es. 1 Dato il sistema in forma I/O del II ordine:

$\ddot{y} + 5\dot{y} + 6y = u$

1) Verificare che $\exists\ x(0)$ tale che: se $u(t) = \rho e^{pt}$ $\quad y(t) = G(p)\rho e^{pt}$ $\quad \rho = 2,\ p = -1$

$y(t) = y_L(t) + y_F(t) = G(p)\, e^{pt} \rho$

a) FdT ? $\quad \mathcal{L}[y(t)](s) = Y(s)$ $\qquad \mathcal{L}[\dot{y}(t)](s) = sY(s) - y(0)$ $\qquad \mathcal{L}[\ddot{y}(t)](s) = s(sY(s) - y(0)) - \dot{y}(0)$

$s(sY(s) - \cancel{y(0)}) - \cancel{\dot{y}(0)} + 5(sY(s) - \cancel{y(0)}) + 6Y(s) = U(s)$

$s^2 Y(s) + 5sY(s) + 6Y(s) = U(s)$, $\quad Y(s) = \dfrac{1}{s^2+5s+6} U(s) = \underbrace{\dfrac{1}{(s+2)(s+3)}}_{G(s)} U(s)$

b) $y_F(t)$: $\quad u(t) = 2e^{-t}$ $\quad U(s) = \dfrac{2}{s+1} \longrightarrow Y(s) = G(s)U(s) \longrightarrow y_F(t)$

$Y(s) = G(s)U(s) = \dfrac{1}{(s+2)(s+3)} \cdot \dfrac{2}{s+1} = \dfrac{2}{(s+1)(s+2)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3} \Rightarrow \dfrac{A(s^2+5s+6) + B(s^2+4s+3) + C(s^2+3s+2)}{(s+1)(s+2)(s+3)}$

$$\begin{cases} (A+B+C) = 0 \\ (5A+4B+3C) = 0 \\ 6A+3B+2C = 2 \end{cases} \qquad \begin{cases} A = -B-C \\ -5B-5C+4B+3C = 0 \longrightarrow -B-2C = 0 \longrightarrow B = -2C \longrightarrow B = -2 \\ -6B-6C+3B+2C = 2 \longrightarrow -3B-4C = 2 \longrightarrow C = 1 \end{cases} \qquad A = 1$$

DEF: le var. di stato sono le componenti per cui compaiono le derivate rispetto al tempo

$Y(s) = \dfrac{1}{s+1} - \dfrac{2}{s+2} + \dfrac{1}{s+3}$ $\qquad y_F(t) = e^{-t} - 2e^{-2t} + 3e^{-3t}$ $\quad t \geq 0$

$G(s) = \dfrac{1}{(s+2)(s+3)}$ $\quad \lambda_1 = -2$ $\quad \lambda_2 = -3$ autov. $\Rightarrow$ modi e^{-2t}, e^{-3t} ... y_L è comb lin dei modi

c) $y_L = Ce^{At}x(0) = \underline{\underline{K}}e^{-2t} + \underline{\underline{\ell}}e^{-3t}$ ci basta trovare K e ℓ $\qquad G(p) = G(-1) = \dfrac{1}{\underbrace{(-1+2)}_{1}\underbrace{(-1+3)}_{2}} = \dfrac{1}{2}$

$y(t) = y_L(t) + y_F(t) = Ke^{-2t} + \ell e^{-3t} + e^{-t} - 2e^{-2t} + e^{-3t}$

$y(t) = G(p)\rho e^{pt} = \dfrac{1}{2} \cdot 2 \cdot e^{-t} = e^{-t}$ $\quad t \geq 0$ $\quad \Rightarrow y(t) = e^{-t} + \underbrace{(K-2)}_{K=2} e^{-2t} + \underbrace{(\ell+1)}_{\ell=-1} e^{-3t} = e^{-t}$

Siccome esistono K e ℓ, allora $\underline{\exists\ x(0)}$

2) Cosa accadrebbe se la fdt avesse forma $G(s) = \dfrac{s+1}{(s+2)(s+3)}$?

$G(p) = G(-1) = 0$, $\quad y(t) = 0$ nonostante $u(t) = 2e^{-t}$ ed è la Proprietà bloccante degli zeri

es. 2 Dato S: $\begin{cases} \dot{x}_1 = -2x_1 + x_2 + u \\ \dot{x}_2 = -3x_2 + 3u \\ y = x_2 \end{cases}$ $\quad A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad C = [0 \ \ 1]$ $\quad D = 0$

1) Mov. forzato di $y(t)$ con $u(t) = 2$ $\quad t \geq 0$ $\qquad [u(t) \equiv 2\,\mathrm{sca}(t)]$

Noto che $\dot{x}_2(t)$ non dipende da x_1, in più nemmeno l'uscita dipende da x_1 per cui non devo calcolarne la dinamica

$y_F(t) = \displaystyle\int_0^t e^{-3(t-\tau)} \cdot 3 \cdot u(\tau)\, d\tau = 6e^{-3t} \int_0^t e^{3\tau} d\tau = \cancel{6}^{2} e^{-3t} \left[\frac{e^{3\tau}}{3}\right]_0^t = 2(1 - e^{-3t})$ $\quad t \geq 0$

2) Calcolare la fdt da u a y: 3 metodi

a) propr. trasformate: poichè A è triangolare,

so già che $Y(s) = X_2(s) \longrightarrow sX_2(s) = -3X_2(s) + 3U(s)$ $\qquad \overset{Y(s)}{\overset{\parallel}{X_2(s)}} = \frac{3}{s+3}U(s)$ $\qquad G(s) = \frac{3}{s+3}$

1 solo polo: c'è una parte nascosta perchè X_2 non "vede" ciò che succede a x_1, ma sappiamo che $\lambda_1 = -2$ e $\lambda_2 = -3$ con $\underline{-2}$ parte nascosta

b) Applicando la formula della fdT $\quad G(s) = C(sI - A)^{-1}B + D$

c) $y_F(t) = 2 - 2e^{-3t} = 2\,sca(t) - 2e^{-3t} \qquad t \geq 0$

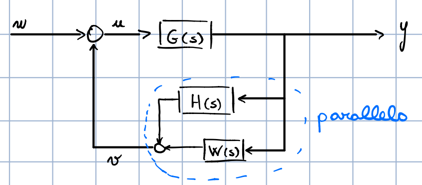
$$Y(s) = \frac{2}{s} - \frac{2}{s+3} = \frac{2(s+3) - 2s}{(s+3)\cdot s} = \frac{6}{s(s+3)} = \underbrace{\frac{3}{s+3}}_{G(s)} \cdot \underbrace{\frac{2}{s}}_{U(s)}$$

3) $H(s) = -\frac{K}{s+2} \qquad K \in \mathbb{R}$

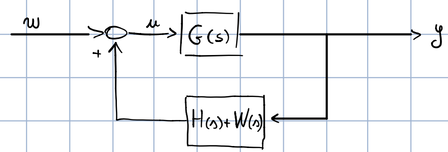
$W(s) = -\frac{K}{s+4}$ $\qquad$ Calcolare fdt da w a y

2 SYS II ORD

a) Schemi a blocchi



$\Longrightarrow$



$$S(s) = \frac{G(s)}{1 - G(s)(H(s)+W(s))} = \frac{\frac{3}{s+3}}{1 - K\left(\frac{1}{s+2} + \frac{1}{s+4}\right)\frac{3}{s+3}} = \frac{\frac{3}{s+3}}{1 - 2K\left(\frac{\cancel{s+3}}{(s+2)(s+4)}\right)\frac{3}{\cancel{s+3}}} = \frac{\frac{3}{s+3}}{1 + \frac{6K}{(s+2)(s+4)}} =$$

$$= \frac{3(s+2)(s+4)}{(s+3)(s^2+6s+8+6K)} = S(s)$$

b) $u = w + v \qquad \Rightarrow u = w + (H(s) + W(s))\,y$

$v = H(s)\,y + W(s)\,y = (H(s) + W(s))\,y$

$y = G(s)\,u \qquad\qquad y = G(s)\,w + G(s)\,(H(s) + W(s))\,y$

4) AS. STAB. AL VARIARE DI $K \in \mathbb{R}$

$S(s) = \frac{3(s+2)(s+4)}{(s+3)(s^2+6s+8+6K)} \longrightarrow$ 3 poli

stesso segno $\Rightarrow Re < 0$

se $K > -\frac{4}{3}$

$\lambda_2 = -3$

$Re < 0$

$H(s)$ I ord — 1 polo

$W(s)$ I ord — 1 polo

$\left.\right\}$ 3 poli

$\left\{ \begin{array}{l} \dot{x}_1 \\ \dot{x}_2 \\ y \end{array} \right.$ II ord / IV ord $\longleftrightarrow$ $G(s)$ 1 polo

Stesso # di poli

$\boxed{\lambda_1 = -2}$ $\qquad Re < 0$

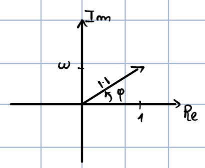
AS. STABILE, se K è più piccolo di $\frac{4}{3}$ diventa instabile

5) Ponendo $K = -1/2$ $w(t) = 5 + 10\,\mathrm{sen}(100t) - \cos(0{,}1t)$ determinare $y_\infty(t)$

$a, b \in \mathbb{C}$ $|a \cdot b| = |a| \cdot |b|$ $\measuredangle(ab) = \measuredangle(a) + \measuredangle(b)$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $\measuredangle\left(\frac{a}{b}\right) = \measuredangle a - \measuredangle b$

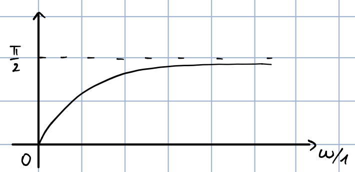
$$S(s) = \frac{3(s+2)(s+4)}{(s+3)(s^2+6s+8+6K)}$$



$|1 + j\omega| = \sqrt{1+\omega^2}$ $\begin{cases} \omega \gg 1 & |1+j\omega| \approx \sqrt{\omega^2} = \omega \\ \omega \ll 1 & |1+j\omega| \approx \sqrt{1} = 1 \end{cases}$

$\omega > 0$

$\measuredangle(1+j\omega) = \tan^{-1}\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right)$ $\begin{cases} \omega \gg 1 & \measuredangle(1+j\omega) \simeq \frac{\pi}{2} \text{ rad} \\ \omega \ll 1 & \measuredangle(1+j\omega) \simeq 0 \text{ rad} \end{cases}$



$\mathcal{L}[w_1(t)](s) = \frac{5}{s}$

$w_1(t) = 5\,\mathrm{sca}(t) \longrightarrow y_{1,\infty}(t)$ TVF : $y_{1,\infty}(t) = \lim_{s\to 0}\left[s\,Y(s) = \cancel{s}\,S(s)\,\frac{5}{\cancel{s}}\right] = \frac{5 \cdot 3 \cdot 2 \cdot 4}{3 \cdot 5} = 8$

$w_2(t) = 10\,\mathrm{sen}(100t) \longrightarrow y_{2,\infty}(t)$
$w_3(t) = -\cos(0{,}1t) \longrightarrow y_{3,\infty}(t)$ $\Big\}$ TRF

$$S(s) = \frac{8}{5} \cdot \frac{(1+s/2)(1+s/4)}{(1+s)(1+s/3)(1+s/5)}$$

$y_{2,\infty}(t) = 10\,|S(j100)|\,\mathrm{sen}(100t + \measuredangle(S(j100)))$

$$|S(j100)| = \left|\frac{8}{5}\,\frac{(1+j\frac{100}{2})(1+j\frac{100}{4})}{(1+j100)(1+j\frac{100}{3})(1+j\frac{100}{5})}\right| = \frac{8}{5}\,\frac{|1+j50| \cdot |1+j25|}{|1+j100| \cdot |1+j\frac{100}{3}| \cdot |1+j20|} \approx \frac{8}{5}\,\frac{50 \cdot 25}{100 \cdot \frac{100}{3}\,20} = \frac{3}{100}$$

$$\measuredangle S(j100) = \underbrace{\measuredangle\left(\frac{8}{5}\right)}_{\emptyset} + \underbrace{\measuredangle(1+j50)}_{+\frac{\pi}{2}} + \underbrace{\measuredangle(1+j25)}_{+\frac{\pi}{2}} - \underbrace{\measuredangle(1+j100)}_{-\frac{\pi}{2}} - \underbrace{\measuredangle(1+j\tfrac{100}{3})}_{-\frac{\pi}{2}} - \underbrace{\measuredangle(1+j20)}_{-\frac{\pi}{2}} \approx -\frac{\pi}{2}$$

$y_{3,\infty}(t) = (-1)\,|S(j0{,}1)|\cos(0{,}1t + \measuredangle(S(j0{,}1)))$

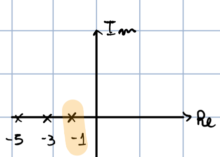
$$|S(j0{,}1)| = \frac{8}{5}\,\frac{|1+j\frac{0{,}1}{2}| \cdot |1+j\frac{0{,}1}{4}|}{|1+j0{,}1| \cdot |1+j\frac{0{,}1}{3}| \cdot |1+j\frac{0{,}1}{5}|} \approx \frac{8}{5}\,\frac{1 \cdot 1}{1 \cdot 1 \cdot 1} = \frac{8}{5} \Rightarrow y_{3,\infty} = -\frac{8}{5}\cos(0{,}1t)$$

$\measuredangle(S(j0{,}1)) \approx -\frac{\pi}{2}$

$y_{2,\infty}(t) = \frac{3}{10}\,\mathrm{sen}\left(100t - \frac{\pi}{2}\right)$ $y_{1,\infty}(t) = 8$

$y_\infty(t) = 8 + \frac{3}{10}\,\mathrm{sen}(100t - \frac{\pi}{2}) - \frac{8}{5}\cos(0{,}1t)$

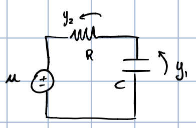
6) In quanto tempo $y(t) \to y_\infty(t)$? $S(s) \to \begin{matrix} \lambda_1 = -3 \\ \lambda_2 = -1 \\ \lambda_3 = -5 \end{matrix}$ con $\lambda_1 = -2$ nascosto che <u>NON</u> <u>INFLUENZA</u> il T_a



$T_A = 5\tau$ $\tau = -1/\lambda_3 = 1$

$= 5$ udt

es. 3



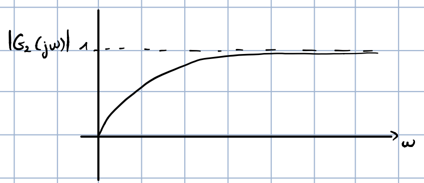
$$\begin{cases} \dot{x} = -\frac{1}{RC}x + \frac{1}{RC}u \\ y_1 = x \\ y_2 = u - x \end{cases}$$

$$G_1(s) = \frac{Y_1(s)}{U(s)} = \frac{1/RC}{s + 1/RC}$$

$$G_2 = \frac{Y_2(s)}{U(s)} = \frac{s}{s + 1/RC} = \frac{sRC}{1+sRC}$$

$$|G_1(j\omega)| = \frac{1}{|1+jRC|} = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$|G_2(j\omega)| = \frac{|j\omega RC|}{|1+j\omega RC|} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$$



$G_1(s)$ < "ω alta" $|G_1(j\omega)| \approx 0$; "ω bassa" $|G_1(j\omega)| \approx 1$

$u(t) = \cos(\omega t)$

$G_2(s)$ < "ω alta" $|G_2(j\omega)| \approx 1$; "ω bassa" $|G_2(j\omega)| \approx 0$

$y_1(t) \approx$ < $u(t)$ ω bassa ; 0 ω alta

$y_2(t) \approx$ < 0 ω bassa ; $u(t)$ ω alta