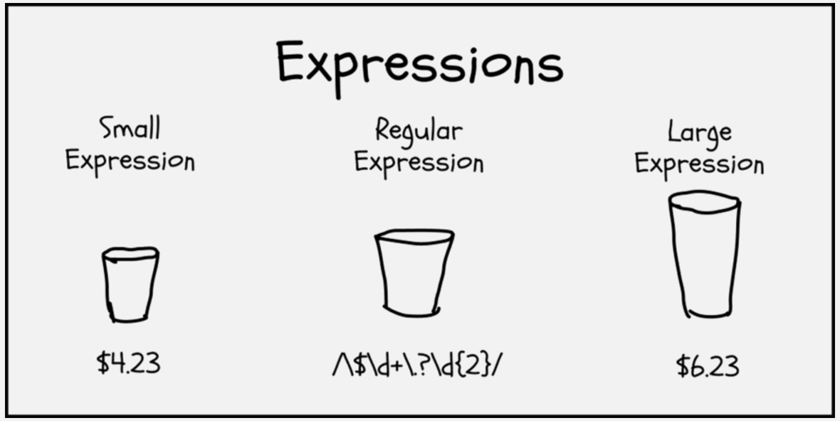
UMB CS 420

Regular Expressions

Tuesday, October 4, 2022



Announcements

- HW 2 in
  - ~~Due Sun 10/2 11:59pm EST~~
- HW 3 out
  - Due Sun 10/9 11:59pm EST
- Sean's office hours
  - Mon 4-5pm EST (McCormack 3rd floor room 139)
- HW 1 issues – many submitted solutions do not answer the question
  - Example Question: "Prove that language L is regular"
  - Example Good Answer: "Language L is regular because …"
  - Example Bad Answer: "Here are some sets of stuff, called Q, Σ, …"

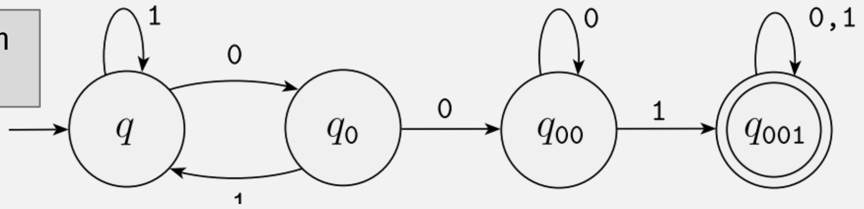
Last Time: Why These (Closed) Operations?

- Union
- Concat
- Kleene star

<u>All</u> regular languages can be <u>constructed</u> from:
- single-char strings, and
- these operations!

So Far: Regular Language Representations

1. State diagram (NFA/DFA)



Actually, it's a real programming language: for **text search**

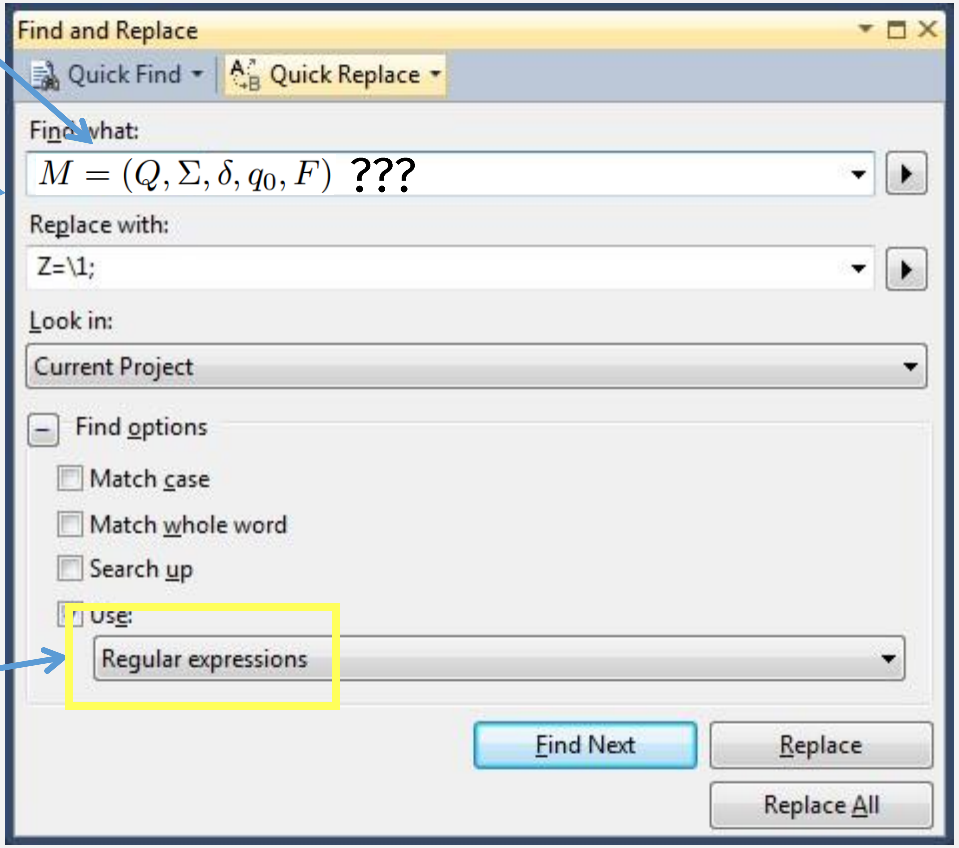
2. Formal description

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

4. q_1 is the start state
5. $F = \{q_2\}$

(doesn't fit)



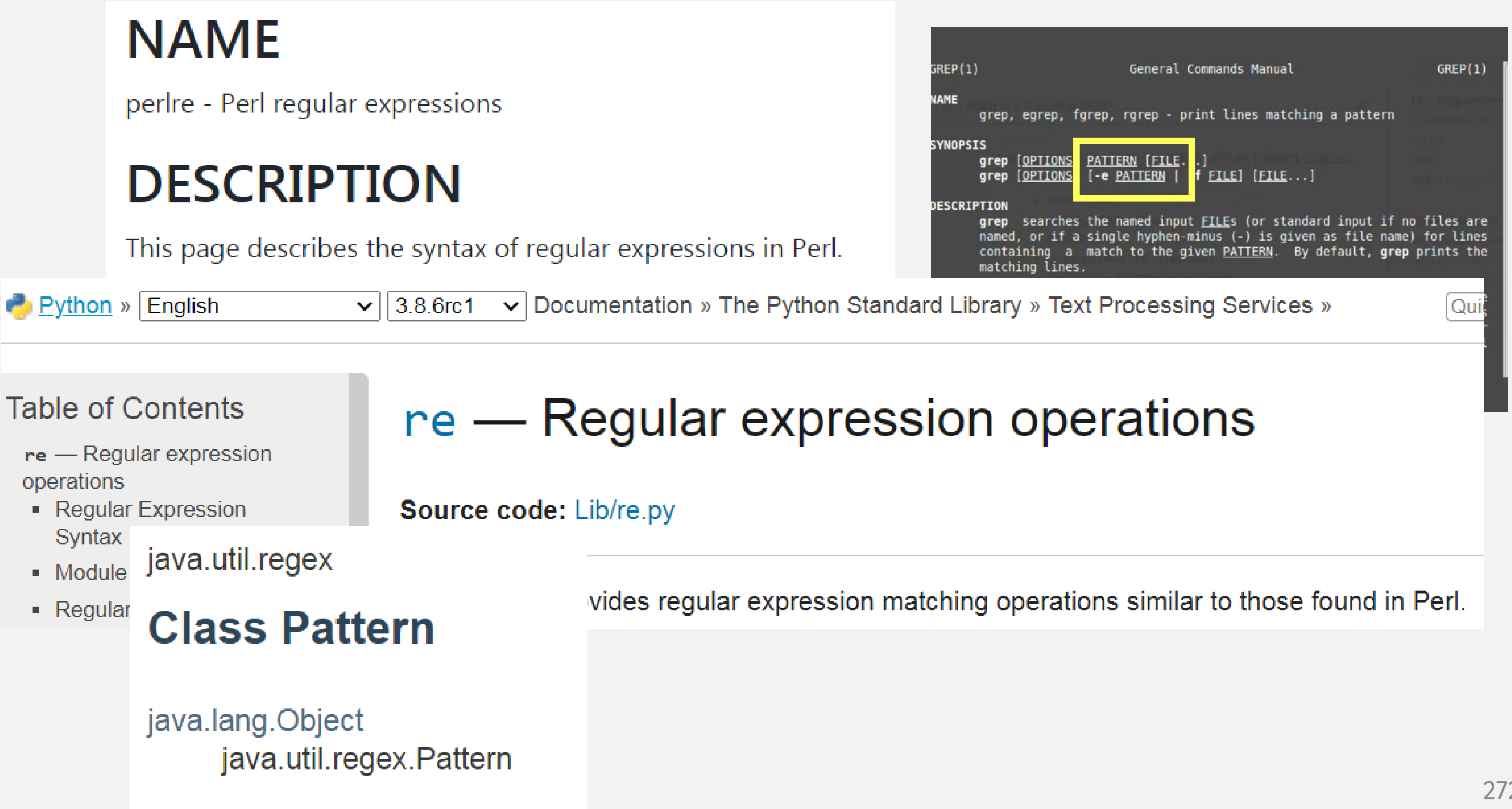
<u>Our Running Analogy:</u>
- <u>Class</u> of **regular languages** ~ a "programming language"
- <u>One</u> **regular language** ~ a "program"

? 3. $\Sigma^*001\Sigma^*$

Need a more concise (textual) notation??

Regular Expressions:
A Widely Used Programming Language
(inside other programming languages)

- Unix
- Perl
- Python
- Java



Why These (Closed) Operations?

- Union
- Concat
- Kleene star

All regular languages can be constructed from:
- single-char strings, and
- these operations!

The are used to define **regular expressions**!

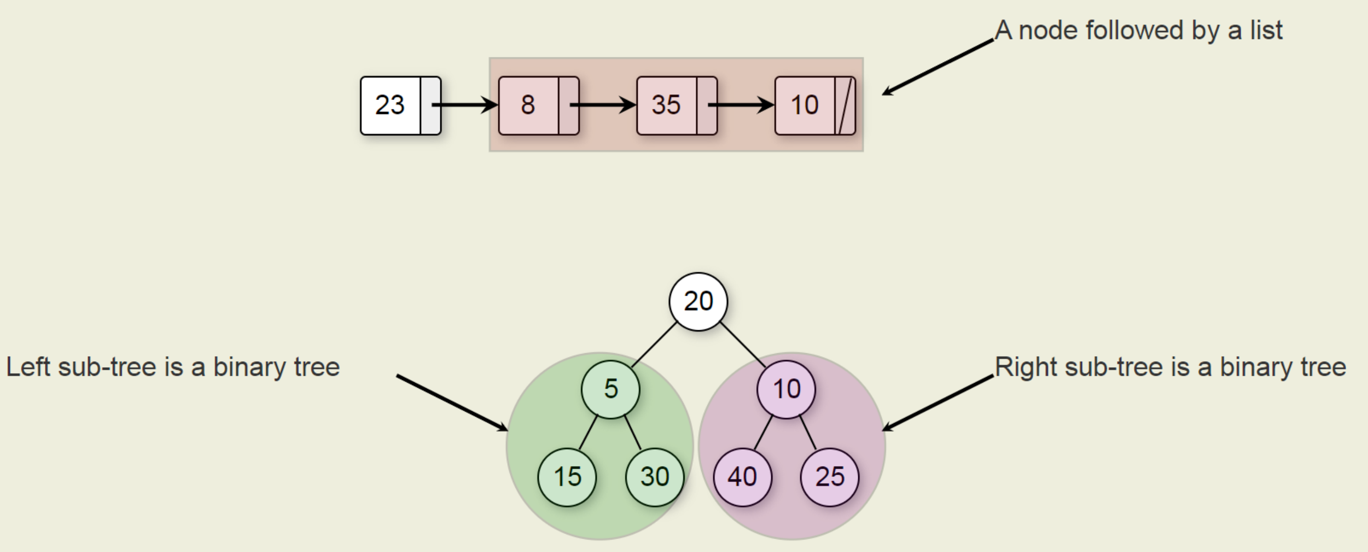
Regular Expressions: Formal Definition

R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, or
6. (R_1^*), where R_1 is a regular expression.

This is a <u>recursive definition</u>

Recursive Definitions



Recursive definitions have:
- <u>base case</u> and
- <u>recursive case</u>
  (with a “smaller” object)

```
/* Linked list Node*/
class Node {
    int data;
    Node next;
}
```

This is a <u>recursive definition:</u>
Node used before it's defined
(but must be “smaller”)

Regular Expressions: Formal Definition

R is a ***regular expression*** if R is

3 Base Cases

1. a for some a in the alphabet Σ, (A lang containing a) **length-1 string**
2. ε, (A lang containing) **the empty string**
3. $\emptyset$, **The empty set** (i.e., a lang containing no strings)

union — 4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,

concat — 5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, or

star — 6. (R_1^*), where R_1 is a regular expression.

3 Recursive Cases

Regular Expression: Concrete Example

Entire regular expr: language whose strings come from these languages concat'ed (implicit) together

the language {“0”, ”1”}

$(0 \cup 1)0^*$

the language {“”, ”0”, “00”, …}

the language {“0”}

the language {“1”}

- Operator Precedence:
  - Parentheses
  - Kleene Star
  - Concat (sometimes use $\circ$, sometimes implicit)
  - Union

R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ,
2. ε,
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4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, or
6. (R_1^*), where R_1 is a regular expression.

Regular Expressions = Regular Langs?

3 Base Cases

3 Recursive Cases

R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, or
6. (R_1^*), where R_1 is a regular expression.

Any regular language can be constructed from: base cases + union, concat, and Kleene star

(But we have to prove it)

Thm: A Lang is Regular **iff** Some Reg Expr Describes It

⇒ If **a language is regular,** it **is described by a reg expression**

⇐ If **a language is described by a reg expression,** it **is regular**

(Easier)

- To prove this part: convert reg expr → equivalent NFA!
- (Hint: we mostly did this already when discussing closed ops)

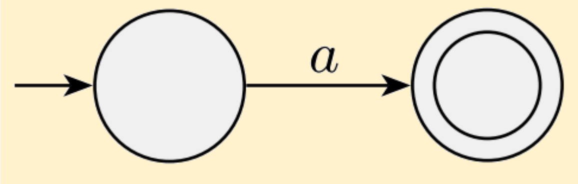
How to show that a language is regular?

Construct a DFA *or* NFA!

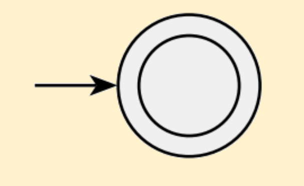
RegExpr→NFA

R is a ***regular expression*** if R is

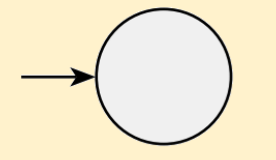
1. a for some a in the alphabet Σ,



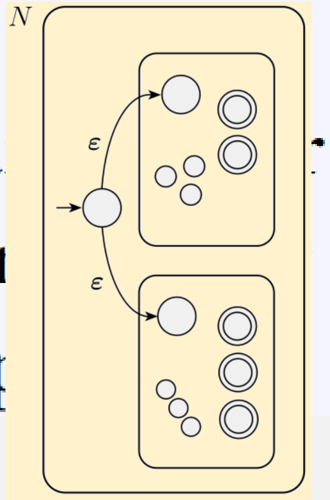
2. ε,

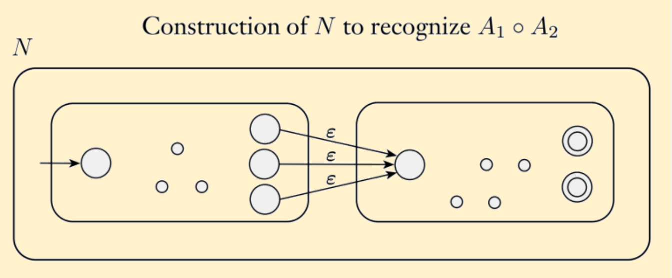


3. $\emptyset$,

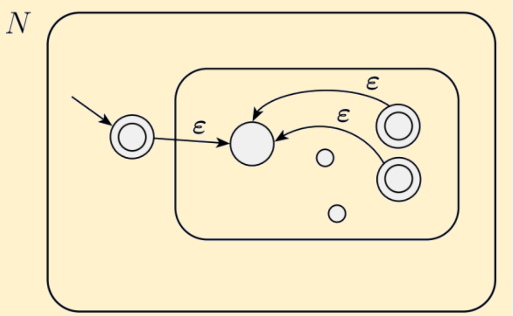


4. $(R_1 \cup R_2)$, where R_1 and R_2 a





5. $(R_1 \circ R_2)$, where R_1 and R_2 ar expre



6. (R_1^*), where R_1 is a regular exp

Thm: A Lang is Regular **iff** Some Reg Expr Describes It

⇒ If **a language is regular,** it **is described by a reg expression**

(Harder)

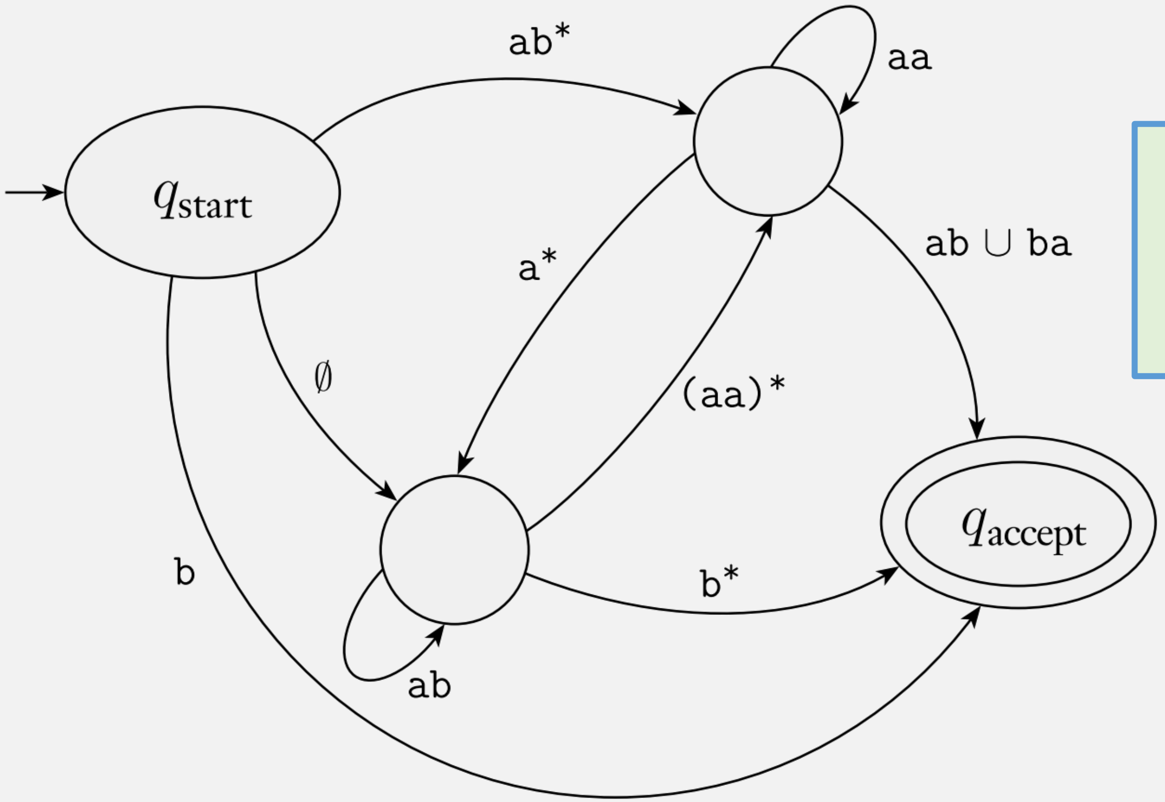
- To prove this part: Convert an DFA or NFA → equivalent Regular Expression
- To do so, we first need another kind of finite automata: a **GNFA**

⇐ If **a language is described by a reg expression,** it **is regular**

(Easier)

☑ • Convert the regular expression → an equivalent NFA!

Generalized NFAs (GNFAs)



plain NFA
= GNFA with single char
regular expr transitions

Goal: **convert GNFAs to Regular Exprs**

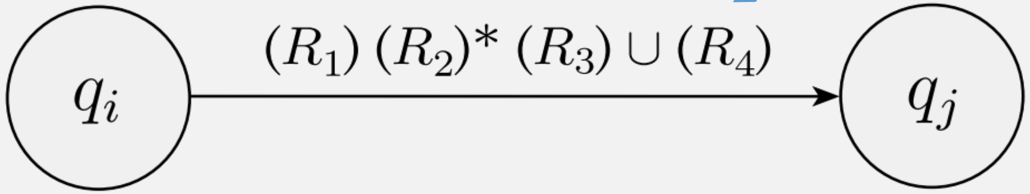
- GNFA = NFA with regular expression transitions

GNFA→RegExpr function

On **GNFA** <u>input</u> G:

- If G **has 2 states, `return` the regular expression** (on transition), e.g.:

Equivalent regular expression



GNFA

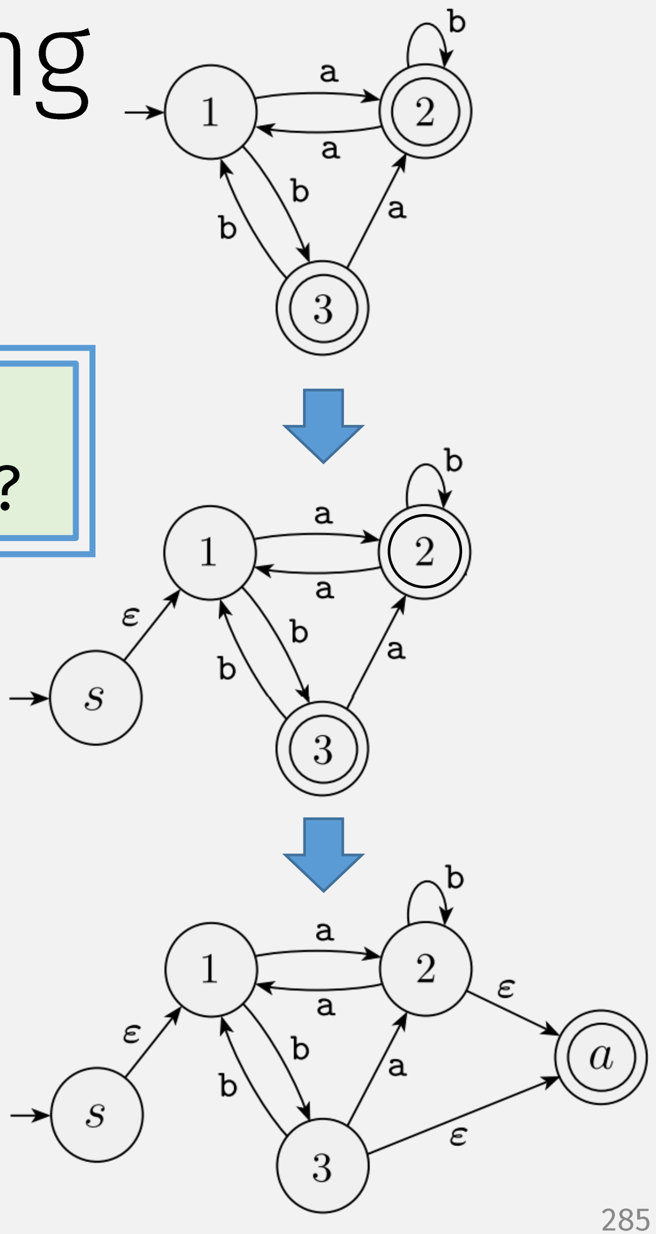
Could there be less than 2 states?

GNFA→RegExpr Preprocessing

- First, modify input machine to have:

Does this change the language of the machine?

  - New start state:
    - No incoming transitions
    - ε transition to old start state
  - New, single accept state:
    - With ε transitions from old accept states

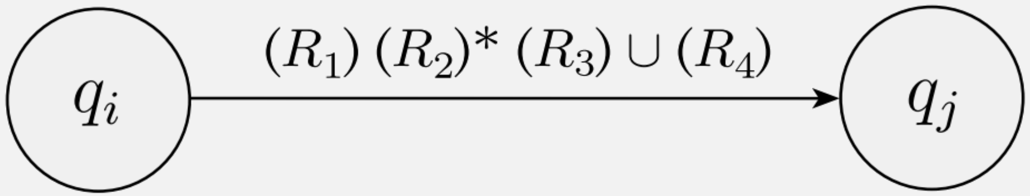


GNFA→`RegExpr` function (recursive)

On GNFA <u>input</u> G:

Base Case

- If G has 2 states, **`return`** the regular expression (from transition), e.g.:

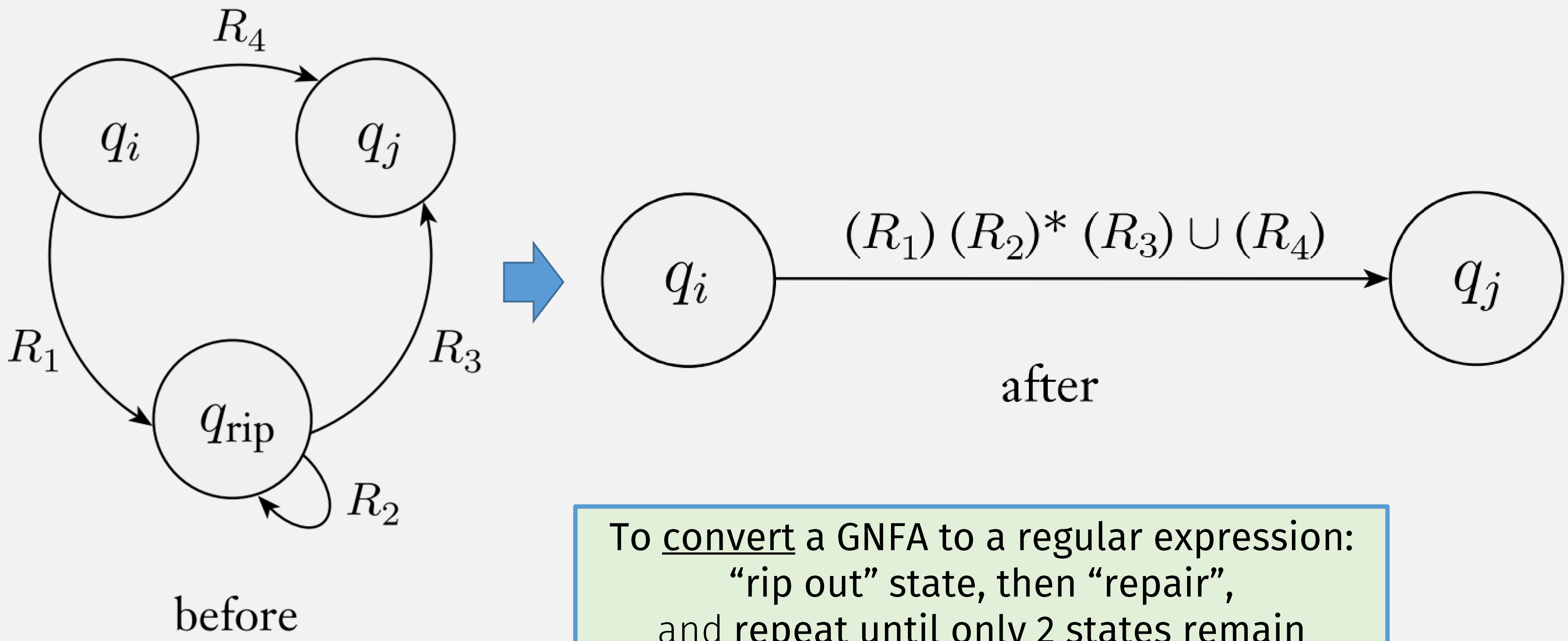


Recursive Case

- Else:
  - “Rip out” one state
  - “Repair” the machine to get an <u>equivalent</u> GNFA G'
  - <u>Recursively</u> call GNFA→`RegExpr`(G')

Recursive definitions have:
- <u>base case</u> and
- <u>recursive case</u>
  (with a “smaller” object)

GNFA→RegExpr: "Rip/Repair" step

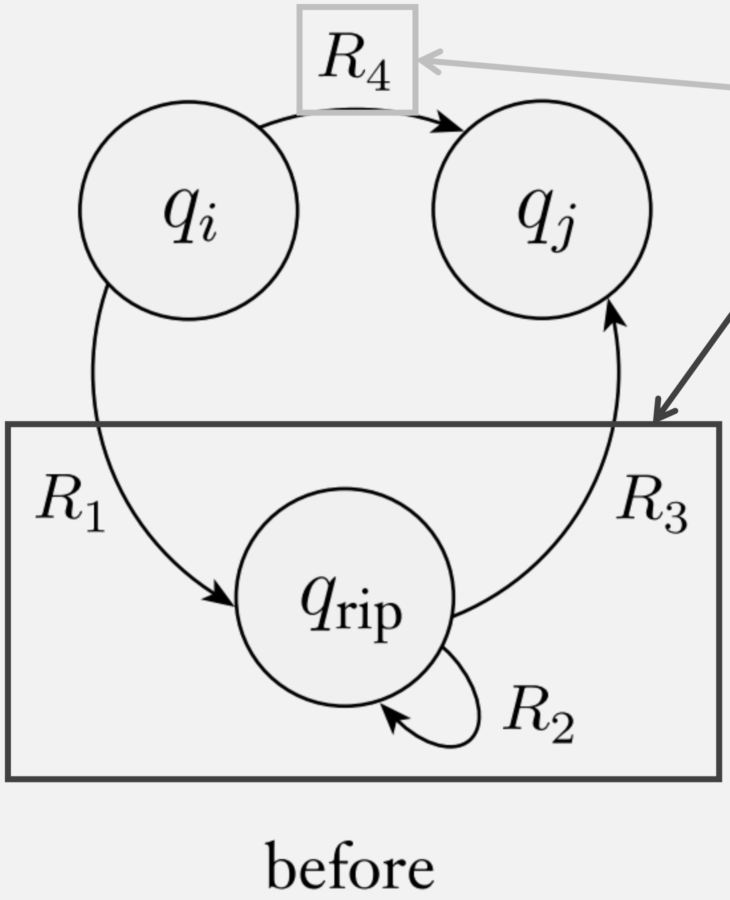


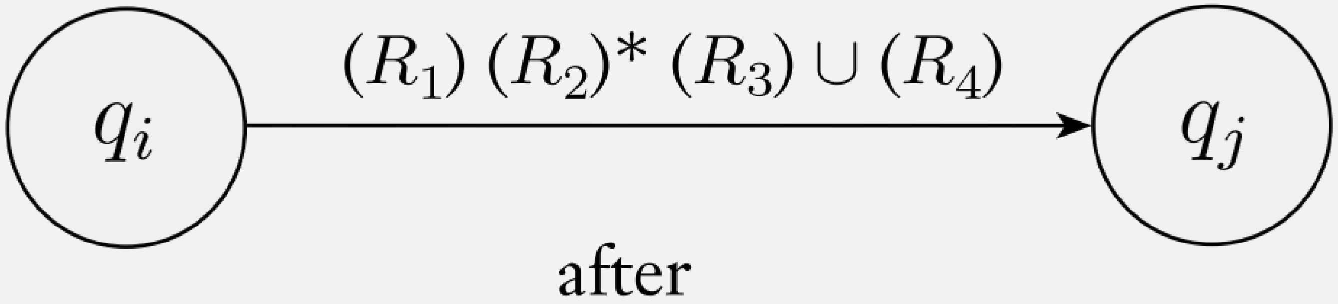
To convert a GNFA to a regular expression:
"rip out" state, then "repair",
and repeat until only 2 states remain

GNFA→RegExpr: "Rip/Repair" step

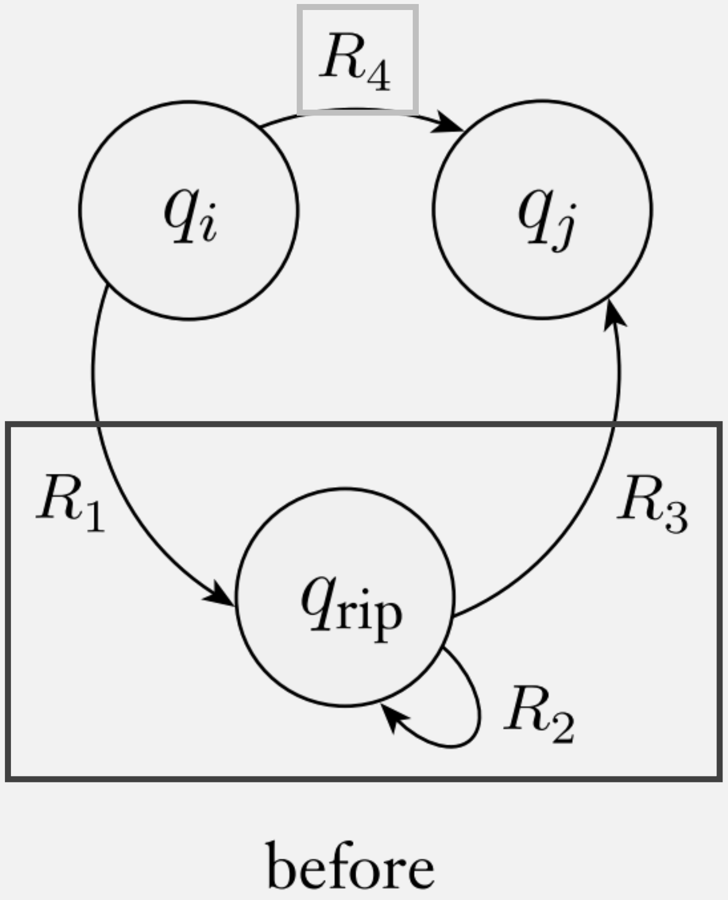
<u>Before</u>: two paths from q_i to q_j:

1. Not through q_{rip}
2. Through q_{rip}



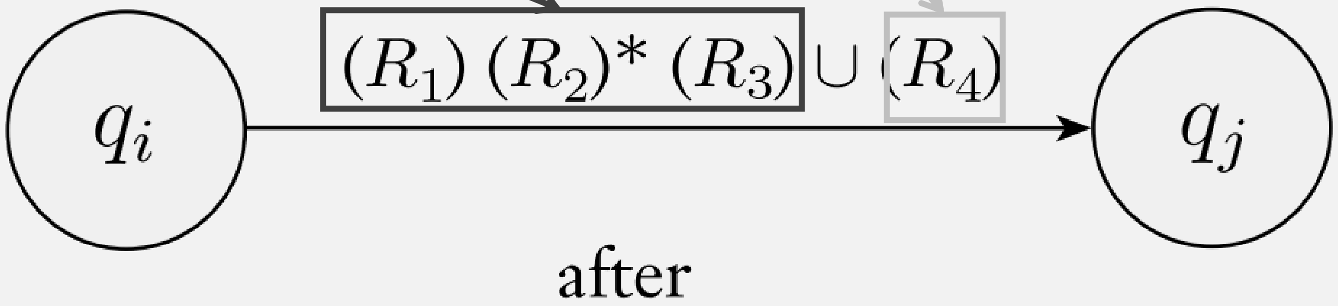


GNFA→RegExpr: "Rip/Repair" step

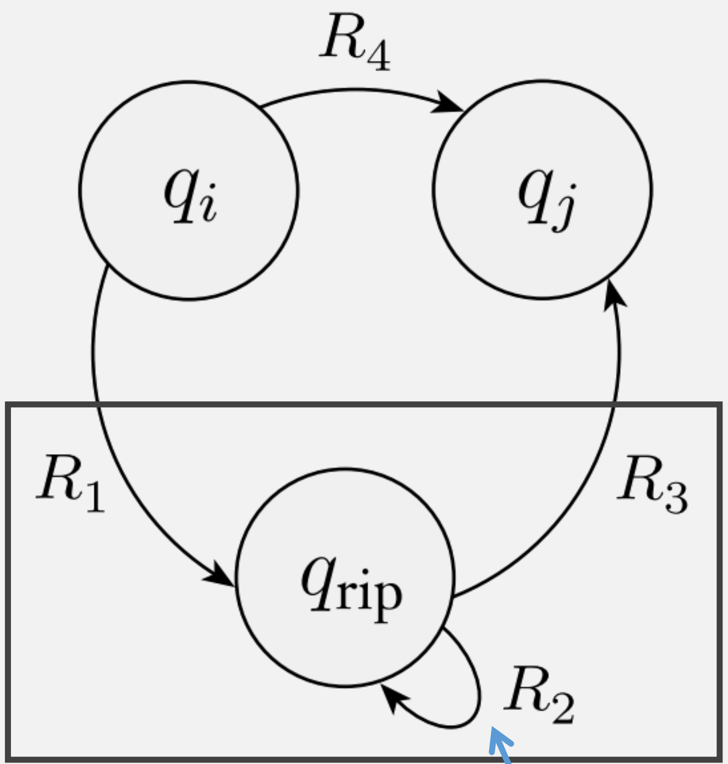


After: still two "paths" from q_i to q_j

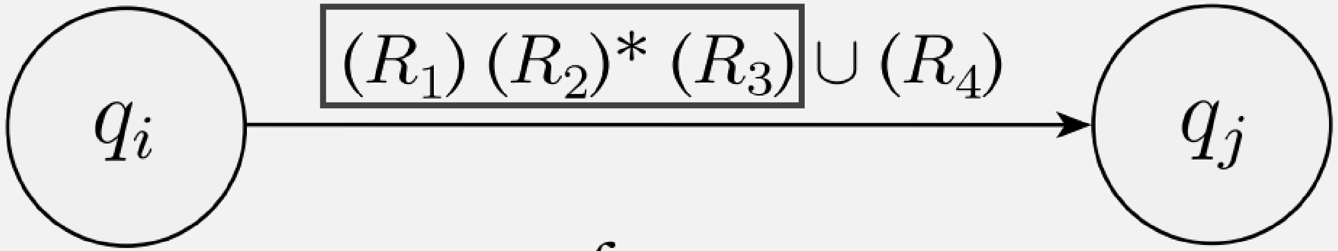
1. Not through q_{rip}
2. Through q_{rip}



GNFA→RegExpr: “Rip/Repair” step



before

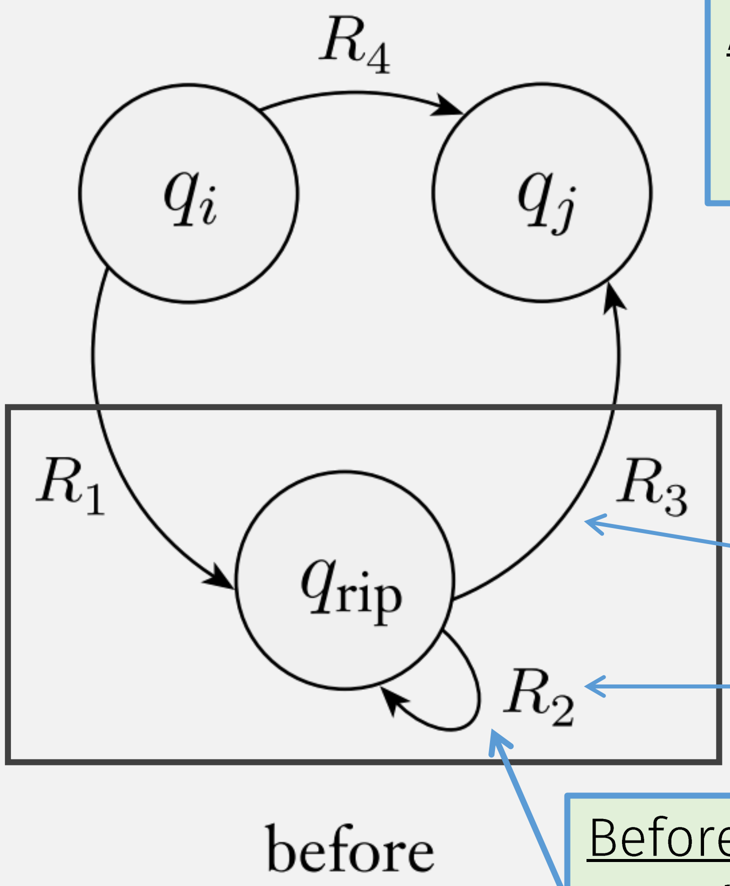


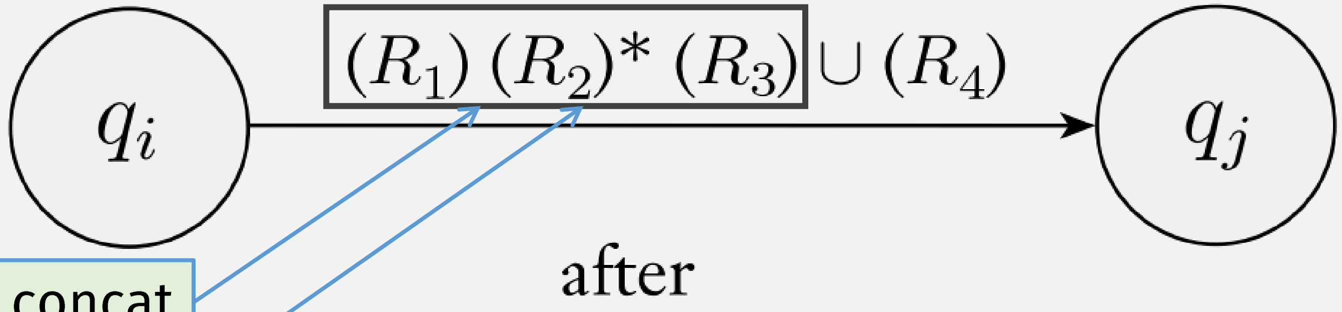
after

<u>Before</u>:
- path through q_{rip} has 3 transitions
- One is self loop

GNFA→RegExpr: “Rip/Repair” step

<u>After</u>:
- Self loop becomes star operation
- Others are concat’ed together



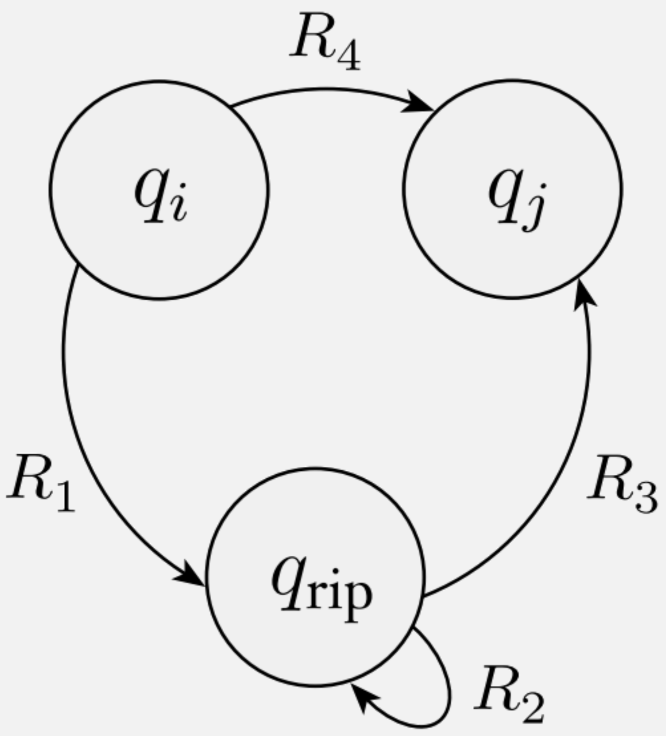


concat

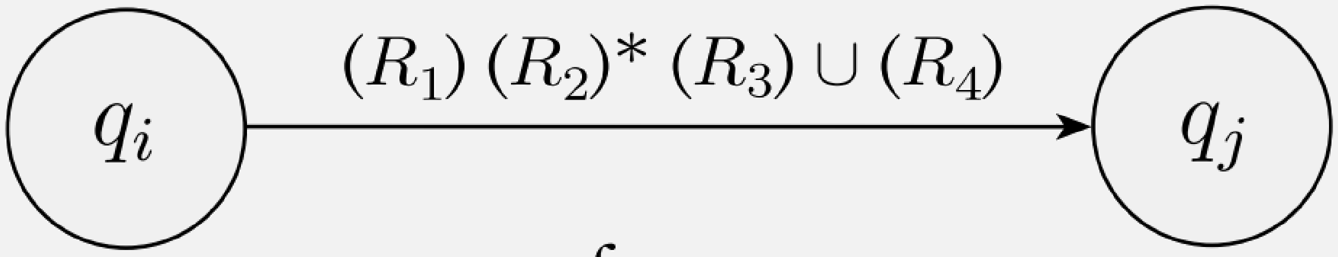
Star operation

<u>Before</u>:
- path through q_{rip} has 3 transitions
- One is self loop

GNFA→RegExpr: Rip/Repair "Correctness"



before



after

Must show these are <u>equivalent</u>

GNFA→RegExpr "Correctness"

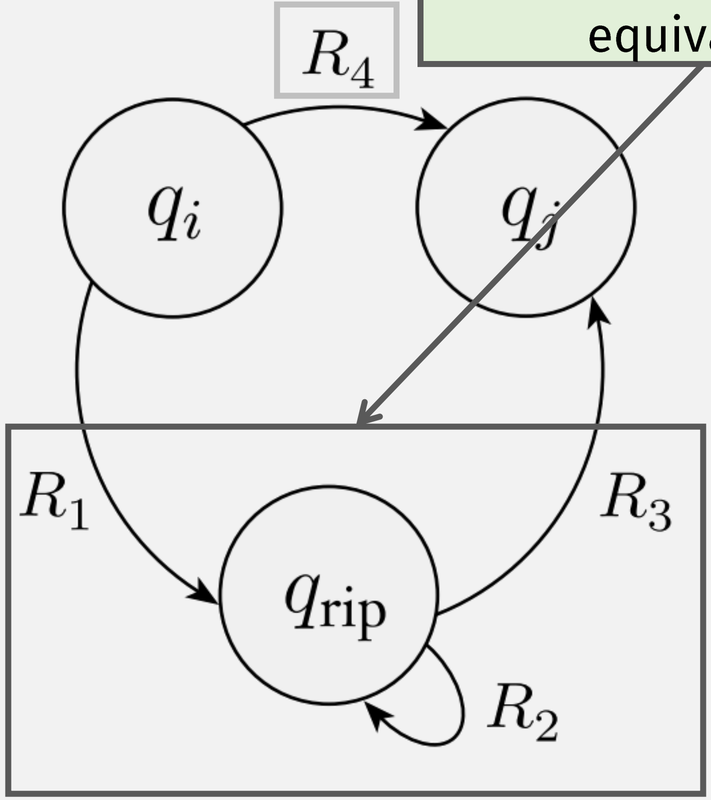
- "Correct" / "Equivalent" means:

$$\text{LangOf}\,(\,G\,) = \text{LangOf}\,(\,\textbf{GNFA→RegExpr}(\,G\,)\,)$$

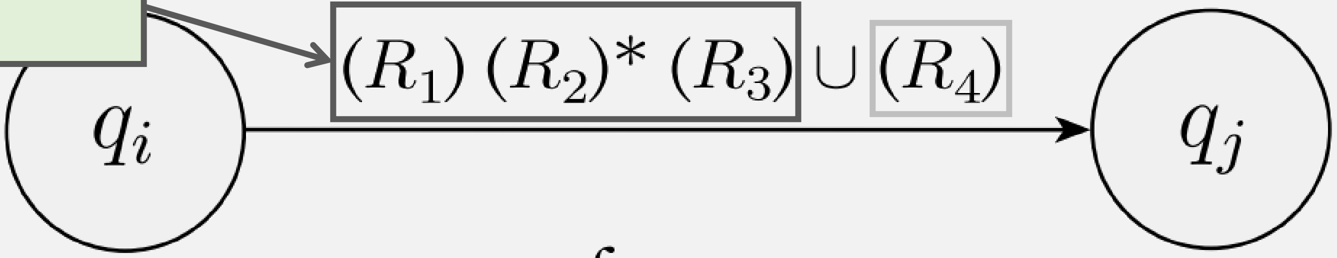
- i.e., **GNFA→RegExpr** must not change the language!
    - Key step: the rip/repair step

GNFA→RegExpr: Rip/Repair "Correctness"

Must show these are equivalent



before



after

Must <u>prove</u>:

- Every string accepted <u>before</u>, is accepted <u>after</u>
- <u>2 cases:</u>
  1. Accepted string does not go through q_{rip}
     - ☑ Acceptance unchanged (both use R_4 transition part)
  2. String goes through q_{rip}
     - Acceptance unchanged?
     - ☑ Yes, via our previous reasoning

Thm: A Lang is Regular **iff** Some Reg Expr Describes It

⇒ If **a language is regular,** it **is described by a regular expr**

Need to convert DFA or NFA to Regular Expression …

☑ • Use **GNFA→RegExpr** to convert GNFA → equiv regular expression!

⇐ If **a language is described by a regular expr,** it **is regular**

☑ • Convert regular expression → equiv NFA!

∎

Now we may use regular expressions to represent regular langs.

So a regular language has these equivalent representations:

- DFA
- NFA
- Regular Expression

So we also have another way to prove things about regular languages!

How to Prove A Language Is Regular?

- Construct DFA
- Construct NFA
- Create Regular Expression

Slightly different because of recursive definition

R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
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6. (R_1^*), where R_1 is a regular expression.

Kinds of Mathematical Proof

- **Deductive proof** (from before)
  - Make logical inferences
- **Inductive proof** (now)
  - Use this when working with <u>recursive</u> definitions

In-Class quiz 9/29

See gradescope