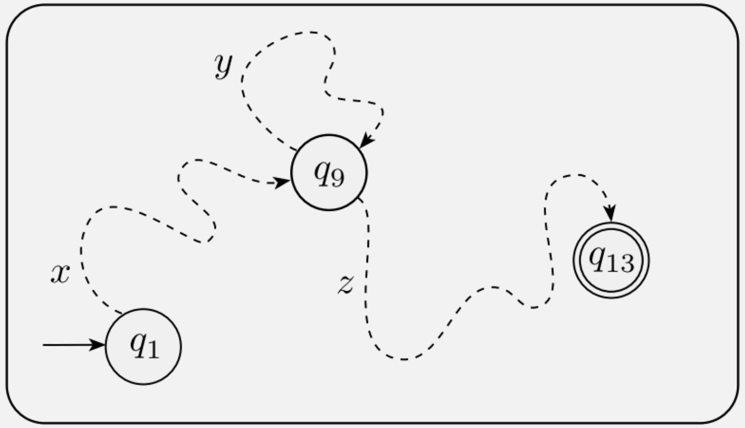
Examples with the Pumping Lemma

Wed Feb 24, 2021



Logistics

- HW3 solutions posted (soon)
- HW4 due Sunday 2/28 11:59pm EST
- Questions?

<u>Last time</u>: The Pumping Lemma says:

For all strings in a regular language that are "long enough" (i.e., length p) …

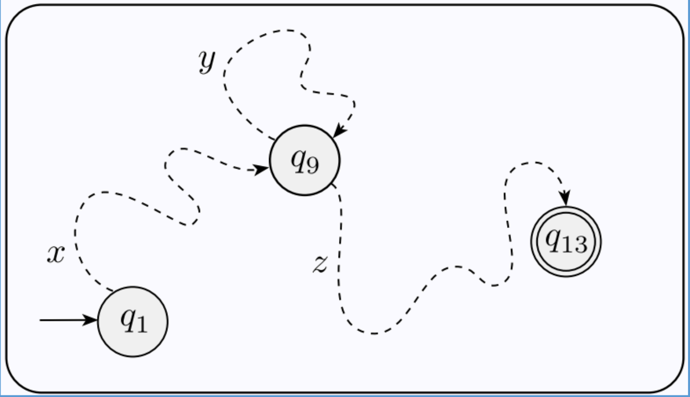
Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

… these strings must be divisible into three pieces (call them x, y, and z) …

… where repeating the middle piece y results in a "pumped" string is also in the language

Also, repeating part:
- can't be empty string
- must be in the first p characters



tl;dr:
Long enough strings means repeated states

<u>Last time</u>: Equivalence of Contrapositive

- "If X then Y" is equivalent to … ?
  - "If Y then X" (converse)
    - No!
  - "If not X then not Y" (inverse)
    - No!
  - ✓"If not Y then not X" (contrapositive)
    - **Yes**!
    - Proof by contradiction uses this equivalence

The Pumping Lemma is an If-Then Stmt

... then the language is **not** regular

Pumping lemma $\boxed{\text{If}}$ A is a regular language, $\boxed{\text{then}}$ there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Just need one counterexample!

Contrapositive: If (**any** of) these are **not** true ...

IMPORTANT NOTE:
The pumping lemma **cannot** be used to show that a language is regular, only that is is non-regular

Pumping Lemma: Non-Regularity Example

Let B be the language $\{\mathtt{0}^n\mathtt{1}^n | n \geq 0\}$. We use the pumping lemma to prove that B is not regular. The proof is by contradiction.

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
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Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Possible Split: y = all 0s

- Assumption: 0^n1^n is a regular language (must satisfy pumpi
- Counterexample = 0^p1^p

But pumping lemma requires **only one** pumpable splitting

- If xyz chosen so y contains
  - all 0s

$$\overbrace{\underbrace{00}_{x}\underbrace{\ldots 0}_{y}}^{p \text{ 0s}}\overbrace{\underbrace{11 \ldots 1}_{z}}^{p \text{ 1s}}$$

So we must show that **every splitting** produces a contradiction

- Pumping y: produces a string with more 0s than 1s
  - This string is not in the language 0^n1^n
  - This means that 0^n1^n does not satisfy the pumping lemma
  - Which means that that 0^n1^n is a not regular lang
  - This is a **contradiction** of the assumption!

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Possible Split: y = all 1s

- Assumption: 0^n1^n is a regular language (must satisfy pumping lemma)
- Counterexample = 0^p1^p

$$\underbrace{\overbrace{00\ldots0}^{p\text{ 0s}}}_{x}\overbrace{\underbrace{11}_{y}\underbrace{\ldots1}_{z}}^{p\text{ 1s}}$$

- If xyz chosen so y contains
  - all 1s
- Is this string pumpable?
  - No!
  - By the same reasoning as in the previous slide

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Possible Split: y = 0s and 1s

- Assumption: 0^n1^n is a regular language (must satisfy pumping lemma)
- Counterexample = 0^p1^p

$$\overbrace{00\ldots0}^{p\text{ 0s}}\overbrace{11\ldots1}^{p\text{ 1s}}$$

$$\underbrace{00}_{x}\ \underbrace{\ldots011}_{y}\ \underbrace{\ldots1}_{z}$$

- If xyz chosen so y contains
  - both 0s and 1s

Did we examine every possible splitting?

Yes. But maybe we don't have to.

- Is this string pumpable?
  - No!
  - Pumped string will have equal 0s and 1s
  - But they will be in the wrong order: so there is still a **contradiction**!

Last time: The Pumping Lemma says:

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Also, repeating part y:
- can't be empty string
- must be in the first p characters

$$\overbrace{00 \ldots 0}^{p \text{ 0s}}11 \ldots 1$$

$$0\underbrace{0 \ldots 0}_{y \text{ must be in here!}}11 \ldots 1$$

Pumping Lemma: How to use Condition 3

Let $F = \{ww |\ w \in \{0,1\}^*\}$. We show that F is nonregular

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
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3. $|xy| \leq p$.

Pumping Lemma: Pumping Down

use the pumping lemma to show that $E = \{0^i 1^j \mid i > j\}$ is not regular.

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Check-in Quiz 2/24

On gradescope