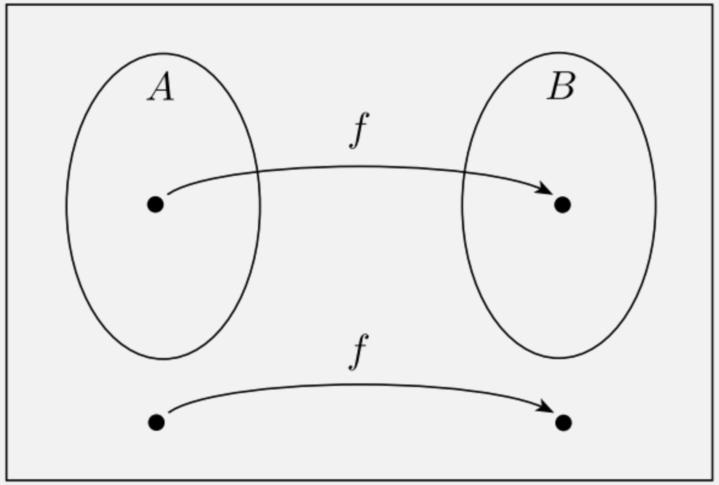
UMB CS622

Mapping Reducibility & Unrecognizability

Wednesday, October 27, 2021



Announcements

- HW6 due date extended
  - Due Wed 11/3 11:59pm

Last Time: Algorithms For CFLs

- $A_{\mathsf{CFG}} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}$ Decidable
- $E_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$ ← Why is this **decidable?** Decidable
- $ALL_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ ← But this is **undecidable?** **Undecidable**

Last time: Exploring the Limits of CFLs

- This is a CFL: $\{w_1\#w_2 \mid w_1 \neq w_2\}$
  - PDA nondeterministically checks matching positions in 1st/2nd parts
  - And rejects if **any** pair of chars are not the same
  - I.e., Each branch is "context free"

This is like the TM config-rejecting PDA used to prove ALL_{CFG} undecidable

- This is not a CFL: $\{w_1\#w_2 \mid w_1 = w_2\}$
  - Can nondeterministically check matching positions
  - But needs to accept only if **all** branches match
  - I.e., each branch is not "context free"

There's no TM config-accepting PDA because this language is not a CFL! So it's ok that E_{CFG} is decidable

This is similar to the *ww* language (not pumpable)

Summary: CFLs cannot do (stack-based) nondet. computation where a branch depends on other branch results

(This is also why **union is closed for CFLs** but **intersection is not**)

Last time: Algorithms For CFLs

- $A_{\text{CFG}} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}$ — Decidable
- $E_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$ — Decidable
- $ALL_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ — **Undecidable**
- $EQ_{\text{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$ — **Undecidable?**

(Still need to prove this is undecidable)

Theorem: EQ_{CFG} is undecidable

$EQ_{\text{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$

Proof by contradiction: Assume EQ_{CFG} has a decider R

- Use R to create a decider for ALL_{CFG}:

On input <G>:

- Construct a CFG G_{ALL} which generates all possible strings
- Run R with G and G_{ALL}
- Accept G if R accepts, else reject

The Post Correspondence Problem (PCP)

A Non-Formal Languages Undecidable Problem: *PCP*

- Let P be a set of "**dominos**" $\left\{ \left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right] \right\}$
  - Where each t_i and b_i are strings
  - E.g., $P = \left\{ \left[\frac{\mathtt{b}}{\mathtt{ca}}\right], \left[\frac{\mathtt{a}}{\mathtt{ab}}\right], \left[\frac{\mathtt{ca}}{\mathtt{a}}\right], \left[\frac{\mathtt{abc}}{\mathtt{c}}\right] \right\}$

- A **match** is:
  - A sequence of dominos with the same top and bottom strings

    Repeats allowed
  - E.g., $\left[\frac{\mathtt{a}}{\mathtt{ab}}\right]\left[\frac{\mathtt{b}}{\mathtt{ca}}\right]\left[\frac{\mathtt{ca}}{\mathtt{a}}\right]\left[\frac{\mathtt{a}}{\mathtt{ab}}\right]\left[\frac{\mathtt{abc}}{\mathtt{c}}\right]$ ➡ $\begin{array}{cccccccc} \mathtt{a} & \mathtt{b} & \mathtt{c} & \mathtt{a} & \mathtt{a} & \mathtt{a} & \mathtt{b} & \mathtt{c} \\ \mathtt{a} & \mathtt{b} & \mathtt{c} & \mathtt{a} & \mathtt{a} & \mathtt{a} & \mathtt{b} & \mathtt{c} \end{array}$

- Then: $PCP = \{ \langle P \rangle \mid P \text{ is a set of dominos with a match} \}$

Theorem: PCP is undecidable

Proof by contradiction:

Assume PCP has a decider R and use to create decider for A_{TM}:

On input $<M, w>$:
1. Construct a set of dominos P that has a match only when M accepts w
2. Run R with P as input
3. Accept if R accepts, else reject

P has M's TM configurations as its domino strings

A match is a sequence of configs showing M accepting w!

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

PCP Dominos

- First domino: $\left[\dfrac{\#}{\# q_0 w_1 w_2 \cdots w_n \#}\right]$

- <u>Key idea:</u> the remaining dominos allow the top to "catch up" (i.e., create a match) only when the bottom is a valid TM step:

$$\text{if } \delta(q,a) = (r,b,\mathrm{R}),\ \text{put } \left[\frac{qa}{br}\right] \text{ into } P$$

$$\text{if } \delta(q,a) = (r,b,\mathrm{L}),\ \text{put } \left[\frac{cqa}{rcb}\right] \text{ into } P$$

- For the tape cells that don't change: put $\left[\dfrac{a}{a}\right]$ into P

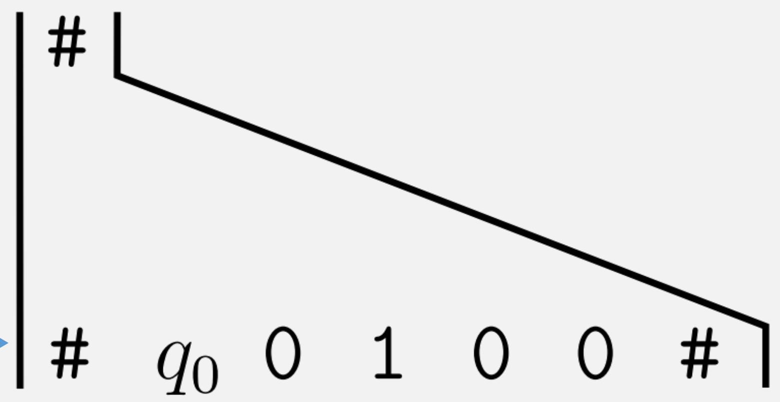
PCP Example

- Let w = `0100` and $\delta(q_0, 0) = (q_7, 2, \mathrm{R})$ so $\left[\dfrac{q_0 0}{2 q_7}\right]$ in P

Top starts to catch up

Bottom is valid transition

Fill in with unchanged tape



PCP Dominos (accepting)

- When accept state reached, let top "catch" up:

For every $a \in \Gamma$,

put $\left[\frac{a\, q_{\text{accept}}}{q_{\text{accept}}}\right]$ and $\left[\frac{q_{\text{accept}}\, a}{q_{\text{accept}}}\right]$ into P

Bottom "eats" one char

Only possible match is accepting sequence of TM configs

$\#$

$\ldots$

$\#\ 2\ 1\ q_{\text{accept}}\ 0\ 2\ \#$

"eat" one char

Flashback: "Reduced"

$A_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$

$\Downarrow$

$HALT_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ halts on input } w\}$

Thm: $HALT_{\mathsf{TM}}$ is undecidable

Proof, by contradiction:

PROBLEM: What if it takes forever to create this decider?

- Assume $HALT_{\mathsf{TM}}$ has *decider* R; use to create A_{TM} *decider*:

S = "On input $\langle M, w\rangle$, an encoding of a TM M and a string w:

1. Run TM R on input $\langle M, w\rangle$. ← Use R to first check if M will loop on w
2. If R rejects, *reject*.
3. If R accepts, simulate M on w until it halts. ← Then run M on w knowing it won't loop
4. If M has accepted, *accept*; if M has rejected, *reject*."

- Contradiction: A_{TM} is undecidable and has no decider!

We need a formal definition of "reducibility"

Flashback: A_{NFA} is a decidable language

$A_{\mathsf{NFA}} = \{\langle B, w\rangle |\ B \text{ is an NFA that accepts input string } w\}$

Decider for A_{NFA} :

$N =$ "On input $\langle B, w\rangle$, where B is an NFA and w is a string:

1. Convert NFA B to an equivalent DFA C, using the procedure `NFA→DFA`
2. Run TM M on input $\langle C, w\rangle$.
3. If M accepts, *accept*; otherwise, *reject*."

We said this **NFA→DFA** algorithm is a TM, but it doesn't accept/reject?

More generally, we've been saying
"**programs = TMs**",
but programs do more than accept/reject?

Computable Functions

- A TM that, instead of accept/reject, "outputs" final tape contents

> A function $f: \Sigma^* \longrightarrow \Sigma^*$ is a ***computable function*** if some Turing machine M, on every input w, halts with just $f(w)$ on its tape.

- Example 1: All arithmetic operations

- Example 2: Converting between machines, like **DFA→NFA**
  - E.g., adding states, changing transitions, wrapping TM in TM, etc.

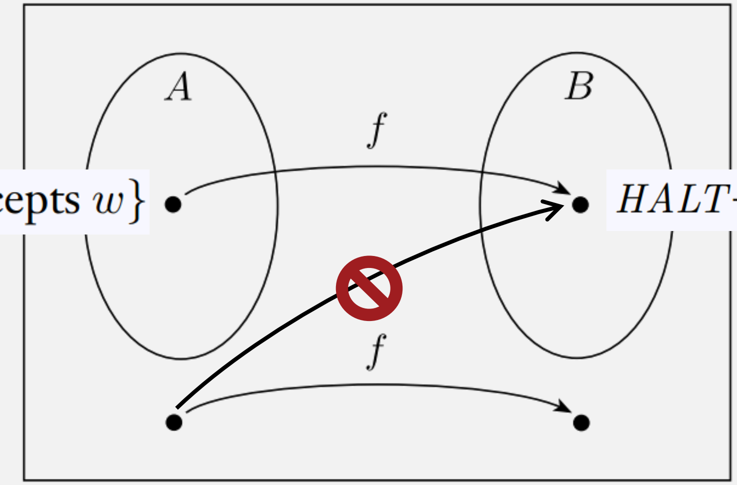
Mapping Reducibility

Language A is ***mapping reducible*** to language B, written $\boxed{A \leq_{\mathrm{m}} B}$, if there is a $\boxed{\text{computable function } f\colon \Sigma^* \longrightarrow \Sigma^*}$, where for every w,

$$\boxed{w \in A \iff f(w) \in B.}$$

The function f is called the ***reduction*** from A to B.

$A_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a } \mathsf{TM} \text{ and } M \text{ accepts } w\}$



$HALT_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a } \mathsf{TM} \text{ and } M \text{ halts on input } w\}$

A function $f\colon \Sigma^* \longrightarrow \Sigma^*$ is a ***computable function*** if some Turing machine M, on every input w, halts with just $f(w)$ on its tape.

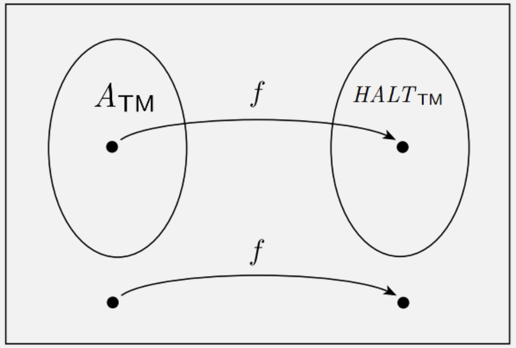
<u>Thm</u>: A_{TM} is mapping reducible to $HALT_{\mathsf{TM}}$

$A_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$

$HALT_{\mathsf{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ halts on input } w\}$

- <u>To show</u>: $A_{\mathsf{TM}} \leq_{\mathrm{m}} HALT_{\mathsf{TM}}$
- <u>Want</u>: computable fn f : $\langle M, w\rangle \rightarrow \langle M', w'\rangle$ where:

$$\langle M, w\rangle \in A_{\mathsf{TM}} \text{ if and only if } \langle M', w'\rangle \in HALT_{\mathsf{TM}}$$



The following machine F computes a reduction f.

$F =$ "On input $\langle M, w\rangle$:

1. Construct the following machine M'.
   $M' =$ "On input x:
   1. Run M on x.
   2. If M accepts, *accept*.
   3. If M rejects, enter a loop."
2. Output $\langle M', w\rangle$."

Converts *M* to *M'*

Still need to show:
M accepts *w*
if and only if
M' halts on *w*

Output new *M'*

M' is like *M*, except it always loops when it doesn't accept

Language A is ***mapping reducible*** to language B, written $A \leq_{\mathrm{m}} B$, if there is a computable function $f\colon \Sigma^* \longrightarrow \Sigma^*$, where for every w,

$$w \in A \iff f(w) \in B.$$

The function f is called the ***reduction*** from A to B.

A function $f\colon \Sigma^* \longrightarrow \Sigma^*$ is a ***computable function*** if some Turing machine M, on every input w, halts with just $f(w)$ on its tape.

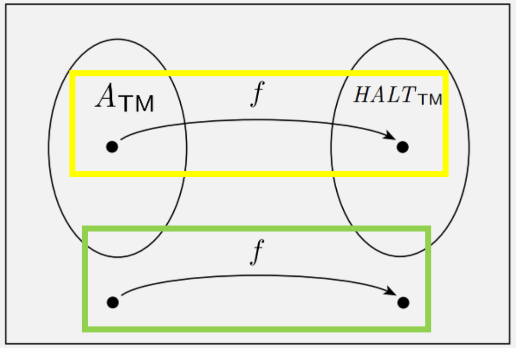
$\Rightarrow$ If M accepts w, then M' halts on w

- M' accepts (and thus halts) if M accepts

$\Leftarrow$ If M' halts on w, then M accepts w

$\Leftarrow$ (Alternatively) If M doesn't accept w, then M' doesn't halt on w (contrapositive)

- Two possibilities
    1. M loops: M' loops and doesn't halt
    2. M rejects: M' loops and doesn't halt



The following machine F computes a reduction f.

$F =$ "On input $\langle M, w \rangle$:

1. Construct the following machine M'.
   $M' =$ "On input x:
   1. Run M on x.
   2. If M accepts, *accept*.
   3. If M rejects, enter a loop."
2. Output $\langle M', w \rangle$."

Use Mapping Reducibility to Prove …

- Decidability
- Undecidability

<u>Thm</u>: If $A \leq_{\mathrm{m}} B$ and B is decidable, then A is decidable.

Has a decider

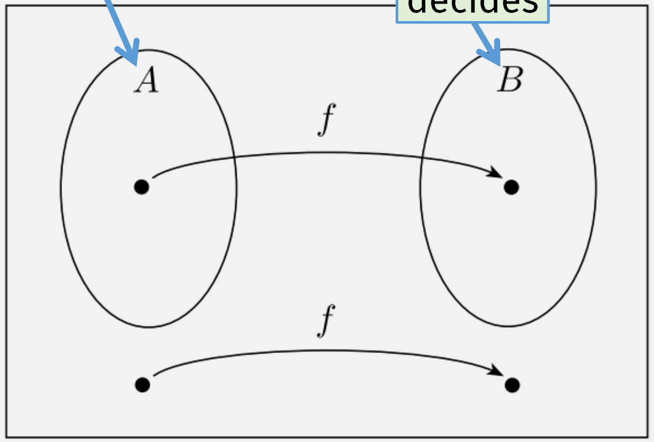
PROOF We let M be the decider for B and f be the reduction from A to B. We describe a decider N for A as follows.

$N =$ "On input w:

1. Compute $f(w)$.
2. Run M on input $f(w)$ and output whatever M outputs."

decides

decides



Language A is ***mapping reducible*** to language B, written $A \leq_{\mathrm{m}} B$, if there is a computable function $f\colon \Sigma^* \longrightarrow \Sigma^*$, where for every w,

$$\boxed{w \in A \iff f(w) \in B.}$$

The function f is called the ***reduction*** from A to B.

Coro: If $A \leq_m B$ and A is undecidable, then B is undecidable.

- Proof by contradiction.
- Assume B is decidable.
- Then A is decidable (by the previous thm).
- Contradiction: we already said A is undecidable

If $A \leq_m B$ and B is decidable, then A is decidable.

Summary: Mapping Reducibility Theorems

- If $A \leq_{\mathrm{m}} B$ and B is decidable, then A is decidable.

Known Unknown

- If $A \leq_{\mathrm{m}} B$ and A is undecidable, then B is undecidable.

Alternate Proof: The Halting Problem

$HALT_{\mathsf{TM}}$ is undecidable

- If $A \leq_{\mathrm{m}} B$ and A is undecidable, then B is undecidable.
- $A_{\mathsf{TM}} \leq_{\mathrm{m}} HALT_{\mathsf{TM}}$
- Since A_{TM} is undecidable, then $HALT_{\mathsf{TM}}$ is undecidable

Flashback: EQ_{TM} is undecidable

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

Proof by contradiction:

- Assume EQ_{TM} has *decider* R; use to create E_{TM} *decider*:

$= \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$

$S =$ "On input $\langle M \rangle$, where M is a TM:

1. Run R on input $\langle M, M_1 \rangle$, where M_1 is a TM that rejects all inputs.

2. If R accepts, *accept*; if R rejects, *reject*."

Alternate proof: Show: $E_{\mathsf{TM}} \leq_{\mathrm{m}} EQ_{\mathsf{TM}}$

- Computable fn f : $\langle M \rangle \rightarrow \langle M, M_1 \rangle$

Last step: show iff requirements of mapping reducibility (exercise)

Reducing to complement: E_{TM} is undecidable

$E_{\mathsf{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$

Proof, by contradiction:

- Assume E_{TM} has *decider* R; use to create A_{TM} *decider*:

S = "On input $\langle M, w\rangle$, an encoding of a TM M and a string w:

1. Use the description of M and w to construct the TM M_1

   M_1 = "On input x:
   1. If $x \neq w$, *reject*.
   2. If $x = w$, run M on input w and *accept* if M does."

2. Run R on input $\langle M_1\rangle$.
3. If R accepts, *reject*; if R rejects, *accept*."

If M accepts w, M_1 not in E_{TM}!

Alternate proof: computable fn: $\langle M, w\rangle \rightarrow \langle M_1 \rangle$???

Last step: show iff requirements of mapping reducibility (exercise)

- So this only reduces A_{TM} to $\overline{E_{\mathsf{TM}}}$
- It's good enough! Still proves E_{TM} is undecidable
  - Because undecidable langs are closed under complement

Undecidable Langs Closed under Complement

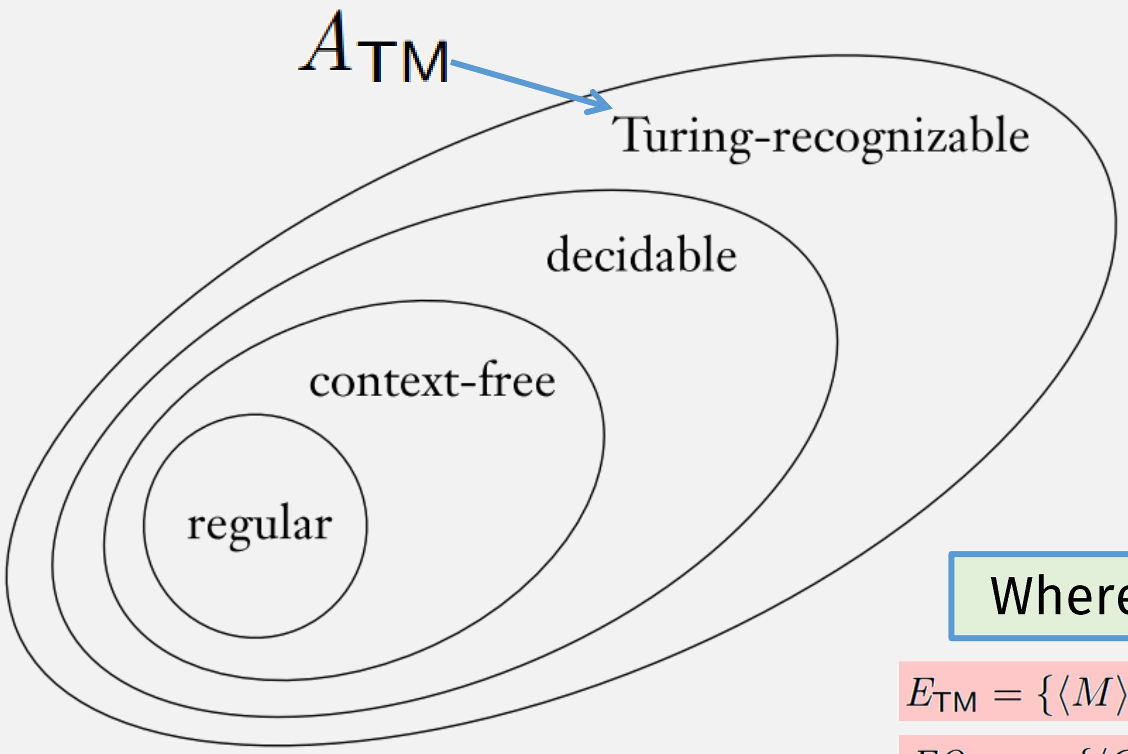
- E.g., if L is undecidable and $\overline{L}$ is decidable …
- … then we can create decider for L from decider for $\overline{L}$ …
- … which is a contradiction!

- Because decidable languages are closed under complement!

Unrecognizability

Turing Unrecognizable?

Is there anything out here?



Where do these go?

$E_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$

$EQ_{\text{CFG}} = \{\langle G, H \rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$

$EQ_{\text{TM}} = \{\langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

Thm: Some langs are not Turing-recognizable

Proof: requires 2 lemmas

- Lemma 1: The **set of all languages** is *uncountable*
  - Proof: Show there is a bijection with another uncountable set …
    - … The set of all infinite binary sequences

- Lemma 2: The **set of all TMs** is *countable*

- Therefore, some language is not recognized by a TM
  (pigeonhole principle)

Mapping a Language to a Binary Sequence

All Possible Strings

Some Language
(subset of above)

Its (unique)
Binary Sequence

$$
\begin{array}{rcl}
\Sigma^* &=& \{\ \varepsilon,\ 0,\ 1,\ 00,\ 01,\ 10,\ 11,\ 000,\ 001,\ \cdots\ \} \\
A &=& \{\ \ 0,\ \ 00,\ 01,\ \ 000,\ 001,\ \cdots\ \} \\
\chi_A &=& \phantom{\{}\ 0\quad 1\quad 0\quad 1\quad 1\quad 0\quad 0\quad 1\quad 1\quad \cdots
\end{array}
$$

Each digit represents one possible string:
- 1 if lang has that string,
- 0 otherwise

Thm: Some langs are not Turing-recognizable

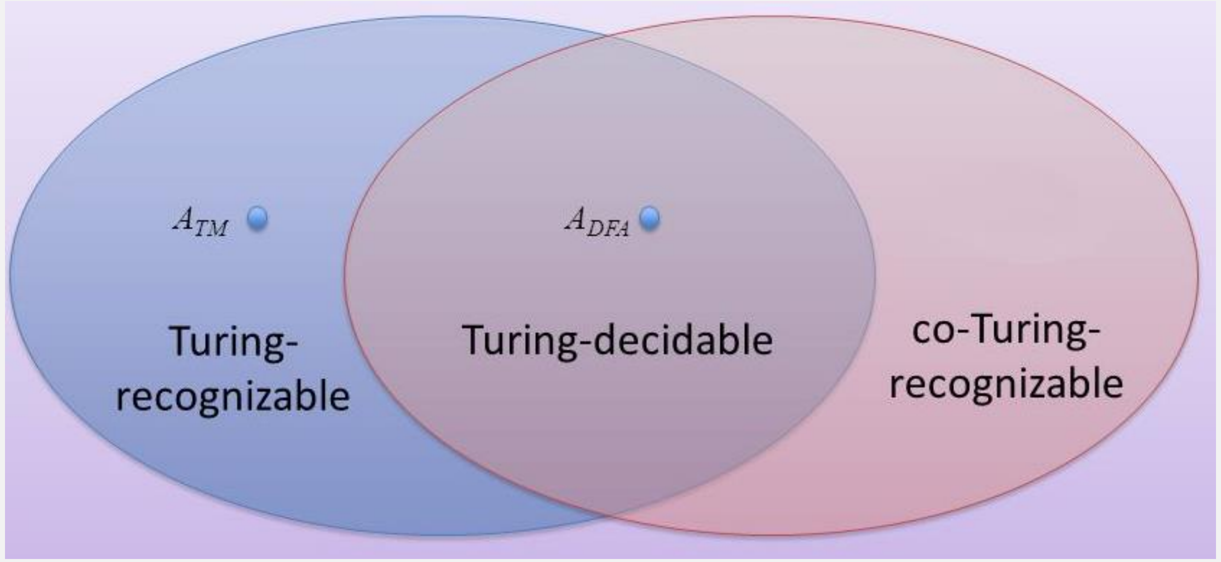
Proof: **requires 2 lemmas**

- Lemma 1: The **set of all languages** is *uncountable*
  - Proof: Show there is a bijection with another uncountable set …
    - … The set of all infinite binary sequences
    - ➢ Now just prove set of infinite binary sequences is uncountable (diagonalization)
- Lemma 2: The **set of all TMs** is *countable*
  - Because every TM M can be encoded as a string $<M>$
  - And set of all strings is countable
- Therefore, some language is not recognized by a TM ∎

Co-Turing-Recognizability

- A language is **co-Turing-recognizable** if …
- … it is the complement of a Turing-recognizable language.

Thm: Decidable ⇔ Recognizable & co-Recognizable



<u>Thm</u>: Decidable ⇔ Recognizable & co-Recognizable

⇒ <u>If</u> a language is **decidable,** <u>then</u> it is **recognizable** and **co-recognizable**

- Decidable => Recognizable (hw5):
  - A decider is just a recognizer that halts
- Decidable => Co-Recognizable:
  - To create co-decider from a decider … switch reject/accept of all inputs
  - A co-decider is a co-recognizer, for same reason as above

⇐ <u>If</u> a language is **recognizable** and **co-recognizable,** <u>then</u> it is **decidable**

Thm: Decidable ⇔ Recognizable & co-Recognizable

⇒ If a language is **decidable**, then it is **recognizable** and **co-recognizable**

- Decidable => Recognizable:
  - A decider is just a recognizer that halts
- Decidable => Co-Recognizable:
  - To create co-decider from a decider … switch reject/accept of all inputs
  - A co-decider is a co-recognizer, for same reason as above

⇐ If a language is **recognizable** and **co-recognizable**, then it is **decidable**

- Let M_1 = recognizer for the language,
- and M_2 = recognizer for its complement
- Decider M:
  - Run 1 step on M_1,
  - Run 1 step on M_2,
  - Repeat, until one machine accepts. If it's M_1, accept. If it's M_2, reject

Termination Arg: Either M_1 or M_2 must accept and halt, so M halts and is a decider

A Turing-unrecognizable language

$\boxed{\text{Recognizable \& co-recognizable implies decidable}}$

- We've proved:

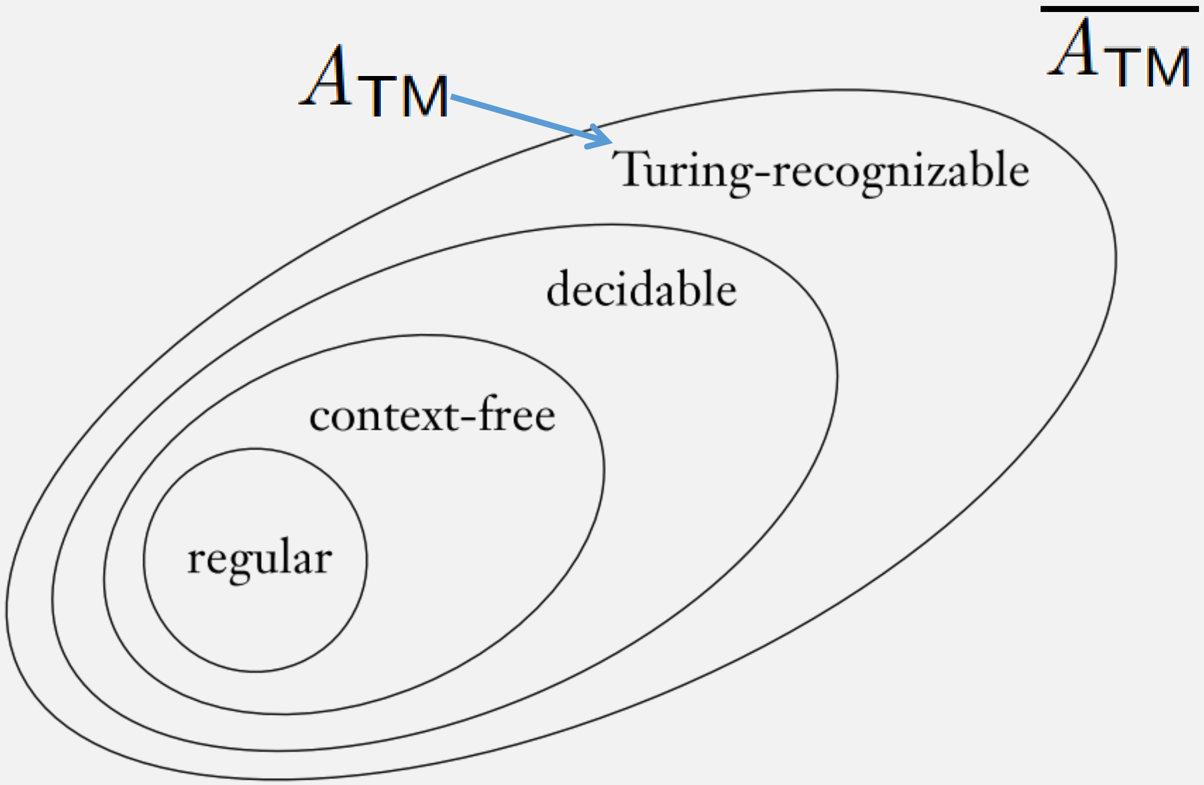
  A_{TM} is Turing-recognizable

  A_{TM} is undecidable

- So:

  $\overline{A_{\mathsf{TM}}}$ is not Turing-recognizable

Is there anything out here?



Use Mapping Reducibility to Prove …

- Decidability
- Undecidability
- Recognizability
- Unrecognizability

More Helpful Theorems

If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.

If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.

- Same proofs as:

If $A \leq_m B$ and B is decidable, then A is decidable.

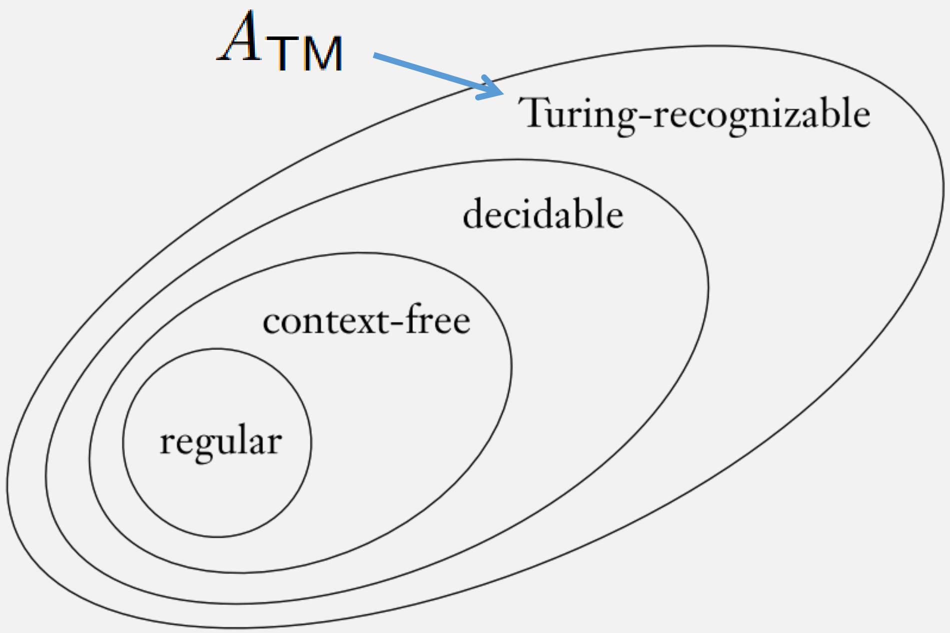
If $A \leq_m B$ and A is undecidable, then B is undecidable.

Thm: EQ_{TM} is neither Turing-recognizable nor co-Turing-recognizable.

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

1. EQ_{TM} is not Turing-recognizable

$\overline{A_{\mathsf{TM}}}$



$\overline{A_{\mathsf{TM}}} \leq_{\mathrm{m}} EQ_{\mathsf{TM}}$ A is not Turing-recognizable, th EQ_{TM} not Turing-recognizable.

Mapping Reducibility implies Mapping Red. of Complements

Language A is ***mapping reducible*** to language B, written $A \leq_{\mathrm{m}} B$, if there is a computable function $f\colon \Sigma^* \longrightarrow \Sigma^*$, where for every w,

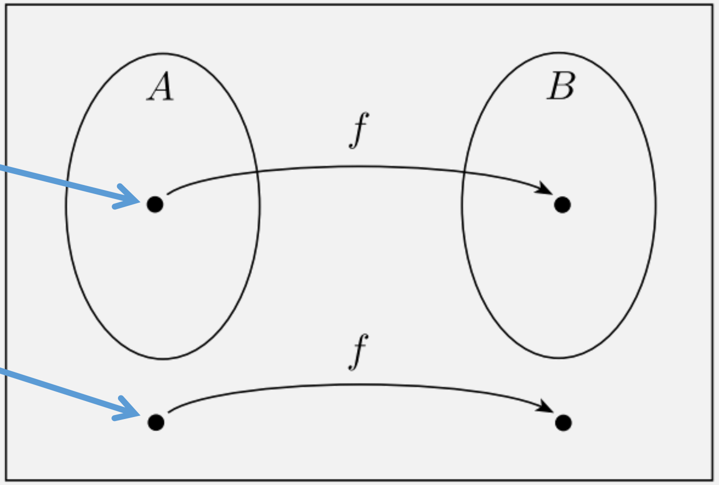
$$\boxed{w \in A \iff f(w) \in B.}$$

The function f is called the ***reduction*** from A to B.

$A \leq_{\mathrm{m}} B$

implies

$\overline{A} \leq_{\mathrm{m}} \overline{B}$



Thm: EQ_{TM} is neither Turing-recognizable nor co-Turing-recognizable.

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

1. EQ_{TM} is not Turing-recognizable

Two Choices:

- Create Computable fn: $\overline{A_{\mathsf{TM}}} \rightarrow EQ_{\mathsf{TM}}$
- Or Computable fn: $A_{\mathsf{TM}} \rightarrow \overline{EQ_{\mathsf{TM}}}$

Thm: EQ_{TM} is not Turing-recognizable

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

- Create Computable fn: A_{TM} → $\overline{EQ_{\mathsf{TM}}}$
- $\langle M, w\rangle$ → $\langle M_1, M_2\rangle$ M_1 and M_2 are TMs and $L(M_1) \neq L(M_2)$

F = "On input $\langle M, w\rangle$, where M is a TM and w a string:

1. Construct the following two machines, M_1 and M_2.

   M_1 = "On any input: ← Accepts nothing

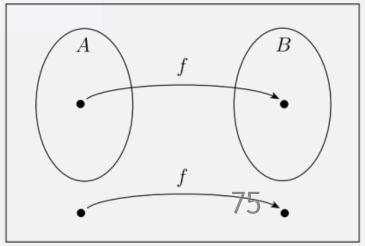
   1. *Reject.*"

   M_2 = "On any input: ← Accepts nothing or everything

   1. Run M on w. If it accepts, *accept*."

2. Output $\langle M_1, M_2\rangle$."

- If M accepts w, M_1 not equal to M_2
- If M does not accept w, M_1 equal to M_2



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$EQ_{\mathsf{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

1. EQ_{TM} is not Turing-recognizable

- Create Computable fn: $\overline{A_{\mathsf{TM}}} \rightarrow EQ_{\mathsf{TM}}$
- Or Computable fn: $A_{\mathsf{TM}} \rightarrow \overline{EQ_{\mathsf{TM}}}$
- **DONE!**

2. $\overline{EQ_{\mathsf{TM}}}$ is not ~~co~~-Turing-recognizable

- (A lang is co-Turing-recog. if it is complement of Turing-recog. lang)

<u>Prev</u>: EQ_{TM} is not Turing-recognizable

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

- Create <u>Computable fn</u>: A_{TM} → $\overline{EQ_{\mathsf{TM}}}$
- $\langle M, w\rangle$ → $\langle M_1, M_2\rangle$ M_1 and M_2 are TMs and $L(M_1) \neq L(M_2)$

F = "On input $\langle M, w\rangle$, where M is a TM and w a string:

1. Construct the following two machines, M_1 and M_2.

M_1 = "On any input: ← Accepts nothing

1. *Reject*."

M_2 = "On any input: ← Accepts nothing or everything

1. Run M on w. If it accepts, *accept*."

2. Output $\langle M_1, M_2\rangle$."

DONE!

Now: $\overline{EQ}_{\mathsf{TM}}$ is not Turing-recognizable

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

- Create Computable fn: $A_{\mathsf{TM}} \rightarrow \overline{EQ}_{\mathsf{TM}}$
- $\langle M, w\rangle \rightarrow \langle M_1, M_2\rangle$ M_1 and M_2 are TMs and $L(M_1) \neq L(M_2)$

$F =$ "On input $\langle M, w\rangle$, where M is a TM and w a string:

1. Construct the following two machines, M_1 and M_2.

$M_1 =$ "On any input: ← Accepts ~~nothing~~ everything

1. *Accept*."

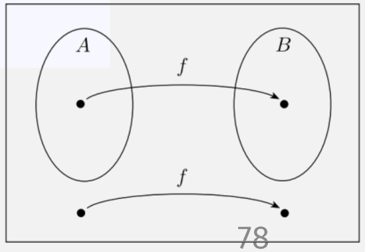
$M_2 =$ "On any input: ← Accepts nothing or everything

1. Run M on w. If it accepts, *accept*."

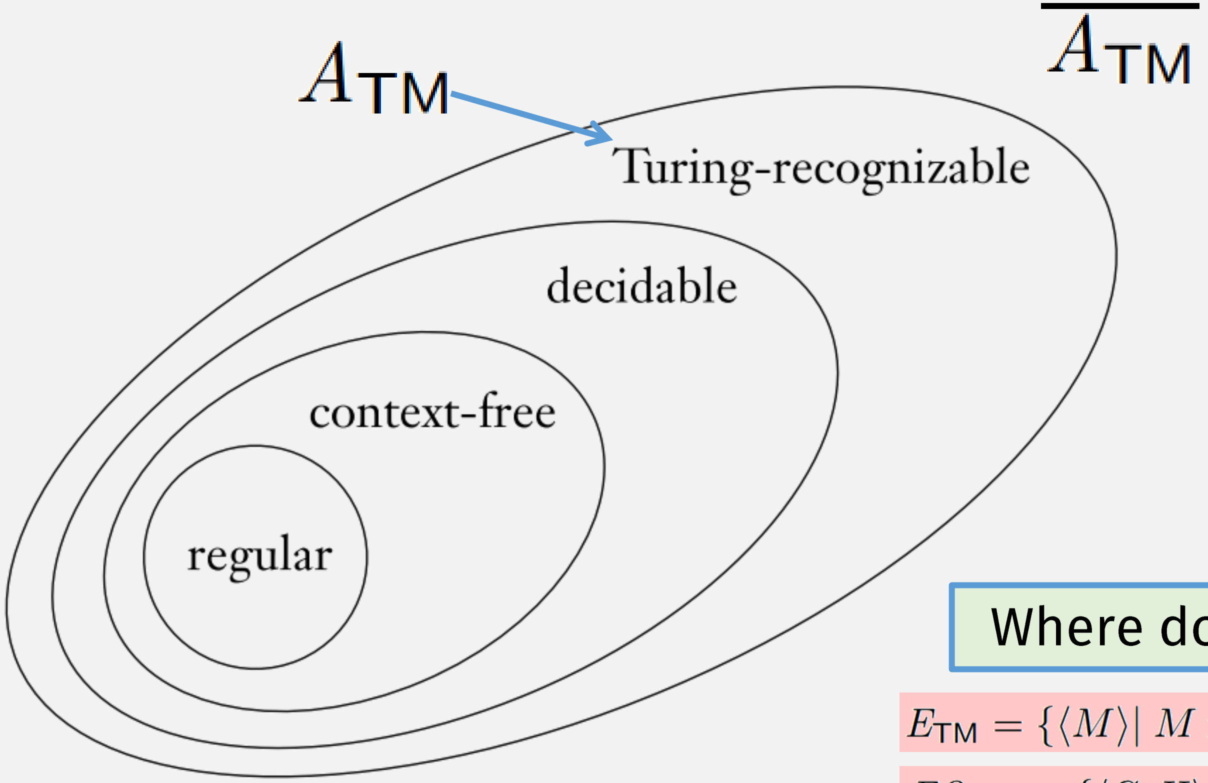
2. Output $\langle M_1, M_2\rangle$."

- If M accepts w, M_1 equals to M_2
- If M does not accept w, M_1 not equal to M_2

DONE!



Unrecognizable Languages?



Where do these go?

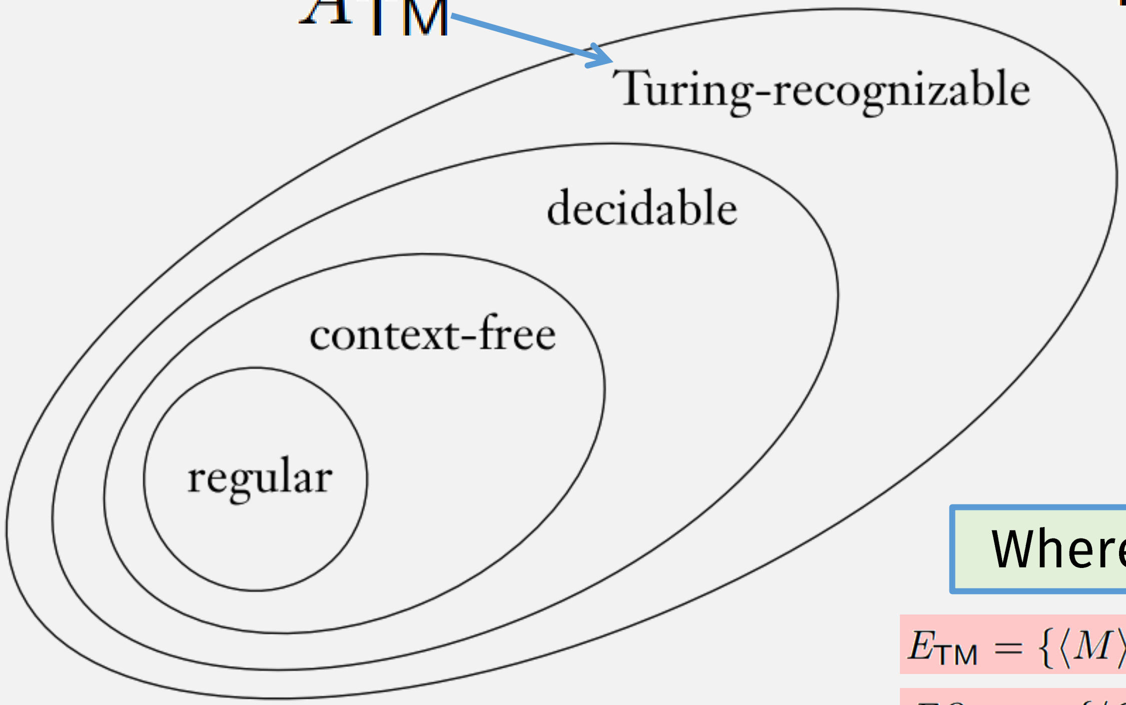
$E_{\mathsf{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$

$EQ_{\mathsf{CFG}} = \{\langle G, H \rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$

$EQ_{\mathsf{TM}} = \{\langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$

Unrecognizable Languages

A_{TM}



$\overline{A_{\mathsf{TM}}}$

EQ_{TM}

$\overline{EQ_{\mathsf{TM}}}$

Where do these go?

$E_{\mathsf{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$

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Check-in Quiz 10/27

On gradescope