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UNIVERSALITY OF THE ISOSPECIAL RELATIVITY FOR THE INVARIANT DESCRIPTION OF ARBITRARY SPEEDS OF LIGHT

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Abstract

We review the recent experimental evidence on arbitrary values of the speed of light within physical media (interior dynamical problems); we recall that classical (operator) theories with speeds of light different than that in vacuum are noncanonically (nonunitarily) related to the special relativity, thus losing the original invariant character with consequential rather serious problems of physical consistency; we show that the isominkowskian geometry, the isopoincaré symmetry and the isospecial relativity provide a formulation of arbitrary speeds of light within physical media as invariant as that of the special relativity for the propagation of light in vacuum; we prove that the isospecial relativity is directly universal, in the sense of holding for all possible spacetime models with a symmetric metric directly in the frame of the observer; we outline the broader genotopic and hyperstructural formulations which are directly universal for single- and multiple-valued irreversible systems, respectively, with a generally nonsymmetric metric; we review the anti-isomorphic isodual formulations for the classical and operator representation of antimatter and the prediction of the new "isodual photon"; we finally point out a few representative connections between the invariant isotopic formulations and a rather large variety of generalized theories existing in the literature.

1. Local character of the speed of light.

One of the most majestic achievements of this century for mathematical beauty, axiomatic consistency and experimental verifications has been the *special relativity* (SR) [1].

Nevertheless, science is a discipline that will never admit "final theories". No matter how valid a given theory may appear at a given point in time, its generalization for broader physical conditions should be expected as inevitable.

Along the latter lines, the SR has been challenged during the recent decades on various grounds. That primarily addressed in this note is the experimental evidence of the *local* character of the speed of light c , i.e., its dependence on the local spacetime coordinates x , the local density μ , the frequency ω , and any other needed variable, according to the familiar law

$$c = c_o/n(x, \mu, \omega, \dots), \quad (1)$$

where n is the index of refraction, with the propagation in vacuum at the constant speed c_o admitted as a particular case for $n = 1$.

In fact, speeds $c = c_o/n < c_o, n > 1$ (also called *subluminal speeds*), are known to exist in our Newtonian environment since the discovery of the refraction of light in the transition from air to water. Lesser known is the fact that one of the first invariance studies of speeds $c < c_o$ was done by Lorentz [2a] (see the related mention in Pauli's book [2b]).

In short, incontrovertible experimental evidence establishes that electromagnetic waves have all locally different speeds when propagating in water, glass, oil, or other media.

Speeds $c = c_o/n > c_o, n < 1$ (also called *superluminal speeds*), have been recently measured by A. Enders and G. Nimtz [2c] in the tunneling of photons between certain guides. Speeds $c = c_o/n > c_o$, have also been measured for large masses in certain astrophysical explosions [2e-2g]. The literature on the experimental measurements of superluminal speeds of electromagnetic waves is nowadays rather vast and it is herein assumed as known (see also review [2d], the recent data [2h] and the comprehensive review [2i]).

The lack of exact validity of the SR for all these different local speeds of light $c < c_o$ and $c > c_o$ is then beyond scientific or otherwise credible doubt, the only scientific issue being the identification of a more adequate theory.

Note that the old hopes of regaining the exact validity of the SR by reducing light to photons scattering among molecules are no longer viable because: 1) the reduction, e.g., of a *classical* electromagnetic wave in our atmosphere with one meter wavelength to photons in *second quantization* cannot evidently eliminate the need for a classical representation prior to quantum forms acquiring credibility; 2) the reduction does not permit quantitative studies of superluminal speeds; and 3) the reduction eliminates the representation of the inhomogeneity and anisotropy of physical media, which have apparent, experimentally measurable effects (see below).

Numerous additional experimental data have been identified in recent times which provide further conditions beyond those of validity of the SR. It is important to see that they are all reducible or, equivalently, expressible via the arbitrary character of the speed of electromagnetic waves.

Recall that hadrons are not ideal spheres with isolated points in them, but are instead some of the densest media measured in laboratory until now. If spacetime anomalies have been experimentally detected for media of relatively low density (such as our atmosphere), the hypothesis that the SR can be *exactly* valid within hadrons has little scientific credibility. As one among the numerous arguments, the basic topology of the SR (the Zeeman topology of the Minkowskian spacetime) is purely local-differential, and, as such, it cannot be exactly valid under the notorious *nonlocal* conditions in the interior of hadrons caused by deep mutual penetration- overlapping of the *wavepackets* of the hadronic constituents (a feature totally independent from their *point-like charges*).

One of the first quantitative studies of the above setting was done by D. L. Blokhintsev [3a] in 1964, followed by L. B. Redei [3b], D. Y. Kim [3c] and others. It should be stressed that the exact validity of the SR for the *center-of-mass behavior* of a hadron, e.g., in a particle accelerator is beyond scientific doubt. Therefore, Refs. [3a-3c] solely studied the interior structural problem of hadrons and argued that a possibility for internal anomalies due to nonlocal and other effects to manifest themselves in the outside is given by *deviations from the conventional Minkowskian behavior of the meanlives of unstable hadrons with the speed v (or energy E)*.

The reduction of the latter deviations to arbitrary speeds of light is then consequential. In fact, the Minkowski metric was originally written by Minkowski [1d] $\eta = \text{Diag.}(1, 1, 1, -c_o^2)$. Therefore, *any deviation $\hat{\eta}$ from η necessarily implies a deviation from c_o* , as one can see by altering any component of the metric and then using Lorentz transforms.

Along these lines, R. M. Santilli [3d] submitted in 1982 the hypothesis that *contact-nonpotential interactions in general, and those in the interior of hadrons in particular, can accelerate ordinary (positive) masses at speed bigger than the speed of light in vacuum, much along the subsequent astrophysical measures [2e-2g]*. The above hypothesis implies that *photons travel inside the hyperdense hadrons at speeds bigger than that in vacuum*. V. de Sabbata and M. Gasperini [3e] conducted the first phenomenological verification within the context of the conventional gauge theories supporting the hypothesis of Ref. [3d], and actually reaching limit speeds up to $75c_o$ for superheavy hadrons.

The hypothesis of Ref. [3d] is also supported by the phenomenological calculations conducted by H. B. Nielsen and I. Picek [3f] via the spontaneous symmetry breaking in the Higgs sector of conventional gauge theories, which have resulted in the anomalous Minkowskian metrics (here written in the notation above)

$$\pi : \hat{\eta} = \text{Diag.}[(1 + 1.2 \times 10^{-3}), (1 + 1.2 \times 10^{-3}), (1 + 1.2 \times 10^{-3}), -c_o^2(1 - 3.79 \times 10^{-3})], \quad (2)$$

$$K : \hat{\eta} = \text{Diag.}[(1 - 2.0 \times 10^{-4}), (1 - 2.0 \times 10^{-4}), (1 - 2.0 \times 10^{-4}), -c_o^2(1 + 6.00 \times 10^{-4})]. \quad (3)$$

As one can see, calculations [3f] confirm speeds of photons $c = c_o/n > c_o, n < 1$ for the interior of kaons, precisely as conjectured in Ref. [3d]. Recall that: spacetime anomalies are expected to increase with the density; all hadrons have approximately the same size; and

hadrons have densities increasing with mass. Therefore, results similar to (3) are expected for all hadrons *heavier* than kaons, as supported by phenomenological studies [3e].

The first direct experimental measurements on the behavior of the meanlife of K_S^0 with energy, $\tau(E)$, were done by S. H. Aronson *et al.* [3g] at Fermilab and they suggested *deviations* from the Minkowskian spacetime in the energy range of 30 to 100 GeV. More recent elaborations [3j,3k] of these measurements have also confirmed superluminal speeds inside kaons. Subsequent direct measurements also for K_S^0 were done by S. H. Aronson *et al.* [3h] also at Fermilab, suggesting instead *no deviations* of $\tau(E)$ from the Minkowskian form in the *different* energy range of 100 to 400 GeV. Nevertheless, fits [3j,3k] of both data [3g,3h] have confirmed the superluminal character of the speed, despite the conventional character for the latter data.

More recently, a test of the decay law at short decay times was made by the OPAL group at LEP [3i]. In the latter experiment the ratio of number of events $Z^0 \rightarrow \tau^+\tau^-$ with deviations of τ from the conventional law to number of "normal" events was $(1.1 \pm 1.4 \pm 3.5)\%$.

In conclusion, the conceptual, epistemological, phenomenological, and experimental evidence on deviations from the SR within hyperdense media such as hadrons or the interior of stars is so overwhelming to render mandatory the study of a more adequate theory. In particular, all experimental evidence suggest a local value of the maximal causal speed arbitrarily bigger than that in vacuum.

We should stress that all the above anomalies have been and are hereon strictly and solely referred to *ordinary photons* (those with positive energy) and *not* to tachyons. In fact, tachyons are ruled out as possible candidates here, e.g., because the photons for experiments [2b,2c] are ordinary tardyons before and after the test, thus preventing the possibility that they acquire a tachyonic state when passing thru the guides. A similar situation occurs for all other tests here considered, which all suggest conventional particles moving at speeds greater than that of light in vacuum.

The experimental evidence considered herein therefore requires *a new definition of tachyons*, as the particles traveling within physical media at speeds bigger than the local maximal causal speed. This study is not considered here for brevity.

By no means experimental evidence [2,3] exhausts all physical conditions beyond the SR, and numerous others exist in the literature. Several of them are somewhat hidden in *ad hoc* parameters which, in reality, measure the *deviation* from the axioms of the SR. This is the case for the Bose-Einstein correlation where the fit of experimental data requires the introduction of the so-called "caoticity parameters" of unknown physical origin and motivation. A study of this case [4] has revealed that these experimental data *cannot* be represented via the strict use of the SR. Therefore, the "caoticity parameters" provide a *bona fide* quantitative representation of the *deviations* from the SR.

Deviations from the SR are also hidden in theories which reduce experimental data on the hadronic structure to *hypothetical particles* (namely, particles which cannot be directly

detected on an individual basis), such as the quarks, whose characteristics *cannot* be defined in our spacetime, and are solely definable in the mathematical unitary space. It is evident that, under the reduction of real experimental measurements in our spacetime to hypothetical particles outside our spacetime, the characteristics of the latter can always be adapted to verify the SR.

However, the lack of a possible, direct and independent verifications of said characteristics of undetectable quarks, combined with the serious problematic aspects of quark theories still basically open (such as the impossibility for a scientific definition of gravity for all matter made up of quarks [15c]) and other factors essentially prevent quark theories from reaching truly scientific conclusions.

At any rate, studies [5] have shown that Minkowskian anomalies: A) are fully admitted by quark theories; B) they merely imply a rescaling of certain characteristics; and C) rather than "destroying" unitary models, they resolve some of the open problems (such as confinement), besides permitting otherwise impossible advances (such as *convergent* perturbative expansions for *strong* interactions). These results then turn quark theories into one of the most compelling evidence of *deviations* from (rather than verification of) the SR in the interior of hadrons.

The above entire scientific scene can be represented via the following historical distinction, which was well known in the early part of this century and thereafter ignored:

- 1) *Exterior dynamical problems in vacuum*, in which the SR is here assumed as valid; and
- 2) *Interior dynamical problems within physical media*, in which the SR is "inapplicable (and not "violated" because not constructed for that scope).

Under the latter perspectives, there is no need for new experimental data to reach general physical conclusions. In fact, the conventional ten conservation laws for exterior problems imply the exact validity of the fundamental Poincaré symmetry, with consequential validity of the SR. The same features then imply the impossibility for the SR to be exactly valid for interior problems because it would imply, e.g., that electrons orbit in the core of a star with conserved angular momentum, and other nonscientific beliefs of the type of the perpetual motion.

Equivalently, exterior problems are solely characterized by "action-at-a-distance" interactions which, as such, are local-differential, thus implying the applicability of the mathematical foundations of the SR. Moreover, these interactions are entirely representable via a Lagrangian or a Hamiltonian, thus implying the applicability of the physical foundation of the theory.

By comparison, whether within our atmosphere or within a star, interior dynamical problems require the additional presence of *contact interactions* as experienced by a space-shuttle during re-entry in our atmosphere or, along much of the same lines, a protons moving in the core of a star. It is well known that contact interactions can only occur among

extended objects and have *zero range*, thus implying the lack of exact applicability, first, of the mathematical foundations of the SR and, then, of its physical structure.

In conclusion, the validity of the SR for exterior problems and its inapplicability for interior problems can be established beyond scientific doubt via the mere observation (and admission) of physical reality. Experiments are only needed to establish the *amount of deviations* from the SR for each given interior conditions. As a result, any expectation of the necessarily exact validity of the SR for interior dynamical problems is outside the boundaries of science, because the only scientifically open issue is the selection of a more adequate theory for interior dynamical problems.

It should be mentioned that, by no means, the above comments exhaust all criticisms of the SR. An additional serious criticism is that concerning the *ether* to be a "universal substratum", which is necessary not only to propagate electromagnetic waves, but also for the existence of elementary particles (such as the electron) which are known to be localized oscillation of the same medium. It is evident that a universal substratum requires the existence of a universal privileged reference frame which is strictly prohibited by the SR.

Arbitrary values of the speed of light combined to the need for a universal substratum and other insufficiencies, have stimulated the return to *Galilean* forms of relativities for *relativistic* conditions. For brevity, we here indicate Ref. [6] and papers quoted therein.

For completeness we should also mention that, contrary to the majestic axiomatic consistency of the SR, the *general relativity* GR [7] has remained afflicted throughout this century by serious problematic aspects at both classical and quantum levels, when already in its conventional formulation for exterior gravitational problems in vacuum. As a matter of fact, the unresolved basic problems are so numerous, to discourage even a partial outline in this note (one may consult the recent article [8a], Sect. 3 of paper [8b], and the literature quoted therein).

When passing to interior gravitational problems, all limitations of the SR carry over in their entirety to the GR, thus implying its lack of exact character for the conditions considered (because GR is locally Minkowskian). The inapplicability begins with the underlying Riemannian geometry because it is strictly local-differential and Lagrangian as compared to the generally nonlocal and nonlagrangian interior gravitational problems. The inapplicability then extends to numerous other aspects, from the limited velocity-dependence of the GR (which is insufficient to represent the notoriously arbitrary velocity-dependence of interior problems), to the field equations (because of the lack of full verification of the Freud identity, lack of torsion, and other aspects).

Finally, we should mention major insufficiencies of both the special and general relativities for the *classical* representation of *antiparticles* in a way truly compatible with their operator counterpart.

2. Problematic aspects of existing generalized theories.

The insufficiencies of contemporary relativities outlined in Sect. 1 have stimulated the construction of numerous generalized theories in an attempt to represent broader physical conditions. Unfortunately, most of them are afflicted by problems of physical consistency so serious to prevent their application to experiments.

The problems of physical consistency of broader theories have been studied in details in memoir [9]. Since they are still vastly unknown to the general physics audience, an outline appears recommendable here.

One of the reasons for the majestic axiomatic consistency of the SR is its *invariant structure*, that is, a formulation based on a line element which is invariant under the rotational, Lorentz and Poincaré symmetries. In turn, this invariance implies:

- 1) *The invariance of the fundamental units of space, time, energy, etc.* which is an evident pre-requisite for all valid measurements. In fact, the basic invariant of the theory, the unit of the Poincaré symmetry $I = \text{Diag.}([1, 1, 1], 1)$, represents in a dimensionless way the units $I = \text{Diag.}([1\text{cm}, 1\text{cm}, 1\text{cm}], 1\text{cm}/1\text{sec})$.
- 2) *The preservation in time of basic notions such as that of Hermiticity*, which then implies a consistent representation of observables.
- 3) *The uniqueness and invariance of the numerical predictions;*
- 4) *The verification of rigorous causality and probability laws;*
- 5) *the verification of basic axioms of unquestionable consistency*; and other known features all essential for physical consistency.

The main point of the problems of physical consistency under consideration here can be expressed via the following:

LEMMA 1: All classical (operator) theories with speed of light c different than that in vacuum c_o are noncanonically (nonunitarily) related to the special relativity

In fact, by their very conception, the theories here considered are based on a structural change of the Minkowski metric which can only be achieved via a noncanonical-nonunitary transform

$$\eta \rightarrow \hat{\eta} = U \times \eta \times U^\dagger, U \times U^\dagger \neq I, \quad (4)$$

where \times is the conventional associative product.

When formulated on conventional spaces over conventional fields, the above noncanonical-nonunitary structure of broader theories has then disastrous consequences for their physical consistency. In fact, it implies (see Ref. [9] for all details and original references):

- 1') *The lack of invariance of the basic units of space, time, energy, etc.*, as an evident consequence of the very notion of noncanonical-nonunitary transforms, with consequential lack of consistent applicability to real measurements.
- 2') *The lack of preservation in time of Hermiticity and other basic features*, with consequential absence of physically acceptable observables.

- 3') *The lack of uniqueness and invariance of the numerical predictions;*
- 4') *The general violation of causality and probability laws;*
- 5') *The clear violation of the basic axioms of Einstein's special relativity*, with consequential rather robust problems of identifying new covering axioms, proving their mathematical consistency and then establishing them experimentally; and other rather serious shortcomings.

In general, the above physical inconsistencies remain hidden in the existing literature for various reasons. For instance, a number of theories with arbitrary speeds of light are formulated via a seemingly conventional Minkowski space with metric $\eta = \text{Diag.}(1, 1, 1, 1)$ and speeds of light embedded in the fourth coordinate $x^4 = ct$. However, the lack of uniqueness of the value c implies consequential problematic aspects and ambiguities in the definition of time t .

Generalized theories which exhibit the above physical shortcomings because possessing a nonunitary time evolution or for other reasons (e.g., nonlinearity in the wavefunction which implies the collapse of Mackay impriimitivity theorem with consequential invalidity of the SR, or theories with nonassociative envelopes which 'violate Okubo "no quantization theorem"') are rather numerous. Among them we mention (see Ref. [9] for details): dissipative nuclear models with nonhermitean Hamiltonians [17]; statistical models with external collision terms [18]; nonlinear theories [19]; theories with nonassociative envelopes [20]; q-deformations [21]; k-deformations [22]; *-deformations [23]; deformed creation-annihilation algebras [24]; nonunitary statistics [25]; Lie-admissible interior dynamics [26]; noncanonical time theories [27]; supersymmetries [28]; Kac-Moody superalgebras [29] and others.

The reader is encouraged to inspect any of the above generalized theories, prove their lack of unitarity on conventional Hilbert spaces over conventional fields and verify the occurrence of physical shortcomings 1') to 5').

Other generalized theories appear as consistent as the conventional formulation of the SR. The problematic aspects emerge in their full light when identifying the necessarily noncanonical-nonunitary relation to the SR.

This is particularly the case when Darboux's transforms are used with the reduction of nonhamiltonian-noncanonical theories in the given coordinate system x of the observer to seemingly Hamiltonian-canonical theories in a mathematical frame x' . However, Darboux's transforms are not only strictly noncanonical, but above all highly nonlinear, thus implying the loss of the original inertial character of the reference frame, thus implying the evident loss of the SR. At any rate, there is the impossibility of conducting actual measurements in the mathematical frame x' . Under these conditions, the validity of the SR in the new mathematical reference frame x' has no physical value.

The problematic aspects of Darboux's and other transforms are the reason for the insistence in achieving first a "direct representation", i.e., a representation in the fixed reference frame of the experimenter, before the transformation theory may acquire a physical meaning.

It should be stressed to avoid possible misrepresentation that, by no means, *all* broader theories verify shortcomings 1') to 5'). As an illustration, we indicate the theories worked out by Ahluwalia [3a], Dvoeglazov [10b] and others which do have a fully consistent axiomatic structure which bypasses said shortcomings.

3. The novel iso-, geno- and hyper-mathematics and their isodual.

Studies conducted by the author during the past decades have indicated that the primary reason for the physical problematic aspects is the formulation of generalized theories via the *conventional mathematics of the SR*, namely, via conventional numbers and fields, conventional vector and metric spaces, conventional Lie algebras and groups, etc.

To our best knowledge, the occurrence admits no alternatives. On one side there is the need for noncanonical-nonunitary theories as a mandatory condition for novelty over existing theories. On the other side, when formulated via conventional mathematics, noncanonical-nonunitary theories have no physical meaning known to this author because of the unavoidability of the problematic aspects of Sect. 2.

The above occurrence left no other choice than the construction of a *new mathematics*, specifically conceived for the invariant formulation of a broader relativity representing arbitrary speeds of light via noncanonical-nonunitary structures.

After laborious trials and errors, this author proposed back in 1978 [10] the construction of a new mathematics with nonsingular well behaved, Hermitean, generalized, $n \times n$ units with an unrestricted functional dependence

$$\hat{I} = \hat{I}(x, \mu, \omega, \dots) = 1/\hat{T}, \quad (5)$$

and then the reformulation of number and fields, vector and metric spaces, Lie algebras and groups, etc. in such a way to admit \hat{I} , rather than I , as the correct left and right new unit.

This first requires the lifting of the conventional associative product $A \times B$ among generic quantities A, B into the form $A \hat{\times} B = A \times \hat{T} \times B$ (namely, the lifting of $A \times B$ by an amount that is the *inverse of \hat{I}*), under which \hat{I} is indeed the correct left and right new unit called the *isounit*, while \hat{T} is called the *isotopic element*. For consistency, the *totality* of the of the conventional mathematics must then be reconstructed, with no exception known to this author.

One reaches in this way the *isounumbers* $\hat{n} = n \times \hat{I}$ with *isoproduct* $\hat{n} \hat{\times} \hat{m} = (n \times m) \times \hat{I}$ and related *isofields* $\hat{F}(\hat{n}, +, \hat{\times})$; *isospaces* defined over \hat{F} with *isometric* $\hat{\eta} = \hat{T} \times \eta$; *isolie algebras and groups*; etc.

Since \hat{I} preserves all the topological properties of I (nonsingularity, positive-definiteness, etc.), the generalized mathematics resulted to be "axiom-preserving" and was therefore proposed under the name of *isomathematics* [10].

Regrettably, we are not in a position to review the isomathematics for brevity. A technical understanding of this paper will however require at least a rudimentary knowledge of Refs. [11].

For the particular case of theories with arbitrary speeds of light c , the generalized unit is given by $\hat{I} = \text{Diag.}(1, 1, 1, c^2)$. Since the unit is the basic invariant of any theory, the construction of a theory with new unit \hat{I} therefore *guarantees* the invariance for arbitrary speeds $c = c_o/n(x, \mu, \omega, \dots)$.

It is however evident that isomathematics permits the invariant formulation of theories dramatically broader than those with arbitrary speeds of light, evidently in view of the arbitrariness of the functional dependence of the generalized unit.

A yet broader mathematics is the *genomathematics* [11], which is characterized by *non-hermitean* generalized units called *genounits*. In this case we have *two* genounits, $I^> = 1/T^>$ and $<I = 1/<T = (I^>)^{\dagger}$, with two corresponding products, one ordered to the right, $A > B = A \times T^> \times B$, and one ordered to the left, $A < B = A \times <T \times B$. Genomathematics is then characterized by two independent liftings of the conventional mathematics, one for each of the two ordered products. This includes *two* classes of genonumbers and genofields, genovector and genohilbert spaces, genolie algebras and groups, etc.

A still broader mathematics is the *hypermathematics* [11], which is characterized by non-hermitean *multivalued* generalized units, $I^> = (I_1^>, I_2^>, I_3^>, \dots)$ and $<I = (<I_1, <I_2, <I_3, \dots) = (I^>)^{\dagger}$, with two corresponding multivalued hyperproducts $A > B = A \times T_1^> \times B + A \times I_2^> \times B + A \times T_3^> \times B \dots$ and $A < B = A \times <T_1^> \times B + A \times <I_2 \times B + A \times <T_3 \times B \dots$, and related hypermathematics.

Once the conventional value of the basic unit +1 dating back to biblical times is abandoned in favor of an arbitrary quantity, additional possibilities emerge besides iso-, geno- and hyper-mathematics. The additional possibility significant for this note is the map from positive into *negative* generalized units or, more generally, via the map called *isoduality* [12b,15]

$$\hat{I} = \hat{I}(x, v, \omega, \dots) \rightarrow \hat{I}^d = -\hat{I}^{\dagger} = -\hat{I}^{\dagger}(-x, -\mu, -\omega, \dots). \quad (6)$$

which characterizes yet novel mathematics called *isodual iso-, geno- and hyper-mathematics*. A particular case is the *isodual mathematics*, that is, the image of the *conventional* mathematics under isoduality with negative unit -1.

The need for all these generalized mathematics is the following. Since it is axiom-preserving, *isomathematics represents closed-isolated systems with Hamiltonian and non-hamiltonian internal effects whose center-of-mass trajectories are reversible in time*, such as the classical structure of Jupiter or the operator structure of a hadron with nonhamiltonian internal effects when the systems are considered as isolated from the rest of the universe. In fact, conventional potential interactions are represented with the Hamiltonian, while all non-hamiltonian effects are represented via the isounit. The reversible character is then ensured

by the Hermiticity of the isounit, $\hat{I} = \hat{I}^\dagger$, which guarantees the identity of the center-of-mass trajectories for motions forward and backward in time.

Despite their generalized structure, by no means the above systems exhaust physical reality. *Genomathematics is then particularly suited for the representation of the broader class of open-nonconservative systems with Hamiltonian and nonhamiltonian interactions whose center-of-mass trajectories are irreversible*, such as the classical representation of the space-shuttle during re-entry in our atmosphere or a proton moving in the core of a star. In fact, potential interactions are again represented with the Hamiltonian and all nonhamiltonian effects are represented by the genounits. An axiomatic representation of irreversibility is then guaranteed by the nonhermiticity of the genounits yielding two conjugate representations, one for the ordered motion forward in time with $I^>$ and $A > B$, and the other for the ordered motion backward in time with ${}^< I$ and $A < B$. The conjugation $I^> = ({}^< I)^\dagger$ then ensures the transition from motion forward to motion backward in time under time reversal.

Despite its further generalized character, by no means genomathematics can represent the entire physical reality because a number of structures, such as biological entities, and even the entire universe are expected to have multivalued structures. *Hypermathematics then permits a representation of multivalued open-nonconservative systems with Hamiltonian and nonhamiltonian internal effects and irreversible structures*, as inherent in the very notion of multivalued hyperunits and hyperproducts.

Despite its further generalized character, hypermathematics too cannot represent the entire universe. In fact, it is now known that conventional, iso-, geno- and hypermathematics cannot provide a consistent *classical* representation of antimatter which is compatible with the established operator counterpart [15].

Isodual maps are anti-isomorphic like the charge conjugation, although the latter solely applies at the level of *second quantization*, while the former apply at all levels *beginning* at the classical level and then continuing at the operator level where they are equivalent to charge conjugation. Moreover, isodual theories have *negative norm*, thus reversing the sign of *all* physical characteristics of matter, including charge. In view of these and other reasons, isodual theories have resulted to be particularly suited for a *classical* representation of *antimatter* in a way which is compatible with their operator counterpart [15].

As a result, conventional, iso-, geno- and hyper-mathematics are used for the representation of *matter* in conditions of increasing complexity, and, respectively for: point-like abstractions of particles in exterior conditions in vacuum; reversible systems of extended particles in interior dynamical conditions; irreversible systems of extended particles in interior conditions; and multivalued irreversible systems of extended particles in interior conditions.

The isoduals of the conventional, iso-, geno- and hypermathematics are used for the characterization of *antimatter* in corresponding conditions of increasing complexity.

For all details, we refer the reader to memoir [11b] and monograph [11f].

4. Isotopic, genotopic and hyperstructural liftings of the special relativity.

The achievement of an invariant formulation of theories with arbitrary speeds of light required the isotopic (that is, axiom-preserving) lifting of the entire formalism of the SR, including the Minkowski space, the Poincaré symmetry and the basic axioms of the SR, whose rudiments were first submitted in Refs. [12]. Operator treatments are available in Refs. [13]. Refs. [14] provides a partial list of directly related presentations among a literature that is rather vast at this writing. For brevity we can evidently review here only the main lines.

The fundamental isotopy for relativistic theories is the lifting of the unit of conventional 4-dimensional theories, $I = \text{diag.}(1, 1, 1, c_o^2)$ of the Minkowski space and of the Poincaré' symmetry, into a well behaved, nowhere singular, Hermitean and positive-definite 4-dimensional matrix $\hat{I} = 1/\hat{T}$ whose elements have an arbitrary dependence on the needed local quantities.

Since $\hat{I} = 1/\hat{T}$ can always be diagonalized, we shall hereon assume the form of the isotopic element $\hat{T} = \text{Diag.}(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33}, \hat{T}_{44}), \hat{T}_{\mu\mu} > 0$.

Let $M(x, \eta, R)$ be the Minkowski space with spacetime coordinates $x = \{x^\mu\} = \{r, x^4\}$, $x^4 = c_0 t$, and metric $\eta = \text{Diag.}(1, 1, 1, -c_o^{-2})$ over the reals $R = R(n, +, \times)$.

The original field $R = R(n, +, \times)$ is then lifted into the isofield $\hat{R} = \hat{R}(\hat{n}, +, \hat{\times})$ characterized by the above isounit \hat{I} for which all operations (multiplication, division roots, etc.) are isotopic. It is easy to see that \hat{R} is locally isomorphic to R by construction and, thus, the lifting $R \rightarrow \hat{R}$ is an isotopy. Despite its simplicity, the lifting is not trivial, e.g., because the notion of primes and other properties of number theory depend on the assumed unit [11a,11g].

Next, we need the lifting of the space M into the *isominkowskian space* (today also called the *Minkowski-Santilli isospace* [14]) $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with the same isounit \hat{I} of the underlying isofield, which was first proposed by Santilli in Ref. [12a], and which should not be confused with "deformed Minkowski spaces" (because the former is based on a new unit and related mathematics, while the latter is based on the conventional unit and conventional mathematics).

The isominkowski space is characterized by the *isocoordinates* $\hat{x} = x \times \hat{I}$ on \hat{R} , and the *isometric* $\hat{\eta} = \hat{T} \times \eta$ although, for consistency, the latter should be defined on \hat{R} , thus having the structure $\hat{N} = (\hat{N}_{\mu\nu}) = \hat{\eta} \times \hat{I} = (\hat{\eta}_{\mu\nu}) \times \hat{I}$. The conventional interval on M is then lifted into the *isointerval* on \hat{M} over \hat{R} [12a]

$$\begin{aligned} (\hat{x} - \hat{y})^2 &= (\hat{x} - \hat{y})^\mu \hat{\times} \hat{N}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu = [(x - y)^\mu \times \hat{\eta}_{\mu\nu} \times (x - y)^\nu] \times \hat{I} = \\ &= [(x^1 - y^1) \times \hat{T}_{11} \times (x^1 - y^1) + (x^2 - y^2) \times \hat{T}_{22} \times (x^2 - y^2) + \\ &\quad + (x^3 - y^3) \times \hat{T}_{33} \times (x^3 - y^3) - (x^4 - y^4) \times \hat{T}_{44} \times (x^4 - y^4)] \times \hat{I}. \end{aligned} \quad (7)$$

It easy to see that \hat{M} is locally isomorphic to M and the lifting $M \rightarrow \hat{M}$ is also an isotopy. In fact, the lifting of the unit $I \rightarrow \hat{I} = 1/\hat{T}$ is compensated by the *inverse* lifting

of the metric, $\eta \rightarrow \hat{\eta} = \hat{T} \times \eta$. Despite this axiom-preserving character, it is evident that the isominkowski space is *noncanonically-nonunitarily* related to the conventional space, as necessary for novelty. For a recent detailed study of the isominkowskian geometry we refer the reader to memoir [12g].

The *isopoincaré symmetry* first submitted for the first time by R. M. Santilli in Refs. [12] and it is today called the *Poincaré-Santilli isosymmetry* [15]. In particular, Ref. [12a] presented the isotopies of the Lorentz symmetry; Ref.s [12b] presented the isotopies of the rotational symmetry; Re.s [12c,12d] presented the isotopies of the SU(2)-spin symmetry; Ref. [12e] presented the isotopies of the Poincaré symmetry; Ref. [12f] presented the isotopies of the spinorial covering of the Poincaré symmetry; and Ref. [11g] provided a systematic study of the underlying geometry. For independent direct treatment of the Poincaré-Santilli isosymmetry we refer the reader to various studies by Kadeisvili, e.g., Ref. [11c].

The Poincaré-Santilli isosymmetry is the image of the conventional symmetry under isotopies or, equivalently, under the condition of admitting \hat{I} , rather than I , as the basic unit. As such, the isopoincaré symmetry leaves invariant by construction the generalized interval (7), thus leaving invariant by construction arbitrary speeds c for the particular case , $\hat{I} = \text{Diag.}(1, 1, 1, n^{-2}), c_o = 1$.

Due to the axiom-preserving character, the Poincaré-Santilli isosymmetry is locally isomorphic to the conventional symmetry. Yet, the two symmetries are noncanonically or nonunitarily related, as established by the different basic units. Thus the local isomorphic occurs only when each symmetry is treated with its own mathematics. Regrettably, we cannot review here for brevity the new isosymmetry.

Isounit $\hat{I} = \text{Diag.}(1, 1, 1, n^{-2}), c_o = 1$ is not sufficiently symmetrized in spacetime and must be further lifted into the expression $\hat{I} = \text{Diag.}(n_1^{-2}, n_2^{-2}, n_3^{-2}, n_4^{-2}), n = n_4$ which can be reached via the application of a conventional (or, more properly, an iso-) Lorentz transform, or just the diagonalization of an arbitrary isounit \hat{I} . In this case n_4 preserves its character as representing the local index of refraction, while the space n_k can represent space characteristics outside any hope of representation via the SR, such as the densities along the three Euclidean axis, extended, nonspherical and deformable shapes of the particles considered and other properties of the interior problem at hand.

The *isospecial relativity* (ISR) [12,13] is the image of the SR under the isotopies. As such, it is based on the isotopic image of the conventional axioms, here expressed for the simpler case $n_1 = n_2 = n_3 = n_s \neq n_4$ (see Vol. II of Refs. [13b] for a detailed presentation):

ISOPOSTULATE I: *The maximal causal speed for interior dynamical problems is given by*

$$\hat{V}_{Max} = c_o \times n_s/n_4. \quad (8)$$

ISOPOSTULATE II: *The addition of speeds for interior dynamical problems follows the*

isotopic law

$$\hat{v}_{Tot} = (u + v) / (1 + u_k \times n_k^{-2} \times v_k / c_o \times n_4^{-2} \times c_o). \quad (9)$$

ISOPOSTULATE III: The dilation of time and the space contraction for interior dynamical problems follow the isotopic laws

$$\hat{t}' = \hat{\gamma} \times t, \hat{L}' = \hat{\gamma}^{-1} \times L, \hat{\gamma} = (1 - \hat{\beta}^2)^{-1}, \hat{\beta} = v_k \times n_k^{-2} \times v_k / c_o \times n_4^{-2} \times c_o \quad (10)$$

ISOPOSTULATE IV: The isodoppler law for interior dynamical problems is given by the expression (for the simple case of null aberration)

$$\hat{\omega}' = \hat{\gamma}^{-1} \times \hat{\omega}. \quad (11)$$

ISOPOSTULATE V: The mass-energy equivalence for interior dynamical problems follows the isotopic law

$$\hat{E} = m \times c^2 = m \times c_o^2 / n_4^2. \quad (12)$$

One should note that in the isorelativity the speed of light *is not*, in general, the maximal causal speed, with the sole exception of motion in vacuum (where the two speeds trivially coincide). This is due to the need of avoiding inconsistencies of the SR for interior conditions such as those occurring for the Cerenkov light in which the assumption of the speed of light as the maximal causal speed in water would imply the violation of the principle of causality (because electrons would travel faster). If causality is somewhat salvaged by assuming that the speed of light *in vacuum* is the maximal causal speed *in water* we would have other inconsistencies, such as the violation of the relativistic law of addition of speeds. For these and other inconsistencies in assuming the speed of light as the maximal causal speed within physical media, we refer the reader for brevity to monographs [13b].

Note that, the SR and the ISR coincide at the abstract, realization-free level by conception and construction, as guaranteed by the axiom-preserving character of the isotopies. Therefore, the isotopies resolve problematic aspect 5') of Sect. 2, i.e., they avoid the robust problems of identifying new axioms and proving their axiomatic consistency. They merely identify *new realizations of the abstract Einsteinian axioms of the SR* which, of course, remained top be verified experimentally (see Sections 5 and 7). In particular, criticisms on the axiomatic structure of the ISR *are* criticism on the structure of the conventional SR.

As an incidental note, tachyons should be defined as particles traveling faster than the local maximal causal speed (8), rather than the local speed of light c . In fact, only the former exit from the causal isocone in isospace [12g]. Also, the assumption of tachyons as particles traveling faster than the local speed of light would imply that the electrons of the Cerenkov light are tachyons which is dramatically disproved by physical evidence (e.g., because they can be proved to be physical particles when exiting water).

The above isotopies of the SR evidently admit sequentially broader liftings under genotopies and hyperstructures which yield the *genospecial relativity* (GSR) and the *hyperspecial relativity* (HSR), respectively. They are essentially characterized by the relaxation, first, of the Hermitean and, then, of the single-valued character of the generalized units. In particular, these further liftings imply the relaxation of the totally symmetric character of line element (7) and then the relaxation of its single-valuedness on \hat{R} . These additional features permit the only invariant formulation of single-valued and multivalued irreversible systems known to this author (the communication of other invariant formulations of irreversibility by the interested reader would be appreciated). These broader relativities are considerably more involved on technical grounds and, therefore, they cannot be treated in this elementary note (for details, see monographs [13b] and the recent update [13c]).

The *isodual isominkowskian spaces, isodual isopoincaré symmetry and isodual isospecial relativity* [12,13,15] are given by the isodual images of the preceding formulations and are used for the representation of *antimatter in interior conditions*.

An important particular case is given by the *isodual Minkowski space, isodual Poincaré symmetry and isodual special relativity* for the characterization of *antimatter in vacuum*. These novel theories predict the existence of the *isodual photon* [15c,15e] which is indistinguishable from the ordinary photon for all interactions except gravitation. If confirmed experimentally, isodual photons would therefore permit in the future quantitative studies as to whether far away galaxies and quasars are made up of matter or of antimatter, a study which is strictly impossible with current theory due to the complete lack in contemporary physics of a *classical theory of antimatter* (let alone the availability of a consistent theory). For additional properties on the latter isodual theories we must also refer the reader to Refs. [12,13,15].

5. Direct universality of the new relativities.

It is important to see that *the isospecial relativity and its isodual are directly universal for all possible spacetime formulations of matter and antimatter possessing a symmetric metric, respectively*, where "universality" is referred to all possible spacetime theories of the indicated type, and "direct universality" is referred to the representation in the given frame of the observer (without any use of the transformation theory). This important property can be easily seen from the arbitrariness of the isounit or, equivalently, of the isometric and related isointerval [7] which includes *all* known symmetric spacetime intervals, including the Minkowski, Riemann, Finsler other intervals.

The reader should be aware that the above direct universality was reached only after proving that *the isogeometries underlying the ISR are directly universality for all possible (well behaved) nonhamiltonian systems* [11b]. This was necessary to avoid the need for a Darboux's transform and related physical problematic aspects mentioned earlier.

To illustrate the direct universality of the ISR, we first note that Galilean-type spacetime are admitted as a particular case of the isospecial relativity for isounits given by the tensorial product of a space and a time component, $\hat{I} = \hat{I}_{space} \times \hat{I}_{time}$, with corresponding, rather intriguing (and mostly unexplored until now) factorization of the isominkowskian geometry and of the Poincaré-Santilli isosymmetry.

As a second illustration, we note that all the several different anomalous time evolution laws of Refs. [3] have resulted to be particular cases of the isotopic law (10). In fact, as shown by Aringazin [11h], the former are obtained from the latter via different expansions in terms of different parameters and with different truncations [13b].

As a third illustration, it is easy to see that *the isominkowski metric admits as a particular case all infinitely possible conventional Riemannian metrics*, $\hat{\eta} = g(x)$. In fact, all Riemannian metrics are locally Minkowskian, thus admitting the decomposition $g(x) = \hat{T}(x) \times \eta$, where \hat{T} is a positive-definite 4×4 matrix, which is precisely the fundamental concept underlying the isominkowskian geometry.

Therefore, the ISR provides the first known formulation of exterior gravity via a Minkowskian-type geometry, with rather important consequences, such has: the geometric unification of the special and general relativities; the first formulation of gravity with a universal symmetry, the Poincaré-Santilli isosymmetry, locally isomorphic to the conventional Poincaré symmetry; the representation of horizon and singularities via the zeros of the isotopic unit and isounit; and other advances.

The isominkowskian formulation of gravity also permits the resolution of some of the controversies in gravitation that are still open following about one century of debates, such as: the compatibility of gravitational and relativistic conservation laws (which is now resolved via the visual inspection that the generators of the related two symmetries coincide); the lack of a consistent *relativistic* limit of gravitation (which is easily achieved via the limit $\hat{I} \rightarrow I$); and other resolutions [12g].

As a fourth illustration, we indicate that the arbitrary functional dependence of the isotopic element \hat{T} in the isometric also permits a direct representation of interior gravitational problems with the desired arbitrarily nonlinear dependence on the velocities, nonlocal-integral effects, and other features of interior problems outside realistic capabilities by the Riemannian geometry.

A fifth illustration of the capability of the ISR is that of having provided the only known axiomatically consistent grand unification of gravitation and electroweak interactions [16] without structural disparities as in other theories (e.g., electromagnetism defined on a *flat* spacetime, with gravitation defined on a *curved* manifold). This result was precisely due to the isominkowskian representation of gravity, that is, its formulation in a way axiomatically compatible with all other interactions.

All the preceding theories are based on a *diagonal spacetime metric*. An additional class of still broader spacetime theories is that characterized by *nondiagonal isometrics* $\hat{\eta}$ or,

equivalently, *nondiagonal isounits* \hat{I} which have far reaching physical implications, such as the first possibility of synthesizing neutrons from protons and electrons as occurring in stars at their first formation [12f].

The geno- and hyper-special relativities are directly universal for all infinitely possible *single- and multi-valued irreversible systems with nonsymmetric metrics*, respectively, as the reader is encouraged to verify.

6. Simple construction of the new relativities.

The need for new mathematics has been a major deterrent for the understanding of the new relativities, to such an extent that various authors have preferred noninvariant and physically inconsistent formulations, rather than entering into the study and use of new mathematics.

It is important to clarify that the use of the new mathematics can be completely eliminated, and the entire formalism of the new relativities can be constructed via truly elementary methods.

First, the ISR can be simply constructed via the systematic application of the following nonunitary transform

$$U \times U^\dagger = \hat{I}, \quad (13)$$

to the *totality* of the formalism of the SR.

In fact, transform (13) yields the isonumbers $U \times n \times U^\dagger = n \times \hat{I}$, the correct form of the isoproduct, as well as *all* conventional operations, special functions and transforms, including the correct structure and representation of the Lie-Santilli isotheory [13a].

Once the structure of the ISR has been achieved in this way, its invariance is easily proved via the reformulation of nonunitary transforms in the *isounitary form*

$$U = \hat{U} \times \hat{T}^{1/2}, U \times U = \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}. \quad (14)$$

The invariance of the ISR then follows. For instance, we have the *numerical invariance of the basic isounit and related local speed of light* $\hat{I} \rightarrow \hat{I}' = \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger = \hat{I}$; the *numerical invariance of the isotopic element in the isoproduct*; and all other invariances of the SR as the reader is encouraged to verify.

Moreover, the above simple method for the construction of the ISR is also particularly valuable to generalized existing models in vacuum into models for arbitrary speeds of lights. In fact, one can merely selects the nonunitary transform $U \times U^\dagger = \text{Diag.}(1, 1, 1, n_4^{-2})$.

The construction of the GSR is equally elementary, and requires the use, this time, of *two* nonunitary transforms

$$U \times U^\dagger \neq I, W \times W^\dagger \neq I, U \times W^\dagger = I^>, W \times U^\dagger = < I, \quad (15)$$

to the *totality* of the formalism of the SR.

In fact, transforms (15) yield the genonumbers $U \times n \times W^\dagger = n \times I^>$, the correct form of the genoproduct, as well as *all* conventional operations, special functions and transforms, including the correct structure and representation of the Lie-Santilli isotheory [13c]. Once the genotheory is reached in this way, its invariance is easily proved by rewriting the nonunitary transforms in their genounitary version.

The HSR can be easily constructed and proved to be invariant via the mere relaxation of the single-valued character of the genounits and its outline is here omitted for brevity [11f]).

7. Experimental verifications, applications to other theories and concluding remarks.

Another aspect of the new relativities outlined in this note that does not appear to have propagated at this writing in the physics community is the existence of numerous applications and experimental verifications among which we quote (for brevity, see memoir [13a] for details and all original references): the exact fit of the experimental data on behavior of the meanlives of unstable hadrons with speed (thanks to Isopostulate III); the exact fit of the experimental data on the Bose-Einstein correlation for proton-antiproton annihilation at high and low energies (thanks to various Isopostulate); the first exact representation of total nuclear magnetic moments; the exact representation of the large differences in cosmological redshifts between quasars and galaxies when physically related according to gamma spectroscopy (thanks to Isopostulate IV); the elimination of the need for a missing mass in the universe (thanks to Isopostulate V); and several other experimental verifications.

The iso-, geno- and hyper-special relativities also permit the invariant formulation of existing theories [17-29] by resolving their physical inconsistencies as outlined in Sect. 2 (see memoir [9] for details).

The new relativities and their isoduals are also directly related to a variety of broader theories existing in the literature, such as the theories by Ahluwalia [30a], Dvoeglazov [30b] and others.....

All in all, the above features of the new relativities are sufficient to warrant further studies.

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