

# Artificial Intelligence

## V10: Probabilistic Learning

Probabilistic modeling  
Example domain: speech processing  
Gaussian Mixture Models

Based on material by

- Stuart Russell, UC Berkeley
- T. Stadelmann, R. Ewerth & B. Freisleben, U Marburg



# Educational objectives

- **Remember** Bayesian learning, especially Bayes' theorem and the Bayes classifier
- **Grasp** how **the concept of probability** is extremely useful **in AI**, especially for learning
- **Explain** how a **Gaussian Mixture Model (GMM)** is **trained** and **evaluated**, given the respective **equations** and the **EM** algorithm
- **Apply** **GMMs** for pattern recognition tasks **on audio** data

*“In which we view learning as a form of uncertain reasoning from observations.”*

➔ Reading: ALMA, ch. 20 (optional: 13-14)



# 1. PROBABILISTIC MODELING

# Probability distributions and density functions

Terminology: its **probability density function (pdf)** is one way to describe a **distribution**.

What does a **pdf** tell about a set of data?

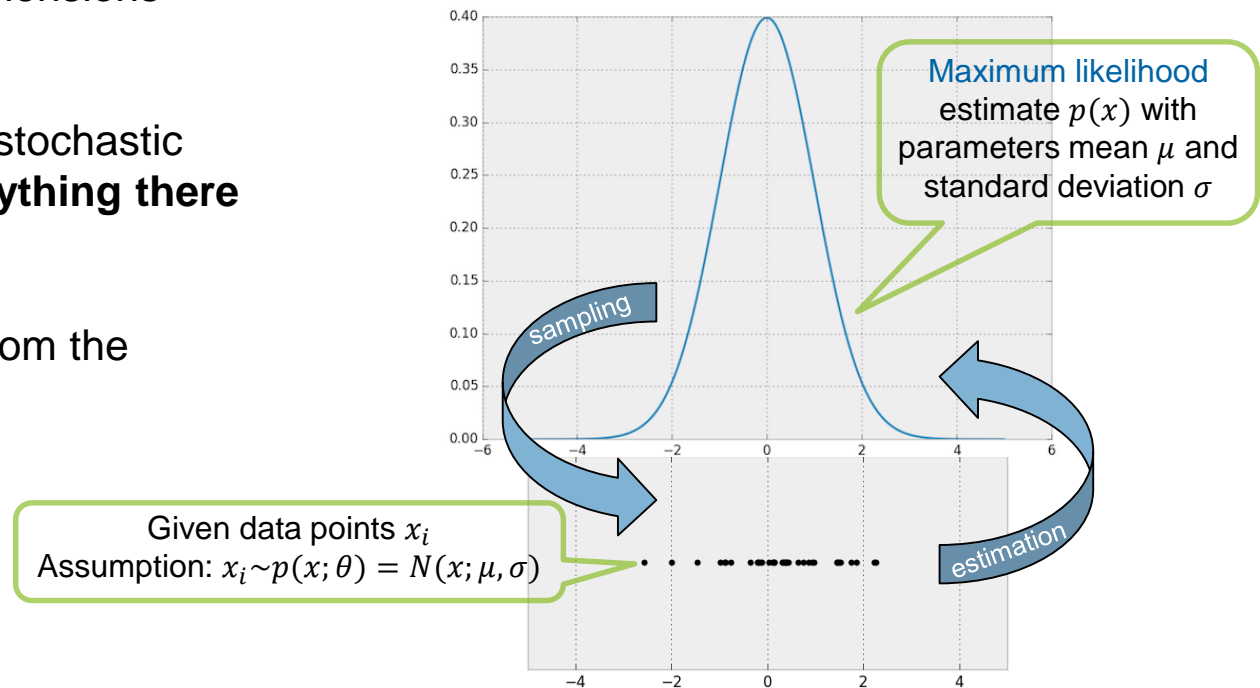
- Where to expect samples  
...with which probability
- Correlation/covariance of dimensions

→ For data coming from some stochastic processes, the pdf tells **everything there is to know** about the data

→ **Allows for sampling** data from the underlying distribution  
(**generative modeling**)

An example generative model

- The univariate Gaussian  
A parametric pdf, recoverable from data  
(Gaussianity given)



Source: Brandon Amos, «Image Completion with Deep Learning in TensorFlow», 2016,  
<https://bamos.github.io/2016/08/09/deep-completion/>

# Bayes' theorem

One of the cornerstones of modern data analysis

$$p(h|X) = \frac{p(X|h) \cdot p(h)}{p(X)}$$

with (in a machine learning context with training data  $X$  and model  $h$ )

- $p(X|h)$  the **likelihood** of the data, given the model  $\rightarrow$  called the **evidence** for  $h$
- $p(X)$  the **a priori** probability of the training data  $X \rightarrow$  this normalization factor is **rarely needed/used**
- $p(h)$  is the **a priori** probability of hypothesis  $h \rightarrow$  **often neglected** in practice due to dominance of evidence



Rev. Thomas Bayes,  
1701-1761

## Use cases

- Generally: **Convert** between **prior** and **posterior** probabilities
- Specific example: **Model selection**
  - $\rightarrow$  Given competing  $h_i \in \mathcal{H}$ , one can calculate the likelihood  $p(X|h_i)$ , then select best  $\hat{h} = \max_{h_i} p(h_i|X) \approx \max_{h_i} p(X|h_i)$

There's a long-standing controversy pro/con Bayesianism in statistics, see e.g. [http://lesswrong.com/lw/1to/what\\_is\\_bayesianism/](http://lesswrong.com/lw/1to/what_is_bayesianism/); for the meaning of Bayesianism in machine learning, see e.g. [https://www.reddit.com/r/MachineLearning/comments/6dbwnf/d\\_what\\_is\\_exactly\\_a\\_bayesian\\_guy\\_in\\_machine/](https://www.reddit.com/r/MachineLearning/comments/6dbwnf/d_what_is_exactly_a_bayesian_guy_in_machine/)



# Bayesian reasoning & learning

Based on [Mitchell, 1997], ch. 6

## Bayesian reasoning

- Built upon Bayes' theorem to convert **prior** probabilities into **posteriors**
- Quantities of interest are governed by probability distributions
- **Optimal decisions** are made by taking them plus observed data into account

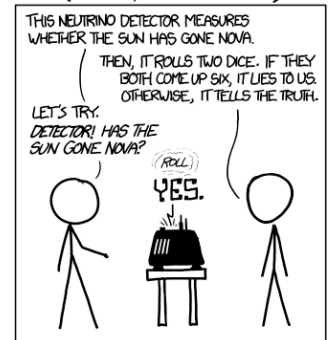
### Pro

- Provides explicit probabilities for hypotheses
- **Helps to understand/analyze algorithms** that don't emit probabilities (e.g., why to minimize sum of squares; what the inductive bias of decision trees is)
- **Everything done probabilistically** (e.g., every training instance contributes to the final hypothesis according to its prior probability; **prior knowledge** can be incorporated as prior probabilities for candidate hypotheses or distributions over training data; predictions can be easily combined)

### Con

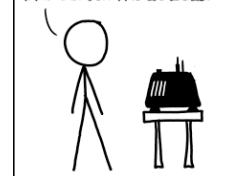
- Many needed **probabilities** are **unknown** in practice (approximations like sampling needed)
- Direct application of Bayes theorem often **computationally intractable**

DID THE SUN JUST EXPLODE?  
(IT'S NIGHT, SO WE'RE NOT SURE.)



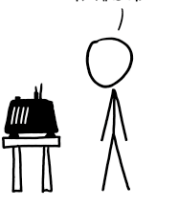
FREQUENTIST STATISTICIAN:

THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS  $\frac{1}{36} = 0.027$ . SINCE  $p < 0.05$ , I CONCLUDE THAT THE SUN HAS EXPLODED.



BAYESIAN STATISTICIAN:

BET YOU \$50 IT HASN'T.



# The Bayes optimal classifier

## Classification's «gold standard»

Theoretically **optimal** (=most probable) classification

- **Combine predictions** of all hypotheses, **weighted by** their **posterior** probabilities:

$$\operatorname{argmax}_{y_j \in Y} \sum_{h_i \in \mathcal{H}} p(y_j | h_i, X) p(h_i | X)$$

(where  $y_j$  is a label from the set  $Y$  of classes,  $h_i$  is a specific hypothesis out of the hypothesis space  $\mathcal{H}$ , and  $p(h_i | X)$  is the posterior of  $h_i$  given the data  $X$ )

- No other method using the same  $\mathcal{H}$  and  $X$  can do better *on average*

The **maximum a posteriori (MAP)** hypothesis is the one with the largest  $p(h|X)$

Pro

- In particular **outperforms** simply taking the classification of the **MAP hypothesis**  
Example: Let 3 classifiers predict tomorrow's weather as  $h_1(x) = \text{sunny}$ ,  $h_2(\text{rainy})$ ,  $h_3(\text{rainy})$  with posterior probabilities of .5, .4 and .1, respectively; let the true weather tomorrow be *rainy*. The MAP hypothesis  $h_1$  wrongly predicts *sunny* weather; the Bayes classifier truly predicts *rainy*.
- Enforces the idea of **ensemble learning**

Con

- Computationally **intractable** (linear in  $|\mathcal{H}| \rightarrow$  see <http://www.cs.cmu.edu/~tom/mlbook/NBayesLogReg.pdf>)

# The EM algorithm

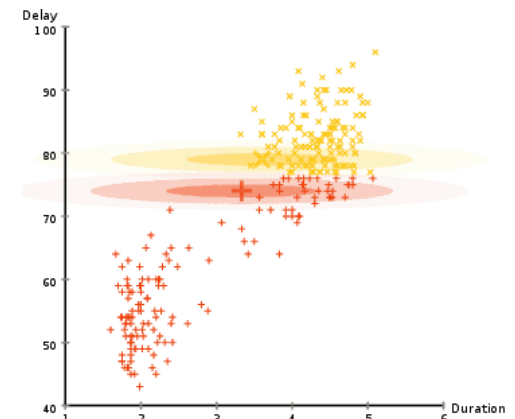
A general-purpose, *unsupervised* learning algorithm

**EM** (expectation maximization)

- Iterative method to **learn in the presence of unobserved variables**
  - A **typical hidden variable** is some sort of **group/cluster membership**
- Good convergence guarantees (finds local maximum)

## Example

- A given dataset is known to be generated by either of 2 Gaussians (with equal probability)
- Only the data is observed
  - **Which Gaussian generated a certain point is unobserved**
  - The Gaussians' **parameters** are unknown
- The means & variances of these Gaussians shall be learned
  - Needs an estimation of the membership probability of each point to either Gaussian



EM algorithm used to iteratively optimize the parameters of 2 Gaussians (animated)  
Source: [https://en.wikipedia.org/wiki/Expectation%E2%80%93maximization\\_algorithm](https://en.wikipedia.org/wiki/Expectation%E2%80%93maximization_algorithm)

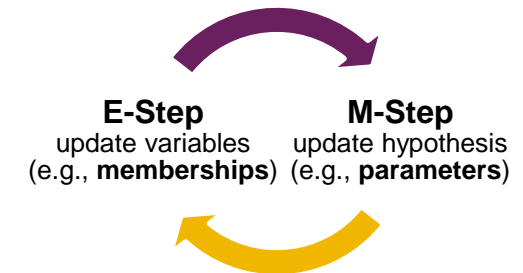


# The EM algorithm (contd.)

## Algorithm

1. Start with a random initial hypothesis

Example: **Pretend to know the parameters**  $\mu, \sigma^2$  of the 2 Gaussians  
(e.g., pick random values)



2. **E-Step**: Estimate **expected values** of unobserved variables,  
*assuming the current hypothesis holds*

Example: **Compute probabilities**  $p_{ti}$  that feature vector  $x_t$  was produced by Gaussian  $i$

(i.e.,  $p_{ti} = p(G = i | x_t) = \frac{p(x_t | G=i)p(G=i)}{p(x_t)} \approx p(x_t | G = i) = g_i(x_t, \mu_i, \sigma_i)$  with  $g_i$  being the Gaussian pdf and  $G$  the unobserved random variable indicating membership to one of the Gaussians)

3. **M-Step**: Calculate new **Maximum Likelihood (ML)** estimate of hypothesis,  
*assuming the expected values from (2) hold*

Example: **Calculate the**  $\mu_i, \sigma_i^2$ , given the currently assigned membership

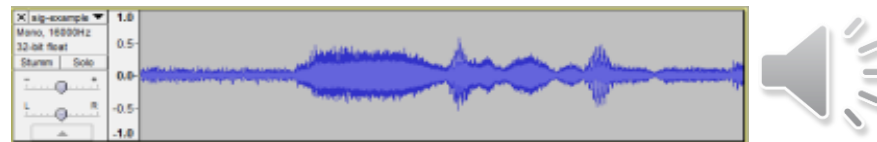
(i.e., using standard ML estimation:  $\mu_i = \frac{1}{T} \sum_{t=1}^T p_{ti} \cdot x_t$ ,  $\sigma_i^2 = \frac{1}{T} \sum_{t=1}^T p_{ti} \cdot (x_t - \mu_i)^2$ )

4. Repeat with step 2 until convergence  
Always replacing old estimates with new ones

## 2. EXAMPLE DOMAIN: SPEECH PROCESSING

# The audio signal

The waveform  $s[n]$  (a 1D array of  $N$  integer samples)

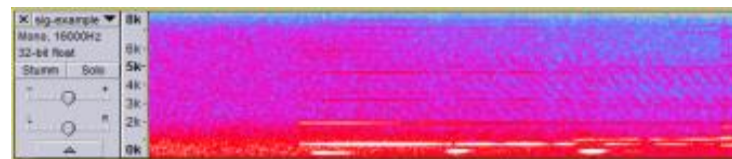


Time domain information (2D: time, amplitude):

- Energy ( $\sim$ loudness):  $NRG = \frac{1}{N} \sum_n s[n]^2$
- Zero crossing rate ( $\sim$ prominent frequency for monophonic signals):  $ZCR = \frac{1}{N} \sum_n I(s[n] \cdot s[n-1] < 0)$

Frequency domain information (3D: time, frequency, amplitude):

- Time frequency representations via **FFT** or **DWT** (phase information typically discarded)



More on signal processing: Smith, *"Digital Signal Processing - A Practical Guide for Engineers and Scientists"*, 2003

# Frame-based processing

## From signal to features

### Feature extraction in general

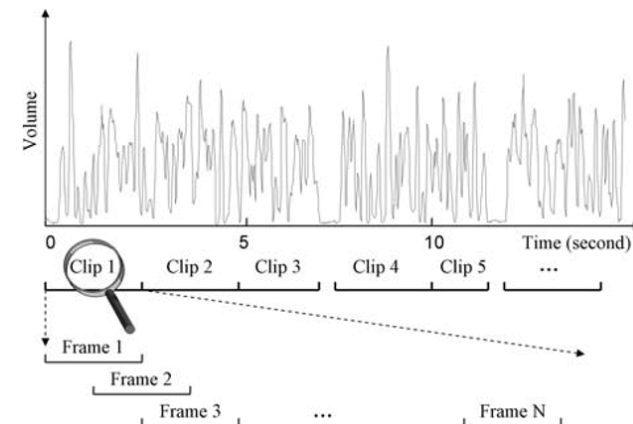
- **Reduction** in **overall** information
- ...**while** maintaining or even **emphasizing** the **useful** information

### Challenging audio signal properties

- Neither **stationary** (i.e., statistical figures change over time)  
→ **problem** with transformations like **Fourier transform** **when analyzed in whole**
- ...nor conveys its meaning in single samples  
→ **problem** when analyzing **per sample**

### Solution

- **Chop into** short, usually overlapping chunks called **frames**  
→ extract features per frame
- Prominent parameters: **32ms frame-size**, **16ms frame-step** (i.e., 50% overlap)  
→ Technically a `double` matrix  $f[T][D]$   
with  $T = 1 + \text{floor}\left(\frac{\text{ceil}(N - \text{frameSize})}{\text{frameStep}}\right)$  the frame count,  $D$  the feature dimensionality



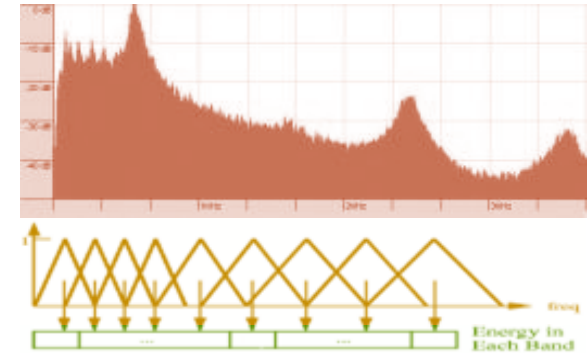
Source: <http://what-when-how.com/video-search-engines/audio-features-audio-processing-video-search-engines/>

# Mel Frequency Cepstral Coefficients (MFCC)

The predominantly used multi-purpose audio feature

## MFCC extraction process

1. **Pre-emphasize**:  $s[n] = s[n] - \alpha \cdot s[n - 1]$   
(**boost high** frequencies to improve **SNR**;  $\alpha$  close to 1, e.g. 0.97)
2. Compute magnitude spectrum:  $|FFT(s[n])|$   
(i.e., **time-frequency decomposition** neglecting phase)
3. Accumulate under triangular **Mel-scaled filter bank**  
(resembles **human ear**)
4. Take **DCT** of filter bank output, discard all coefficients  $> M$   
(i.e., low-pass  $\rightarrow$  **compression**; typically  $M \in [8..24]$ )



Source: <http://developer.nokia.com> &  
<http://phys.unsw.edu.au/~jw>

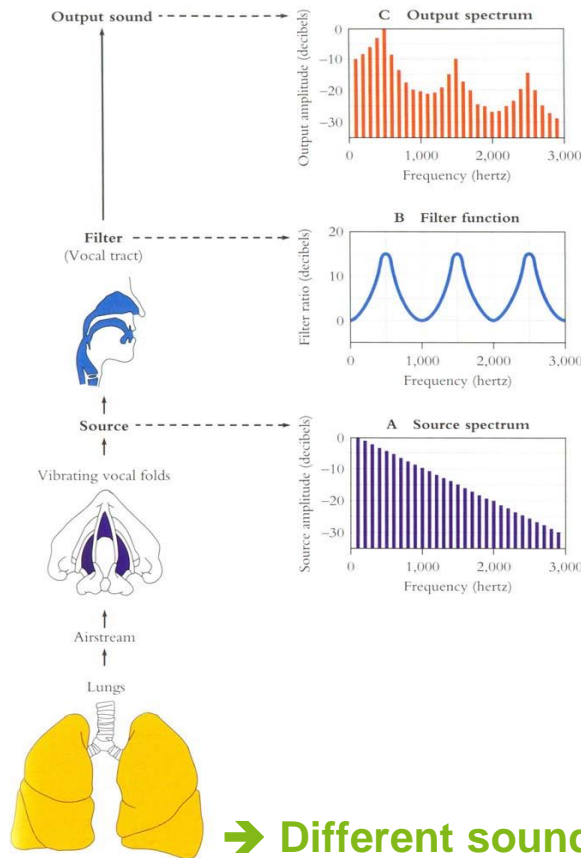
## Content and meaning of MFCCs

- Low-pass filtered spectrum of a spectrum: “**Cepstrum**”
- Intuitively: A **compact representation** of a frame’s **smoothed spectral shape**  
 $\rightarrow$  Convey **most** of the **useful information** in a **speech** or **music** signal, but **no pitch** information

A play with the word “spectrum” and the involved math. operation of **convolution**

Pitch: The perceived tone height (i.e., the tone you would whistle, the melody)

# The source filter model of speech production



## Source

- Air flows from the lungs through the **vocal chords**
- Produces noise-like (**unvoiced**) or periodic (overtone-rich, **voiced**) excitation signal

## Filter

- **Vocal tract** shapes the emitted spectrum

## Important physiological parameters

- Size of the **glottis** determines **fundamental frequency (F0)** range
- Shape of the vocal tract and nasal cavity determines **formant frequencies (F1-5)**, thus "sound"

➔ **Different sounds are produced by changing the source/filter configuration**

The vocal tract; source: DUKE Magazine, Vol. 94, No. 3, 05/06 2008

Source-filter interaction; source: [http://www.spectrum.uni-bielefeld.de/~thies/HTHS\\_WiSe2005-06/session\\_05.html](http://www.spectrum.uni-bielefeld.de/~thies/HTHS_WiSe2005-06/session_05.html)

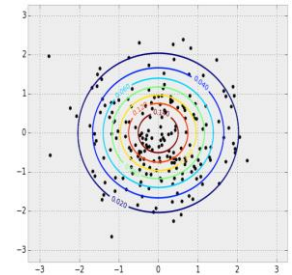
### 3. GAUSSIAN MIXTURE MODELS

# Probabilistic mixture models

## Generative models for unknown, multivariate distributions

### Mixture Models

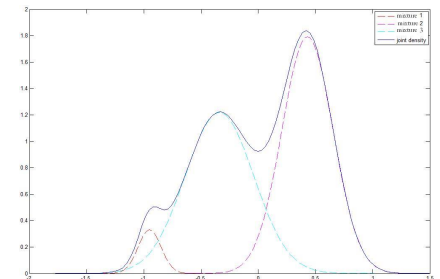
- **Approximate** an **arbitrary** distribution by a **linear combination** of a **simpler**, “well-behaved” distribution
  - Mathematically **tractable**, **compact** formulation, allows **sampling & inference**



Example of a **multivariate** (2D) Gaussian distribution: samples and contour plot.

### The **Gaussian Mixture Model** (GMM)

- **Modeled by** a weighted sum of  $N$  **multivariate Gaussians** ( $N$  being sufficiently large)
- Often used because of “**nice**” mathematical **properties** of Gaussian pdf and **central limit theorem** (~ data from natural phenomena tend to be Gaussian distributed)
- The Gaussians’ **parameters** can be estimated efficiently **using** the **EM** algorithm



Example of a **multimodal** (but univariate) distribution, approximated by a GMM with 3 mixtures.



# GMMs as generative models for voice modeling

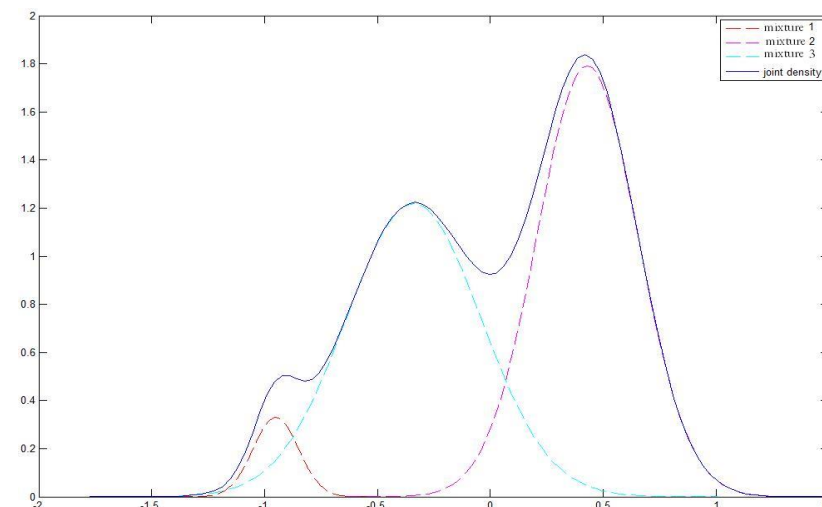
## Reference

- Reynolds, Rose, «*Robust Text-Independent Speaker Identification Using Gaussian Mixture Speaker Models*», 1995



## Key ideas

- Take** the estimated probability density function (**pdf**)  $p(x|h)$  of a speaker's  $D$ -dim. training vectors  $x$  **as a model of his voice**
- Model the **pdf as a weighted sum of  $M$   $D$ -dimensional Gaussians** (e.g.,  $M = 32$ ,  $D = 16$ )



GMM with 3 mixtures in 1 dimension. Solid line shows **GMM density**, dashed lines show **constituting Gaussian densities**.

# GMM rationale

**Hybrid solution** between non-parametric clusters  
(**vector quantization**) and compact smoothing  
(single Gaussian):

- **Smooth approximation** of arbitrary densities
- **Implicit clustering** into broad phonetic classes

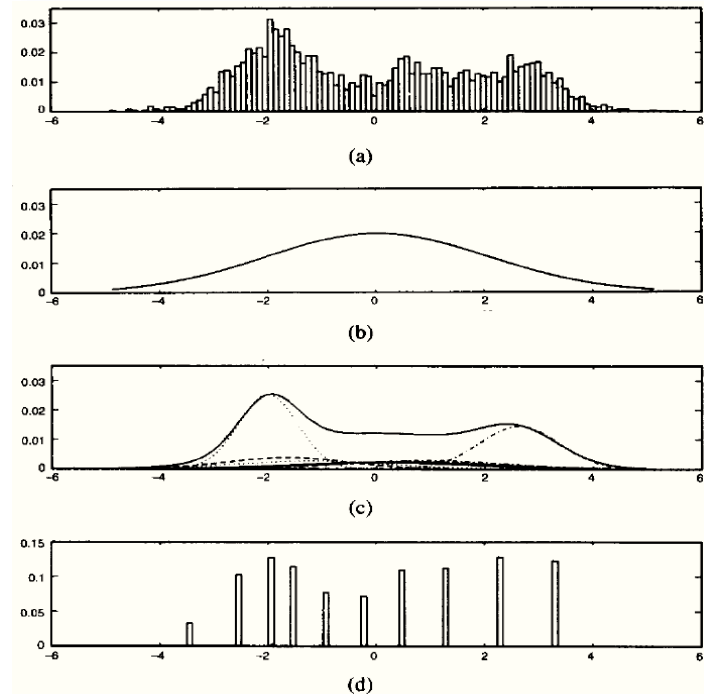


Fig. 3. Comparison of distribution modeling: (a) Histogram of a single cepstral coefficient from a 25 second utterance by a male speaker; (b) maximum likelihood unimodal Gaussian model; (c) GMM and its 10 underlying component densities; (d) histogram of the data assigned to the VQ centroid locations of a 10-element codebook.

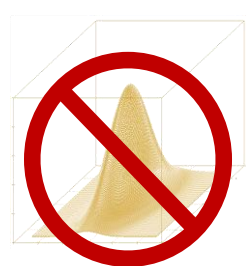
GMM comparison with other techniques; from [Reynolds and Rose, 1995].

# Mathematical formulation of the GMM

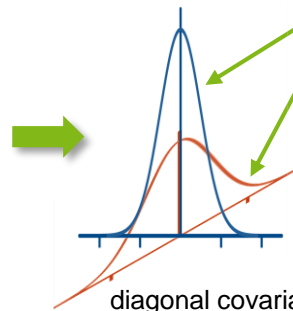
## Using diagonal covariance (→ see appendix for reasons)

### Notation

- $h$ : **model** (GMM)
- $w$ : **weight** (scalar)
- $\mu$ : **mean** vector
- $\sigma^2$ : **variance** vector (the diagonal of the covariance matrix)
- $g_i$ : **Gaussian pdf** of  $i^{\text{th}}$  (out of  $M$ ) mixtures
- $x$ : **feature vector**
- $D$ : **dimensionality** of  $x, \mu, \sigma^2$
- $p$ : **density/likelihood** of a feature vector given the model



full covariance



diagonal covariance

Just the product over the assumedly independent marginals (dimensions)

The univariate Gaussian pdf

### Formulae

- **Model** consists of:  $h = \{w_i, \mu_i, \sigma_i^2\}$   
→ subject to  $i = 1..M$  and  $\sum_{i=1}^M w_i = 1$

Condition on weights to sum up to 1

- The **multimodal Gaussian with diagonal covariance** computes as

$$g_i(x, \mu_i, \sigma_i^2) = \prod_{d=1}^D \frac{1}{\sqrt{2\pi\sigma_{id}^2}} \cdot e^{-\frac{(x-\mu_{id})^2}{2\sigma_{id}^2}}$$

- **Model evaluation:**

$$p(x|h) = \sum_{i=1}^M w_i \cdot g_i(x, \mu_i, \Sigma_i)$$

# GMM training via the EM algorithm

## Maximum likelihood training

- Initialize model  $h = \{w_i, \mu_i, \sigma_i^2\}$  using data  $X = \{x_1 \dots x_T\}$   
 → Instead of pure random initialization, find good start values via clustering (e.g., with *k-means*)
- E-Step:**

→ see appendix

$$p_{ti}(i|x_t, h) = \frac{w_i \cdot g_i(x_t, \mu_i, I_D \cdot \sigma_i^2)}{\sum_{i=1}^M w_i \cdot g_i(x_t, \mu_i, I_D \cdot \sigma_i^2)}$$

The (properly normalized) probability of  $x_t$  being issued by mixture  $i$

- M-Step:**

$$w_i = \frac{1}{T} \sum_{t=1}^T p_{ti}(i|x_t, h)$$

$$\mu_i = \frac{1}{T \cdot w_i} \sum_{t=1}^T p_{ti}(i|x_t, h) \cdot x_t$$

$$\sigma_i^2 = \left( \frac{1}{T \cdot w_i} \sum_{t=1}^T p_{ti}(i|x_t, h) \cdot x_t^2 \right) - \mu_i^2$$

Mixture  $i$ 's weight is just the mean probability of all training vectors being assigned to it

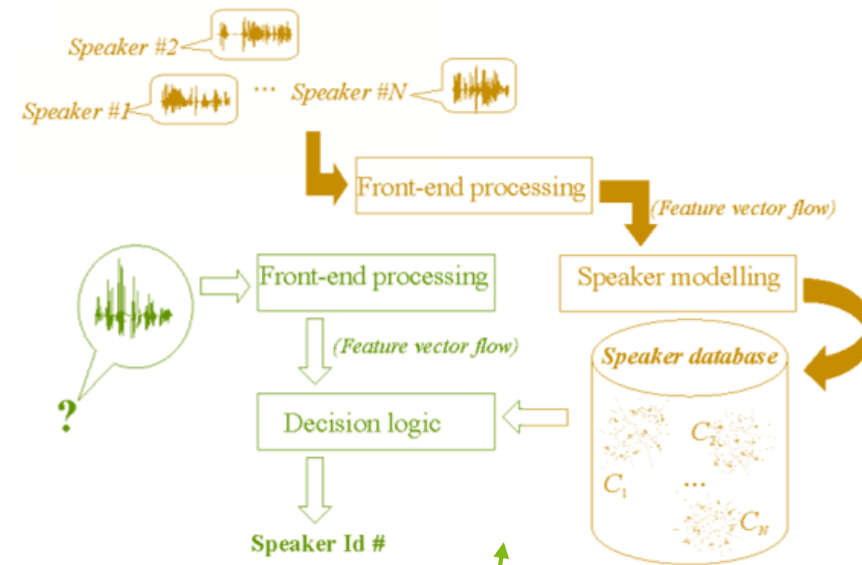
Alternative: Training via maximum a posteriori (MAP) adaptation (i.e. uses a priori knowledge)

→ see Reynolds, Quatieri, Dunn, «*Speaker Verification Using Adapted Gaussian Mixture Models*», 2000

# The task of speaker recognition

## Speaker recognition

- **Tell identity** of an **utterances'** speaker
- Typical: score feature-sequence against a speaker model



## Three subsequently more complex settings

- **Verification**: Verify that a **given utterance fits** a **claimed identity** (model) or not
- **Identification**: **Find** the actual **speaker among** a list of **prebuild models** (or declare as unknown: **open set** identification)
- **Diarization** (a.k.a. tracking, **clustering**): **Segment** an audio-stream by **voice identity** (who spoke when, no prior knowledge of any kind)

# Doing speaker identification

Finding the speaker  $s$  of a new utterance, given a set of trained speaker models

- Utterance represented by its feature vector sequence  $X = \{x_1..x_T\}$
- Speakers models given by  $\{h_1..h_s\}$

$$\begin{aligned} s &= \arg \max_s p(X|h_s) \\ &= \arg \max_s \prod_{t=1}^T p(x_t|h_s) \\ &= \arg \max_s \sum_{t=1}^T \log p(x_t|h_s) \end{aligned}$$

The prob. of a set of feature vectors is the product of the individual probs  
(**independence assumed**)

Using the **log** turns the product into a sum → makes the computation **numerically stable**

Model comparison via **generalized likelihood ration (GLR)**

- Absolute likelihood values are not meaningful, but their ratios are
  - To decide if given models  $h_1, h_2$  trained on utterances  $X_1, X_2$  are actually of the same speaker, threshold GLR **distance measure**:

$$GLR(h_1, h_2) = \log \left( \frac{p(X_1|h_1) \cdot p(X_2|h_2)}{p(X_1 \cup X_2|h_{1 \cup 2})} \right)$$

# What GMMs do not capture

## Re-synthesizing speech from intermediate stages of the speaker modeling pipeline

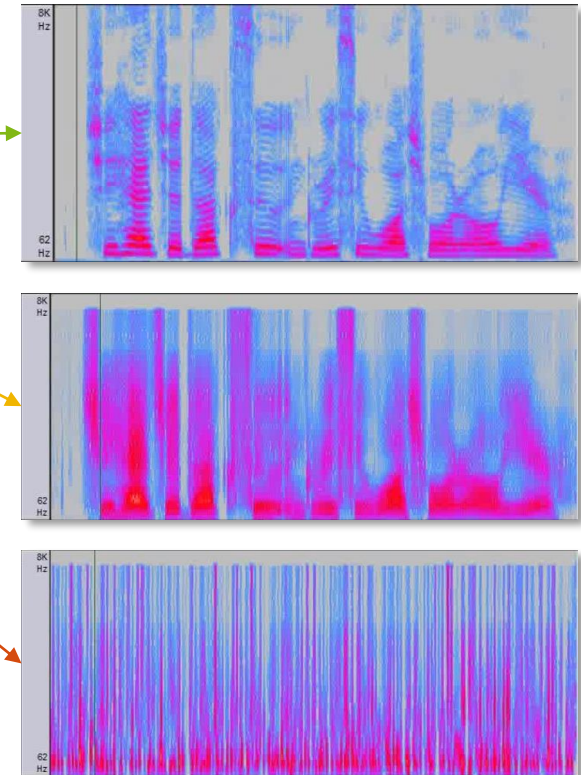
- Original utterance
- Resynthesized feature vectors (MFCCs)
- Resynthesized MFCCs from GMM

### Implication

- **Temporal context isn't modeled** by GMMs

### More on temporal context modeling:

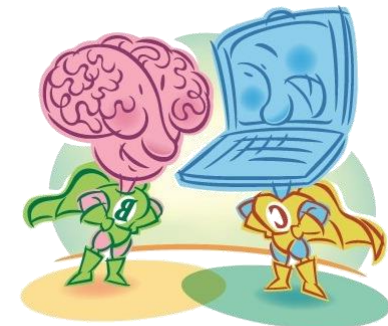
- Friedland, Vinyals, Huang, Müller, «*Prosodic and other Long-Term Features for Speaker Diarization*», 2009
- Stadelmann, Freisleben, «*Unfolding Speaker Clustering Potential – A Biomimetic Approach*», 2009
- **Lukic, Vogt, Dürr, Stadelmann, «*Speaker Identification and Clustering using Convolutional Neural Networks*», 2016**



# Where's the intelligence?

## Man vs. machine

- Using **probability** theory and statistics to make an agent work in a world of uncertain events **is a very good idea**
  - ➔ But: As we already saw with logic, full implementation **without heuristics** is computationally **intractable**
- Particularly in speech processing, **simplifying assumptions** like independence among subsequent feature vectors are **utterly unrealistic**
  - ➔ Results of respective systems are clearly worse than human performance
  - ➔ But: We have been able to **work around** this using **deep feature learning** [Lukic et al., 2016]





# Review

- Understanding uncertain events as random variables gives us a potent arsenal of tools for modeling: E.g., **probability density function** (pdf) of a random variable **tells us everything** there is to know about this function
- Thus, **estimating** the **pdf** is a rewarding **target for** (unsupervised) **learning**
- **Bayes' theorem** is used to turn priors (i.e., prior knowledge) into posteriors (i.e., taking all evidence & priors into account)
- **Speaker recognition** comes in the flavors of **verification**, **identification** or **diarization**
- The classic approach is **MFCC** features and **GMM** models
- Optimal parameters are best found using **best practices** (→ see appendix)
- **EM** training **iterates between** estimating updates values of hidden variables (based on assumed parameters of the sought distribution – **E-step**), and updating these parameters (based on these new estimates – **M-step**)





## APPENDIX

# Other forms of Bayesian learning

## The Naïve Bayes classifier

### Basic idea

- The straightforward way of applying Bayes' theorem to yield a MAP hypothesis is intractable (too many conditional probability terms need to be estimated)
  - Simplification:** Assume **conditional independence** among features given target value
- $$h(x_i) = \operatorname{argmax}_{y_j \in Y} P(y_j | x_{i1}, x_{i2}, \dots, x_{iD}) = \operatorname{argmax}_{y_j \in Y} P(x_{i1}, x_{i2}, \dots, x_{iD} | y_j) \cdot P(y_j) = \operatorname{argmax}_{y_j \in Y} P(y_j) \cdot \prod_{d=1..D} P(x_{id} | y_j)$$
- Very successful in text classification (e.g., SPAM filtering, news classification)



### Example (from <https://alexn.org/blog/2012/02/09/howto-build-naive-bayes-classifier.html>)

- Imagine 74 emails: 30 are SPAM; 51 contain “*penis*” (of which 20 are SPAM); 25 contain “*Viagra*” (24 are SPAM)

- Bayes classifier:  $p(SPAM | penis, viagra) = \frac{p(penis | SPAM \cap viagra) \cdot p(viagra | SPAM) \cdot p(SPAM)}{p(penis | viagra) \cdot p(viagra)}$  = ...

$$p(A \cap B) = P(B|A) \cdot P(A)$$

→ **intractable** with more words because of **cond. prob. terms** also get numerically small

- Naïve Bayes classifier:  $p(SPAM | penis, viagra) = \frac{p(penis | SPAM) \cdot p(viagra | SPAM) \cdot p(SPAM)}{p(penis) \cdot p(viagra)} = \frac{\frac{20}{30} \cdot \frac{24}{30} \cdot \frac{30}{74}}{\frac{51}{74} \cdot \frac{25}{74}} = 0.928$

# Other forms of Bayesian learning (contd.)

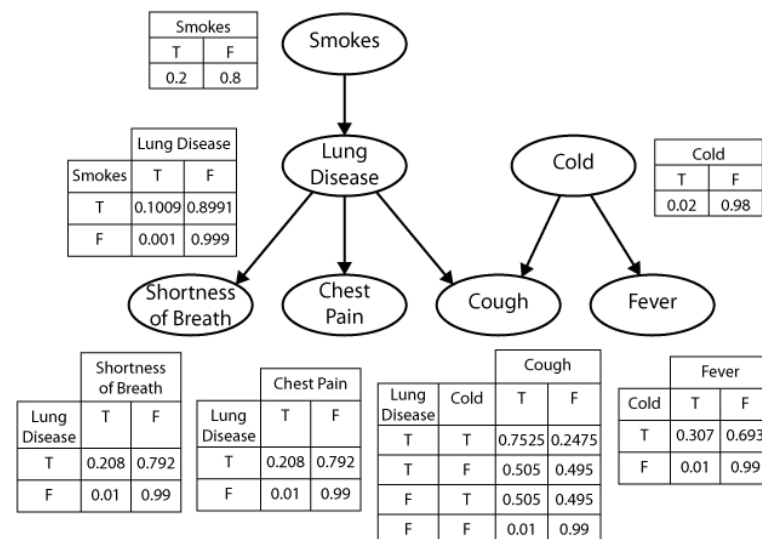
## The Bayes net (or Bayesian belief network)

In a nutshell

- **Loosens naïve Bayes constraint:** Assumes only conditional independence among certain sets of features
- Model of joint probability distribution of features (also unobserved ones):  
→ a **directed acyclic graph** for independence assumptions and local conditional probabilities
- Inference possible for any feature / target, based on any set of observed variables  
→ has to be done approximately to be tractable (NP-hard)
- **Use case:** conveniently **encode prior causal knowledge** in form of conditional (in)dependencies

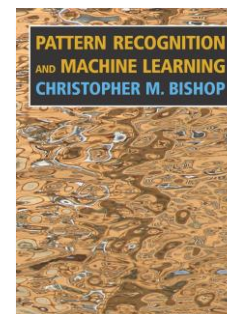
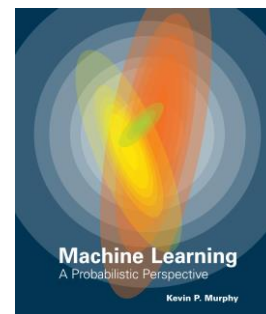
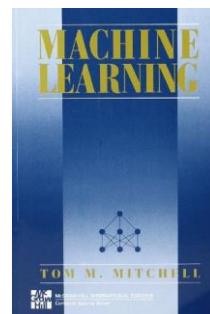
Example (from Goodman and Tenenbaum, “*Probabilistic Models of Cognition*”, <http://probmods.org>)

- A simple Bayes net for medical diagnosis
- One node per random variable  
→ Attached is a conditional probability table with the distribution of that node's values given its parents
- A Link between 2 nodes if there is a direct conditional (causal) dependence



# More on Bayesian learning

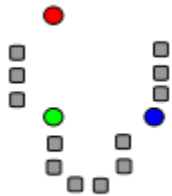
- <http://fastml.com/bayesian-machine-learning/>: Brief overview, explanations and references
- [Mitchell, 1997], ch. 6: Concise introduction to Bayesian learning
- <http://www.cs.cmu.edu/~tom/mlbook/NBayesLogReg.pdf>: New chapter for [Mitchell, 1997]
- [Murphy, 2012] and [Bishop, 2006]: Two text books embracing the Bayesian perspective
- Reynolds, Rose, «*Robust Text-Independent Speaker Identification using Gaussian Mixture Speaker Models*», 1995



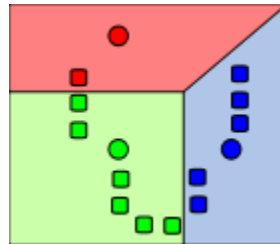
# $k$ -means clustering in a nutshell

Source: [https://en.wikipedia.org/wiki/K-means\\_clustering](https://en.wikipedia.org/wiki/K-means_clustering)

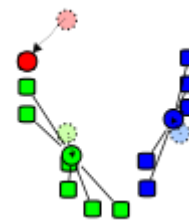
The standard algorithm: non-probabilistic EM



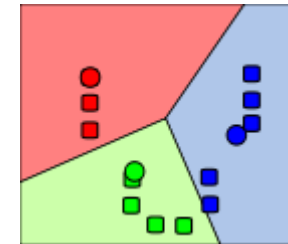
1.  $k$  initial "means" (in this case  $k = 3$ ) are randomly generated within the data domain (shown in color).



2.  $k$  clusters are created by associating every observation with the nearest mean. The partitions here represent the Voronoi diagram generated by the means.



3. The centroid of each of the  $k$  clusters becomes the new mean.



4. Steps 2 and 3 are repeated until convergence has been reached.

## Properties

- Problems: Very sensitive to choice of  $k$ ; even with correct  $k$  it may converge to wrong local minimum



- Variants:  $k$ -medoids (centroid to be member of data set),  $k$ -maxoids (for extremes rather than means)

# Glossary of abbreviations

## FFT – fast Fourier transform

- Standard algorithm to transform a time-domain (time-amplitude) signal into the frequency domain (frequency-amplitude)

## DWT – discrete wavelet transform

- Another transformation to the frequency domain, with higher resolution for higher frequencies

## DFT – discrete Fourier transform

- The theoretical basis for the FFT algorithm on an array of samples

## SNR – signal to noise ratio

- Amplitude of actual signal (what I want to hear) divided by amplitude of any noise (e.g., background music)

## DCT – discrete cosine transform

- As DWT, but decomposes the signal solely based on cosine terms (DFT: sine & cosine)

## Mel – from the word “melody”

- Unit to measure the pitch of a sound on a scale where an increase in Mel corresponds to the same increase in perceived pitch

## SVM – support vector machine

- An often very well-performing supervised machine learning method: give it data (in form of independent feature vectors) of two classes and it learns the discriminative boundary between them

# Glossary of abbreviations (contd.)

ATC – audio type classification

BIC – Bayesian information criterion

- Single-value measure to automatically trade-off model complexity and recognition performance

$\mu$  – mean vector

- As estimated on a set of vectors

$\Sigma$  – covariance matrix

- As estimated on a set of vectors;  $\mu$  and  $\Sigma$  together determine the multivariate Gaussian distribution

$\delta$  – delta coefficients vector

- First temporal derivative of some feature, e.g., a MFCC coefficient:  $\delta_{d_t} = MFCC_{d_t} - MFCC_{d_{t-1}}$

$\delta\delta$  – delta delta coefficients vector

- Second temporal derivative, i.e.  $\delta\delta_{d_t} = \delta_{d_t} - \delta_{d_{t-1}}$  for the  $d^{\text{th}}$  dimension and  $t^{\text{th}}$  time step

AANN – Auto-associative neural network (a.k.a. autoencoder)

- Supervised machine learning method that learns to reproduce its input on the output through a sort of bottleneck (e.g., compression) layer



# Glossary of abbreviations (contd.)

## LPC – linear predictive coding

- Representing a value of a time series as a linear combination of the last few samples

## dB – Dezibel

- Logarithmic unit to express the ratio of two physical quantities, e.g. power or intensity with reference to a “zero” level; a Dezibel is a tenth of a Bel. [after Wikipedia]

## mp3 – MPEG-1 or MPEG-2, Audio Layer III

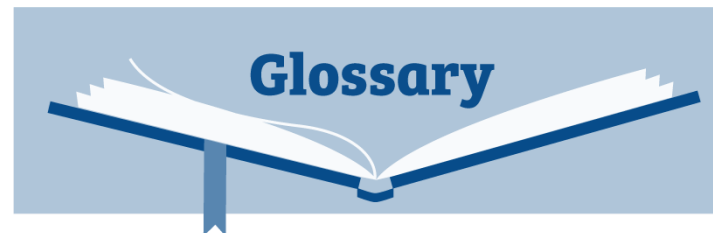
- Lossy audio compression relying heavily on results of psychoacoustics (e.g., masking effects): what can’t be heard doesn’t need to be coded

## ASR – automatic speech recognition

- Joint name for all technologies used to analyze and comprehend human speech with machines

## VQ – vector quantization

- A method to represent a set of vectors by a few «representative» vectors (called the «codebook»)



# Properties of audio signals

## Their content and segmentation

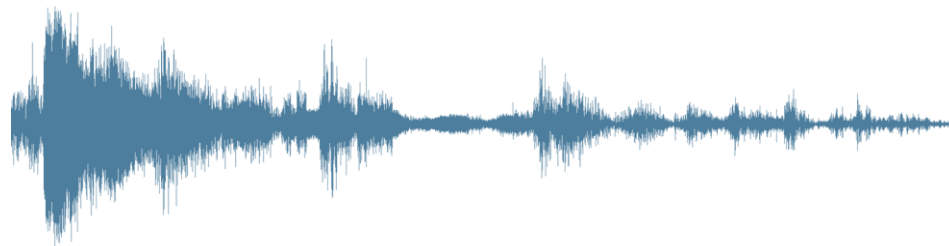
The sample array  $s[n]$  is just 1D

But: Sound still carries **information on** many **different layers** or "dimensions"

- Silence  $\Leftrightarrow$  non-silence
- Speech  $\Leftrightarrow$  music  $\Leftrightarrow$  noise
- Voiced speech  $\Leftrightarrow$  unvoiced speech
- Different musical genres, speakers, dialects, linguistic units, polyphony, emotions, . . .

### Definition of audio **segmentation**

- Temporally separate one or more of the above types from each other into consecutive segments by more or less specialized algorithms



# Properties of the speech signal

Slowly time-varying

- **stationary** over sufficiently **short period** (5-100ms, phoneme)

Speech range: 100 - 6800Hz (telephone: 300 - 3400Hz)

- 8kHz sample rate sufficient, 16kHz optimal

Speech frames convey **multiple information**:

- Linguistic (phonemes, syllables, words, sentences, phrases, ...)
- Identity
- Gender
- Dialect
- ...

➔ fractal structure



# Properties of the human auditory system

**High dynamic range** ( $120dB$ ,  $q_{dB} = 10 \cdot \log_{10}(q/q_{ref})$  for some quantity  $q$ )

- Work in the log domain (increase in  $3dB \rightarrow$  loudness doubled)

Performs **short-time spectral analysis** with log-frequency resolution

- Similar to wavelet-/Fourier-transform  $\rightarrow$  Mel filter bank

**Masking** effects

- That's what makes mp3 successful in compressing audio

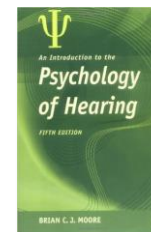
Channel decomposition via "**auditory object recognition**"

- That's what a machine can not do (except **Melodyne**, and nobody knows why)



...and lots of further interesting material

- But no direct/simple applicability to ASR at the moment



$\rightarrow$  More on the auditory system: Moore, "*An Introduction to the Psychology of Hearing*", 2004

# More speech features

## Directly from source-filter decomposition

Represent source characteristics via **pitch & noise**

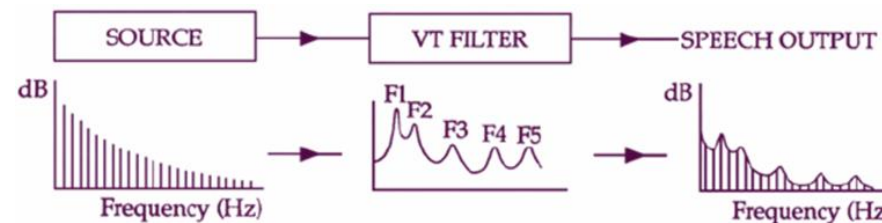
- 1 double per frame

Represent filter characteristics with filter coefficients  $a_k$  from **LPC analysis**

- 8 – 10 double per frame
- $s[n] = \sum_{k=1}^p a[k] \cdot s[n - k] + e[n]$  ( $e[n]$  being the **residual**)
- Btw.: This is the way it is done in mobile phones

LPC coefficients are also applied as speaker specific features

- Sometimes after further processing
- But **typically**, **MFCCs** are used



Source: Keller, "The Analysis of Voice Quality in Speech Processing", 2004

# GMM best practices

- Use **log-likelihoods** instead of likelihoods
  - Likelihoods become so small that one ends up with numerical instabilities otherwise
- Use a **diagonal covariance** matrix
  - Simpler/faster training, same/better results due to more compact model (with more mixtures)
- Use a **variance limit** and beware of **curse of dimensionality**
  - Prohibit artifacts through underestimation of components
- Use **16-32 mixtures** and a minimum of **30s of speech** (ML)
- Adapt only means from 512-1024 mixtures per gender (MAP)
  - Score only with top-scoring mixtures
- **Find optimal number of mixtures** for data via brute force and BIC
- **Compare** models via
  - **Score-wise** (more precise): [Generalized Likelihood Ratio](#) (GLR)
  - **Parameter-wise** (faster): [Earth Mover's Distance](#) (EMD) or this paper:  
Beigi, Maes, Sorensen, «*A distance measure between collections of distributions and its application to speaker recognition*», 1998