

# Gauge theory of things alive and universal dynamics \*

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## Abstract

Positing complex adaptive systems made of agents with relations between them that can be composed, it follows that they can be described by gauge theories similar to elementary particle theory and general relativity. By definition, a universal dynamics is able to determine the time development of any such system without need for further specification. The possibilities are limited, but one of them - reproduction fork dynamics - describes DNA replication and is the basis of biological life on earth. It is a universal copy machine and a renormalization group fixed point. A universal equation of motion in continuous time is also presented.

All known interactions between fundamental particles are described by gauge theories, as we know, including Einsteins theory of gravity, general relativity. Here we argue that the scope of gauge theory is much wider, including

- physical systems
- biological organisms
- Organizations of human society
- spiritual edifices: Minsky's "society of mind" [1], languages, evolutionary algorithms, etc.

*Example:* Gauge theory of swimming of micro-organisms [2].

Gauge theory can describe *complex adaptive systems*, i.e. anything alive in the widest sense, especially *autopoietic systems* which "make themselves" in an approximately *autonomous* fashion [3]. All these systems consist of *agents and their relations*. Both evolve in time. Agents organize themselves into larger structures as a consequence of the relations between them which determine their interaction.

I will give the argument for gauge theory, present examples for various of its aspects, and add remarks on important ramifications.

An expanded discussion of the contents of the first two sections is found in [4].

## 1 Structure: What is a thing?

The state of a system at time  $t$  may be considered as a category  $K$ . In this way structure can be described.

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Object:  Arrow: 

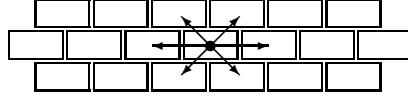


Figure 1: The structure of a brick wall determines a category

*Agents* become objects  $X$  of a category,

*Relations* become arrows  $f : X \mapsto Y$  of a category

The *basic postulates* of mathematical category theory are

1. Identity arrows  $\iota_X : X \mapsto X$  exist,
2. Arrows can be composed,

$$f : X \mapsto Y, g : Y \mapsto Z \text{ defines } g \circ f : X \mapsto Z.$$

Composition is associative;  $\iota_Y \circ f = f = f \circ \iota_X$ .

*Example:* *brick wall.* Its objects are the bricks and the fundamental arrows specify the translation of a brick to the position of a nearest neighbour. These arrows specify the structure of the wall. They can be composed to yield translations to other bricks' positions.

*Composability of relations* is our central postulate. This postulate *leads to gauge theory*.

*Examples:* friend of a friend, husband of a sister = brother-in-law.

In a  $*$ -category a relation  $f$  of  $X$  to  $Y$  defines a possible relation  $f^*$  of  $Y$  to  $X$  (adjoint arrow).

An arrow  $f$  is *invertible* if an arrow  $f^{-1}$  exists such that  $f \circ f^{-1} = \iota_Y$ ,  $f^{-1} \circ f = \iota_X$ .

*Functors*  $\mathcal{F} : K \mapsto K'$  are maps of categories which preserve identity and composition law.

We call  $K'$  a representation of  $K$  if  $\mathcal{F}$  is onto.

The following special features will be important.

**Locality:** Certain arrows are declared as fundamental, and called *links*, all others are composed from them. Dynamics will be such that an agent can be influenced by another one if they are related by a link.  $\iota_X$  is fundamental.

**Emergence:** Nonlocal phenomena should emerge from local relations as in physics.

*Example:* *Cognition.* The salient features of a picture (long smooth curves) can be detected by local interaction among low level cortical neurons which represent neighbouring picture elements [5].

From links  $b_i : X_{i-1} \mapsto X_i$  one composes

**Paths:**  $b_n \circ \dots \circ b_2 \circ b_1 = C : X_0 \mapsto X_n$ .

$K$  is *connected* if there exists a path connecting  $X$  to  $Y$  for every pair  $(X, Y)$ . Closed paths are called *loops*.

*Example:* *Lattice Gauge Theory* [6] furnishes a discretized version of the gauge theory of elementary particles. The *objects* are indexed by sites  $x$  of a lattice. They specify matter fields  $\Psi(x) \in \Omega_x$ , where  $\Omega_x$  are vector spaces of some dimension  $n$ . The *fundamental relations* are parallel transporters  $U[b] : \Omega_x \mapsto \Omega_y$  along links  $b = (y, x)$  of a lattice. They are unitary linear maps, possibly respecting additional invariant structure such as antisymmetric products  $v_1 \wedge \dots \wedge v_n$ .

**Representation theorem:** *Every finite category admits a faithful representation as a communication network as follows: There are spaces  $\Omega_X$  associated with objects  $X$  and arrows act as maps  $f : \Omega_X \mapsto \Omega_Y$ , with  $\iota_X = id$ .*

The construction of the space  $\Omega_X$  uses the sets of all arrows to and from  $X$ . Details are given in Appendix A. We talk of one time. The maps  $f$  represent channels, not acts, of communication. Time development is considered later.

The maps  $f$  need not be linear. *Apart from this, the setup is as in lattice gauge theory.* The lattice may be irregular, but irregular lattices were considered before, and standard constructs such as covariant Laplacians carry over to this case. Nonlinearity introduces features of neural nets [7]. There is a fundamental difference to neural net theory and to any cybernetic approach, though. Because of the gauge principle it is in general not appropriate to characterize an object at some time  $t$  by what is true about it.

The representation is not unique. Select for every  $X$  an invertible loop  $g_X$  and substitute  $g_Y \circ f \circ g_X^{-1}$  for  $f : X \mapsto Y$ . This produces another, equivalent representation. These transformations are called *gauge transformations*. They will be considered in the next section.

*Remark:* According to Heidegger [8], the question "what is a thing?" "*is an old one. Always new about it is only that it must be asked again and again.*" I did not answer it; I merely explained how to describe a thing. The description is in the spirit of Wittgenstein's isomorphism theory [12] (see Appendix C). In the last section of this paper, the question will be tied to the problem of how to choose block spins in renormalization group theory [13], and to a basic feature (autonomy) of Maturana and Varela's theory of autopoietic systems [3].

## 2 Gauge theory

We generalize the basic notions of gauge theory to the general setup.

### Curvature = Field Strength = Frustration

exists unless there is at most one arrow  $f$  from  $X$  to  $Y$  for every  $X, Y$ . States in different  $\Omega_X$  cannot be compared (without parallel transport which depends on the path ) if there is frustration. Without frustration, all paths from a given object  $X_0$  to a given  $X_n$  define the same arrow..

*Example:* *Gossip* results from frustrated communication. When it returns from a loop, the story is no longer the same. Human communication is not necessarily rational, i.e. limited to exchange of information in terms which have the same meaning to everybody. Poetry, political propaganda and all kinds of seduction exploit this. Gauge theory avoids a priori rationality posits.

**Invariants:**  $F(\xi_1, \dots, \xi_n)$ ,  $\xi_i \in \Omega_X$  is an invariant if  $F(C\xi_1, \dots, C\xi_n)$  is independent of the path  $C : X \mapsto Y$  for every  $Y$ . It follows that  $F(C\xi_1, \dots, C\xi_n) = F(\xi_1, \dots, \xi_n)$  for all loops  $C$ , since  $C = \iota_X$  is one of them.

Invariant functions of loops  $s : \Omega_X \mapsto \Omega_X$  are similarly defined. More generally, invariants of  $K$  can be defined as *functors*  $\mathcal{F}$  from the network  $K$  or from categories derived from  $K$  to *unfrustrated categories*; for details see [4].

*Remark:* The totality of such functors we may call *meaning* of  $K$ , and the image of objects, *concepts*. Kant, in contradistinction, proposed to transcend *from* concepts *to* objects in his critique of pure reason [8]. It is of interest to consider extended categories which include besides  $K$  also concepts and their relations. Besides images of relations in  $K$  there are relations from objects to concepts through the functors  $\mathcal{F}$ , and there may be additional relations between concepts from intertwiners between different  $\mathcal{F}$ 's. One may ask how such extended categories can grow from  $K$  dynamically. (The intertwining relation reads like this.  $T_{12} : \Omega_{\mathcal{F}_1(X)}^1 \mapsto \Omega_{\mathcal{F}_2(X)}^2$  is defined, but possibly zero, for every pair of concepts  $\mathcal{F}_i(X)$  which are related to the same

object  $X$ . If  $f : X \mapsto Y$  is an arrow then  $T\mathcal{F}_1(f) = \mathcal{F}_2(f)T$ . In group theory such intertwiners are made of Clebsch Gordan coefficients.)

*Example: Money* [9]. Money in a money-based economy is a prototypical example of an invariant, and it serves to give an invariant meaning to certain transfers in society, viz. acquisition by purchase. According to Luhmann [9], p.69, "the medium [money] assures that in the realm of economy, actions have approximately the same meaning for him who acts as for the observer."

**Consensus** on the meaning of invariants can be achieved among all agents  $Y$  by synchronization = path independent parallel transport to  $Y$ .

*Example:* In general relativity, and more generally in any curved (pseudo) Riemannian manifold  $M$  there is no global notion of parallelism of tangent vectors  $\xi \in TM$ , i.e. no consensus on the meaning of the direction of a vector is possible. But the length of a vector is an invariant.

### Gauge group $G=\{G_X\}=\{\text{holonomy groups}\}$

$G_X$  contains all invertible arrows  $f : X \mapsto X$  (loops). These gauge transformations map  $\Omega_X$  into  $\Omega_X$  and  $f : \Omega_X \mapsto \Omega_Y$  into  $g_Y \circ f \circ g_X^{-1}$ . So they are invertible maps of objects and relations. They are isomorphisms of representations of the same category  $K$  and leave invariants unchanged. The invariants are the observables in lattice gauge theory.

We could instead define a gauge group  $G' \supset G$  as the group of all those invertible transformatons of the spaces  $\Omega_X$  which leave all invariants invariant. The holonomy group has the advantage that it is intrinsic to the category.

*Remark:* For human minds we call the totality of fundamental loops *consciousness*. They make the agent aware of himself (because they transform output into input), and they can serve as a short time memory (because dynamics will be such that a fundamental arrow propagates a signal in one time step). These features characterize consciousness according to Crick and Koch. The totality of all loops is related to *perception*.

**Composite objects with internal structure:** Categories can be objects of new categories. Conversely, a connected unfrustrated subcategory  $K_O \subset K$  may be reinterpreted as a single new composite object. This defines a new category  $K'$  which is a representation of  $K$ . The required new composition laws are of the form  $g \circ f$  (in  $K'$ ) =  $g \circ C \circ f$  (in  $K$ ),  $C$  a path in  $K_O$ .

*Example: Gauge covariant block spins* in lattice gauge theory [14].

## 3 Gauge transformations in linguistics

were described by Quine before the gauge theory of elementary particles was found [11]. He says

"the infinite totality of sentences of any given speaker's language can be so permuted or mapped onto itself that

(a) the totality of speakers disposition to verbal behavior remains invariant, and yet

(b) the mapping is no mere correlation of sentences with equivalent sentences, in any plausible sense of equivalence however loose. Sentences without numbers can diverge drastically from their respective correlates yet the divergences can systematically so offset one another that the overall pattern of associations of sentences with one another and with non-verbal stimulation is preserved".

The disposition to verbal behavior in the presence of a nonverbal stimulus is the linguists observable. Observables are invariant. The pattern of associations is preserved - i.e. the map

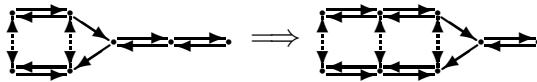


Figure 2: Reproduction fork dynamics

is an isomorphism.

About the role of language, Quine has this to say: "*This structure of interconnected sentences is a single connected fabric including all sciences and indeed everything we ever say about the world*". Thus, if gauge transformations are here, they are everywhere.

Quine's connected fabric is interpreted as a category. The agents are sentences of the language, and the relations are interconnections or associations between them. In the spirit of Wittgenstein [12] we may regard sentences as translations (functorial images) of the world - or at least an attempt at that.

Translations need not exist. Quine hints at that by giving an example. A tentative english translation "*all rabbits are men reincarnate*" of a native sentence would violate the rules, because a native would assent (it is imagined) in the presence of any nonverbal stimulus whatever, while an Englishman would not.

Next we turn to time development  $t \mapsto K_t$ . Sorin Solomon proposed to call it "drama".

## 4 Universal Dynamics

is local dynamics which is defined for *every category*. A state should contain all necessary information about its time development in itself, without need for further extrinsic specification. Gauge invariance is an automatic consequence.

If such a dynamics operates at any level, it will also operate at more composite levels if it has renormalization group fix point properties. An example will be shown.

We consider first dynamics in discrete time. The following moves are defined for every (local  $*$ -) category

1. death of an object or arrow,
2. replication of an object or arrow,
3. fusion of indistinguishable arrows, or objects (inverse of 2),
4. restitution of a missing adjoint arrow in a  $*$ -category,
5. declaration as fundamental of a composite arrow.

It is easy to describe the new categories also in formulas, but I omit them.

Dynamics fixes which moves take place next, given the present state (1st order dynamics) or the state and the moves in the previous time interval (2nd order dynamics).

There are two types of replication of an object:

- 2a) with replication of arrows
- 2b) dividing arrows among duplicates.

*Example 1: Reproduction fork dynamics* is a version of 2b combined with 4. It is first order. The forks are made of arrows without adjoints. They designate the objects next to be split, see figure 2. The arrows are divided like this: ingoing to one copy, outgoing to the other, the other way round for arrows in forks. The dotted arrows are optional. This kind of dynamics is the basis of biological life on earth. It governs DNA replication during cell division [15]. It is a universal copy machine not only for linear chains or helices, but for **any** structure with any

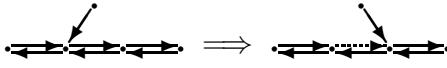


Figure 3: Move 5 interpreted as motion

number of incident arrows on each object and with substructures of any scale. Therefore it is a renormalization group fix point [13]. Take any connected \*-category without missing adjoints. Make a single object replicate. This creates forks which travel until the whole category has been duplicated, including any substructure. The dynamics is the same on all scales.

Moreover, the fix point dynamics is insensitive to small perturbations. Rare random death of arrows or objects will in general not jeopardize ability to replicate. Excessive death of arrows leads to random forks, though, and too many of them can result in cancerous growth.

A universal copy machine is an optimal prerequisite for evolution. Programming just a few sterilizations and refertilizations of objects, elaborate structures can be grown, for instance hypercubic lattices with periodic boundary conditions, multigrids etc.. A  $C^{++}$ -class library was written to implement this [18]. Deviant first replication steps can in some cases induce formation of double sheeted covers in place of two copies.

Material properties - embedding of material particles in space - did not enter here. An encyme - topoisomerase type II - is known to exist which relieves DNA from the most serious burden of its material existence. It enables strings of DNA to pass through each other [15]. It is thought to operate by cutting one of the strings, holding on to the ends while passing them beyond the other string, and then rejoining them.

*Remark 1:* It might be objected that this categorical introduction of replication begs the question of how the replication of anything works in the first place. Autocatalytic replication of real molecules is known [16] and there are simple models. Suppose the potential energy of unbound "molecule"  $a$  in the field of unbound  $b$  has a minimum at binding energy  $E$  and a high maximum at activation energy  $A$ , while bound or unbound molecules in the field of bound ones have binding energy  $E/2$  and no activation energy. If any  $X = ab$  is present initially, it replicates in an environment where  $a$  and  $b$  abound,  $ab + a + b \mapsto abab \mapsto ab + ab ; E \mapsto 3E/2 \mapsto 2E$ .

*Remark 2:* Deterministic replication starting from a single object will yield indistinguishable clones inside clones. Stochasticity will alter that, leading to "*identity breaking*" in two stages: from strong identity, where an object is identical only to itself, to indistinguishability, and from there to distinguishability. Regarding strong identity as strongest order, each step would decrease the order.

Move 5) is the prototype of motion.

*Example 2:* *Quantum mechanical motion of particles in space* can be described by a universal dynamics. We may picture a category of objects ("space points") linked by bidirectional arrows plus additional objects ("point particles") linked to the former by one unidirectional arrow each. The Schrödinger equation for the complex amplitude of a category  $K$  reads

$$i\dot{\Psi}(K) = -\Delta\Psi(K) \equiv -\sum_{\mu}[\Psi(\mu K) - \Psi(K)] .$$

Summation is over moves  $\mu$  of individual particles as in figure 3. For a single particle of mass  $m$  on a cubic lattice of space points, this is the standard discretization of the Schrödinger equation for free motion. Units of time  $\hbar/2m$  are set to 1. The "used up" dashed arrow is immediately restituted

*Example 3: Universal equation of motion in continuous time.* Consider fundamental arrows  $f : \Omega_X \mapsto \Omega_Y$ , including fundamental loops regarded as attributes of objects. Assume spaces  $\Omega_X$  possess tangent spaces  $T\Omega_X$  and maps  $f$  are invertible and have derivatives  $f' : T\Omega_X \mapsto T\Omega_Y$ . Demand

$$\frac{d}{dt}(\dot{f} \circ f^{-1}) = \beta \dot{f}' \circ \sum_{g=g_1 \circ g_2 : X \mapsto Y} \left\{ (f^{-1'} \circ (\dot{g} \circ g^{-1}) - (g^{-1'} \circ \dot{g}) \circ f^{-1}) \right\} \quad (1)$$

This is meaningful, i.e. generally covariant (see Appendix B). It is remarkable that a meaningful equation exists at all.

Summations are over pairs  $(g_1, g_2)$  of fundamental arrows so that  $g$  is defined.  $\beta \in \mathbf{R}$  could depend on whether  $f$  is a loop or not.

The equation is quadratic in velocities like Einsteins equation for the motion of a massive particle in a gravitational field,  $\ddot{x}^\mu = -\Gamma^\mu_{\nu\rho}(x)\dot{x}^\nu\dot{x}^\rho$ . It retains its form when  $f^{-1}$  is substituted for  $f$ , and induces similar equations for composite arrows. In this sense it scales.

Note that the expression in  $\{\}$  is only nonzero if there is nonvanishing frustration, i.e. path dependence, in a generalized sense as appropriate for initial states  $\{K, \bar{K}\}$ .

## 5 Dynamics of composite objects

should be determined by a *renormalization group transformation* with dynamically determined coarse grained variables=blockspins [13].

Grabowski showed recently how to determine blockspins dynamically from locality demands on effective actions in quantum field theory [17]. The locality demand amounts to making the composite objects *as autonomous as possible*.

The limits of resolution of observations will enter into the specification of the block spins (they fix the scale): *What is a (composite) thing is in the eye of the observer.* Our mind organizes the world into things which are stable and capable of an approximately autonomous existence.

At composite levels, unfrustrated communication - i.e. communication by exchange of invariants - may tend to dominate, because of destructive interference effects for others.

*Remark:* This can be seen from random walk expansions.

According to Luhmann [9], a system determines its own medium of [unfrustrated] communication. "The most important effect of the medium money at the level of the whole society arises from the fact that the payment acquiesces third parties. Although they are themselves interested in scarce goods (or could be interested in the future) they are able to look on [peacefully] while somebody takes posession of scarce goods, because he pays for them." Basically, the intrinsically frustrated character of human perception is partially overcome by assignment of invariant meaning.

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## Appendix A: Proof of the main representation theorem

A slightly more elaborate version of the representation theorem was proven by the author in [4]. For the convenience of the reader, the statement and proof of this theorem is reproduced here, and it is shown how the present version, which is closer to lattice gauge theory, is obtained as a corollary.

We use a slightly more elaborate notation than in the main text. Given the category  $K$  with objects  $X, Y, \dots$ , we denote by  $Mor(Y, X)$  the set of arrows  $f : X \mapsto Y$  and  $g \circ_Y f : X \mapsto Z$  for the composite of  $f \in Mor(Y, X)$  with  $g \in Mor(Z, Y)$ .

**Representation theorem 1** (Representation of a category as a communication network) *Every category  $K$  permits a faithful representation with the following properties*

*To every object  $X$  there exists an input space  $A_X$  and an output space  $\Omega_X$ . The input space contains a distinguished element  $\emptyset$  ("empty input"). Arrows  $f \in Mor(Y, X), g \in Mor(Z, Y)$  and objects  $X$  act as maps*

$$X : A_X \mapsto \Omega_X, \tag{2}$$

$$\iota_X : \Omega_X \mapsto A_X \tag{3}$$

$$f : \Omega_X \mapsto A_Y \tag{4}$$

with the properties

$$X\iota_X = id : \Omega_X \mapsto \Omega_X , \quad \iota_X X = id : A_X \mapsto A_X , \tag{5}$$

$$g \circ_Y f = gYf : \Omega_X \mapsto A_Z . \tag{6}$$

It should be noted that  $\iota_X$  does not act as the identity map in general in this context.

Given this version of the representation theorem, we restrict attention to the output spaces  $\Omega_X$  and to maps  $\hat{f} = Y \circ f : \Omega_X \mapsto \Omega_Y$ . Renaming  $\hat{f}$  into  $f$  we obtain the representation theorem of the main text.

*Remark:* In some applications, the distinguished empty input is mapped into a distinguished output which is distinct from all other output. In this case it may be ignored. This happens in pure gauge theory without matter fields. In other cases,  $X\emptyset = \Psi_X \in \Omega_X$  is dynamically determined. Matter fields  $\Psi_x$  in lattice gauge theory are an example.

## Proof of the representation theorem 1 for categories

Given a category  $K$ , we write  $Mor(Y, *)$  for the set of all its arrows to  $Y$  etc.. We define

$$In(Y) = Mor(Y, *) , \quad Out(Y) = Mor(*, Y) .$$

We write  $X = \alpha(f)$  if  $f \in Mor(Y, X) \subset In(Y)$ , and correspondingly  $Z = \omega(f)$  if  $f \in Mor(Z, Y) \subset Out(Y)$ . The output space will be defined as a subspace  $\Omega_Y$  of  $\Omega_Y^{virt}$ .  $\Omega_Y^{virt}$  consists of maps

$$\zeta : Out_Y \mapsto Mor(*, *)$$

with the property  $\zeta(f) \in Mor_K(\omega(f), *)$ .

An object  $Y$  will act as a map

$$Y : In(Y) \mapsto \Omega_Y .$$

according to

$$Yf(g) = g \circ_Y f \quad (g \in Out(Y)).$$

The output space is defined as the image of  $Y$ , and the input space as space of equivalence classes (if necessary) of elements of  $In_K(Y)$ , which  $Y$  maps into the same  $\zeta \in \Omega_Y^{virt}$ .

$$\Omega_Y = IM Y \subset \Omega_Y^{virt}, \quad (7)$$

$$A_Y = In(Y)/KER Y. \quad (8)$$

$Y$  is invertible as a map from  $A_Y$  to  $\Omega_Y$ . Its inverse is  $\iota_Y$ . The empty input  $\emptyset \in A_Y$  is defined as the equivalence class of  $\iota_Y \in Mor(Y, Y) \subset In(Y)$ .

An arrow  $f \in Mor(Y, X)$  is defined as a map  $\Omega_X \mapsto A_Y$  by use of the map  $\iota_X : \Omega_Y \mapsto A_Y$ , as follows.

$$f = \hat{f} \circ_X \iota_X, \quad (9)$$

$$\hat{f}(g) = f \circ_X g \quad \text{for } g \in Mor(X, *) . \quad (10)$$

The last formula defines  $\hat{f}$  as a map from  $In(X)$  to  $In(Y)$ . This map passes to equivalence classes (8) thereby defining a map  $A_X \mapsto A_Y$ , The composition rule (6) holds.

## Appendix B: Covariance of the universal equation of motion

To show that the equation of motion (1) has a coordinate independent meaning, we exhibit both sides as elements of a space which has a coordinate independent meaning.

Let  $E_X = T\Omega_X$  the tangent space of  $\Omega_X$ . It is a fibre bundle over  $\Omega_X$  with projector  $\pi_X$ . Then  $\dot{f} : \Omega_X \mapsto T\Omega_Y$  with  $\pi_Y \circ \dot{f} = f$ .

The tangent space  $TE_X = TT\Omega_X$  is a fibre bundle over  $E_X$  with projector  $\pi_{TX}$ . Like the tangent space of every bundle it has a vertical subspace  $VE_X$  which is naturally isomorphic to a factor space  $VE_X \simeq TE_X/\pi_X^*T\Omega_X$  where  $\pi_X^* : T\Omega_X \mapsto TE_X$  is the pullback of  $\pi_X$ .

The map  $f$  induces a map of curves on  $\Omega_X$  into curves on  $\Omega_Y$  and thereby a map  $f'$ , called its derivative,

$$f' : TO_X \mapsto T\Omega_Y \quad \text{obeying } \pi_Y \circ f' = f \circ \pi_X.$$

Similarly,  $\dot{f}$  maps curves in  $\Omega_X$  into curves in  $E_Y$ . Its time derivative  $\ddot{f}$  is therefore a map from  $\Omega_X$  to  $TE_X$  which obeys  $\pi_{TY}\ddot{f} = \dot{f}$ . This map passes to the quotient, also denoted by  $\ddot{f}$ . Since  $\pi_{TY}$  also passes to the quotient,  $h = \ddot{f}$  is a map

$$h : \Omega_X \mapsto VE_Y \quad \text{obeying } \pi_{TY} \circ h = \dot{f}. \quad (11)$$

**R**-linear combinations of such maps are defined and share the same property.

The derivative  $\dot{f}'$  of  $\dot{f}$  is naturally defined as a map from  $\Omega_X$  to  $TE_Y$ . It passes to the quotient, also denoted by  $\dot{f}'$ , viz.

$$\dot{f}' : \Omega_X \mapsto VE_Y \quad \text{obeying } \pi_{TY} \circ \dot{f}' = \dot{f} \circ \pi.$$

Using these identities one verifies that the equation of motion is consistent with eq.(11) for  $h = \ddot{f}$  and is therefore meaningful.

In holonomic coordinates  $\{\eta^\alpha, \dot{\eta}^\alpha\}$  on  $E_Y$ ,

$$\ddot{f}(t, \xi) = \frac{\partial^2}{\partial t^2} f^\alpha(t, \xi) \frac{\partial}{\partial \dot{\eta}^\alpha} \in VE_Y.$$

There is no term proportional to  $\frac{\partial}{\partial \eta^\alpha}$  here because  $\pi_Y^* T\Omega_Y$  consists of such terms.

*Remark:* In the notation used here, the chain rule reads  $\frac{d}{dt} g \circ f = \dot{g} \circ f + g' \circ \dot{f}$ .

## Appendix C: Wittgensteins isomorphy theory [12]

Wittgensteins propositions are numbered, therefore every reader may find them in his own edition of the *tractatus* in his own language. I give a selection of those propositions which are particularly relevant here, in german.

*Comment 1:* The english translation is problematic; it hides something that I wish to emphasize. (This only goes to confirm that translations to other languages may not exist) Wittgenstein defines *Sachverhalt* as a "link (*Verbindung*) between objects (entities, things)" in proposition 2.01. *Verbindung* comes from *binding*. This crucial connection is suppressed by the english translation of *Verbindung* as "combination" instead of link or bond. *Sachverhalt* is translated as "atomic fact". This is an interpretation due to B. Russell which is motivated by what is said later in the text. In the same spirit, I propose to compose general arrows from fundamental ones, called links.

*Comment 2:* Wittgensteins remark 5.5303 on identity is superseded by quantum mechanics. In his introduction to [12], Russell states (p.16/17) [One has] "sought to find such a property [which must belong to every thing by a logical necessity] *in self identity* ... [but] *accidental characteristics of the world must, of course, not be admitted into the structure of logic*. Mr. Wittgenstein accordingly banishes identity ...". Particles which are identical in the sense of indistinguishable are very important in quantum mechanics, and quantum fluctuations introduce the accidental into the result of measurements.

The issue is important in the general context of this paper. Replication of objects, composite or whatever, produces only indistinguishable copies. Variability comes only from stochasticity. Mutations such as random death of a component can make formerly indistinguishable composite objects distinguishable. Sensitive dependence on initial conditions (chaos) can magnify small random fluctuations. But how do small fluctuations arise in the first place? From quantum fluctuations. According to the views expressed in this article, things are in the eye of the observer. The mental organization of the world into things requires an act of observation. This link to observation brings in quantum fluctuations, because results of observations on systems in the same quantum state can fluctuate.

### Wittgensteins propositions

1. Die Welt ist alles, was der Fall ist.
  - 1.1 Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.
2. Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten.
  - 2.01 Der Sachverhalt ist eine Verbindung von Gegenständen (Sachen, Dingen).
  - 2.0121 ... so können wir uns *keinen* Gegenstand ausserhalb der Möglichkeit seiner Verbindung mit andern denken.

- 2.013 Jedes Ding ist, gleichsam, in einem Raum möglicher Sachverhalte. Diesen Raum kann ich mir leer denken, nicht aber das Ding ohne den Raum.
- 2.021 Jede Aussage über Komplexe lässt sich in eine Aussage über deren Bestandteile und in diejenigen Sätze zerlegen, welche die Komplexe vollständig beschreiben.
- 2.024 Die Substanz ist das, was unabhängig von dem, was der Fall ist, besteht.
- 2.032 Die Art und Weise, wie die Gegenstände in Sachverhalten zusammenhängen, ist die Struktur der Sachverhalte.
- 2.033 Die Form ist die Möglichkeit der Struktur.

2.1 Wir machen uns Bilder der Tatsachen.

- 2.12 Das Bild ist ein Modell der Wirklichkeit.
- 2.14 Das Bild besteht darin, daß sich seine Elemente in bestimmter Art und Weise zueinander verhalten.
- 2.15 Daß sich die Elemente des Bildes in bestimmter Art und Weise zueinander verhalten stellt vor, daß sich die Sachen so zueinander verhalten.  
Dieser Zusammenhang der Elemente des Bildes heiße seine Struktur, und ihre Möglichkeit seine Form der Abbildung.
- 2.1513 Nach dieser Auffassung gehört also zum Bilde auch noch die abbildende Beziehung, die es zum Bild macht.

3. Das logische Bild der Tatsachen ist der Gedanke

3.1 Im Satz drückt sich der Gedanke sinnlich wahrnehmbar aus.

- 3.12 Das Zeichen, durch welches wir den Gedanken ausdrücken, nenne ich das Satzzeichen. Und der Satz ist das Satzzeichen in seiner projektiven Beziehung zur Welt.
- 3.13 Zum Satz gehört alles, was zur Projektion gehört; aber nicht das Projezierte ...
- 3.14 Das Satzzeichen besteht darin, daß sich seine Elemente, die Wörter, in ihm auf bestimmte Art und Weise zueinander verhalten.
- 3.144 ..  
(Namen gleichen Punkten, Sätze Pfeilen ...)

4. Der Gedanke ist der sinnvolle Satz.

- 4.001 Die Gesamtheit der Sätze ist die Sprache.
- 4.01 Der Satz ist ein Bild der Wirklichkeit.  
Der Satz ist ein Modell der Wirklichkeit, so wie wir sie uns denken.
- 4.1 Der Satz stellt das Bestehen oder Nichtbestehen von Sachverhalten dar.
- 4.21 Der einfachste Satz, der Elementarsatz, behauptet das Bestehen eines Sachverhalts.

5. Der Satz ist eine Wahrheitsfunktion des Elementarsatzes.

- 5.526 Man kann die Welt vollständig durch vollkommen verallgemeinerte Sätze beschreiben, das heißt also, ohne einen bestimmten Namen von vornherein einem bestimmten Gegenstand zuzuordnen.

Um dann auf die gewöhnliche Ausdrucksweise zu kommen, muß man einfach nach dem Ausdruck "es gibt ein und nur ein  $x$ , welches ..." sagen: Und dies  $x$  ist  $a$ .

- 5.5303 Beiläufig gesprochen: Von zwei Dingen zu sagen, sie seien identisch, ist ein Unsinn, und von einem zu sagen, es sei identisch mit sich selbst, sagt gar nichts.
- 6.3432 Wir dürfen nicht vergessen, daß die Weltbeschreibung durch die Mechanik immer die ganz allgemeine ist. Es ist in ihr z.B. nie von einem *bestimmten* materiellen Punkte die Rede, sondern immer nur von *irgendwelchen*.
- 6.361 In der Ausdrucksweise Hertz's könnte man sagen: Nur gesetzmäßige Zusammenhänge sind denkbar.

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