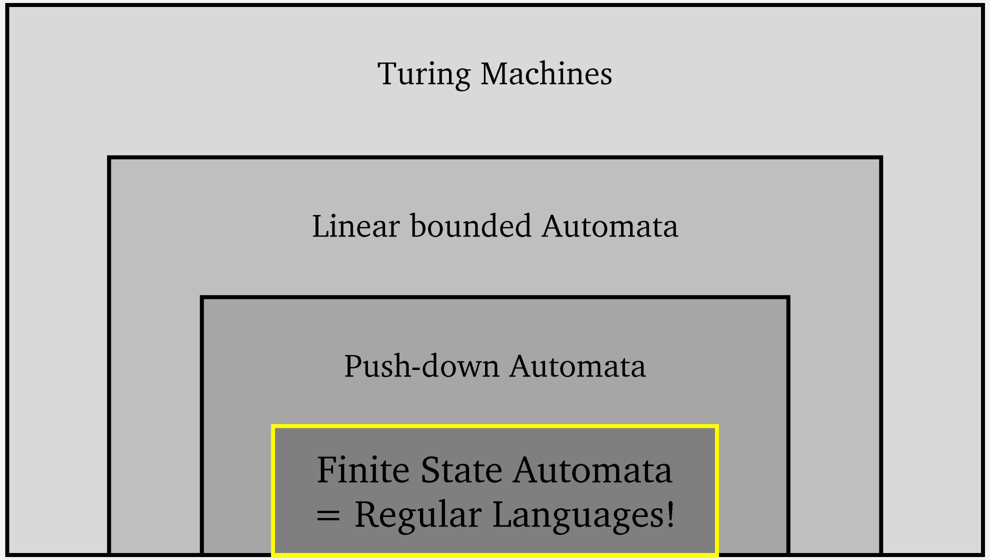
CS420

Regular Languages

Wednesday, February 1, 2023

UMass Boston Computer Science



Announcements

- HW 0 in
  - ~~Due Tues 1/31 11:59pm EST~~
- HW 1 out
  - Due Tues 2/7 11:59pm EST
- Quiz preview:
  Why do we know that a language is a regular language if it has an FSM recognizing it?

M ***accepts*** w if $\hat{\delta}(q_0, w) \in F$

Last Time: Computation and Languages

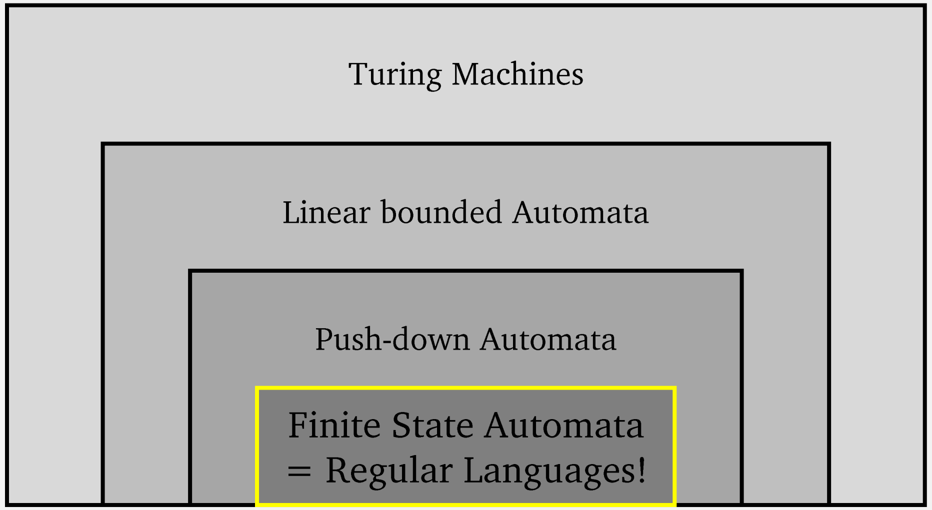
- The **language of a machine** is the <u>set of all strings that it accepts</u>

- A **computation model** is equivalent to the <u>set of machines</u> it defines
  - E.g., all possible Finite State Automata are a computation model

DEFINITION

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta \colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



- Thus: a **computation model** is also equivalent to a <u>set of languages</u>

Last Time: Regular Languages: Definition

If a finite automaton (FSM) **recognizes** a language,

then **that language is called a regular language**.

A ***language*** is a set of strings.

M ***recognizes language*** A
if $A = \{w \mid M \text{ accepts } w\}$

Last Time: A Language, Regular or Not?

- If given: **a Finite Automaton M**
  - We know: $L(M)$, **the language recognized by M, is a regular language**
  - Because:

> If **a finite automaton** (FSM) **recognizes** a language,
> then **that language is called a regular language.**

(and modus ponens)

- If given: **a Language A**
  - **Is A is a regular language?**
    - **Not necessarily!**
  - **How do we determine,** i.e., ***prove*, that A is a regular language?**

An Inference Rule: Modus Ponens

Premises

- If P then Q
- P is true

Conclusion

- Q is true

<u>Example</u> **Premises**

We know this
(definition of regular language)

- If **there is an FSM recognizes language** A**,** then A **is a regular language**
- **There is an FSM** M **where** $L(M) = A$

... then we need to show

Conclusion

- A **is a regular language!**

If we want to prove this ...

Proving a Language is Regular: Example

Prove that the following language is regular:

$$L = \{ \, w \mid w \text{ is a string with an odd \# of } \mathtt{1}\text{s} \, \}$$

$\Sigma = \{ \mathtt{0}, \mathtt{1} \}$

Proving a Language is Regular: Example

Statements	**Justifications**
1. If an FSM recognizes L, then L is a regular language	1. Def. of a Regular Language
2. $M = (Q, \Sigma, \delta, q_0, F)$ is an FSM (todo)	2. Definition of an FSM
3. M recognizes L	3. This is hard problem! In this class, we use tests.
4. $L = \{ w \mid w$ is string with odd # of `1`s$\}$ is a regular language	4. Stmt #1 & #3 (modus ponens)

When programming, how do you “prove” your program does what it is supposed to do?

Tips on Designing Finite Automata

Analogy
Finite Automata ~ "Programs" ::
Designing Finite Automata ~ "Programming"!

In programming, to "understand" a problem, create examples!

1. Confirm understanding of the problem
   - Create tests: examples and expected results (`accept` / `reject`)

FSM M Examples: accept strs with odd # **1**s

- On input `1`:
  - **Accept**
- On input `0`:
  - **Reject**
- On input `01`:
  - **Accept**
- On input `11`:
  - **Reject**
- On input `1101`:
  - **Accept**
- On input ε
  - **Reject**

Tips on Designing Finite Automata

Analogy
Finite Automata ~ "Programs" ::
Designing Finite Automata ~ "Programming"!

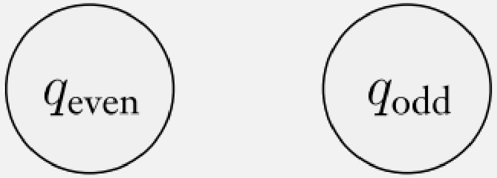
1. Confirm understanding of the problem
   - Create tests: examples and expected results (`accept` / `reject`)

2. Decide information to "remember"
   - These are the machine states: some are accept states; one is start state

3. Determine transitions between states

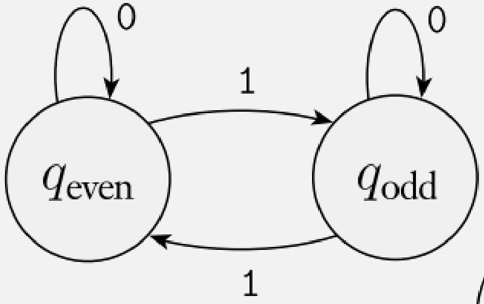
Designing FSM M: accept strs with odd # **1**s

- States:
  - 2 states:
    - seen even **1**s so far
    - seen odds **1**s so far

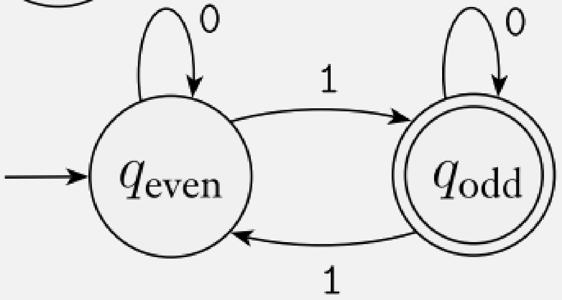


- Alphabet: **0** and **1**

- Transitions:



- Start / Accept states:



Tips on Designing Finite Automata

Analogy
Finite Automata ~ "Programs" ::
Designing Finite Automata ~ "Programming"!

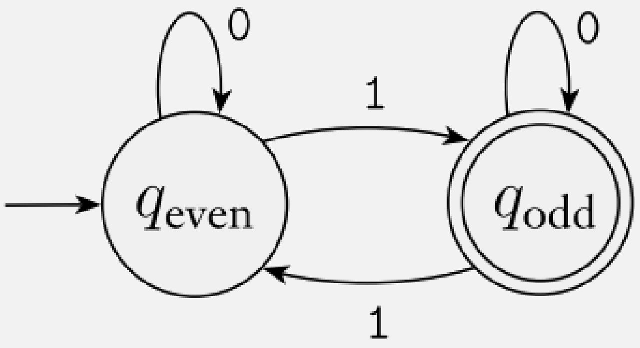
1. Confirm understanding of the problem
   - Create tests: examples and expected results (`accept` / `reject`)

2. Decide information to "remember"
   - These are the machine states: some are accept states; one is start state

3. Determine transitions between states

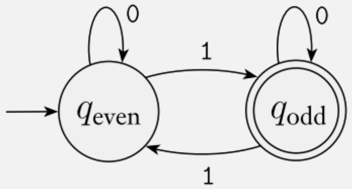
4. Test machine behaves as expected
   - Use initial examples; and create additional tests if needed

Does the Machine Accept Expected Strings?



- On input `1`: **??**
  - **Accept**
- On input `0`:
  - **Reject**
- On input `01`:
  - **Accept**
- On input `11`:
  - **Reject**
- On input `1101`:
  - **Accept**
- On input ε
  - **Reject**

Proving a Language is Regular: Example

Statements	**Justifications**
1. If an FSM recognizes L, then L is a regular language	1. Def. of a Regular Language
2. M =  is an FSM	2. Definition of an FSM
3. M recognizes L	3. See examples. This isn't a proof, but good enough for programmers(?), and CS 420
4. $L = \{ w \mid w$ is string with odd # of `1`s$\}$ is a regular language	4. Stmt #1 & #3 (modus ponens)

∎

In-class exercise

- <u>Prove:</u> the following language is a regular language:
  - $A = \{w \mid w \text{ has exactly three } \mathbf{1}\text{'s}\}$
  - Key step: **design a finite automata that recognizes it!**

Come up with examples first!

- Where $\Sigma = \{\mathbf{0}, \mathbf{1}\}$

- Remember:

DEFINITION

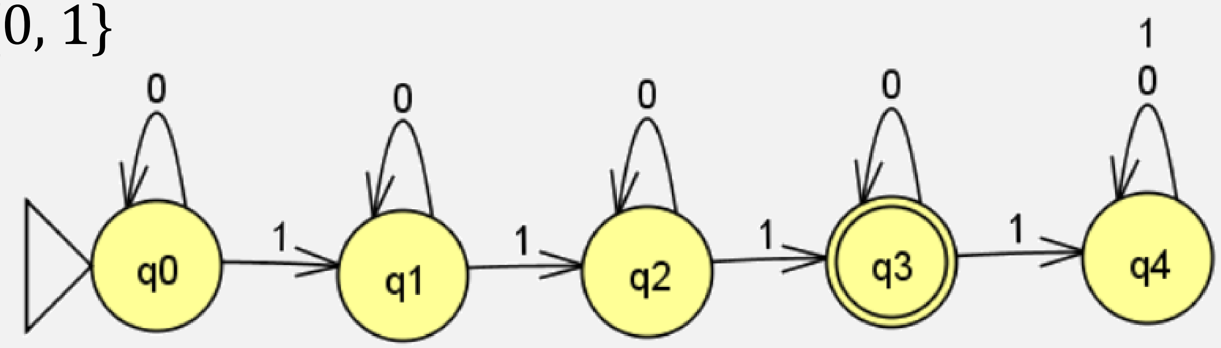
A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

In-class exercise Solution

- Design finite automata recognizing:
  - $\{w \mid w \text{ has exactly three 1's}\}$

- *States*:
  - Need one state to represent how many 1's seen so far
  - $Q = \{q_0, q_1, q_2, q_3, q_{4+}\}$
- *Alphabet*: $\Sigma = \{0, 1\}$
- *Transitions*:



- *Start state*:
  - q_0
- *Accept states*:
  - $\{q_3\}$

So finite automata are used to **recognize** <u>simple string patterns</u>?

Yes!

Do you know a "programming language" to <u>recognize simple string patterns</u>?

Make sure to test this with your examples!

So Far: Finite State Automaton, a.k.a. DFAs

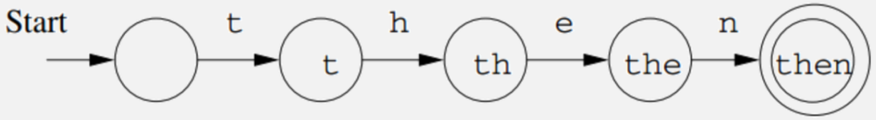
deterministic

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a $\boxed{\text{finite}}$ set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

- Key characteristic:
  - Has a finite number of states
  - I.e., a "program" with access to only a single cell of memory,
    - Where: states = the possible values that can be written to memory

- Often used for **text matching**



Combining DFAs?

Password Requirements

DFA

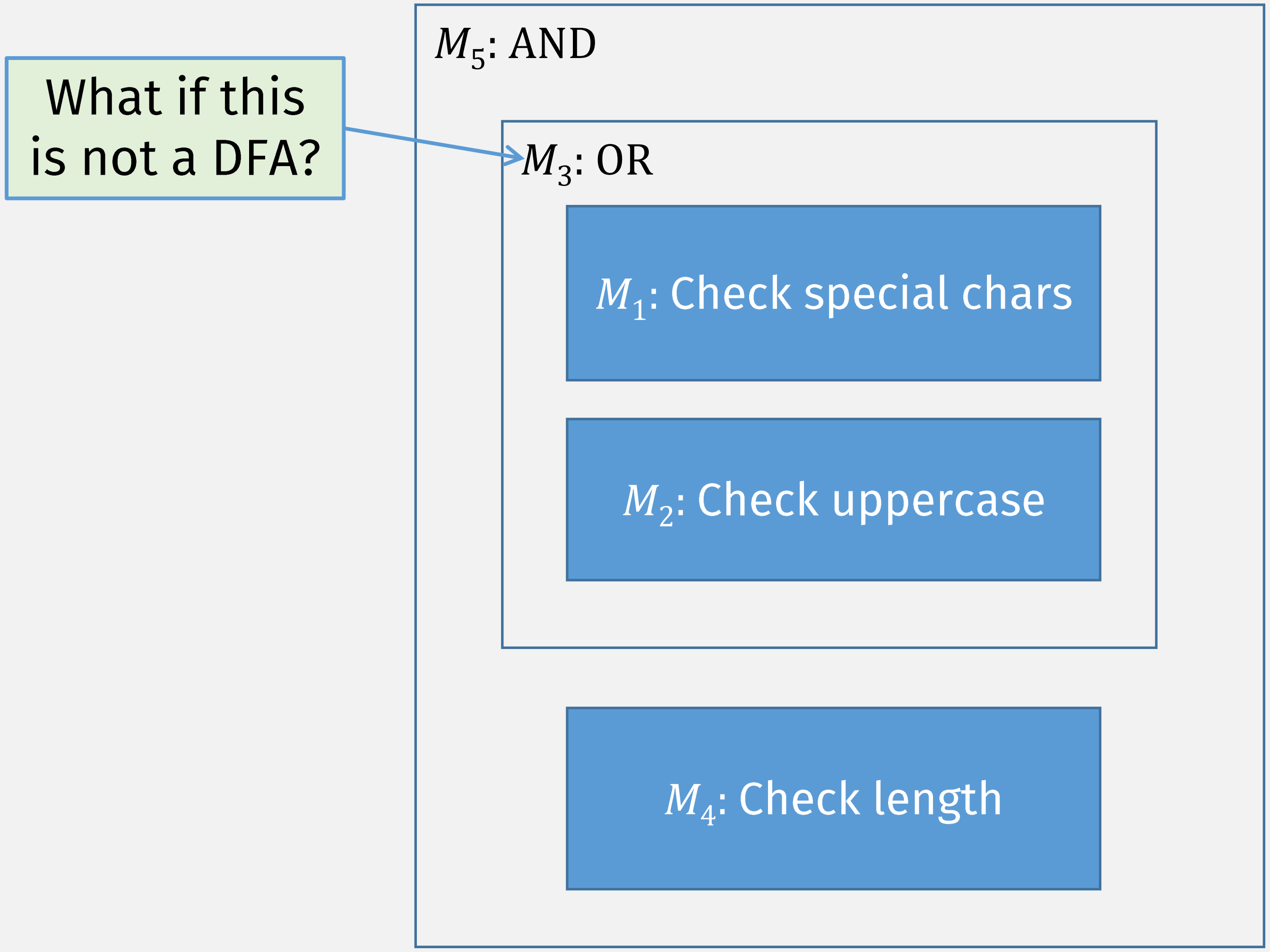
- Passwords must have a minimum length of ten (10) characters - but more is better!
- Passwords **must include at least 3** different types of characters:
  - upper-case letters (A-Z) DFA
  - DFA lower-case letters (a-z)
  - symbols or special characters (%, &, *, $, etc.) DFA
  - numbers (0-9) DFA
- Passwords cannot contain all or part of your email address DFA
- Passwords cannot be re-used DFA

To match <u>all</u> requirements, <u>combine</u> smaller DFAs into one big DFA?

https://www.umb.edu/it/password

(We do this with programs all the time)

Password Checker DFAs



Want to be able to easily combine DFAs, i.e., composability

We want these operations:

$$\text{OR} : \text{DFA} \times \text{DFA} \rightarrow \text{DFA}$$

$$\text{AND} : \text{DFA} \times \text{DFA} \rightarrow \text{DFA}$$

To combine more than once, operations must be **closed**!

"Closed" Operations

A set is **closed** under an operation if:
the result of applying the operation to
members of the set is in the same set

- Set of Natural numbers = $\{0, 1, 2, \ldots\}$
  - Closed under addition:
    - if x and y are Natural numbers,
    - then $z = x + y$ is a Natural number
  - Closed under multiplication?
    - **yes**
  - Closed under subtraction?
    - **no**
- Integers = $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
  - Closed under addition and multiplication
  - Closed under subtraction?
    - **yes**
  - Closed under division?
    - **no**
- Rational numbers = $\{x \mid x = y/z, y$ and z are Integers$\}$
  - Closed under division?
    - **No**?
    - **Yes** if z !=0

Why Care About Closed Ops on Reg Langs?

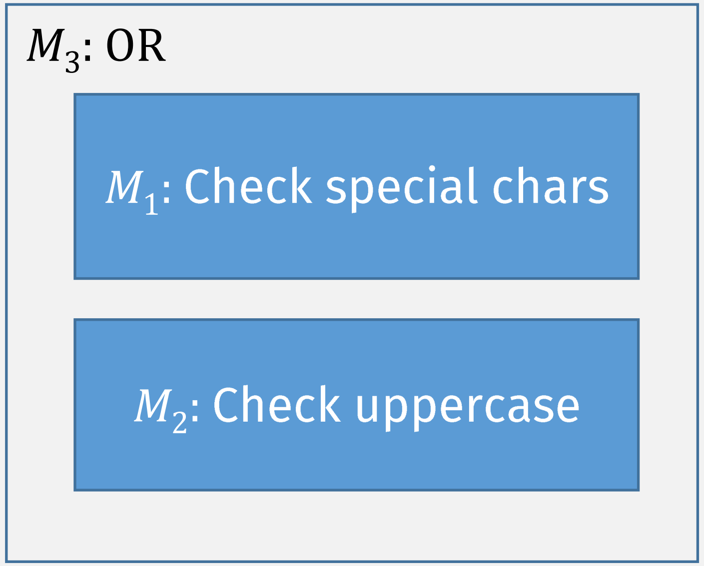
- Closed operations preserve "regularness"
- I.e., it preserves the same computation model!
- This way, a "combined" machine can be "combined" again!

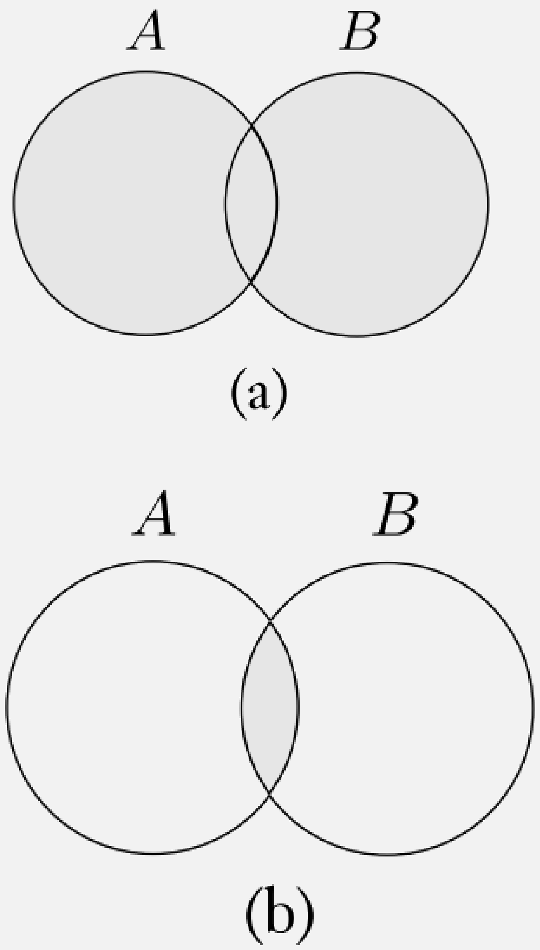
<u>We want:</u>

OR, AND : DFA $\times$ DFA $\rightarrow$ DFA

- So this semester, we will look for operations that are <u>closed</u>!

Password Checker: "**OR**" = "Union"





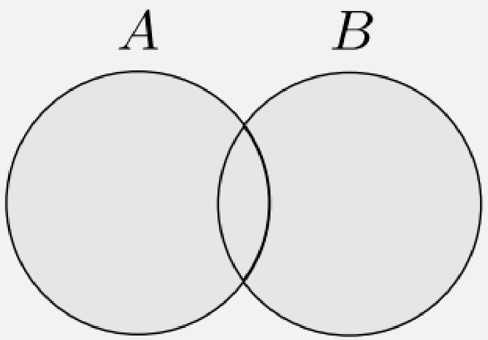
???

Password Checker: “**OR**” = “Union”

M_3: OR

M_1: Check special chars

M_2: Check uppercase



(a)

Union: $A \cup B = \{x | \ x \in A \text{ or } x \in B\}$

Union of Languages

Let the alphabet Σ be the standard 26 letters $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\}$.

If $A = \{\mathtt{good}, \mathtt{bad}\}$ and $B = \{\mathtt{boy}, \mathtt{girl}\}$, then

$$A \cup B = \{\mathtt{good}, \mathtt{bad}, \mathtt{boy}, \mathtt{girl}\}$$

Check-in Quiz 2/1

On gradescope