DeFi Stylized facts

A. Christian Silva[1], Next author, etc[2]

[1]idatafactory

Abstract

List and model in the crypto world. Outline of paper.

Introduction

Cryptocurrencies, and in particular Bitcoin (BTC), have become very popular and are making their way into the portfolio of many investors. Furthermore, there apears to be increasing interest in the financial industry illustrated by the recent attempts to create ETF based on Bitcoin (reference). Despite the attention given to Bitcoin in the media, Bitcoin it is only the tip of the iceberg. The introduction of the etherium network in 2015 (reference) allowed the creation of an ecosystem of decentralized applications that is now called decentralized finance (DeFi). The advent of DeFi further extended the cryptocurrencies offering from simple Bitcoin clones to essential components of decentralized applications. The functions of such cryptocurrency is so broad that the better term is to call these tokens. One such example is ETH. ETH is the tocken used in the etherium network in order to pay for the execution of code (smart contracts). An other example are liquidity provider tokens such as UNI which allow the owner to receive fees for providing liquidity on Uniswap but which can also be traded much like ETH or BTC. The expansion of DeFi in the last 5 years has been fast with applications that address a series of financial needs. For further discussion and examples see (Harvey et al. 2021).

Here we perform statitical analysis of the different tokes in DeFi. We report a series of stylized facts and contrast these with the extensively reported stylized facts in equities (Mantegna and Stanley 1999),(Cont 2001),(Bouchaud and Potters 2003),(Anirban Chakraborti and Abergel 2011). Our goal is to provide a characterization of the main tokens in circulation which can guide models and potentially help further development of DeFi. In addition, we contrast CDX with DEX and its derivative products such as perpetuals and options.

We start with a brief introduction to the technology behind DeFi focusing on some operation aspects which are relevant for this article.

Blockchain and tokens

In this section we describe the technical aspects which allow for DeFi to function. For a more in depth discussion, please see (Lipton and Treccani 2021) or the original articles by Nakamoto and Buterin (references).

Blockchain is a chain of information (blocks) connected and protected by cryptography keys. A copy of this chain resides on many computers across the globe which makes the storage of information decentralized and fully transparent. In the case of Bitcon, the information content is essentially a ledger (Mary has 10 BTC, John has 1 BTC, etc) similar to one found in a bank. Therefore the function of a bank is now performed by this blockchain which records all transactions. One of the main challenges of such a decentralized architecture is to make sure that the transactions are unique. For instance, that John can not buy two pizzas with the same 0.0001 BTC. In order for that to happen, the transaction has to be recorded on all ledgers in the entire network while the funds are dispersed (consensus mechanism). This is where miners come in and the mechanism of "proof-of-work" or "proof-of-stake" is utilized. In the case of ETH, the chain has a double function of tracking ETH balance and also to execute code (more here).

What to address in this section:

- What is the function of the token?
- What powers does different tokens provide?
- Examples of different tokens and a list of what tokens we will look into.
- Contrast CDX with DEX

More in depth examples of tokens see (Harvey et al. 2021).

Stylized facts

These are the potential stylized fact list:

- Heavy tails in particular for short return horizons: excess kurtosis
- Returns become Gaussian
- Losses are larger than gains: skewness is significantly negative
- Returns do not show autocorrelations
- Volatility clustering: slow decay of autocorrelations
- Leverage effect
- Volume/volatility relation
- Intraday trading pattern
- Lack of time reversal when looking at volatilities (Zumbach 2009),(Guyon and Lekeufack 2023)
- Very particular price process for Uniswap. Can model by Ju-Yi Yen be address in a second paper?
- Cross-token correlations: RMT and related
- Subordination: activity clock

What do we model?

It is common in finance to model log-returns. We look at the difference between relative and log-returns because cryptocurrencies have substantial volatility.

```
Unable to display output for mime type(s): text/html
```

```
Unable to display output for mime type(s): text/html
```

Probability distribution

The analysis here consist on producing a series of plots that focus mainly on the tails. The idea is to show the heavy tail nature of the assets, the scalling with volatility and convergence to a gaussian for larger returns. In order to achive this we will use a combination of figures and statistics such as the skewness and kurtosis and other (see (Cont 2001)).

The next set of figures illustrate the type of presentation needed.

ETH and BTC

The longest time-series are BTC and ETH. We will look a the unconditional PDF/CDF of these two tockens first. We will add SPX to the figures as well. These are few questions we need to address:

- Evolution of the PDF from Minutes to Months. Does it converge to a gaussian?
- Compare the different definition of "day" (15h UTC vs 15h UTC or 23H vs 23H). Is it relevant where we slice?
- What if we define the time-series in since with the SPX trading times?
- Does the unconditional PDF/CDF change through history? We can break the history in equal bucket and also "economic" buckets (recessions etc). We should also break it according to data available of the other tokens in preparation for next section.

Other tockens ...

Most tokens do not have plenty of history. We first start with the group of tokens that have at least 3 years of history and then we increase the number of tokens by looking at different time periods. Questions to address:

- Is the unconditional distribution essentially the same if we account for scale?
- Is the volatility of these different tokens substantially different?

DEX vs CEX

Is there a substantial difference between DEX and CEX with regard to unconditional probabilities? Here we have less data per token.

	pair	fD
1	ALGO	2019-08-15T16:00:00Z
2	BCH	2017-12-20T01:00:00Z
3	BTC	2016-01-01T00:00:00Z
4	DASH	2019-09-17T16:00:00Z
5	EOS	2019-04-09T16:00:00Z
6	ETC	2018-08-08T18:00:00Z
7	ETH	2016-05-18T00:00:00Z
8	LINK	2019-06-27T16:00:00Z
9	LTC	2016-08-17T04:00:00Z
10	OXT	2019-12-16T15:00:00Z
11	REP	2019-04-09T16:00:00Z
12	XLM	2019-03-14T16:00:00Z
13	XRP	2019-02-26T17:00:00Z
14	XTZ	2019-08-06T16:00:00Z
15	ZRX	2018-10-12T14:00:00Z

Figure 1: Tokens with more than 3 years of history available in coinbase

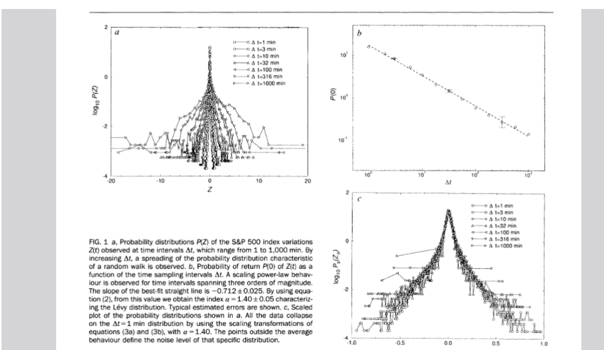

Figure 2: PDF 2

Are the returns IID?

Auto-correlation of returns and the absolute returns for both ETH and SPX. Next section shows only ETH per exchange.

ETH

Are the returns auto-correlated? On a first approximation they are uncorrelated except on a very short time scale for certain exchanges. The variance is well

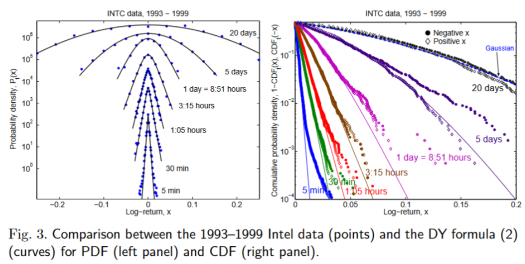

Figure 3: PDF and CDF

fit by a linear function (Figure 4) which implies no memory. Nevertheless the autocorrelation plot (Figure 5) shows that returns of less than one hour is significantly auto-correlated for Uniswap.

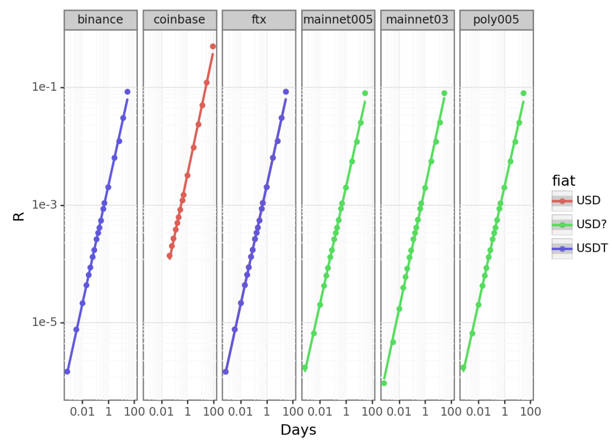

Figure 4: Volatility as a function of log-return horizon

Looking at the auto-correlation:

ETH and SPX

Side by side in order to compare .... that also helps with the limited data availability for ETH is com-

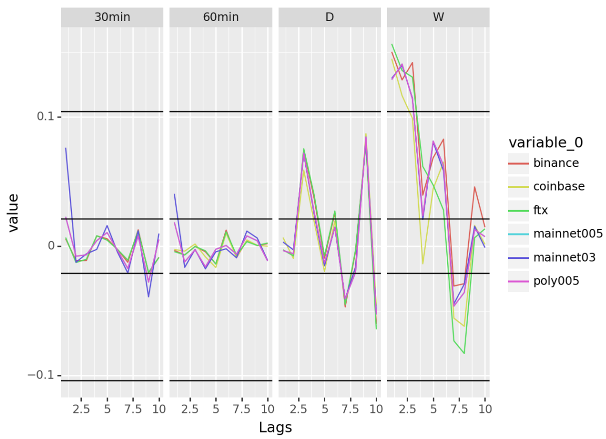

Figure 5: Auto-correlation: error bar for 60min

pared to SPX.

ACF daily data

Notice that the start and the end of the day is different for both assets. More work is needed.

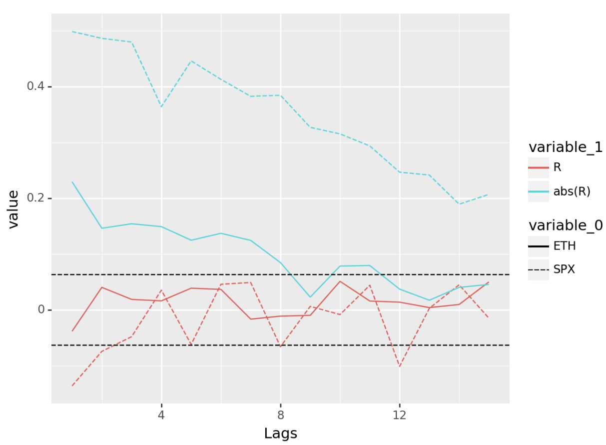

Figure 6: ACF day

ACF for hour

Notice that this is not that simple because hourly returns for the spx do not run for 24 hours. More work is needed. There is clear seasonality for SPX...

ACF of abs(R):

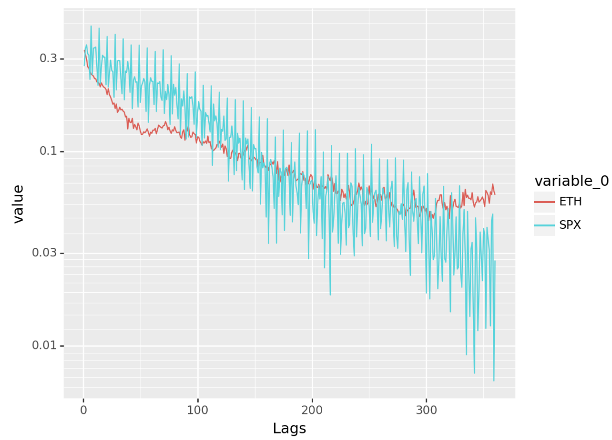

Figure 7: ACF hour for abs(R)

ACF of R:

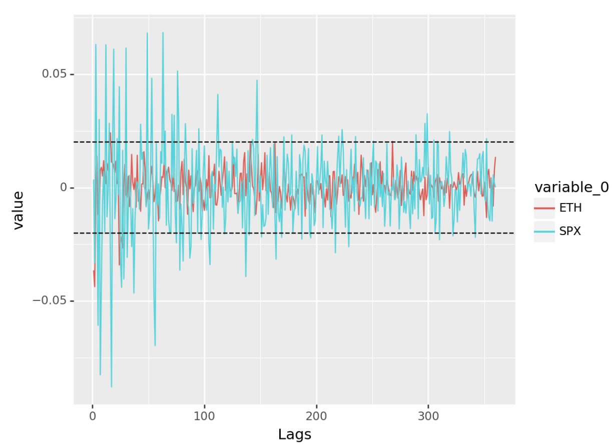

Figure 8: ACF hour for R

Time-series

Figure (Figure 9) shows the time series for ETH and for the SPX index for approximately the same period.

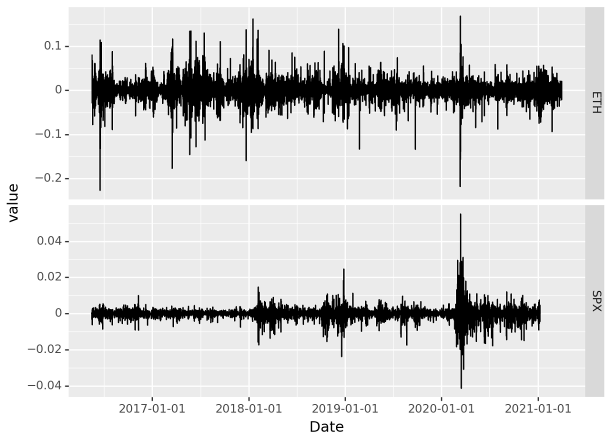

Figure 9: Returns on a hour interval

Volume pattern

To address here:

- Daily volume patter that should be similar to volatility pattern. We need to check this further.
- The weekly volume pattern. Possible graph could show daily pattern over day of the week.

Volatility

Volatility is the main scale of the price process. It has been shown that for traditional assets (stocks, etc) the volatility presents leverage effect and time reversal assymetriy. These results should be tested for the different crypto assets.

The next figures illustrate the leverage effect and volatility clustering (can be also shown by simple autocorrelation, see ETH). The time reversal asymetric is less of an obvious result: see (Zumbach 2009) for the statistical procedure to show such result.

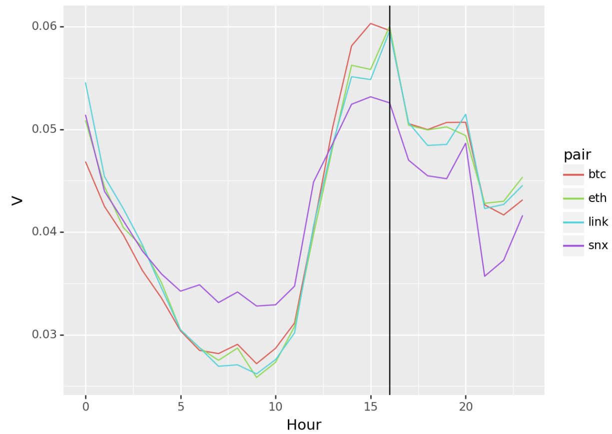

Figure 10: Volume seasonality

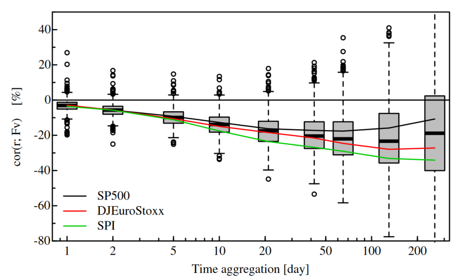

Figure 7: *Scaling box plot for the the leverage effect, namely correlation between the return and the forward volatility.*

Figure 11: Leverage effect

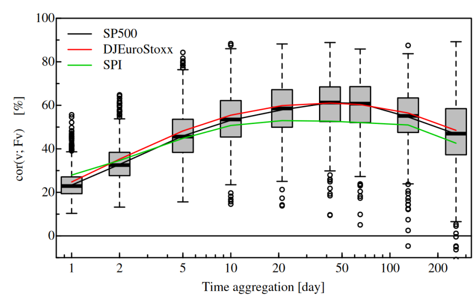

Figure 2: *Scaling box plot for the heteroskedasticity, measured by the historical–realized volatility correlation* $\rho(\sigma[\Delta T], F\sigma[\Delta T])$.

Figure 12: Heteroskedastocity

Uniswap price process

More work needed but this should be presented and more theoretical work can be added. See next figure for a possible illustration of the process.

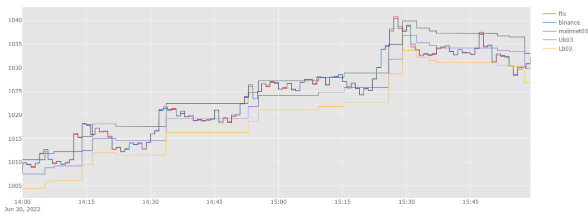

Figure 13: Uniswap Price process

References

Anirban Chakraborti, Marco Patriarca, Ioane Muni Toke, and Frédéric Abergel. 2011. "Econophysics Review: I. Empirical Facts." *Quantitative Finance* 11 (7): 991–1012. https://doi.org/10.1080/14697688.2010.539248.

Bouchaud, J. P., and M. Potters. 2003. *Theory of Financial Risk and Derivative Pricing: From Statistical Physics to Risk Management*. Cambridge University Press. https://books.google.com/books?id=neT6jVyMtlIC.

Cont, R. 2001. "Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues." *Quantitative Finance* 1 (2): 223–36. https://doi.org/10.1080/713665670.

Guyon, Julien, and Jordan Lekeufack. 2023. "Volatility Is (Mostly) Path-Dependent." *Quantitative Finance* 23 (9): 1221–58. https://doi.org/10.1080/14697688.2023.2221281.

Harvey, C. R., A. Ramachandran, J. Santoro, F. Ehrsam, and V. Buterin. 2021. *DeFi and the Future of Finance*. Wiley. https://books.google.com/books?id=CGM4EAAAQBAJ.

Lipton, A., and A. Treccani. 2021. *Blockchain and Distributed Ledgers: Mathematics, Technology, and Economics*. G - Reference,information and Interdisciplinary Subjects Series. World Scientific. https://books.google.com/books?id=GAx6zQEACAAJ.

Mantegna, Rosario N., and H. Eugene Stanley. 1999. *Introduction to Econophysics: Correlations and Complexity in Finance*. Cambridge University Press. https://doi.org/10.1017/CBO9780511755767.

Zumbach, Gilles. 2009. "Time Reversal Invariance in Finance." *Quantitative Finance* 9 (5): 505–15. https://doi.org/10.1080/14697680802616712.