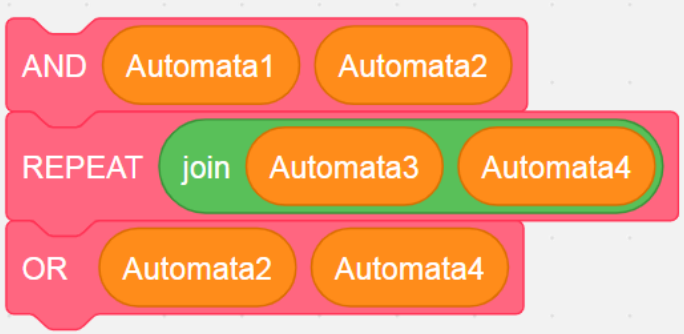
CS420
Combining Regular Languages

Monday January 31, 2022



Announcements

- HW 0 in
- HW 1 out
  - Due Sun 2/6 11:59pm

Last Time: Regular Languages

A language is called a ***regular language*** if some finite automaton recognizes it.

Last Time: Finite State Automaton, a.k.a. DFAs

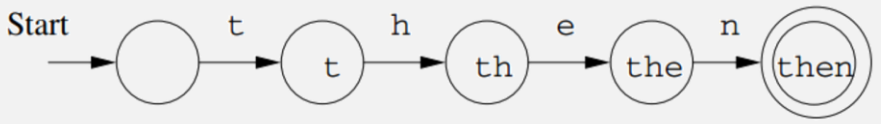
deterministic

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

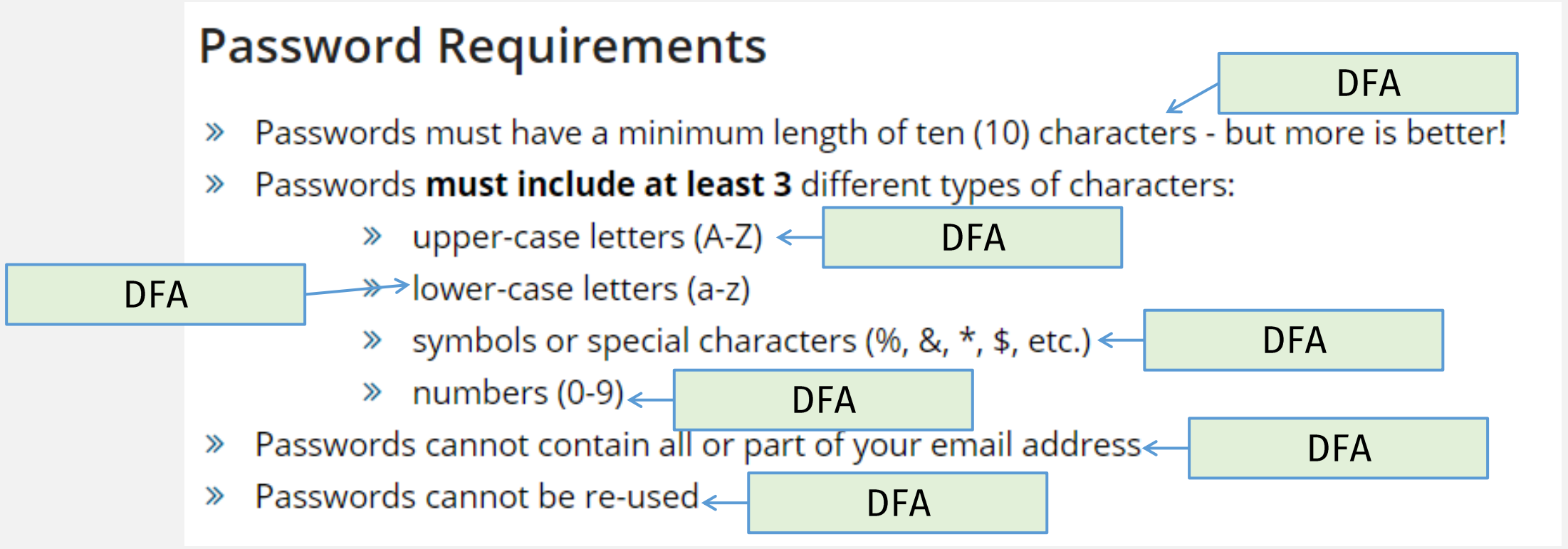
1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

- Key characteristic:
  - Has a **finite** number of states
  - I.e., a computer or program with access to a single cell of memory,
    - Where: # states = the possible symbols that can be written to memory

- Often used for **text matching**



Combining DFAs?

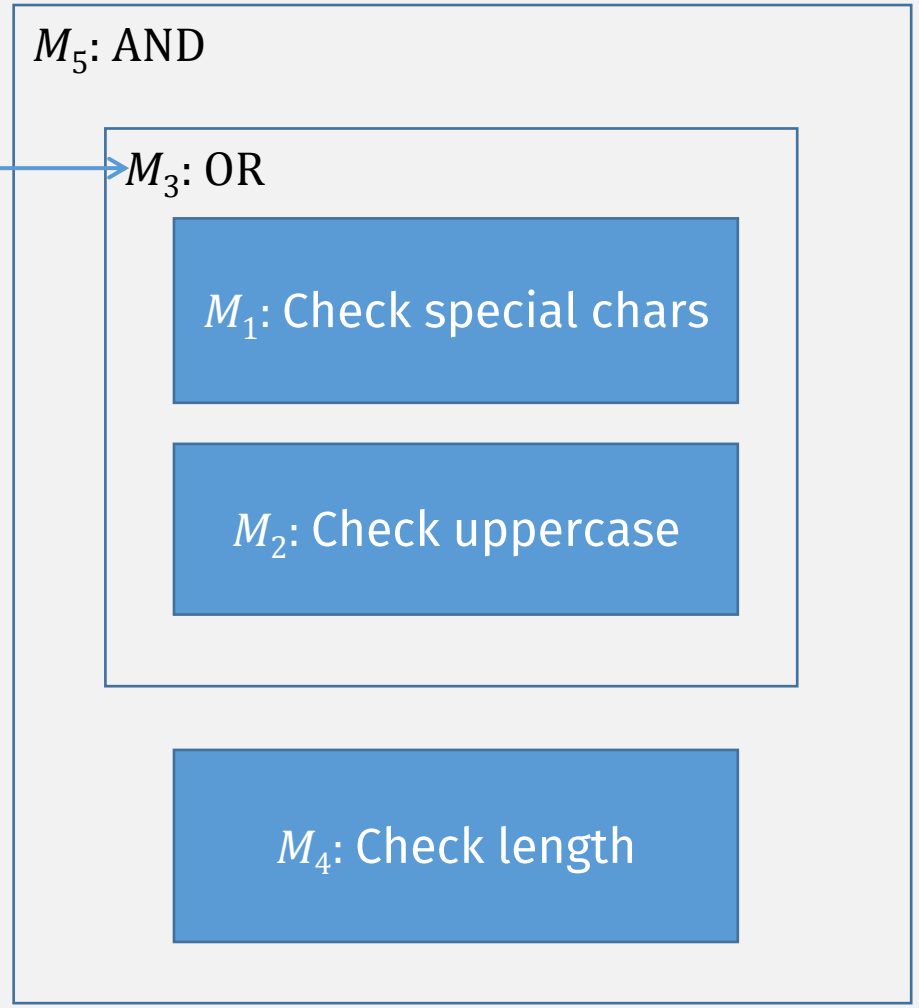


https://www.umb.edu/it/password

To match all requirements, combine smaller DFAs into one big DFA?

Password Checker DFAs

But what if this is not a DFA?



Want to be able to easily combine DFAs

We want:

$$\text{OR, AND} : \text{DFA} \times \text{DFA} \rightarrow \text{DFA}$$

To combine more than once, operations must be **closed**!

"Closed" Operations

A set is **closed** under an operation if:
the result of applying the operation to members of the set is in the same set

- Set of Natural numbers = {0, 1, 2, …}
  - Closed under addition:
    - if x and y are Natural numbers,
    - then $z = x + y$ is a Natural number
  - Closed under multiplication?
    - **yes**
  - Closed under subtraction?
    - **no**
- Integers = {…, -2, -1, 0, 1, 2, …}
  - Closed under addition and multiplication
  - Closed under subtraction?
    - **yes**
  - Closed under division?
    - **no**
- Rational numbers = $\{x \mid x = y/z, y$ and z are Integers$\}$
  - Closed under division?
    - **No**?
    - **Yes** if z !=0

Why Care About Closed Ops on Reg Langs?

- Closed operations preserve “regularness”

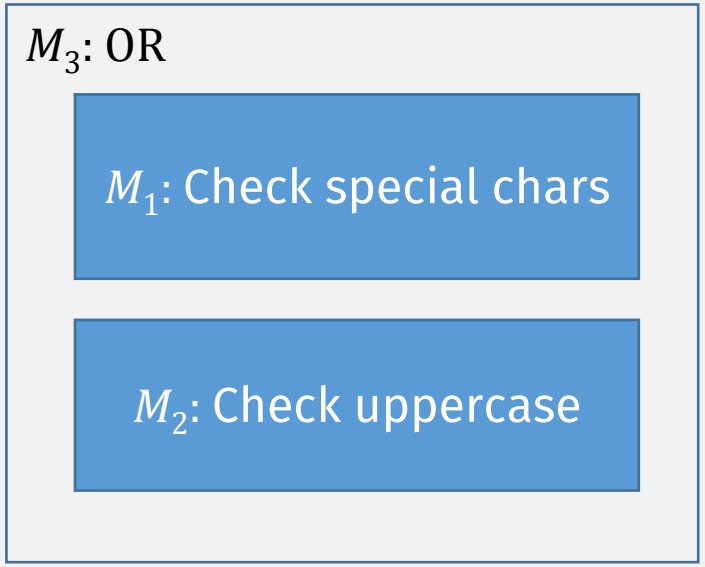
We want:

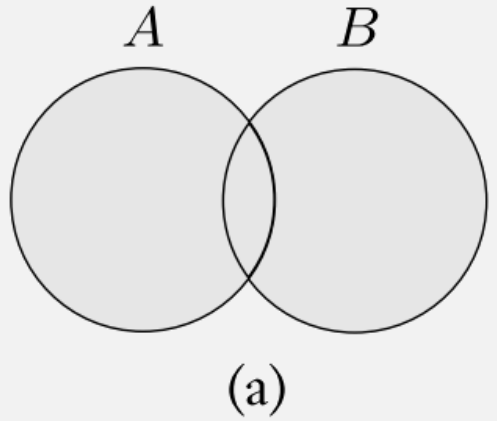
$$\text{OR, AND} : \text{DFA} \times \text{DFA} \rightarrow \text{DFA}$$

- I.e., it preserves the same computation model!

- This way, a “combined” machine can be “combined” again!

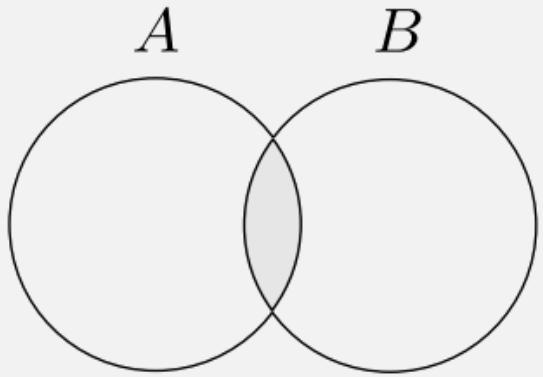
Password Checker: “OR” = “Union”





(a)

???



(b)

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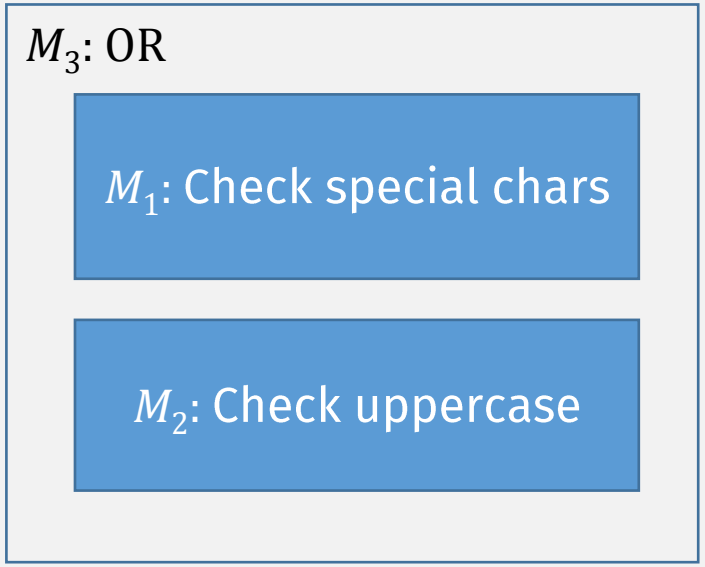
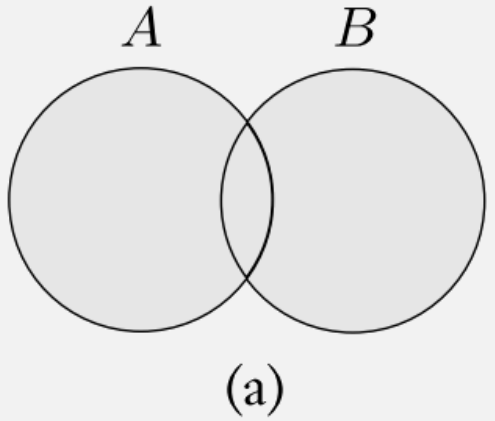




Union: $A \cup B = \{x | \ x \in A \text{ or } x \in B\}$

Union of Languages

Let the alphabet Σ be the standard 26 letters $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\}$.

If $A = \{\mathtt{good}, \mathtt{bad}\}$ and $B = \{\mathtt{boy}, \mathtt{girl}\}$, then

$$A \cup B = \{\mathtt{good}, \mathtt{bad}, \mathtt{boy}, \mathtt{girl}\}$$

A Closed Operation: Union

(A set is **closed** under an operation if the result of applying the operation to members of the set is in the same set)

Set of languages

THEOREM

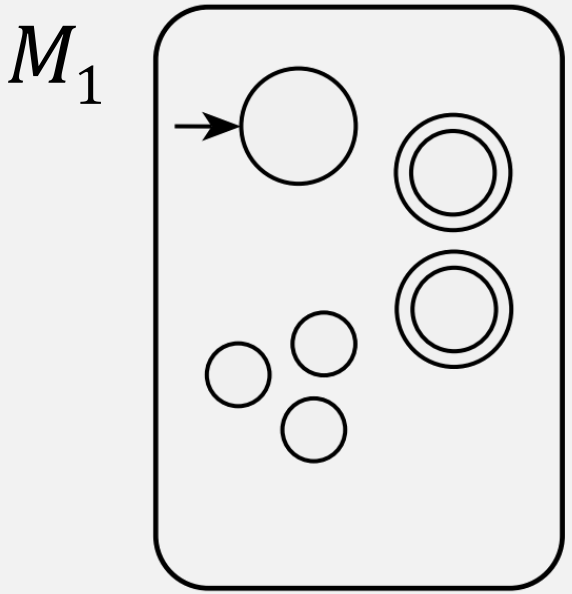
The class of regular languages is closed under the union operation.

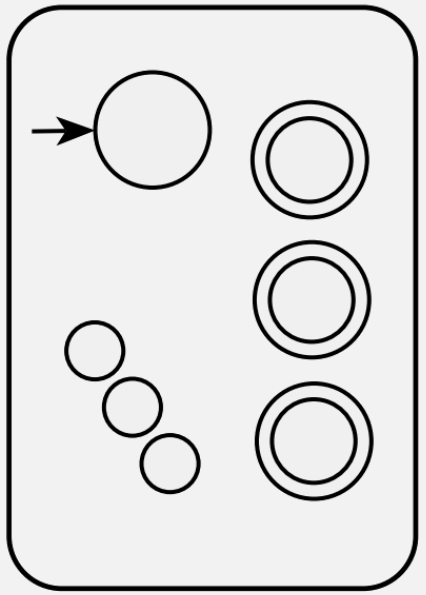
In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

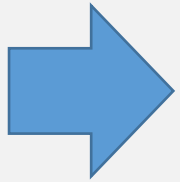
- How do we prove that a language is regular?
  - Create a DFA recognizing it!
- So to prove this theorem …
  create a DFA that "combines" the machines of A_1 and A_2

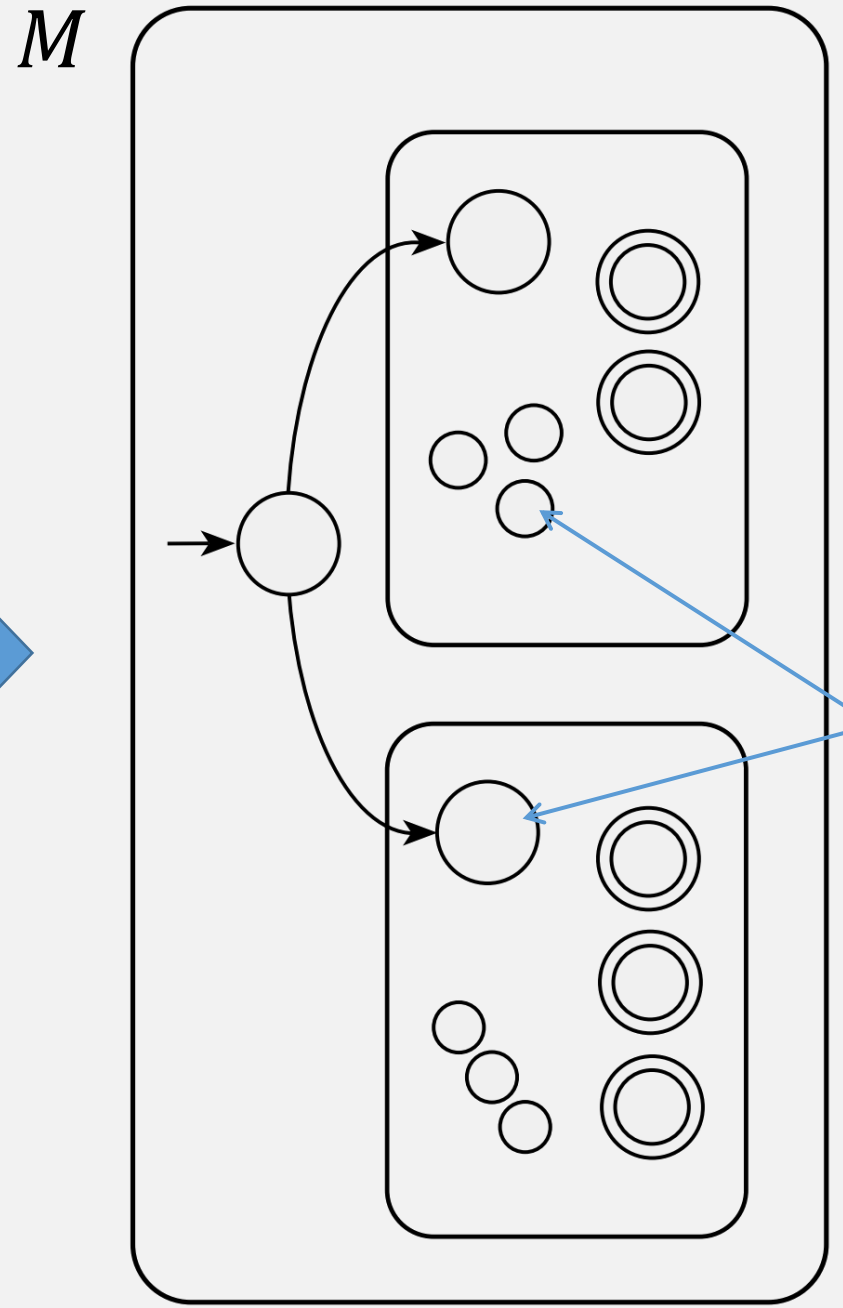
A language is called a ***regular language*** if some finite automaton recognizes it.

Union









Rough sketch Idea:
M "runs" its input on both M_1 and M_2 in parallel

M needs to be "in" both an M_1 and M_2 state simultaneously

And then accept if either accepts

Union is Closed For Regular Languages

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In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Proof

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,

Idea: M "runs" its input on both M_1 and M_2 in parallel

- Construct: a new machine $M = (Q, \Sigma, \delta, q_0, F)$ using M_1 and M_2
- states of M:

$$Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2\} \quad = Q_1 \times Q_2$$

This set is the ***Cartesian product*** of sets Q_1 and Q_2

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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$a) = (\delta_1(r_1, a), \delta_2(r_2, a))$ a step in M_1, a step in M_2

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- Construct: a new machine $M = (Q, \Sigma, \delta, q_0, F)$ using M_1 and M_2
- states of M: $Q = \{(r_1, r_2) | \, r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$
  This set is the ***Cartesian product*** of sets Q_1 and Q_2
- M transition fn: $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$ a step in M_1, a step in M_2
- M start state: (q_1, q_2)

Union is Closed For Regular Languages

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- M start state: (q_1, q_2)
- M accept states: $F = \{(r_1, r_2) | \, r_1 \in F_1 \text{ or } r_2 \in F_2\}$

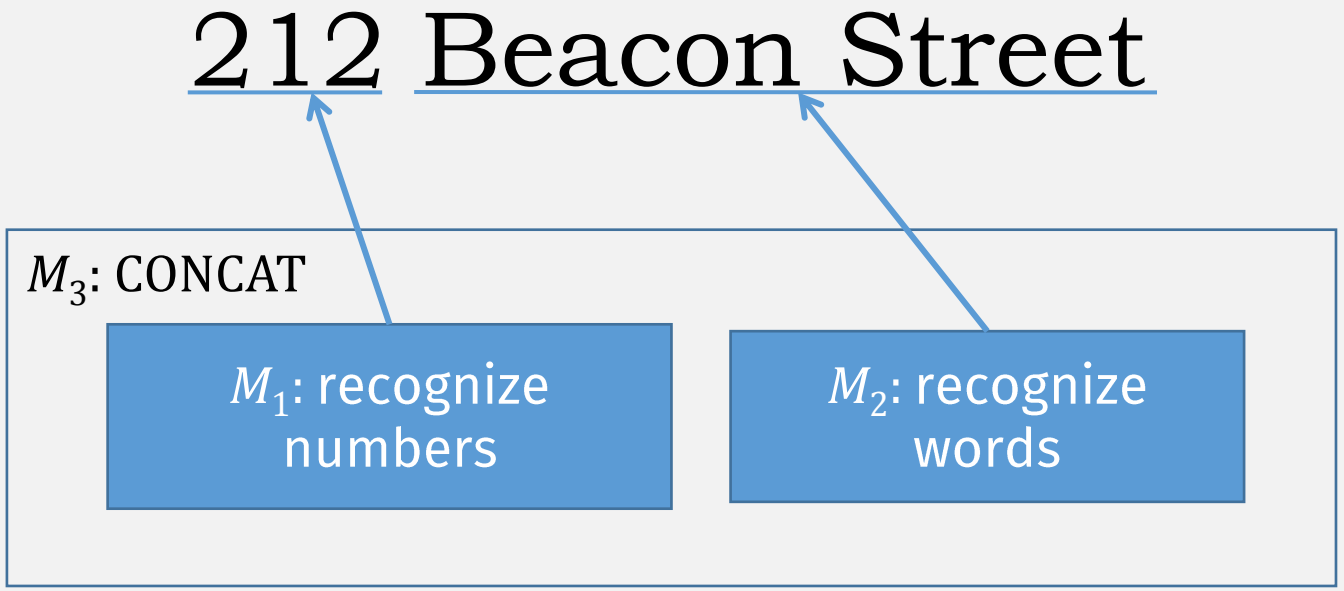
Accept if either M_1 or M_2 accept

Remember:
Accept states must be subset of Q

(Q.E.D.) ■

Another operation: Concatenation

Example: **Recognizing street addresses**



Concatenation: $A \circ B = \{xy | \, x \in A \text{ and } y \in B\}$

Concatenation of Languages

Let the alphabet Σ be the standard 26 letters $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\}$.

If $A = \{\mathtt{good}, \mathtt{bad}\}$ and $B = \{\mathtt{boy}, \mathtt{girl}\}$, then

$$A \circ B = \{\mathtt{goodboy}, \mathtt{goodgirl}, \mathtt{badboy}, \mathtt{badgirl}\}$$

Is Concatenation Closed?

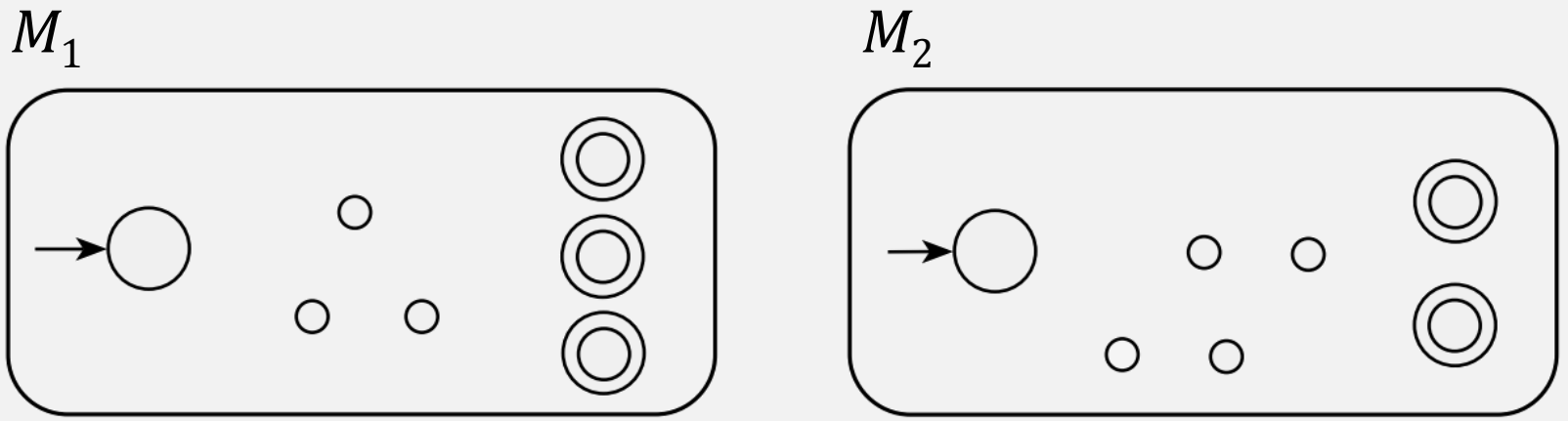
THEOREM

The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

- Construct a new machine M? (like union)
  - From DFA M_1 (which recognizes A_1),
  - and DFA M_2 (which recognizes A_2)

Concatentation

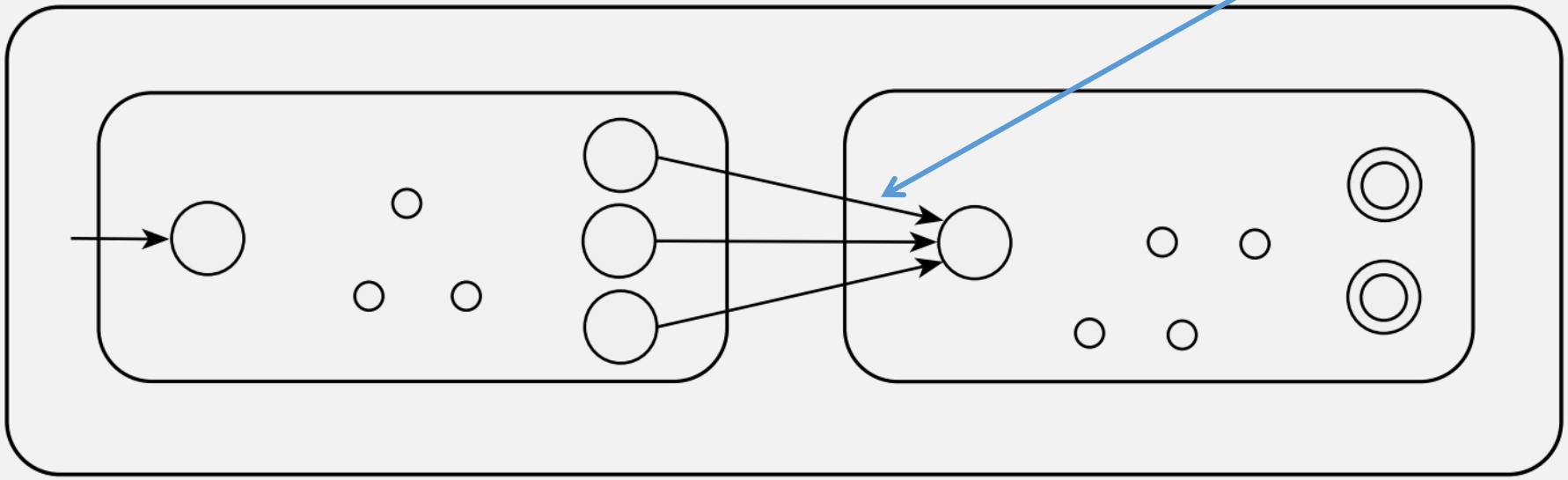


PROBLEM: Can only read input once, can't backtrack

Let M_1 recognize A_1, and M_2 recognize A_2.

Want: Construction of M to recognize $A_1 \circ A_2$

Need to switch machines at some point, but when?



Concatenation: $A \circ B = \{xy | \, x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 recognize language A = {**ab**, **abc**}
- And M_2 recognize language B = {**cde**}
- Want: Construct M to recognize $A \circ B$ = {**abcde**, **abccde**}

- But if M sees **ab** as first part of input …
- M must decide to either:
  - stay in M_1 (correct, if full input is **abccde**)

Concatenation: $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

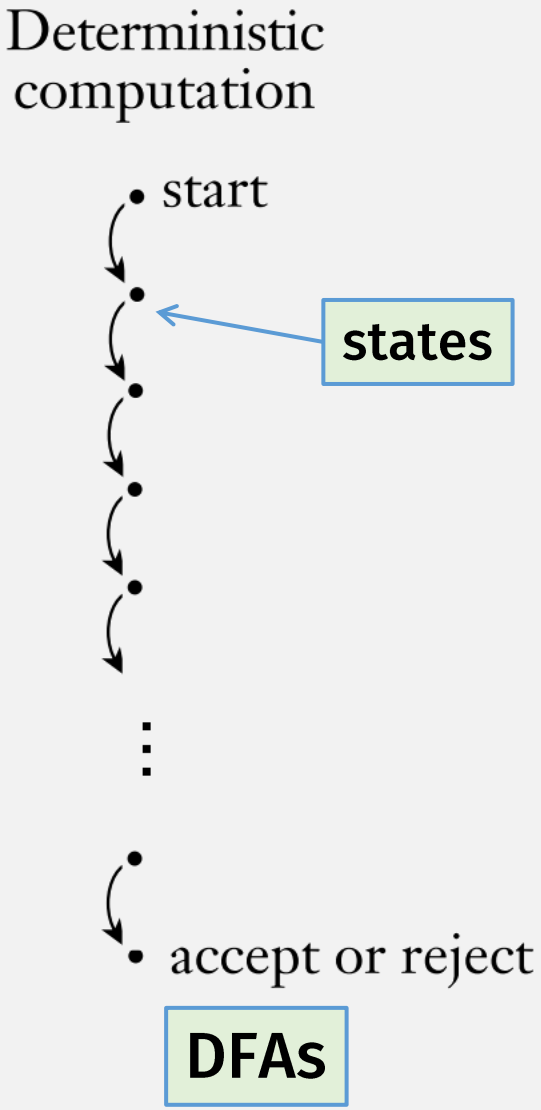
- Let M_1 recognize language A = {**ab**, **abc**}
- And M_2 recognize language B = {**cde**}
- Want: Construct M to recognize $A \circ B$ = {**abcde**, **abccde**}

- But if M sees **ab** as first part of input …
- M must decide to either:
  - stay in M_1 (correct, if full input is **abccde**)
  - or switch to M_2 (correct, if full input is **abcde**)
- But it needs to do <u>both</u>!

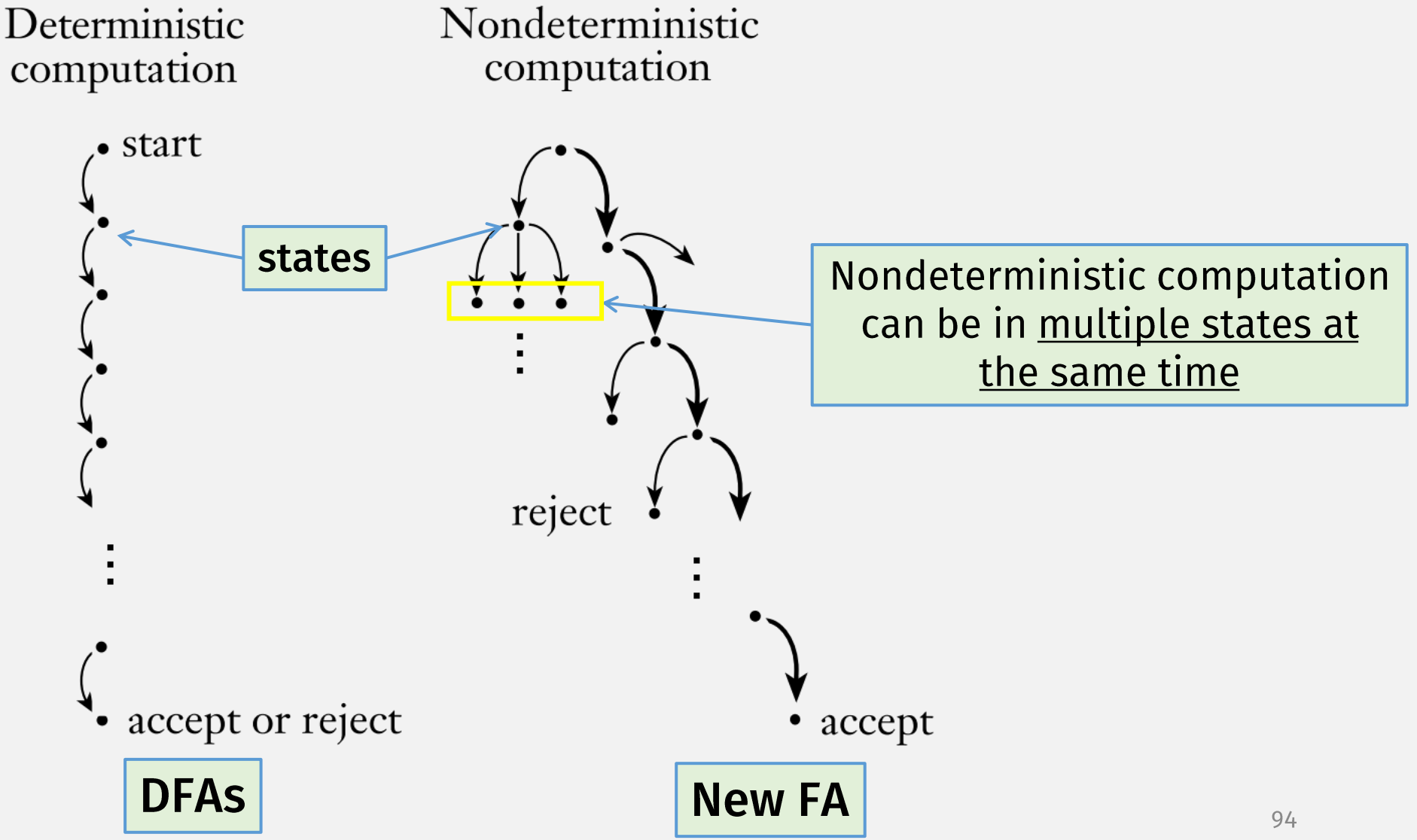
A DFA can't do this!
(We need a new kind of machine)

Nondeterminism

Deterministic vs Nondeterministic



Deterministic vs Nondeterministic



Nondeterministic Finite Automata (NFA)

DEFINITION

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta: Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

Compare with DFA:

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Difference

Power set, i.e. a transition results in set of states

Power Sets

- A power set is the set of all subsets of a set
- Example: S = {a, b, c}
- Power set of S =
  - {{ }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}
  - Note: includes the empty set!

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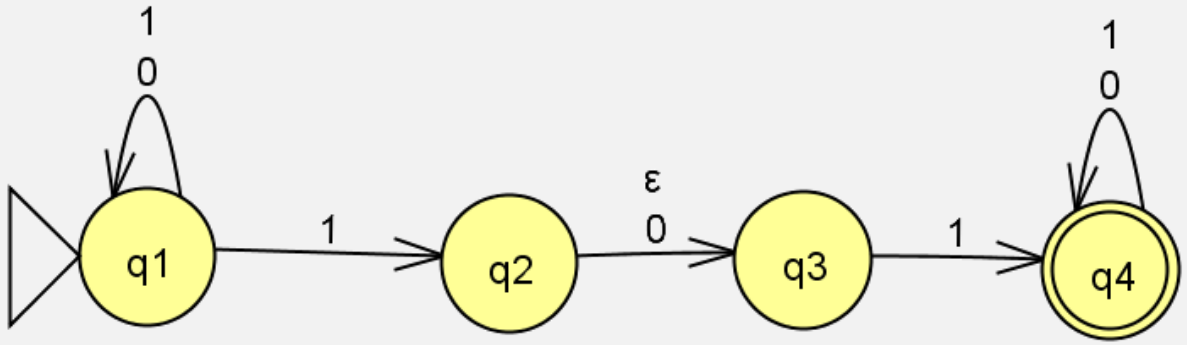
5. $F \subseteq Q$ is the set of accept states.

Transition label can be "empty", i.e., machine can transition without reading input

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

NFA Example

- Come up with a formal description of the following NFA:



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The formal description of N_1 is $(Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3, q_4\}$,
2. $\Sigma = \{0,1\}$,
3. δ is given as

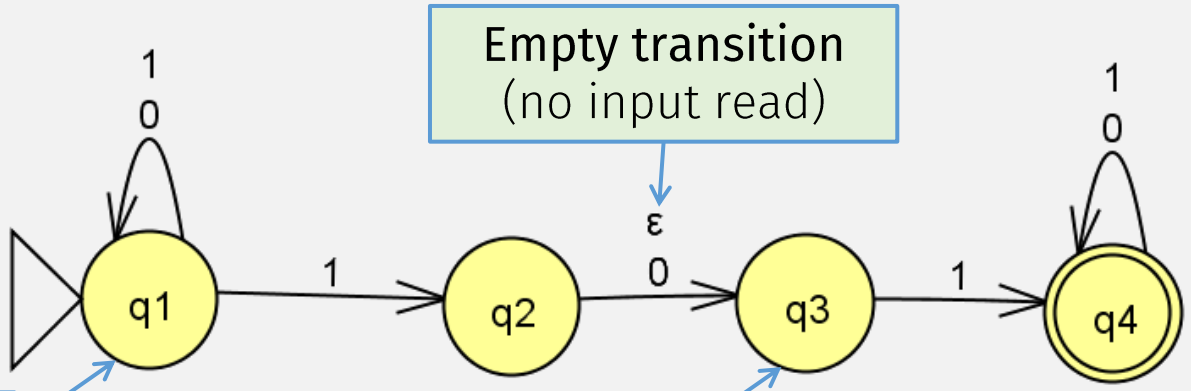
$$\delta: Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$$

Empty transition (no input read)

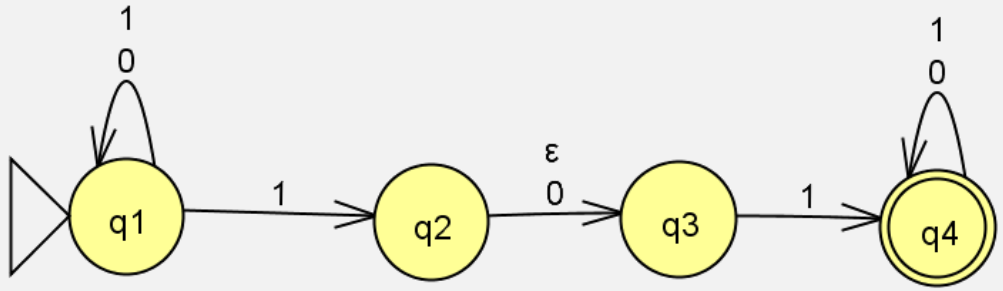
Result of transition is a set

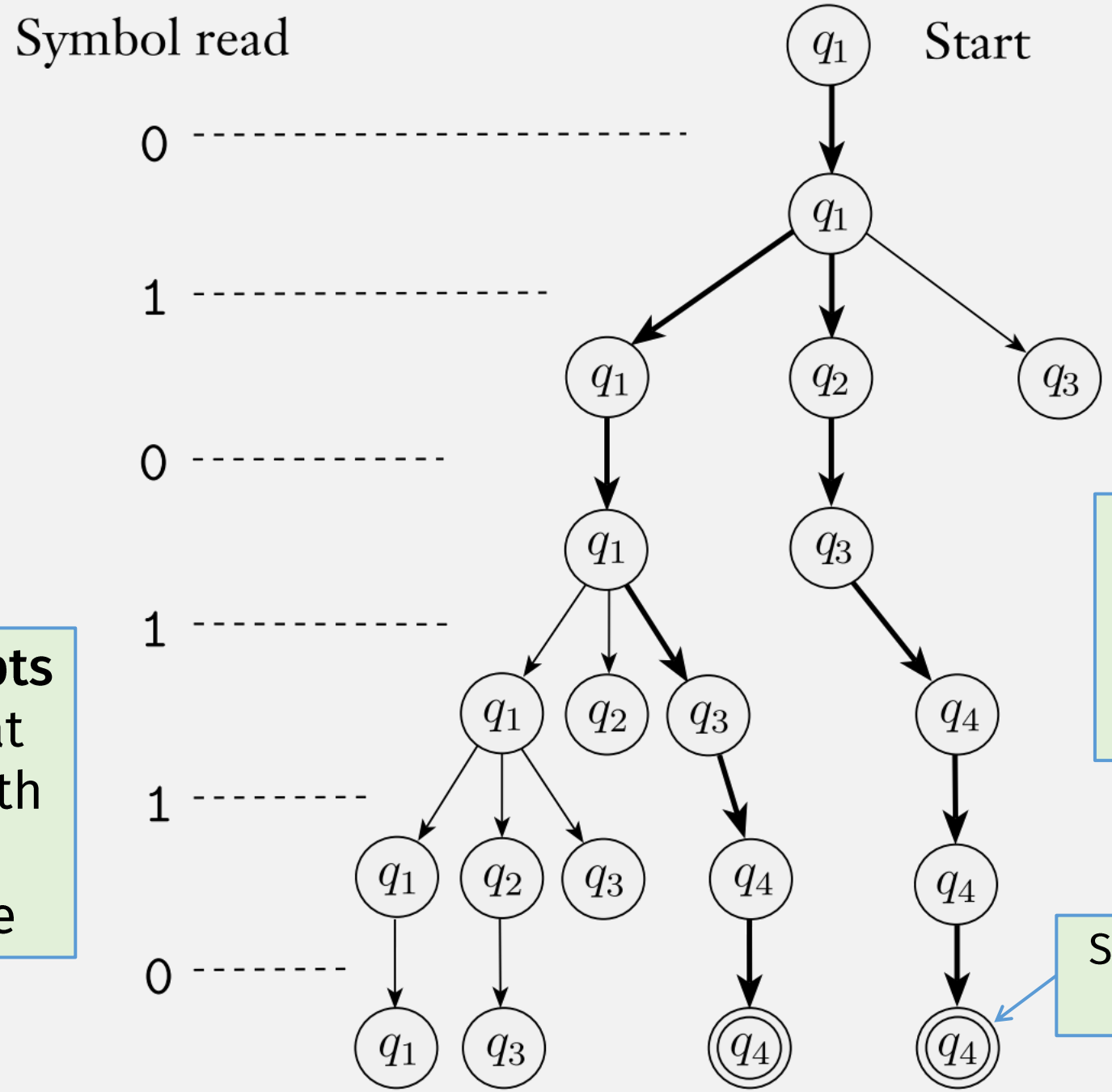
	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$,

4. q_1 is the start state, and
5. $F = \{q_4\}$.



Running Programs, NFAs (JFLAP demo): **010110**





An NFA **accepts** its input if at least one path ends in an accept state

A nondeterministic machine can be in <u>multiple states at the same time</u>!

So this is an **accepting computation**

Flashback: DFA Computation Model

Informally

- Machine = a DFA
- Input = string of chars

Machine accepts input if:

- Start in "start state"
- Repeat: Read 1 char, change state according to transition table
- Result =
  - "Accept" if last state is "Accept" state
  - "Reject" otherwise

Formally

- $M = (Q, \Sigma, \delta, q_0, F)$
- $w = w_1 w_2 \cdots w_n$

M ***accepts*** w if
sequence of states $r_0, r_1, \ldots, r_n$ in Q exists with

- $r_0 = q_0$
- $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$
- $r_n \in F$

NFA

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- $r_0 = q_0$
- ~~$\delta(r_i, w_{i+1}) = r_{i+1}$,~~ for $i = 0, \ldots, n-1$

$$r_{i+1} \in \delta(r_i, w_{i+1})$$

- $r_n \in F$

This is now a set

DEFINITION

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta: Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

Flashback: DFA Extended Transition Fn

Define the extended transition function: $\hat{\delta} : Q \times \Sigma^* \to Q$

- *Inputs*:
  - Beginning state $q \in Q$
  - Input string $w = w_1 w_2 \cdots w_n$ where $w_i \in \Sigma$
- *Output*:
  - Ending state

(Defined recursively)

- Base case: $\hat{\delta}(q, \varepsilon) = q$

Empty string

- Recursive case: $\hat{\delta}(q, w) = \hat{\delta}(\delta(q, w_1), w_2 \cdots w_n)$

First char

Remaining chars

Recursive call

Single transition step

NFA

~~Flashback:~~ ~~DFA~~ Extended Transition Fn

Define the extended transition function: ~~$\hat{\delta} : Q \times \Sigma^* \to Q$~~ $\hat{\delta} : Q \times \Sigma^* \to \mathcal{P}(Q)$

- *Inputs*:
  - Beginning state $q \in Q$
  - Input string $w = w_1 w_2 \cdots w_n$ where $w_i \in \Sigma$
- *Output*:
  - ~~Ending state~~ Set of ending states

Result is set of states

(Defined recursively)

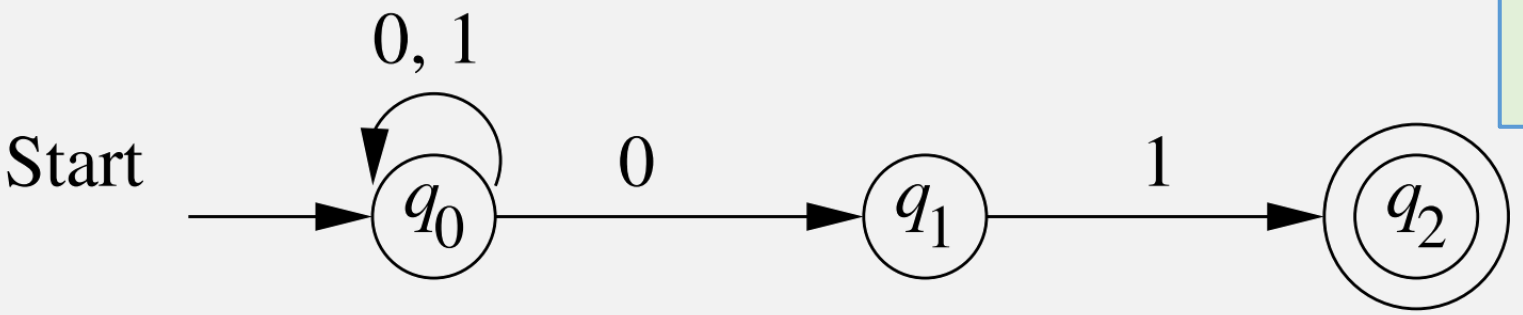
- Base case: ~~$\hat{\delta}(q, \varepsilon) = q$~~ $\hat{\delta}(q, \epsilon) = \{q\}$
- Recursive case: ~~$\hat{\delta}(q, w) = \hat{\delta}(\delta(q, w_1), u$~~ $\hat{\delta}(q, w) = \bigcup_{i=1}^{k} \hat{\delta}(q_i, w_2 \cdots w_n)$

First char

$\delta(q, w_1) = \{q_1, \ldots, q_k\}$

Combine results of recursive calls for each q_i

NFA Extended delta Example



We haven't considered empty transitions!

- $\hat{\delta}(q_0, \epsilon) = \{q_0\}$
- $\hat{\delta}(q_0, 0) = \delta(q_0, 0) = \{q_0, q_1\}$
- $\hat{\delta}(q_0, 00) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$
- $\hat{\delta}(q_0, 001) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$

Read one char, do recursive call for each result, then combine

Adding Empty Transitions

- Define the set $\varepsilon\text{-REACHABLE}(q)$
  - … to be all states reachable from q via one or more empty transitions

(Defined recursively)

- Base case: $q \in \varepsilon\text{-REACHABLE}(q)$

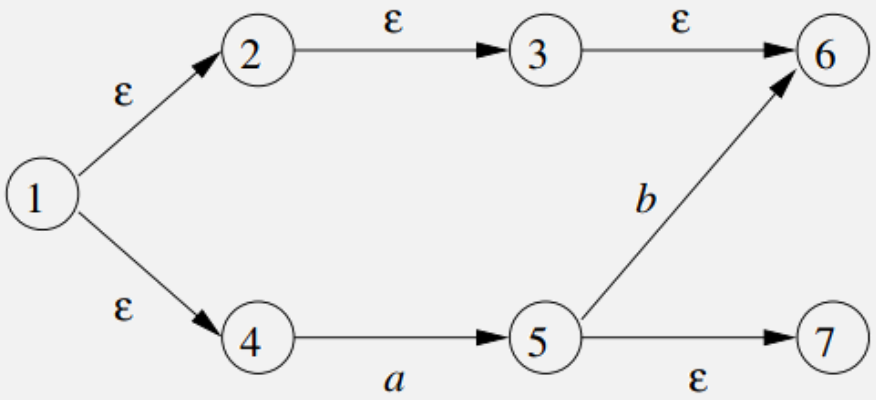
- Inductive case:

$$\varepsilon\text{-REACHABLE}(q) = \{r \mid p \in \varepsilon\text{-REACHABLE}(q) \text{ and } r \in \delta(p, \varepsilon)\}$$

A state is in the reachable set if …

… there is an empty transition to it from another state in the reachable set

ε-REACHABLE Example



$$\varepsilon\text{-REACHABLE}(1) = \{1, 2, 3, 4, 6\}$$

No empty transitions

NFA Extended Transition Function

Define the extended transition function: $\hat{\delta} : Q \times \Sigma^* \to \mathcal{P}(Q)$

- *Inputs*:
  - Some beginning state q
  - Input string $w = w_1 w_2 \cdots w_n$
- *Output*:
  - Set of ending states

(Defined recursively)

- Base case: $\hat{\delta}(q, \epsilon) = \{q\}$
- Recursive case:
  - If: $\delta(q, w_1) = \{q_1, \ldots, q_k\}$
  - Then: $\hat{\delta}(q, w) = \bigcup_{i=1}^{k} \hat{\delta}(q_i, w_2 \cdots w_n)$

With empty transitions

NFA Extended Transition Function

Define the extended transition function: $\hat{\delta} : Q \times \Sigma^* \to \mathcal{P}(Q)$

- *Inputs*:
  - Some beginning state q
  - Input string $w = w_1 w_2 \cdots w_n$
- *Output*:
  - Set of ending states

(Defined recursively)

- Base case: ~~$\hat{\delta}(q, \epsilon) = \{q\}$~~ $\varepsilon\text{-REACHABLE}(q)$
- Recursive case:
  - If: $\delta(q, w_1) = \{q_1, \ldots, q_k\}$ ➡ $\bigcup_{i=1}^{k} \varepsilon\text{-REACHABLE}(q_i) = \{r_1 \ldots, r_m\}$
  - Then: $\hat{\delta}(q, w) = \bigcup_{i=1}^{~~k~~\ m} \hat{\delta}(~~q_i~~\ r_i, w_2 \cdots w_n)$

NFAs vs DFAs

DFAs

- Can only be in one state
- Transition:
  - Must read 1 char
- Acceptance:
  - If final state is accept state

NFAs

- Can be in multiple states
- Transition
  - Can read no chars
  - i.e., empty transition
- Acceptance:
  - If one of final states is accept state

Concatenation: $A \circ B = \{xy | \, x \in A \text{ and } y \in B\}$

Back to: Concatenation is Closed?

THEOREM

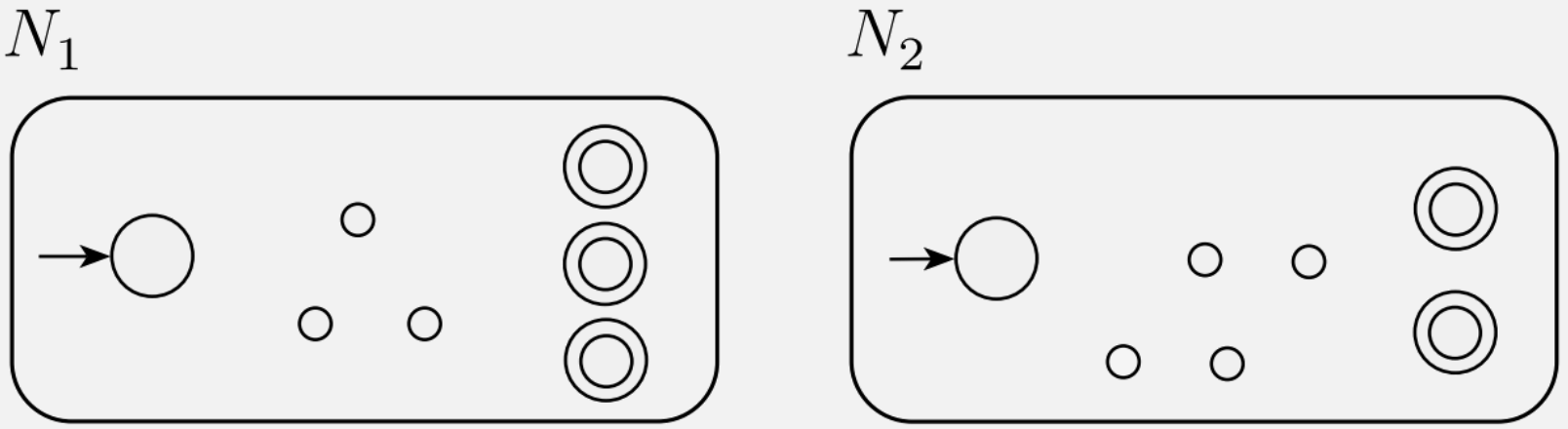
The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

Proof: Construct a new machine?

- How does it know when to switch machines
  - Can only read input once

Concatentation



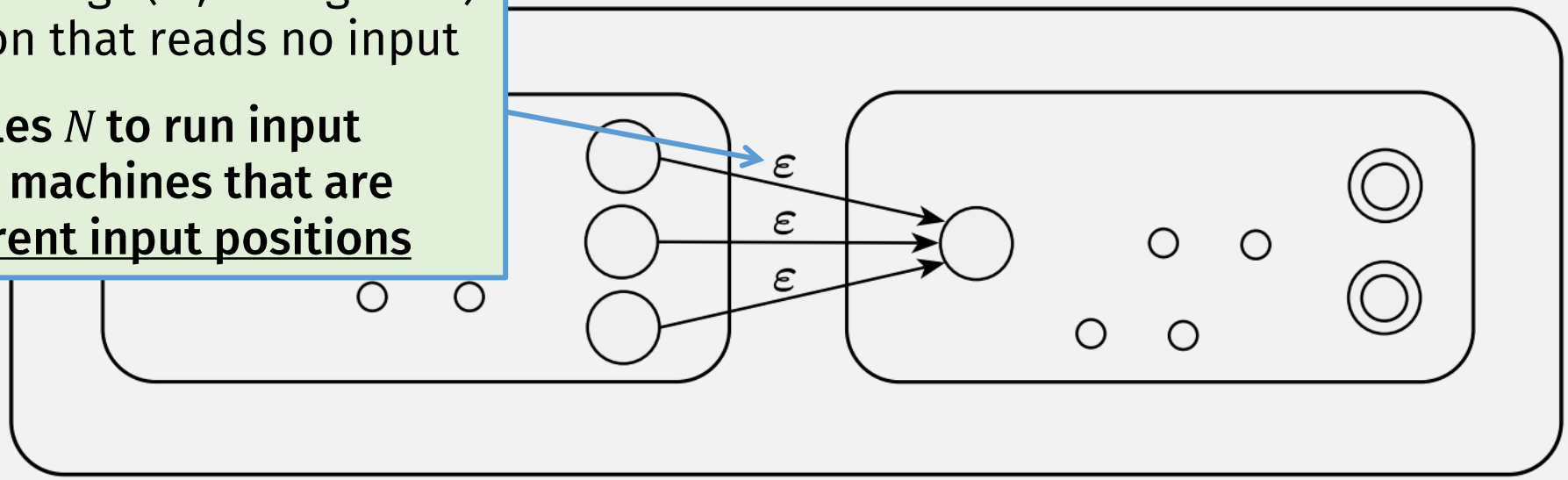
Let N_1 recognize A_1, and N_2 recognize A_2.

Want: Construction of N to recognize $A_1 \circ A_2$

N must simultaneously:
- Keep checking with N_1 **and**
- Move to N_2 to check 2nd part

ε = "empty string" (ie, 0 length str)
= transition that reads no input

Enables N to run input on two machines that are at different input positions



Concatenation is Closed for Regular Langs

PROOF

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

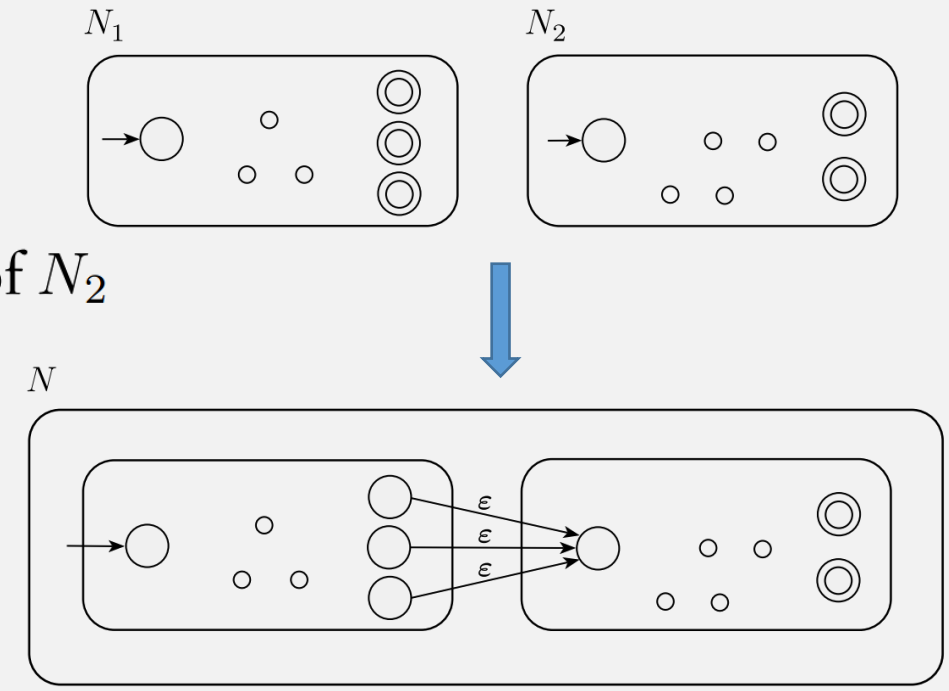
Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$

1. $Q = Q_1 \cup Q_2$

2. The state q_1 is the same as the start state of N_1

3. The accept states F_2 are the same as the accept states of N_2

4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,



Concatenation is Closed for Regular Langs

Wait, is this true?

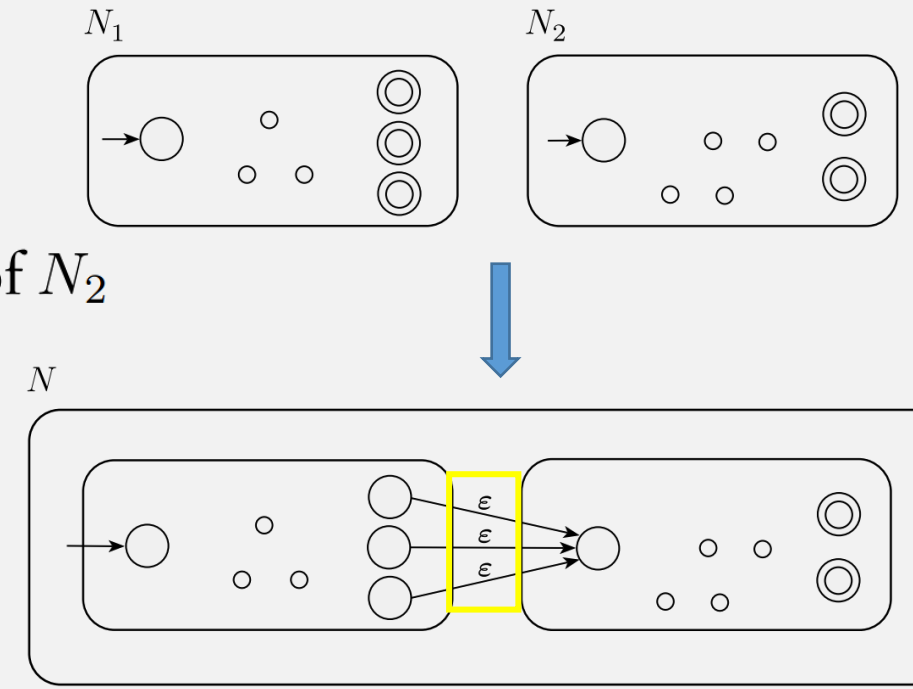
PROOF

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$

1. $Q = Q_1 \cup Q_2$
2. The state q_1 is the same as the start state of N_1
3. The accept states F_2 are the same as the accept states of N_2
4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q\ ?\ a) \\ \delta_1(q\ ?\ a) \\ \delta_1(q\ ?\ a) \cup \{q_2\} \\ \delta_2(q\ ?\ a) \end{cases}$$



???

Flashback: A DFA's Language

- For DFA $M = (Q, \Sigma, \delta, q_0, F)$
- M ***accepts*** w if $\hat{\delta}(q_0, w) \in F$
- M ***recognizes language*** A if $A = \{w | M \text{ accepts } w\}$
- A language is a **regular language** if a DFA recognizes it

An NFA's Language

- For NFA $N = (Q, \Sigma, \delta, q_0, F)$
- N ***accepts*** w if $\hat{\delta}(q_0, w) \cap F \neq \emptyset$
  - i.e., if the final states have at least one accept state
- Language of $N = L(M) = \{w \mid \hat{\delta}(q_0, w) \cap F \neq \emptyset\}$
- Q: How does an NFA's language relate to regular languages
  - Reminder: A language is regular if a **DFA** recognizes it

So is Concatenation Closed for Reg Langs?

- Concatenation of DFAs produces an NFA
- But:

> A language is called a ***regular language*** if some DFA recognizes it.

- To finish the proof,
  we must <u>prove</u> that NFAs *also* recognize regular languages.
- Specifically, we must <u>prove</u>:
  - NFAs ⇔ regular languages

How to Prove a Theorem: $X \Leftrightarrow Y$

- $X \Leftrightarrow Y$ = "X if and only if Y" = X iff Y = X <=> Y
- Proof <u>at minimum</u> has 2 parts:

1. $\Rightarrow$ if X, then Y
   - "forward" direction
   - assume X, then use it to prove Y
2. $\Leftarrow$ if Y, then X
   - "reverse" direction
   - assume Y, then use it to prove X

Proving NFAs Recognize Regular Langs

Theorem:

A language L is regular **if and only if** some NFA N recognizes L.

Proof:

$\Rightarrow$ If L is regular, then some NFA N recognizes it.

- Easier
- We know: if A is regular, then a **DFA** recognizes it.
- Easy to convert DFA to an NFA! (how?)

$\Leftarrow$ If an NFA N recognizes L, then L is regular.

- Harder
- Idea: Convert NFA to DFA

Check-in Quiz 1/31

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