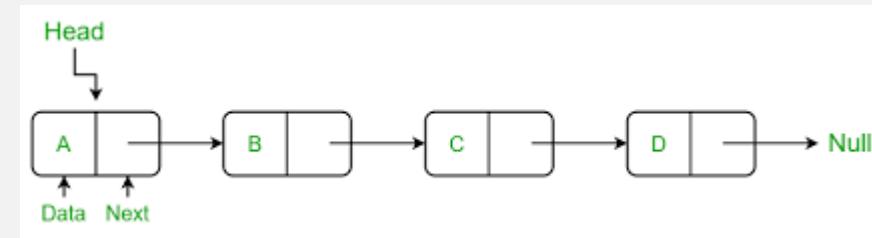
Last
Time

Arbitrary Size Data - Lists

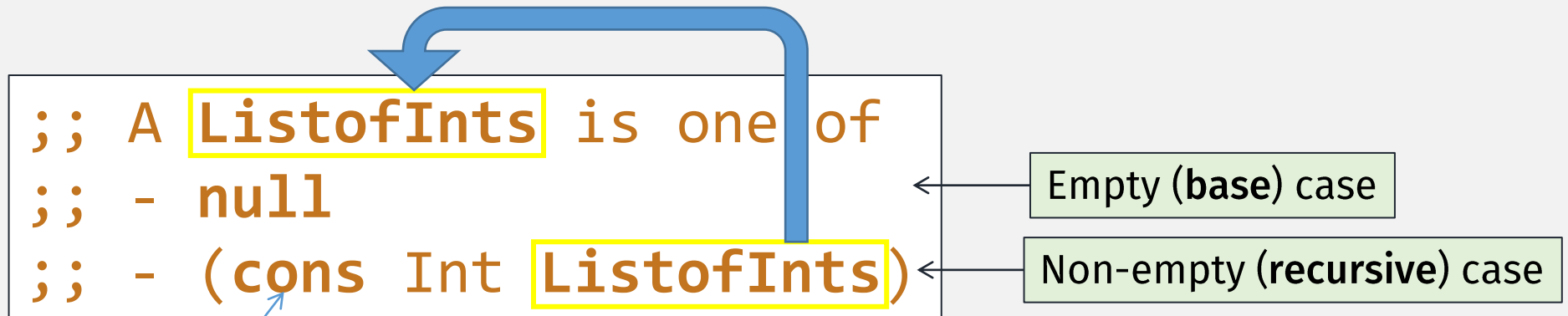
In C

```
struct node  
{ int data;  
  struct node *next; } *head;
```

This is a **self-referential**
(i.e., **recursive!**) definition!



Racket List Data Definition Example



cons = "node"

Recursive!

(using a definition to define itself)

TEMPLATE??

(how can we use a list of ints
to define a list of ints?!?)

Recursion is only valid if there is both

- A **base** case
- A **recursive** case

Racket List Data Definition Example

```
;; A ListofInts is one of  
;; - null  
;; - (cons Int ListofInts)
```

Empty (base) case

Non-empty (recursive) case

This is both **itemization** and **compound** data, so template has both **cond** and **getters**

TEMPLATE??

The shape of the function matches the shape of the data definition!

Wait, where is the **recursion**???

Empty (base) case

Non-empty (recursive) case

```
; TEMPLATE for list-fn  
;; list-fn : ListofInts -> ???  
(define (list-fn lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (rest lst) ....]))
```

Racket List Data Definition Example

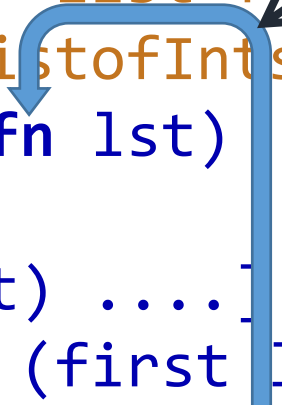
```
;; A ListofInts is one of  
;; - null  
;; - (cons Int ListofInts)
```



The shape of the function
matches the shape of the
data definition!

TEMPLATE??

```
;; TEMPLATE for list-fn  
;; list-fn : ListofInts -> ???  
(define (list-fn lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (list-fn (rest lst)) ....]))
```

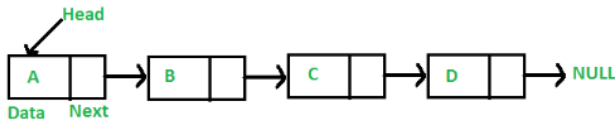


So recursion in the data definition
... means recursion in the
(template) function!

... is also recursive!

Racket Recursive List Fn Example: sum

Given a singly linked list. The task is to find the sum of nodes of the given linked list.



Task is to do $A + B + C + D$.

Examples:

[geeksforgeeks.com](https://www.geeksforgeeks.com)

Input: 7->6->8->4->1

Output: 26

Sum of nodes:

$7 + 6 + 8 + 4 + 1 = 26$

Input: 1->7->3->9->11->5

Output: 36

Examples!

```
;; TEMPLATE for list-fn
;; list-fn : ListofInts -> ???
(define (list-fn lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (list-fn (rest lst)) ....])))
```

Racket Recursive List Fn Example: sum

Design Recipe:
Now fill in
template!
(with arithmetic)

```
;; Returns sum of list of ints  
;; sum-1st: ListofInts -> Int  
(define (sum-1st lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (sum-1st (rest lst)) ....]))
```

Racket Recursive List Fn Example: sum

```
;; Returns sum of list of ints
;; sum-lst: ListofInts -> Int
(define (sum-lst lst)
  (cond
    [(null? lst) 0]
    [else .... (first lst) ....
               .... (sum-lst (rest lst)) ....]))
```

Racket Recursive List Fn Example: sum

```
;; Returns sum of list of ints
;; sum-lst: ListofInts -> Int
(define (sum-lst lst)
  (cond
    [(null? lst) 0]
    [else (+ (first lst)
              (sum-lst (rest lst)))]))
```

Racket Recursive List Fn Example: rev

```
;; TEMPLATE for list-fn
;; list-fn : ListofInts -> ???
(define (list-fn lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (list-fn (rest lst)) ....])))
```


Racket Recursive List Fn Example: rev

Design Recipe:
Now fill in
template!
(with arithmetic)

```
;; reverses a list of ints  
;; rev: ListofInts -> ListofInts  
(define (rev lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (rev (rest lst)) ....]))
```

Racket Recursive List Fn Example: rev

```
;; reverses a list of ints
;; rev: ListofInts -> ListofInts
(define (rev lst)
  (cond
    [(null? lst) null]
    [else .... (first lst) ....
               .... (rev (rest lst)) ....]))
```

Racket Recursive List Fn Example: rev

`(rev (rest lst)) = (list 5 4 3 2)`

`(check-equal? (rev (list 1 2 3 4 5)) (list 5 4 3 2 1))`

“append” (first lst)

The diagram shows the recursive step of the `rev` function. It illustrates how the result of `(rev (rest lst))` is appended to the first element of the list. The first line shows the recursive call: `(rev (rest lst)) = (list 5 4 3 2)`. The second line shows the check: `(check-equal? (rev (list 1 2 3 4 5)) (list 5 4 3 2 1))`. The third line shows the operation: “append” (first lst). Arrows indicate the flow of data: from the recursive call to the list `(list 5 4 3 2)`, and from the `append` operation to the final list `(list 5 4 3 2 1)`.

```
;; reverses a list of ints
;; rev: ListofInts -> ListofInts
(define (rev lst)
  (cond
    [(null? lst) null]
    [else (append (rev (rest lst))
                  (list (first lst)))])
```

Recursive rev fn, with “temp” vars (preview)

```
;; TEMPLATE for list-fn
;; list-fn : ListofInts -> ???
(define (list-fn lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (list-fn (rest lst)) ....])))
```

Recursive rev fn, with “temp” vars (later)

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (rev (rest lst)) ....]))
```

Recursive rev fn, with “temp” vars (later)

Still follows
design
recipe!

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst rev-lst-so-far)
  (define (rev/tmp lst rev-lst-so-far)
    (cond
      [(null? lst) ....]
      [else .... (first lst) ....
               .... (rev/tmp (rest lst) ....)
               .... rev-lst-so-far ....]))
  (rev/tmp lst null))
```

An internal “**helper**”
function adds a “temp”
variable
(main fn calls helper fn)

← Tmp var = reversed list “so far” (initially null)

Recursive rev fn, with “temp” vars (later)

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst rev-lst-so-far)
  (define (rev/tmp lst rev-lst-so-far)
    (cond
      [(null? lst) rev-lst-so-far]
      [else .... (first lst) ....
                .... (rev/tmp (rest lst)
                              rev-lst-so-far) ....]
      )
    (rev/tmp lst null))
```

Now figure out how to
“combine” these pieces
(with “arithmetic”)

Recursive rev fn, with “temp” vars (later)

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst rev-lst-so-far)
  (define (rev/tmp lst rev-lst-so-far)
    (cond
      [(null? lst) rev-lst-so-far]
      [else (rev/tmp
               (rest lst)
               (cons (first lst) rev-lst-so-far))]))
  (rev/tmp lst null))
```

Add next list item to reversed list “so far”

Multi-ball Animation

Design a **big-bang** animation that:

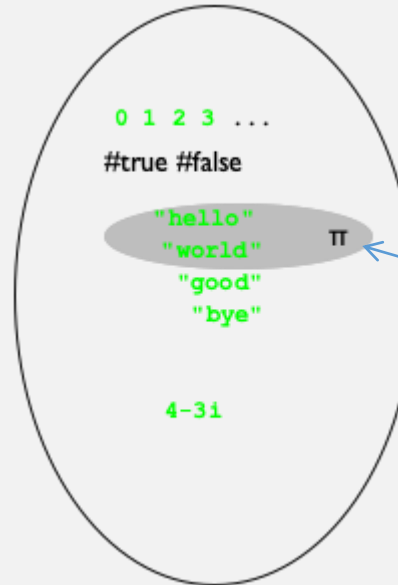
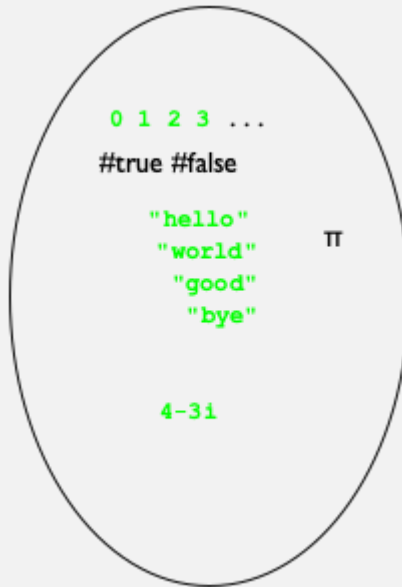
- Start: a single ball, moving with random x and y velocity
- On a click: add a ball at random location, with random velocity
- If any ball “hits” an edge:
 - if it's a vertical edge, the x velocity should flip direction
 - If it's a horizontal edge, the y velocity should flip direction

∴ A `WorldState` is ... an unknown number of balls!

;; A `WorldState` is ... a list of balls!

Interlude: Data Definitions (ch 5.7)

All possible data values



A data definition
= (a named) subset of all possible values

We are defining which data values are valid for our program!

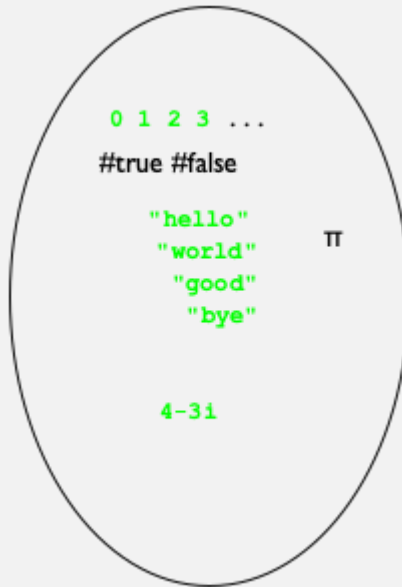
All programs are data manipulators ...

So this must be the first step of programming!

Also makes “error handling” easy

Interlude: Data Definitions (ch 5.7)

All possible basic data values



Possible to expand the universe of values, e.g., new **compound data definitions** (struct, or other data structure)

A Venn diagram with three overlapping ovals. The leftmost oval contains the same basic data values as the first diagram. The middle oval contains the same basic data values plus a list of compound data definitions: (make-posn "hello" 0), (make-posn "world" 1), (make-posn "good" 2), (make-posn "bye" 3), (make-posn (make-posn 0 1) 2), (make-posn 0 3), (make-posn 1 3), (make-posn 2 3), and (make-posn 3 3). The rightmost oval contains a list of compound data definitions: (make-ball -1 0), (make-ball -1 1), (make-ball -1 2), (make-ball -1 3), and (make-ball "bye" #t). The intersection of the middle and right ovals is shaded gray.

(make-posn "hello" 0)
(make-posn "world" 1)
(make-posn "good" 2)
(make-posn "bye" 3)
(make-posn (make-posn 0 1) 2)
(make-posn 0 3)
(make-posn 1 3)
(make-posn 2 3)
(make-posn 3 3)

(make-ball -1 0)
(make-ball -1 1)
(make-ball -1 2)
(make-ball -1 3)
(make-ball "bye" #t)

(list 1)
(list 1 2)
(list 1 2 3)
...

Multi-ball Animation

Design a **big-bang** animation that:

- Start: a single ball, moving with random x and y velocity
- On a click: add a ball at random location, with random velocity
- If any ball “hits” an edge:
 - if it's a vertical edge, the x velocity should flip direction
 - If it's a horizontal edge, the y velocity should flip direction

∴ A `WorldState` is ... an unknown number of balls!

;; A `WorldState` is ... a list of balls!

Ball

```
;; A WorldState is a  
(struct world [x y xvel yvel] #:transparent)  
;; when ball  
;; x: XCoord - represents x coordinate of ball center in animation  
;; y: YCoord - represents y coordinate of ball center in animation  
;; xvel: Integer - represents x velocity, where  
;;                positive = to the right, negative = to the left  
;; yvel: Integer - represents y vel, where  
;;                positive = down, negative = up
```

```
;; A ListofBall is one of  
;; - null  
;; - (cons Ball ListofBall)
```

```
;; A WorldState is a ListofBall
```

```
(define (main)
  (big-bang (list (random-ball))
    [on-mouse mouse-handler]
    [on-tick next-world]
    [to-draw render-world]))
```

These need to be updated to handle new WorldState data def

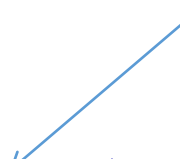
```
;; A WorldState is a ListofBall
```

next-world

List template!

```
;; next-world : WorldState -> WorldState
;; Computes the next world state on a tick
(define (next-world w)
  (cond
    [(null? w) ....]
    [else .... (first w) ....
               .... (next-world (rest w)) ....])))
```

Ball



Create one
function
per “task”

```
(check-equal? (next-world (list (make-ball 0 0 1 1)))
              (list (next-ball (make-ball 0 0 1 1))))
```

next-ball

This was the previous “next-world” function!

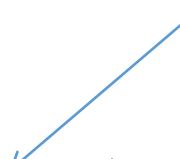
```
(define (next-ball b)
  (match-define (ball x y xvel yvel) b)
  (define new-xvel
    (if (ball-in-scene/x? x) xvel (- xvel)))
  (define new-yvel
    (if (ball-in-scene/y? y) yvel (- yvel)))
  (define new-x (+ x new-xvel))
  (define new-y (+ y new-yvel))
  (ball new-x new-y new-xvel new-yvel))
```


next-world

List template!

```
;; next-world : WorldState -> WorldState
;; Computes the next world state on a tick
(define (next-world w)
  (cond
    [(null? w) ....]
    [else .... (first w) ....
               .... (next-world (rest w)) ....])))
```

Ball



Create one
function
per “task”

```
(check-equal? (next-world (list (make-ball 0 0 1 1)))
              (list (next-ball (make-ball 0 0 1 1))))
```

next-world

```
;; next-world : WorldState -> WorldState
;; Computes the next world state on a tick
(define (next-world w)
  (cond
    [(null? w) null]
    [else .... (first w) ....
               .... (next-world (rest w)) ....]))
```

next-world

```
;; next-world : WorldState -> WorldState  
;; Computes the next world state on a tick  
(define (next-world w)  
  (cond  
    [(null? w) null]  
    [else (cons (next-ball (first w))  
                 (next-world (rest w)))])])
```

render-world

List template!

```
;; render-world : WorldState -> Image
;; Draws the given worldstate as an image
(define (render-world w)
  (cond
    [(null? w) EMPTY-SCENE]
    [else (place-ball (first w) (render-world (rest w)))]))
```

Separate “draw”
function for the ball

For multi-arg function, you choose which (argument's) template to use

Enumeration

```
;; mouseHandler : WorldState XCoord YCoord MouseEvent -> WorldState
;; Inserts a new ball on mouse click
(define (mouse-handler w x y mevt)
  (cond
    [(click? mevt) (cons (make-ball/random-velocity x y) w)]
    [else w ])))
```

Enumeration template
(collapsed)

Check-In Quiz 10/2 on gradescope

(due 1 minute before midnight)