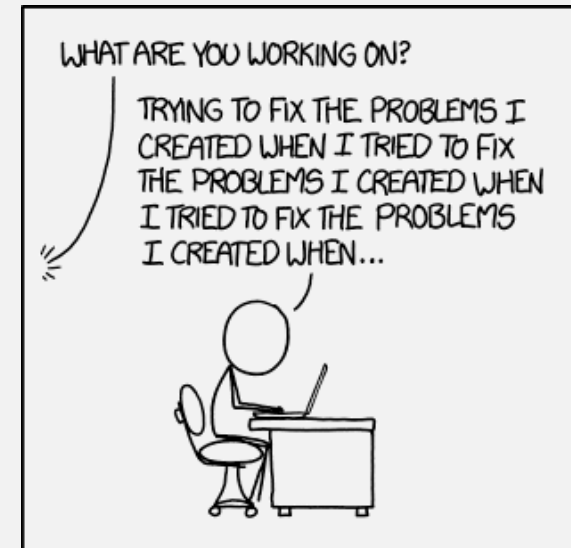


UMass Boston Computer Science  
**CS450 High Level Languages** (section 2)  
**Recursive Data Definitions**

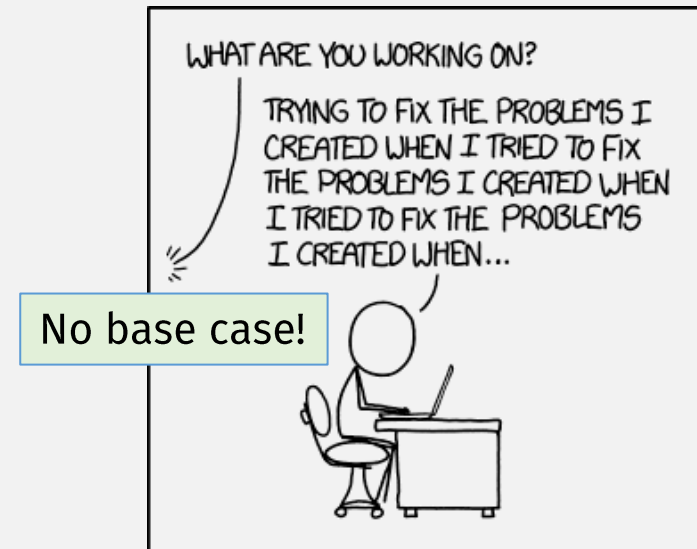
Monday, October 2, 2023



## Logistics

- HW 2 in
  - ~~due: Sun 10/1 11:59 pm EST~~
- HW 3 delayed
  - out: tomorrow
  - due: Sun 10/15 11:59 pm EST
  - (2 weeks)
- No class: next Monday 10/9
  - Indigenous Peoples Day

(What's wrong with this recursion?)



*Last  
Time*

# Bouncing Ball



Last  
Time

# Multi-ball Animation

Design a **big-bang** animation that:

- Start: a single ball, moving with random  $x$  and  $y$  velocity
- If a ball “hits” an edge:
  - for vertical edge, flip  $x$  velocity direction
  - for horizontal edge, flip  $y$  velocity direction

# Randomness

[bracketed args] = optional

```
(random k [rand-gen]) → exact-nonnegative-integer?
```

```
k : (integer-in 1 4294967087)
```

```
rand-gen : pseudo-random-generator?
```

```
= (current-pseudo-random-generator)
```

When called with an integer argument *k*, returns a random exact integer in the range 0 to *k*-1.

Optional arg Default value

```
(random min max [rand-gen]) → exact-integer?
```

```
min : exact-integer?
```

```
max : (integer-in (+ 1 min) (+ 4294967087 min))
```

```
rand-gen : pseudo-random-generator?
```

```
= (current-pseudo-random-generator)
```

When called with two integer arguments *min* and *max*, returns a random exact integer in the range *min* to *max*-1.

What is “random”???

Not secure!  
e.g., for generating  
passwords

A pseudorandom number generator (PRNG), also known as a **deterministic random bit generator (DRBG)**,<sup>[1]</sup> is an **algorithm** for generating a sequence of numbers whose properties approximate the properties of sequences of **random numbers**. The PRNG-generated sequence is **not truly random**, because it is completely determined by an initial value, called the PRNG's **seed**

VS

A **cryptographically secure** pseudorandom number generator (CSPRNG) or **cryptographic pseudorandom number generator (CPRNG)** is a **pseudorandom number generator** (PRNG) with properties that make it suitable for use in **cryptology**.

# Designing Random Functions: Same Recipe!

```
;; A Velocity is a non-negative integer  
;; Interp: reresents pixels/tick change in a ball coordinate  
(define MAX-VELOCITY 10)
```

```
;; random-velocity : -> Velocity  
;; returns a random velocity between 0 and MAX-VELOCITY  
(define (random-velocity)  
  (random MAX-VELOCITY))
```

Functions can  
have zero args

```
;; random-x      : -> ???  
;; random-y      : -> ???  
;; random-ball   : -> ???
```

```
(check-true (< (random-velocity) MAX-VELOCITY))  
(check-true (>= (random-velocity) 0))  
(check-true (integer? (random-velocity)))  
(check-pred (λ (v) (and (integer? v)  
                        (< v MAX-VELOCITY)  
                        (>= v 0))))  
  (random-velocity))
```

Can still **test!**  
Just less precise

Last  
Time

# Multi-ball Animation

Design a **big-bang** animation that:

- Start: a single ball, moving with random  $x$  and  $y$  velocity
- On a click: add a ball at random location with random velocity
- If a ball “hits” an edge:
  - for vertical edge, flip  $x$  velocity direction
  - for horizontal edge, flip  $y$  velocity direction

;; A **WorldState** is ... an unknown number of balls!

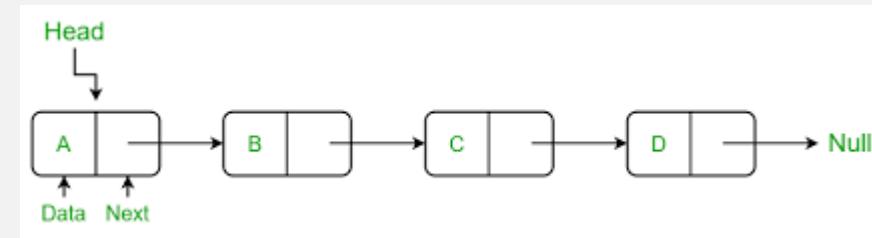
Last  
Time

# Arbitrary Size Data - Lists

In C

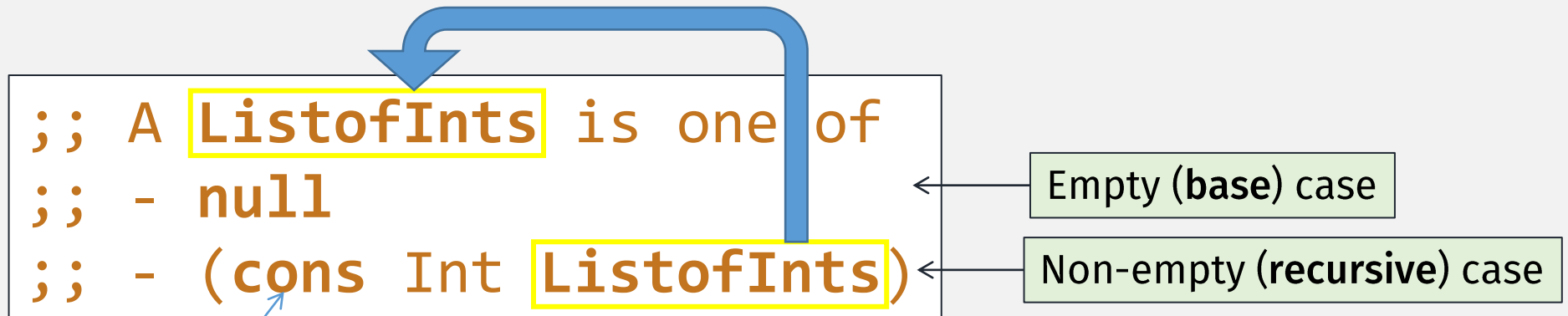
```
struct node
{ int data;
  struct node *next; } *head;
```

This is a **self-referential**  
(i.e., **recursive!**) definition!





# Racket List Data Definition Example



cons = "node"

**Recursive!**

(using a definition to define itself)

TEMPLATE??

(how can we use a list of ints  
to define a list of ints?!?)

**Recursion** is only valid if there is both

- A **base** case
- A **recursive** case

# Racket List Data Definition Example

```
;; A ListofInts is one of  
;; - null  
;; - (cons Int ListofInts)
```

Empty (base) case

Non-empty (recursive) case

This is both **itemization** and **compound** data, so template has both **cond** and **getters**

TEMPLATE??

The shape of the function matches the shape of the data definition!

Wait, where is the **recursion**???

Empty (base) case

Non-empty (recursive) case

```
; TEMPLATE for list-fn  
;; list-fn : ListofInts -> ???  
(define (list-fn lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (rest lst) ....])))
```

# Racket List Data Definition Example

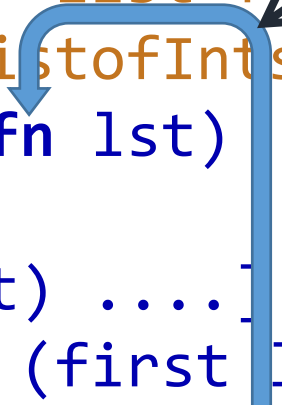
```
;; A ListofInts is one of  
;; - null  
;; - (cons Int ListofInts)
```



The shape of the function  
matches the shape of the  
data definition!

TEMPLATE??

```
;; TEMPLATE for list-fn  
;; list-fn : ListofInts -> ???  
(define (list-fn lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (list-fn (rest lst)) ....]))
```

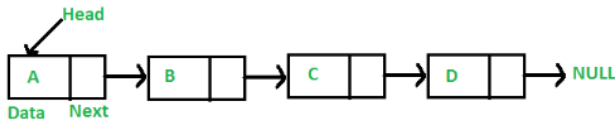


So recursion in the data definition  
... means recursion in the  
(template) function!

... is also recursive!

# Racket Recursive List Fn Example: sum

Given a singly linked list. The task is to find the sum of nodes of the given linked list.



Task is to do  $A + B + C + D$ .

Examples:

[geeksforgeeks.com](https://www.geeksforgeeks.com)

Input: 7->6->8->4->1

Output: 26

Sum of nodes:

$7 + 6 + 8 + 4 + 1 = 26$

Input: 1->7->3->9->11->5

Output: 36

Examples!

```
;; TEMPLATE for list-fn
;; list-fn : ListofInts -> ???
(define (list-fn lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (list-fn (rest lst)) ....])))
```

# Racket Recursive List Fn Example: sum

**Design Recipe:**  
Now fill in  
template!  
(with arithmetic)

```
;; Returns sum of list of ints  
;; sum-lst: ListofInts -> Int  
(define (sum-lst lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (sum-lst (rest lst)) ....]))
```

# Racket Recursive List Fn Example: sum

```
;; Returns sum of list of ints
;; sum-lst: ListofInts -> Int
(define (sum-lst lst)
  (cond
    [(null? lst) 0]
    [else .... (first lst) ....
               .... (sum-lst (rest lst)) ....]))
```

# Racket Recursive List Fn Example: sum

```
;; Returns sum of list of ints
;; sum-lst: ListofInts -> Int
(define (sum-lst lst)
  (cond
    [(null? lst) 0]
    [else (+ (first lst)
              (sum-lst (rest lst)))]))
```

# Racket Recursive List Fn Example: rev

```
;; TEMPLATE for list-fn
;; list-fn : ListofInts -> ???
(define (list-fn lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (list-fn (rest lst)) ....])))
```



# Racket Recursive List Fn Example: rev

**Design Recipe:**  
Now fill in  
template!  
(with arithmetic)

```
;; reverses a list of ints  
;; rev: ListofInts -> ListofInts  
(define (rev lst)  
  (cond  
    [(null? lst) ....]  
    [else .... (first lst) ....  
               .... (rev (rest lst)) ....]))
```

# Racket Recursive List Fn Example: rev

```
;; reverses a list of ints
;; rev: ListofInts -> ListofInts
(define (rev lst)
  (cond
    [(null? lst) null]
    [else .... (first lst) ....
               .... (rev (rest lst)) ....]))
```

# Racket Recursive List Fn Example: rev

```
(rev (rest lst)) = (list 5 4 3 2)
```

```
(check-equal? (rev (list 1 2 3 4 5)) (list 5 4 3 2 1))
```

“append” (first lst)

```
;; reverses a list of ints
;; rev: ListofInts -> ListofInts
(define (rev lst)
  (cond
    [(null? lst) null]
    [else (append (rev (rest lst))
                  (list (first lst)))])
```

# Recursive rev fn, with “temp” vars (preview)

```
;; TEMPLATE for list-fn
;; list-fn : ListofInts -> ???
(define (list-fn lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (list-fn (rest lst)) ....])))
```

# Recursive rev fn, with “temp” vars (later)

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst)
  (cond
    [(null? lst) ....]
    [else .... (first lst) ....
               .... (rev (rest lst)) ....]))
```

# Recursive rev fn, with “temp” vars (later)

Still follows  
design  
recipe!

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst rev-lst-so-far)
  (define (rev/tmp lst rev-lst-so-far)
    (cond
      [(null? lst) ....]
      [else .... (first lst) ....
               .... (rev/tmp (rest lst) ....)
               .... rev-lst-so-far ....]))
  (rev/tmp lst null))
```

An internal “**helper**”  
function adds a “temp”  
variable  
(main fn calls helper fn)

← Tmp var = reversed list “so far” (initially null)

# Recursive rev fn, with “temp” vars (later)

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst rev-lst-so-far)
  (define (rev/tmp lst rev-lst-so-far)
    (cond
      [(null? lst) rev-lst-so-far]
      [else .... (first lst) ....
               .... (rev/tmp (rest lst)
                             rev-lst-so-far) ....]
      )
    (rev/tmp lst null))
```

Now figure out how to  
“combine” these pieces  
(with “arithmetic”)

# Recursive rev fn, with “temp” vars (later)

```
;; reverses a list of ints
;; rev : ListofInts -> ListofInts
(define (rev lst rev-lst-so-far)
  (define (rev/tmp lst rev-lst-so-far)
    (cond
      [(null? lst) rev-lst-so-far]
      [else (rev/tmp
                (rest lst)
                (cons (first lst) rev-lst-so-far))]))
  (rev/tmp lst null))
```

Add next list item to reversed list “so far”



# Multi-ball Animation

Design a **big-bang** animation that:

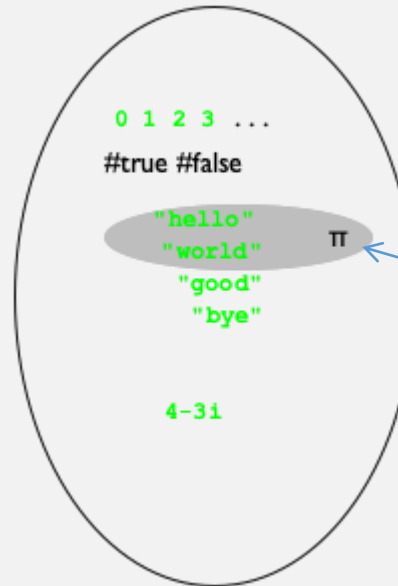
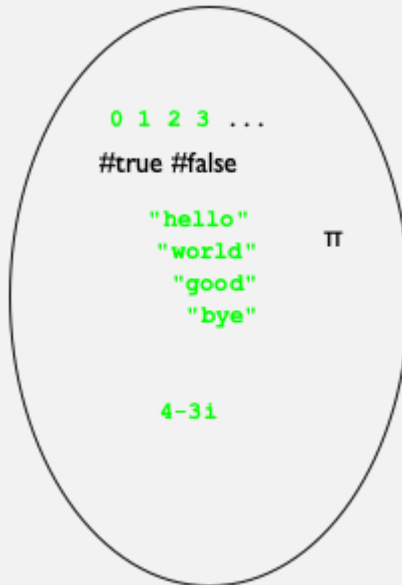
- Start: a single ball, moving with random x and y velocity
- On a click: add a ball at random location, with random velocity
- If any ball “hits” an edge:
  - if it's a vertical edge, the x velocity should flip direction
  - If it's a horizontal edge, the y velocity should flip direction

∴ A `WorldState` is ... an unknown number of balls!

;; A `WorldState` is ... a list of balls!

# Interlude: Data Definitions (ch 5.7)

All possible data values



**A data definition**  
= (a named) subset of all possible values

We are defining which data values are valid for our program!

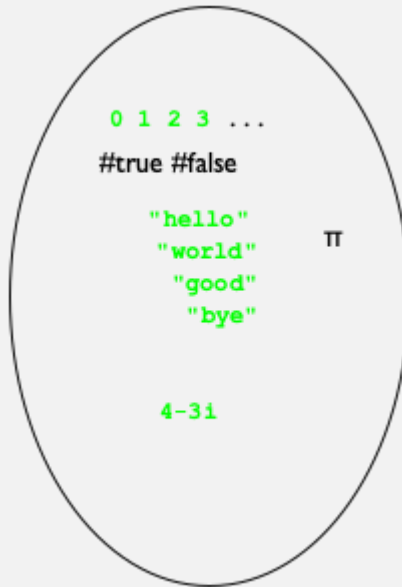
All programs are data manipulators ...

So this must be the first step of programming!

Also makes "error handling" easy

# Interlude: Data Definitions (ch 5.7)

All possible basic data values



Possible to expand the universe of values, e.g., new **compound data definitions** (struct, or other data structure)

A Venn diagram with two overlapping ovals. The left oval contains the same basic data values as the first diagram. The right oval contains compound data definitions. The intersection of the two ovals is shaded gray and contains the following text:

- (make-posn 0 3)
- (make-posn 1 3)
- (make-posn 2 3)
- (make-posn 3 3)

The text in the right oval (outside the intersection) is:

- (make-posn "hello" 0)
- (make-posn "world" 1)
- (make-posn "good" 2)
- (make-posn "bye" 3)
- (make-posn (make-posn 0 1) 2)

A Venn diagram with two overlapping ovals. The left oval contains the same basic data values as the first diagram. The right oval contains compound data definitions. The intersection of the two ovals is shaded gray and contains the following text:

- (make-posn 0 3)
- (make-posn 1 3)
- (make-posn 2 3)
- (make-posn 3 3)

The text in the right oval (outside the intersection) is:

- (make-ball -1 0)
- (make-ball -1 1)
- (make-ball -1 2)
- (make-ball -1 3)
- (make-ball "bye" #t)

A Venn diagram with two overlapping ovals. The left oval contains the same basic data values as the first diagram. The right oval contains compound data definitions. The intersection of the two ovals is shaded gray and contains the following text:

- (make-posn 0 3)
- (make-posn 1 3)
- (make-posn 2 3)
- (make-posn 3 3)

The text in the right oval (outside the intersection) is:

- (list 1)
- (list 1 2)
- (list 1 2 3)
- ...

# Multi-ball Animation

Design a **big-bang** animation that:

- Start: a single ball, moving with random x and y velocity
- On a click: add a ball at random location, with random velocity
- If any ball “hits” an edge:
  - if it's a vertical edge, the x velocity should flip direction
  - If it's a horizontal edge, the y velocity should flip direction

∴ A `WorldState` is ... an unknown number of balls!

;; A `WorldState` is ... a list of balls!

Ball

```
;; A WorldState is a  
(struct world [x y xvel yvel] #:transparent)  
;; when ball  
;; x: XCoord - represents x coordinate of ball center in animation  
;; y: YCoord - represents y coordinate of ball center in animation  
;; xvel: Integer - represents x velocity, where  
;;                positive = to the right, negative = to the left  
;; yvel: Integer - represents y vel, where  
;;                positive = down, negative = up
```

```
;; A ListofBall is one of  
;; - null  
;; - (cons Ball ListofBall)
```

```
;; A WorldState is a ListofBall
```

```
(define (main)
  (big-bang (list (random-ball))
    [on-mouse mouse-handler]
    [on-tick next-world]
    [to-draw render-world]))
```

These need to be updated to handle new WorldState data def

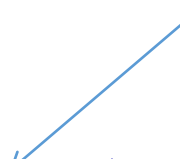
```
;; A WorldState is a ListofBall
```

# next-world

## List template!

```
;; next-world : WorldState -> WorldState
;; Computes the next world state on a tick
(define (next-world w)
  (cond
    [(null? w) ....]
    [else .... (first w) ....
               .... (next-world (rest w)) ....])))
```

Ball



Create one  
function  
per “task”

```
(check-equal? (next-world (list (make-ball 0 0 1 1)))
               (list (next-ball (make-ball 0 0 1 1))))
```

# next-ball

This was the previous “next-world” function!

```
(define (next-ball b)
  (match-define (ball x y xvel yvel) b)
  (define new-xvel
    (if (ball-in-scene/x? x) xvel (- xvel)))
  (define new-yvel
    (if (ball-in-scene/y? y) yvel (- yvel)))
  (define new-x (+ x new-xvel))
  (define new-y (+ y new-yvel))
  (ball new-x new-y new-xvel new-yvel))
```

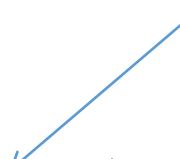


# next-world

## List template!

```
;; next-world : WorldState -> WorldState
;; Computes the next world state on a tick
(define (next-world w)
  (cond
    [(null? w) ....]
    [else .... (first w) ....
               .... (next-world (rest w)) ....])))
```

Ball



Create one  
function  
per “task”

```
(check-equal? (next-world (list (make-ball 0 0 1 1)))
              (list (next-ball (make-ball 0 0 1 1))))
```

# next-world

```
;; next-world : WorldState -> WorldState
;; Computes the next world state on a tick
(define (next-world w)
  (cond
    [(null? w) null]
    [else .... (first w) ....
               .... (next-world (rest w)) ....]))
```

# next-world

```
;; next-world : WorldState -> WorldState  
;; Computes the next world state on a tick  
(define (next-world w)  
  (cond  
    [(null? w) null]  
    [else (cons (next-ball (first w))  
                 (next-world (rest w)))])
```

# render-world

## List template!

```
;; render-world : WorldState -> Image
;; Draws the given worldstate as an image
(define (render-world w)
  (cond
    [(null? w) EMPTY-SCENE]
    [else (place-ball (first w) (render-world (rest w)))]))
```

Separate “draw”  
function for the ball

For multi-arg function, you choose which (argument's) template to use

Enumeration

```
;; mouseHandler : WorldState XCoord YCoord MouseEvent -> WorldState
;; Inserts a new ball on mouse click
(define (mouse-handler w x y mevt)
  (cond
    [(click? mevt) (cons (make-ball/random-velocity x y) w)]
    [else w ])))
```

Enumeration template  
(collapsed)

# Multi-ball Animation: more?

Design a **big-bang** animation that:

- Start: a single ball, moving with random x and y velocity
- On a click: add a ball at random location, with random velocity
  - And random size?
  - And random color?
- If any ball “hits” an edge:
  - if it's a vertical edge, the x velocity should flip direction
  - If it's a horizontal edge, the y velocity should flip direction

**;; A WorldState is ... a list of balls!**

# **Check-In Quiz 10/2** on gradescope

(due 1 minute before midnight)