

UMass Boston Computer Science
CS450 High Level Languages

Implementing Function Calls

Thursday, April 22, 2025

Logistics

- HW 10 in
 - ~~due: Tues 4/22 11am EST~~
- HW 11 out
 - due: Tues 4/29 11am EST

Last Time

The “CS450” Programming Lang!

Programmer writes:



```
;; A Program is one of:  
;; - Atom  
;; - `(+ ,Program ,Program)  
;; - `(× ,Program ,Program)  
;; - `(~= ,Program ,Program)  
;; - `(iffy ,Program ,Program ,Program)
```

Adding Variables

Programmer writes:

```
;; An Variable is a:
... - Symbol
```

```
;; A Program is
;; - Atom
;; - Variable
;; - ...
```

Q₁: What is the “meaning” of a variable?

A₁: Whatever “value” it represents

Q₂: Where do these “values” come from?

A₂: Other parts of the program!

```
;; An AST is one of:
;; - ...
;; - (mk-var Symbol)
```

Hint: Don't use “**var**”
(reserved for a Racket
match pattern)
for struct name

```
;; A Result is one of:
```

The **run** function needs to “remember” these values (with an **accumulator**!)

```
;; - Boolean
;; - String
;; - ???
```

run

(JS semantics)

“meaning” of the program

Design Recipe For Accumulator Functions

When a function needs “extra information”:

1. *Specify* **accumulator**:

- Name
- Signature
- Invariant

2. *Define* internal “helper” fn with extra **accumulator** arg

(Helper fn does not need extra description, statement, or examples, if they are the same ...)

3. *Call* “helper” fn , with initial accumulator value, from original fn

run, with an accumulator

```
;; run: AST -> Result  
;; Computes result of running a CS450 Lang program AST
```

```
(define (run p)  
  ;; accumulator acc : Environment  
  ;; invariant: Contains variable values ... currently in-scope  
  (define (run/acc p acc)  
    (match p  
      [(num n) n]  
      [(add x y) (450+ (run/acc x) (run/acc y))]))  
  (run/acc p ??? ))
```

Environments

- A data structure that “associates” two things (var, val) together
 - E.g., maps, hashes, list-of-pairs



```
;; An Environment is one of:
```

```
;; - empty
```

```
;; - (cons (list Var Result) Environment)
```

```
;; interpretation: a runtime environment for
```

```
;; (i.e., gives meaning to) cs450-lang variables
```

```
;; if there are duplicates,
```

```
;; vars at front of list shadow those in back
```

Environments

- A data structure that “associates” two things (var, val) together
 - E.g., maps, hashes, list-of-pairs

```
;; An Environment is one of:  
;; - empty  
;; - (cons (list Var Result) Environment)
```

- Needed operations:
 - `env-add` : Env Var Result -> Env
 - `env-lookup` : Env Var -> Result

Environments

```
;; An Environment is one of:  
;; - empty  
;; - (cons (list Var Result) Environment)
```

- Needed operations:

- `env-add` : Env Var Result -> Env
- `env-lookup` : Env Var -> Result

```
;; interpretation: a runtime environment  
;; gives meaning to cs450lang variables
```

```
;; for duplicates, vars at front of  
;; list shadow those in back
```

Think about examples where this happens!

env-add examples

```
;; env-add: Env Var Result -> Env
```

```
(check-equal? (env-add '() 'x 1)  
              '((x 1)) ) ; add to empty
```

```
(check-equal? (env-add '((x 1)) 'y 2)  
              '((y 2) (x 1)) ) ; add new var
```

```
(check-equal? (env-add '((x 1)) 'x 3)  
              '((x 3) (x 1)) ) ; add shadowed var
```

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
;; for duplicates, vars at front of  
;; list shadow those in back
```

Env template

2 cases

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

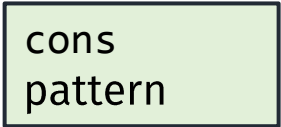
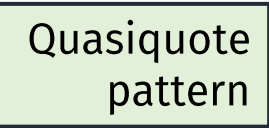
2 cases

```
(define (env-fn env ... )  
  (cond  
    [(empty? env) ... ]  
    [else  
     (match-let  
       ([ (cons (list x result) rest-env) env ])  
       ... x ... result ... (env-fn rest-env ... ) ... ]))
```

2nd case extracts
components of
compound data

Env template

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
(define (env-fn env ... )  
  (cond  
    [(empty? env) ... ]  
    [else  
     (match-let  
          
          
       ([(cons (list x result) rest-env) env])  
       ([`((,x ,result) . ,rest-env) env])  
       ... x ... result ... (env-fn rest-env ... ) ... ]))
```

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
;; env-add: Env Var Result -> Env
```

```
(define (env-add env new-x new-res)  
  (cond  
    [(empty? env) ... ]  
    [else  
     (match-let  
       ([(cons (list x result) rest-env) env])  
       [`((,x ,result) . ,rest-env) env])  
       ... x ... result ...(env-add rest-env ... ) ... ])))
```

Start with template

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```


```
;; env-add: Env Var Result -> Env
```

```
(define (env-add env new-x new-res)  
  (cond  
    [(empty? env) ...]  
    [else ... ]))
```

Examples

```
(check-equal? (env-add '() 'x 1)  
              '((x 1)) ) ; add to empty
```


Base case – empty env



```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
;; env-add: Env Var Result -> Env
```

```
(define (env-add env new-x new-res)  
  (cond  
    [(empty? env) (cons (list new-x new-res) env)]  
    [else ... ]))
```



Base case – empty env

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
;; env-add: Env Var Result -> Env
```

```
(define (env-add env new-x new-res)  
  (cond  
    [(empty? env) (cons (list new-x new-res) env)]  
    [else ...]))
```

recursive case?
(non-empty env)

Examples

```
(check-equal? (env-add '((x 1)) 'y 2)  
              '((y 2) (x 1)) ) ; add new var
```

```
(check-equal? (env-add '((x 1)) 'x 3)  
              '((x 3) (x 1)) ) ; add shadowed var
```



```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
;; env-add: Env Var Result -> Env
```

```
(define (env-add env new-x new-res)  
  (cond  
    [(empty? env) (cons (list new-x new-res) env)]  
    [else (cons (list new-x new-res) env)]))
```

recursive case?
(non-empty env)

Sometimes you start with template ... but don't use all the pieces!

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
;; env-add: Env Var Result -> Env
```

```
(define (env-add env new-x new-res)  
  (cons (list new-x new-res) env))
```

Collapse similar cases

env-lookup examples

```
;; env-lookup: Env Var -> Result
```

```
(check-equal? (env-lookup '((y 2) (x 1)) 'x)  
              1) ; no dup
```

```
(check-equal? (env-lookup '((x 2) (x 1)) 'x)  
              2) ; duplicate
```

```
(check-equal? (env-lookup '() 'x)  
              UNDEFINED-ERROR) ; not found!
```

```
;; A Result is one of:  
;; - Number  
;; ...  
;; - UNDEFINED-ERROR
```

An “error” is a valid program “Result”!

... for now, just represent with special Result value

NOTE: we don't want Racket exception because this is a “CS450 Lang error” ... Racket program runs fine!

env-lookup

```
;; env-lookup: Env Var -> Result
```

```
(define (env-lookup env target-x)
  (cond
    [(empty? env) ... ]
    [else
     (match-let
       ([`((,x ,res) . ,rest-env) env])
       ... x ... res ... (env-lookup rest-env ... ) ... ])))
```

TEMPLATE!

env-lookup: empty (error) case

```
;; env-lookup: Env Var -> Result
```

```
(define (env-lookup env target-x)
  (cond
    [(empty? env) UNDEFINED-ERROR]
    [else
     ... ]))
```

env-lookup: non-empty case

```
;; env-lookup: Env Var -> Result
```

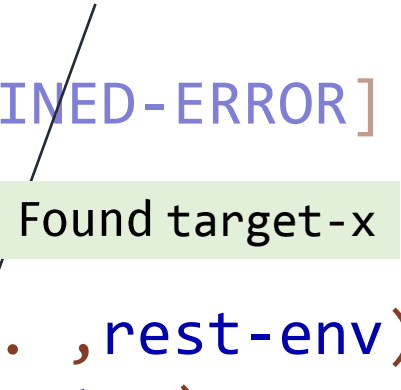
```
(define (env-lookup env target-x)
  (cond
    [(empty? env) UNDEFINED-ERROR]
    [else
     (match-let
      ([`((,x ,res) . ,rest-env) env])
      ... x ... res ... (env-lookup rest-env ... ) ... ]))
```

Extract the pieces

env-lookup: non-empty case

```
;; env-lookup: Env Var -> Result
```

```
(define (env-lookup env target-x)
  (cond
    [(empty? env) UNDEFINED-ERROR]
    [else
     (match-let
       ([`((,x ,res) . ,rest-env) env])
       (if (var=? x target-x)
           res
           ... (env-lookup rest-env ... ) ... ]))
```



The diagram illustrates the execution of the `env-lookup` function in the non-empty case. A green box labeled "Found target-x" has two arrows pointing to the code. One arrow points to the `x` in the `match-let` binding `[`((,x ,res) . ,rest-env) env]`, indicating that `x` is the variable found in the environment. The other arrow points to the `x` in the `if` condition `(var=? x target-x)`, indicating that the function is checking if the found variable matches the target.

env-lookup: non-empty case

```
;; env-lookup: Env Var -> Result
```

```
(define (env-lookup env target-x)
  (cond
    [(empty? env) UNDEFINED-ERROR]
    [else
     (match-let
       ([`((,x ,res) . ,rest-env) env])
       (if (var=? x target-x)
           res
           (env-lookup rest-env target-x)))]))
```

Else, recursive call with remaining env

run, with an Environment accumulator

```
;; run: AST -> Result
```

```
(define (run p)
  ;; accumulator env : Environment
  ;; invariant: contains in-scope var + results
  (define (run/env p env)
    (match p
      [(num n) n]
      [(add x y) (450+ (run/env x) (run/env y))]))
  (run/env p ??? ))
```

run, with an Environment accumulator

```
;; run: AST -> Result
```

```
(define (run p)
  ;; accumulator env : Environment
  ;; invariant: contains in-scope var + results
  (define (run/env p env)
    (match p
      ...
      [(vari x) (env-lookup env x)]
      [(bind x e body) ... (env-add env x (run/env e env)) ...]
      ... ))
  (run/env p ??? ))
```

Don't
name
this var

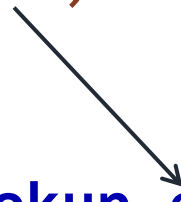
run, with an Environment accumulator

TODO:

- When are variables “added” to environment???
- What is initial environment?

```
;; run: AST -> Result
```

```
(define (run p)
  ;; accumulator env : Environment
  ;; invariant: contains in-scope var + results
  (define (run/env p env)
    (match p
      ...
      [(vari x) (env-lookup env x)]
      [(bind ??? body) ... (env-add env x (run ??? env e env)) ...]
      ...
    ))
  (run/env p ??? ))
```



Programs that Add Variables to Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - ??????
```

Programs that Add Variables to Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - ...
```

(like “let” in other langs)

Programs that Add Variables to Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - ...
```

Need to be more careful parsing

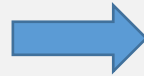
Valid
Program?

```
'(bind)
```

```
'(bind [])
```

```
'(bind [1 2] 3)
```

parse



```
;; An AST is one of:  
;; - ...  
;; - (mk-var Symbol)  
;; - (mk-bind Symbol AST AST)  
;; - ...  
  
;; ...  
(struct vari [name])  
(struct bind [var expr body])  
;; ...
```

Interlude: Racket exceptions

Exceptions are just special structs

Super struct (enables using exception API)



```
(struct exn:fail:syntax:cs450 exn:fail:syntax [])
```

```
(define/contract (parse p)
  (-> Program? AST?)
  (match p
    [(? atom?) (parse-atom p)]
    ...
    [ `(,fn . ,args) ... ]
    [_ (error ... )]))
```

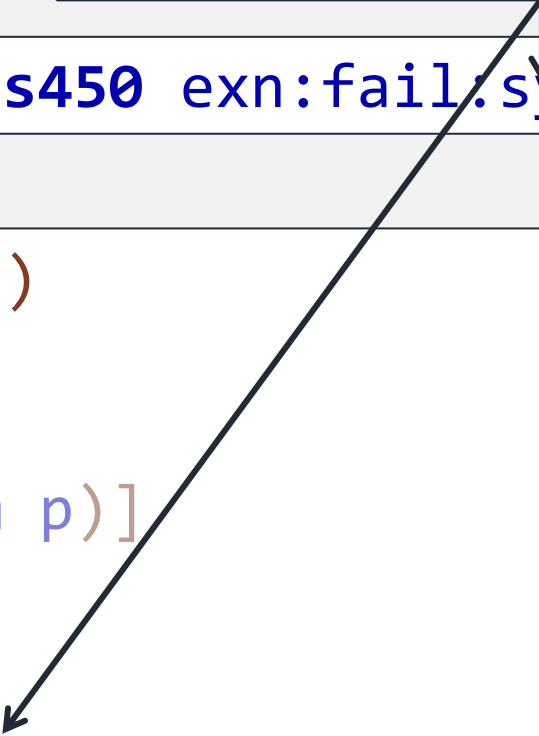
Interlude: Racket exceptions

Exceptions are just special structs

Super struct (enables using exception API)

```
(struct exn:fail:syntax:cs450 exn:fail:syntax [])
```

```
(define/contract (parse p)
  (-> Program? AST?)
  (match p
    [(? atom?) (parse-atom p)]
    ...
    [ `(,fn . ,args) ... ]
    [ _ (raise-syntax-error
          'parse "not a valid CS450 Lang program" p
          #:exn exn:fail:syntax:cs450)])))
```




```
(define/contract (parse p)
  (-> Program? AST?)
  (match p
    ...
    [ `(bind [,(and (? symbol?) x) ,e] ,bod) ... ]
    ...
    [ `(,fn . ,args) ... ]
    [ _ (raise-syntax-error
          'parse "not a valid CS450 Lang program" p
          #:exn exn:fail:syntax:cs450))])))
```

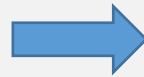
```
(define/contract (parse p)
  (-> Program? AST?)
  (match p
    ...
    [`(bind [,(and (? symbol?) x) ,e] ,bod) ... ]
    [`(bind . _)
      (raise-syntax-error 'parse "invalid bind syntax" p
        #:exn exn:fail:syntax:cs450) ]
    [(,fn . ,args) ... ]
    [_ (raise-syntax-error
      'parse "not a valid CS450 Lang program" p
      #:exn exn:fail:syntax:cs450))]))
```

Bind parse error case

Programs that Add Variables to Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - ...
```

parse



```
;; An AST is one of:  
;; - ...  
;; - (mk-var Symbol)  
;; - (mk-bind Symbol AST AST)  
;; - ...  
  
;; ...  
(struct vari [name])  
(struct bind [var expr body])  
;; ...
```

Need to be more careful parsing

Valid
Program?

```
(check-exn exn:fail:syntax:cs450?  
  (λ () (eval450 '(bind))))
```

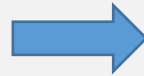
```
(check-exn exn:fail:syntax:cs450?  
  (λ () (eval450 '(bind []))))
```

```
(check-exn exn:fail:syntax:cs450?  
  (λ () (eval450 '(bind [1 2] 3))))
```

Programs that Add Variables to Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - ...
```

parse



```
;; An AST is one of:  
;; - ...  
;; - (mk-var Symbol)  
;; - (mk-bind Symbol AST AST)  
;; - ...  
  
;; ...  
(struct vari [name])  
(struct bind [var expr body])  
;; ...
```



run

???

Bind scoping examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - ...
```

bind obeys “lexical” or “static” scoping

Generally accepted to be “best choice”
for programming language design
(bc it’s determined only by program syntax)



run, with **bind**

```
;; run: AST -> Result
```

```
(define (run p)
```

```
;; accumulator env : Environment
```

```
;; invariant: contains in-scope var + results
```

```
env)
```

```
; An AST is one of:
```

```
; - ...
```

```
; - (mk-bind Symbol AST AST)
```

```
[(vari x) (env-lookup env x)]
```

```
[(bind x e body) ... (env-add env x (run/env e env)) ...]
```

```
... ))
```

```
(run/env p ??? )
```

```
;; An Environment (Env) is one of:
```

```
;; - empty
```

```
;; - (cons (list Var Result) Env)
```

Environment has **Results** (not **AST**)

How to convert **AST** to **Result**?

(From
template!)

Add to environment

Be careful to get correct “**scoping**”
(x not visible in expression e,
so use unmodified input env)

run, with **bind**

run must produce **Result**

;; run: AST -> Result

```
(define (run p)
  ;; accumulator env : Environment
  ;; invariant: contains in-scope var + results
  (run/env p env))
```

; An AST is one of:

; - ...

; - (mk-bind Symbol AST AST)

```
    [(vari x) (env-lookup env x)]
    [(bind x e body) ??? (env-add env x (run/env e env)) ...]
    ... ))
(run/env p ??? )
```

run, with **bind**

```
;; run: AST -> Result
```

```
(define (run p)
  ;; accumulator env : Environment
  ;; invariant: contains in-scope var + results
  (define (run/env p env)
    (match p
      ...
      [(vari x) (env-lookup env x)]
      [(bind x e body) (run/env body (env-add env x (run/env e env)))]
      ... ))
    (run/env p ??? ))
```

(From template!)

run body with new env containing **x**

Initial Environment?

TODO:

- ~~When are variables “added” to environment~~
- What is initial environment? **empty** (for now)

```
;; run: AST -> Result
```

```
(define (run p)
  ;; accumulator env : Environment
  ;; invariant: contains in-scope var + results
  (define (run/env p env)
    (match p
      ...
      [(vari x) (env-lookup env x)]
      [(bind x e body) (run/env body (env-add env x (run/env e env)))]
      ... ))
  (run/env p ??? ))
```

empty ???

(for now)

Initial Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(+,Program ,Program)  
;; - `(×,Program ,Program)
```

These don't need to be separate constructs

Put these into "initial" environment

Initial Environment

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(+,Program,Program)  
;; - `(x ,Program ,Program)
```

Put these into “initial” environment

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
(define INIT-ENV  
  `((+ ,450+)  
    (x ,450*)))
```

+ variable

New kind
of Result

Maps to our
“450+” function

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - (Racket) Function
```

For Program: +

Initial Environment

How do users call these functions???

```
(define INIT-ENV '((+ ,450+) (× ,450*)))
```

```
(define (run p)

  ;; accumulator env : Environment
  (define (run/e p env)
    (match p
      ...
      [(vari x) (lookup env x)]
      [(bind x e body) (run/e body (env-add env x (run/e e env)))]
      ... ))
  (run/e p INIT-ENV ))
```

Function Application in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(fncall ,Program . ,List<Program>)
```

function

arguments

“rest” arg

Allows arbitrary number of args

Function Application in CS450 Lang: Examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(fncall ,Program . ,List<Program>)
```

function

arguments

```
(fncall + 1 2)
```

Programmers shouldn't need to write the explicit "fncall"

Function Application in CS450 Lang: Examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(,Program . ,List<Program>)
```

```
(+ 1 2)
```

Function Application in CS450 Lang: Examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(,Program . ,List<Program>)  
;; - (cons Program List<Program>)
```

Can go back to (simpler?) cons notation

(+ 1 2)

Must be careful when parsing this

Function call case (must be last, why?)

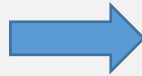
(bind 1 2)

(should not be function call)

Function Application in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(,Program . ,List<Program>)  
;; - (cons Program List<Program>)
```

parse



```
;; An AST is one of:  
;; - ...  
;; - (mk-var Symbol)  
;; - (mk-bind Symbol AST AST)  
;; - (mk-call AST List<AST>)  
  
(struct vari [name])  
(struct bind [var expr body])  
(struct call [fn args])
```

“Running” Function Calls

TEMPLATE: extract pieces of compound data

```
(define (run p)
```

```
  (define (run/e p env)
    (match p
```

...

```
      [(call fn args) (apply
                          (run/e fn env)
                          (map (curryr run/e env) args))])
      ...
    ))
```

```
(run/e p INIT-ENV))
```

```
;; An AST is one of:
```

```
;; - ...
```

```
;; - (mk-var Symbol)
```

```
;; - (mk-bind Symbol AST AST)
```

```
;; - (mk-call AST List<AST>)
```

```
(struct vari [name])
```

```
(struct bind [var expr body])
```

```
(struct call [fn args])
```

“Running” Function Calls

```
(define (run p)
```

```
  (define (run/e p env)
    (match p
```

TEMPLATE: recursive calls

```
      ...
      [(call fn args) (apply
                        (run/e fn env)
                        (map (curry ??? run/e env) args))])
      ...
    ))
```

```
(run/e p INIT-ENV))
```

```
;; An AST is one of:
;; - ...
;; - (mk-var Symbol)
;; - (mk-bind Symbol AST AST)
;; - (mk-call AST List<AST>)
```

“run” args before calling function – “call by value”

“Running” Function Calls

How do we actually run the function?

;; A Result is one of:
;; - Number
;; - UNDEFINED-ERROR
;; - (Racket) Function

```
(define (run p)
```

```
  (define (run/e p env)
    (match p
```

...

```
      [(call fn args) (apply
                        (run/e fn env)
                        (map (curryr run/e env) args))])
      ...
    ))
```

Calls racket fns

???

Does this work???

```
(run/e p INIT-ENV))
```

“Running” Non-Functions

;; A Result is one of:
;; - Number
;; - UNDEFINED-ERROR
;; - (Racket) Function

```
(define (run p)
```

```
  (define (run/e p env)
    (match p
```

Example: (eval450 '(10 10)) ; apply non-fn

```
      ...
      [(call fn args) (apply
                          (run/e fn env)
                          (map (curryr run/e env) args))])
      ...
    ))
  (run/e p INIT-ENV))
```

“Running” Non-Functions

```
(define (run p)
```

```
  (define (run/e p env)
    (match p
```

```
      ...
      [(call fn args) (450apply
                        (run/e fn env)
                        (map (curryr run/e env) args))])
      ...
    ))
```

```
(run/e p INIT-ENV))
```

```
;; A Result is one of:
;; - Number
;; - UNDEFINED-ERROR
;; - NON-FUNCTION-ERROR
;; - (Racket) Function
```

Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) ... ]  
    [(UNDEFINED-ERROR? fn) ... ]  
    [(NON-FUNCTION-ERROR? fn) ... ]  
    [(procedure? fn) ... ])))
```

TEMPLATE?

Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) NON-FUNCTION-ERROR]  
    [(UNDEFINED-ERROR? fn) ... ]  
    [(NON-FUNCTION-ERROR? fn) ... ]  
    [(procedure? fn) ... ]))
```


Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) NON-FUNCTION-ERROR]  
    [(UNDEFINED-ERROR? fn) ...]  
    [(NON-FUNCTION-ERROR? fn) ...]  
    [(procedure? fn) (apply fn args)])))
```

Now this works

Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) NON-FUNCTION-ERROR]  
    [(UNDEFINED-ERROR? fn) ... ]  
    [(NON-FUNCTION-ERROR? fn) NON-FUNCTION-ERROR]  
    [(procedure? fn) (apply fn args)])))
```

Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)
```

```
  (cond
```

```
    [(number? fn) NON-FUNCTION-ERROR]
```

UNDEFINED should have precedence over NON-FN-ERR

```
    [(UNDEFINED-ERROR? fn) UNDEFINED-ERROR]
```

```
    [(NON-FUNCTION-ERROR? fn) NON-FUNCTION-ERROR]
```

```
    [(procedure? fn) (apply fn args)])))
```

Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

Add ARITY-ERROR ???

```
;; 450apply : Result Listof<Result> -> Result
```

For now, we only use variable-arity functions

```
(define (450apply fn args)  
  (cond  
    [(UNDEFINED-ERROR? fn) UNDEFINED-ERROR]  
    [(procedure? fn) (apply fn args)]  
    [else NON-FUNCTION-ERROR]))
```

Combine cases

```
(define INIT-ENV  
  `((+ ,450+)  
    (× ,450*)))
```

Check correct number of arguments???

In-class Coding 4/22: bind + “call” examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [ ,Var ,Program] ,Program)  
;; - `( ,Program . ,List<Program>)
```

Come up with your own examples!

```
(check-equal?  
  (eval450 `(bind [x 10] x))  
  10 ) ; no shadow
```

```
(check-equal?  
  (eval450 `(bind [x 10] (bind [x 20] x)))  
  20 ) ; shadow
```

```
(check-equal?  
  (eval450  
    `(bind [x 10]  
      (+ (bind [x 20]  
        x)  
        x))) ; 2nd x outof scope here  
  30 )
```

```
(check-equal?  
  (eval450  
    `(bind [x 10]  
      (bind [x (+ x 20)] ; x = 10 here  
        x))) ; x = 30 here  
  30 )
```