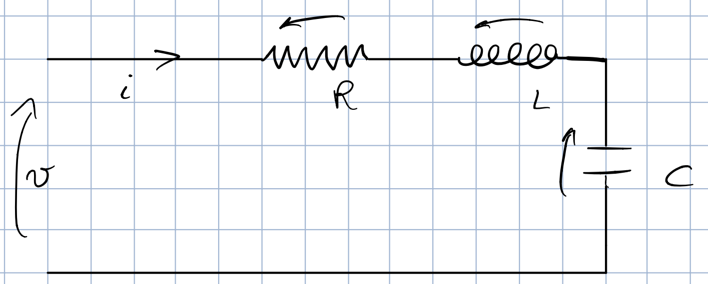
Punto di equilibrio p. 26

RLC



$$i = C\frac{dv_C}{dt}$$

$$v_R = iR$$

$$v_L = L\frac{di_L}{dt}$$

KVL: $v - v_R - v_L - v_C = 0$

$$v - iR - L\frac{di_L}{dt} - v_C = 0 \Rightarrow \frac{di}{dt} = -\frac{R}{L}i - \frac{1}{L}v_C + \frac{1}{L}v$$

$$\frac{dv_C}{dt} = \frac{1}{C}i$$

$$\begin{cases} x_1(t) = i(t) \\ x_2(t) = v_C(t) \end{cases} \qquad \begin{cases} \dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}v \\ \dot{x}_2 = \frac{1}{C}x_1 \end{cases}, \quad v = u(t)$$

$$y(t) = x_2(t) \qquad x(t) = \begin{bmatrix} i(t) \\ v_C(t) \end{bmatrix}$$

$$\dot{x}(t) = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u(t)$$

$$\begin{cases} u(t) = \bar{u} \\ x_1(t) = \bar{x}_1 \\ x_2(t) = \bar{x}_2 \\ y(t) = \bar{y} \end{cases} \qquad \begin{cases} -\frac{R}{L}\bar{x}_1 - \frac{1}{L}\bar{x}_2 + \frac{1}{L}\bar{u} = 0 \\ \frac{1}{C}\bar{x}_1 = 0 \\ \bar{y} = \bar{x}_2 \end{cases} \qquad \begin{cases} \bar{x}_1 = 0 \\ \bar{x}_2 = \bar{u} \end{cases}$$

LAGRANGE p. 51

$$\begin{cases} \dot{x}_1 = -5x_1(t) - 2x_2(t) + u(t) \\ \dot{x}_2 = -x_2(t) + u(t) \\ y(t) = 3x_1(t) \end{cases}$$

Uscita $y(t)$ quando:

$x_1(0) = 1$, $x_2(0) = 10$ $u(t) = 10$ $\forall t \geq 0$

$$x_2(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,d\tau$$

$$= e^{-t}\cdot 10 + \int_0^t e^{-(t-\tau)}\cdot 10\,d\tau =$$

$$= 10\left(e^{-t} + \int_0^t e^{\tau - t}\,d\tau\right) = \underline{10}$$

$$\dot{x}_1 = -5x_1(t) - 20 + 10 = -5x_1 - 10$$

$$x_1(t) = e^{-5t} + \int_0^t e^{-5(t-\tau)}\cdot(-10)\,d\tau = e^{-5t} - 10\int_0^t e^{5(\tau - t)}\,d\tau = e^{-5t} - 10\left[\frac{1}{5}e^{5(\tau-t)}\right]_0^t = e^{-5t} - 2 + 2e^{-5t}$$

$= 3e^{-5t} - 2$

$$y(t) = 8e^{-5t} - 6 \qquad \lim_{t \to \infty} y(t) = -6$$

Eserciziario p. 15 n° 1.5

$$\begin{cases} \dot{x}(t) = x^2(t) - u(t) \\ y(t) = x(t)\,u(t) \end{cases} \qquad u(t) = \bar{u} = 1$$

$$\begin{cases} x^2(t) - 1 = 0 \\ y(t) = x(t) \end{cases} \qquad \begin{matrix} x_a = 1 \\ y_a = 1 \end{matrix} \qquad \begin{matrix} x_b = -1 \\ y_b = -1 \end{matrix}$$

1.6

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2\cos(x_1(t)) + \frac{1}{2}x_2(t) - 4u(t) \end{cases}$$

$u(t) = \bar{u}$

$$\begin{cases} x_2(t) = 0 \\ 2\cos(x_1(t)) - 4\bar{u} = 0 \longrightarrow \cos(x_1(t)) = 2\bar{u} \end{cases}$$

n° 1.7

$$\begin{cases} \dot{x}_1(t) = 4x_1(t)^2 - u(t) \\ \dot{x}_2(t) = x_2(t) + 3\sqrt{x_1(t)} \end{cases} \qquad u(t) = \bar{u} \qquad \begin{cases} 4x_1^2(t) - \bar{u} = 0 \\ x_2(t) + 3\sqrt{x_1(t)} = 0 \end{cases} \qquad \begin{cases} 4\bar{x}_1^2 = \bar{u} \to \bar{x}_1 = -\sqrt{u/4} \\ \bar{x}_2 + 3\sqrt{\bar{x}_1} = 0 \to x_2 = 3\sqrt{\frac{u}{4}} \end{cases}$$

n. 1.8

$$\begin{cases} \dot{x}_1(t) = x_1 x_2 - x_1 + u \\ \dot{x}_2(t) = x_1 x_2 - e^{u(t)} \\ y(t) = \frac{x_1 x_2}{u} \end{cases} \qquad u(t) = \bar{u} \qquad \begin{cases} \bar{x}_1\bar{x}_2 - \bar{x}_1 + \bar{u} = 0 \\ \bar{x}_1\bar{x}_2 - e^{\bar{u}} = 0 \\ y(t) = \frac{\bar{x}_1\bar{x}_2}{\bar{u}} \end{cases} \qquad \begin{cases} \bar{x}_1\bar{x}_2 = e^{\bar{u}} \\ e^{\bar{u}} - \bar{x}_1 + \bar{u} = 0 \\ y(t) = \frac{e^{\bar{u}}}{\bar{u}} \end{cases}$$

2.2

$$\begin{cases} \dot{x}_1 = -2x_1 + x_2 + u \\ \dot{x}_2 = -3x_2 + u(t) \\ y = x_2(t) \end{cases}$$

1) $x(0) = [0 \ \ 1]^T \qquad u(t) = 0 \quad t \geq 0$

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad C = [0 \ \ 1] \quad D = \underline{0}$$

$$x_1(t) = \emptyset + \int_0^t e^{-2(t-\tau)} \cdot e^{-3\tau} \, d\tau = e^{-2t} \int_0^t e^{-\tau} \, d\tau = e^{-2t} \left[e^{-t} - 1 \right] = e^{-3t} - e^{-2t}$$

$$x_2(t) = e^{-3t} + \int_0^t e^{-3(t-\tau)} \cdot \emptyset = e^{-3t}$$

Modo dominante e^{-2t}

2.1

$$A = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{bmatrix} \lambda+1 & -2 \\ +2 & \lambda+1 \end{bmatrix} = (\lambda+1)^2 + 4 = \lambda^2 + 1 + 2\lambda + 4 = \lambda^2 + 2\lambda + 5$$

$$\lambda_{1-2} = -1 \pm \sqrt{1-5} = -1 \pm 2j$$

$$A v_1 = \lambda_1 v_1 \qquad v_1 = [\alpha \ \ \beta] \qquad \begin{cases} -\alpha + 2\beta = (-1+2j)\alpha \\ -2\alpha - \beta = (-1+2j)\beta \end{cases} \qquad \begin{cases} 2\beta = 2\alpha j \\ -2\alpha = 2\beta j \end{cases} \qquad \begin{cases} \beta = \alpha j \\ -\alpha = \beta j \end{cases} \qquad \begin{matrix} \alpha = 1 \\ \beta = j \end{matrix}$$

$$v_2 = [\alpha \ \ \beta]^T \qquad \lambda_2 = -1-2j \qquad \begin{cases} -\alpha + 2\beta = (-1-2j)\alpha \\ -2\alpha - \beta = (-1-2j)\beta \end{cases} \qquad \begin{cases} 2\beta = -2\alpha j \\ -2\alpha = -2\beta j \end{cases} \qquad \begin{cases} \beta = -\alpha j \\ -\alpha = -\beta j \end{cases} \qquad \begin{matrix} \alpha = 1 \\ \beta = -j \end{matrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \qquad T = \frac{1}{-2j} \begin{bmatrix} -j & -1 \\ -j & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix}$$

$$A_d = TAT^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{j}{2} \\ \frac{1}{2} & \frac{j}{2} \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{j}{2} \\ \frac{1}{2} & \frac{j}{2} \end{bmatrix} \begin{bmatrix} -1+2j & -1-2j \\ -2-j & -2+j \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}+j+j-\frac{1}{2} & -\frac{1}{2}-j+j+\frac{1}{2} \\ -\frac{1}{2}+j-j+\frac{1}{2} & -\frac{1}{2}-j-j-\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1+2j & \emptyset \\ \emptyset & -1-2j \end{bmatrix}$$

$$e^{At} = T^{-1} e^{A_d t} T$$

$$\begin{cases} \dot{x}_1 = -2\beta x_1(t) + x_2^2(t) \\ \dot{x}_2 = -x_2(t) + u(t) \\ y = x_1(t) \end{cases} \qquad \beta \neq \emptyset$$

1.1 $\quad u(t) = 2$

$$\begin{cases} 0 = -2\beta \bar{x}_1 + \bar{x}_2^2 \\ 0 = -\bar{x}_2 + 2 \\ \bar{y} = \bar{x}_1 \end{cases} \qquad \begin{cases} \bar{x}_2 = 2 \\ \beta \bar{x}_1 = 2 \\ \bar{y} = \bar{x}_1 \end{cases} \qquad \bar{x} = \begin{bmatrix} 2/\beta \\ 2 \end{bmatrix} \qquad \bar{y} = \frac{2}{\beta}$$

1.2

$$\delta S = \begin{cases} \delta \dot{x}_1 = -2\beta\,\delta x_1 + 2\bar{x}_2\,\delta x_2 \\ \delta \dot{x}_2 = -\delta x_2 + \delta u \\ \delta y = \delta x_1 \end{cases}$$

$\delta x_1 = x_1(t) - \bar{x}_1$ $\qquad \delta u = u(t) - \bar{u}$

$\delta x_2 = x_2(t) - \bar{x}_2$ $\qquad \delta y = y(t) - \bar{y}$

$$A = \begin{bmatrix} -2\beta & 2\bar{x}_2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2\beta & 4 \\ 0 & -1 \end{bmatrix} \qquad \begin{matrix} \lambda_1 = -2\beta \Rightarrow \beta > 0 \\ \lambda_2 = -1 \end{matrix}$$

$\beta = 1 \qquad u(t) = 2$

$x_1(0) = 1.9$

$x_2(0) = 2$

$$\begin{cases} \dot{x}_1 = -2x_1 + x_2^2 \\ \dot{x}_2 = -x_2 + 2 \\ y = x_1 \end{cases}$$

$\dot{x}_2(t) = -x_2(t) + 2$

$$x_2(t) = e^{-t}\cdot 2 + \int_0^t e^{-(t-\tau)}\cdot 2\, d\tau = $$
$$= 2e^{-t} + 2e^{-t}\int_0^t e^{\tau}\, d\tau = 2e^{-t} + 2e^{-t}(e^{t} - 1) = 2e^{-t} + 2 - 2e^{-t} = 2$$

$$x_1(t) = e^{-2t}\cdot 1{,}9 + \int_0^t e^{-2(t-\tau)}\cdot 4\, d\tau =$$
$$= 1{,}9\, e^{-2t} + 4e^{-2t}\int_0^t e^{2\tau}\, d\tau =$$
$$= 1{,}9\, e^{-2t} + 4e^{-2t}\left[\frac{e^{2\tau}}{2}\right]_0^t = 1{,}9\, e^{-2t} - 2e^{-2t} + 2 = 2 - 0{,}1\, e^{-2t}$$

$$y(t) = x_1(t) = 2 - 0{,}1\, e^{-2t}$$

2.

$$\begin{cases} \dot{x}_1(t) = -x_1 + 6x_2 \\ \dot{x}_2(t) = -6x_1 - x_2 + u \\ y(t) = 37x_1 \end{cases} \begin{matrix} \longrightarrow s X_1(s) = -X_1(s) + 6X_2(s) \longrightarrow X_1(s)(1+s) = 6X_2(s) \\ \longrightarrow s X_2(s) = -6X_1(s) - X_2(s) + U(s) \rightarrow X_2(s)(s+1) = -6X_1(s) + U(s) \\ Y(s) = 37 X_1(s) \end{matrix}$$

$$X_2(s) = \frac{-6}{s+1} X_1(s) + \frac{U(s)}{s+1}$$

$$X_1(s)(1+s) = \frac{6}{s+1} U(s) - \frac{36}{s+1} X_1(s)$$

$$X_1(s)\left(1 + s + \frac{36}{s+1}\right) = \frac{6}{s-1} U(s) \longrightarrow X_1(s)\left(\frac{s+1+s^2+s+36}{s-1}\right) = \frac{6}{s-1} U(s)$$

$$X_1(s) = \frac{6}{s^2+2s+37} U(s), \quad \frac{Y(s)}{U(s)} = \frac{222}{s^2+2s+37} \Rightarrow \text{Poli a con Re} < 0\ ✓$$

$y(0),\ \dot{y}(0),\ y_\infty \qquad u(t) = \text{sca}(t)$

$$\mathcal{L}[u(t)](s) = \frac{1}{s} \qquad Y(s) = G(s)U(s) = \frac{222}{s^2+2s+37}\cdot\frac{1}{s}$$

TVI: $\displaystyle\lim_{s\to\infty} s\frac{1}{s}\frac{222}{s^2+2s+37} = \varnothing$

TVF: $\displaystyle\lim_{s\to 0} s\frac{1}{s}\frac{222}{s^2+2s+37} = 6$

TVI $\displaystyle\lim_{s\to\infty} s^2\frac{1}{\cancel{s}}\frac{222}{s^2+2s+37} = \varnothing$

2.3 Poli $\lambda_{1-2} = \dfrac{-2 \pm \sqrt{4 - 37\cdot 4}}{2} = -1 \pm 6i$

$$\omega_n = \sqrt{1+36} = \sqrt{37} = 6{,}08$$

$$\xi = \frac{1}{\omega_n} = 0{,}16 \qquad T_a = \frac{5}{\xi\omega_n} = 5\,\text{udt}$$

2.4 FdT $u \to z$

$$Z(s) = U(s)G(s) + F(s)(U(s) - Z(s)H(s))$$
$$Z(s)(1 + F(s)H(s)) = U(s)G(s) + F(s)U(s)$$
$$V(s) = \frac{Z(s)}{U(s)} = \frac{G(s) + F(s)}{1 + F(s)H(s)}$$

$$\begin{cases} \dot{x}_1(t) = -\alpha x_1 + \beta x_2 + u \\ \dot{x}_2(t) = (\beta+2)x_1 \\ y(t) = x_1 \end{cases} \qquad \alpha, \beta \in \mathbb{R}$$

1.1

$$A = \begin{bmatrix} -\alpha & \beta \\ \beta+2 & 0 \end{bmatrix} \qquad \det(\lambda I - A) = (\lambda+\alpha)\lambda - \beta(\beta+2) = \lambda^2 + \alpha\lambda - \beta(\beta+2)$$

$$\begin{cases} \alpha > 0 \\ \beta(\beta+2) < 0 \to \beta^2 + 2\beta < 0 \to -2 < \beta < 0 \end{cases}$$

$\alpha = 1 \quad \beta = -1$

$$A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \qquad \begin{aligned} sX_1(s) &= -X_1(s) - X_2(s) + U(s) \\ sX_2(s) &= X_1(s) \\ Y(s) &= X_1(s) \end{aligned}$$

$$X_2(s) = \frac{X_1(s)}{s} \to sX_1(s) + X_1(s) + \frac{X_1(s)}{s} = U(s) \to X_1(s)\left(1 + s + \frac{1}{s}\right) = U(s)$$
$$Y(s) = X_1(s) \qquad X_1(s)\left(\frac{s^2+s+1}{s}\right) = U(s)$$

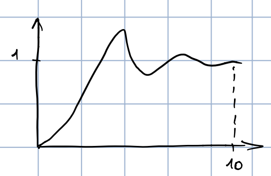
$$\frac{Y}{U} = \frac{s}{s^2+s+1}$$

Y_F
$u(t) = t$

$$Y(s) = \frac{s}{s^2+s+1} \cdot \frac{1}{s^2} = \frac{1}{s(s^2+s+1)} \qquad s^2+s+1 = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$
$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

TVI: $\lim_{s \to \infty} sY(s) = \emptyset$ $\qquad \mu = 1 \qquad T_a = 5 \cdot \frac{1}{\frac{1}{2}} = 10$

TVF: $\lim_{s \to 0} sY(s) = 1$ $\qquad$ TVI: $\lim_{s \to \infty} s(sY(s) - y(0)) = \emptyset$



1.4 $\quad u(t) = \operatorname{sen}(t) + 2\operatorname{sen}(100t)$

$T_a \simeq 10$

$$y_\infty(t) = |F(i)| \operatorname{sen}(t + \measuredangle F(i)) + 2|F(10^2 i)| \operatorname{sen}(100t + \measuredangle F(10^2 i))$$

$$F(i) = \frac{i}{-1+i+1} = 1 \qquad |F(i)| = 1 \qquad \measuredangle F(i) = \emptyset$$

$$F(10^2 i) = \frac{100 i}{-10^4 + 10^2 i + 1} \simeq -\frac{i}{10^2} \longrightarrow |F(i10^2)| \simeq \frac{1}{100} \qquad \measuredangle F(i10^2) = -\frac{\pi}{2}$$

$$y_\infty(t) = \operatorname{sen}(t) + \frac{2}{100}\operatorname{sen}\left(100t - \frac{\pi}{2}\right)$$

1)
$$\begin{cases} \dot{x}_1 = -2x_1 + u(t) \\ \dot{x}_2 = -3x_2 + 3x_3 \\ \dot{x}_3 = x_3 + u \\ y = 2x_1 \end{cases} \qquad A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad C = [2 \;\; 0 \;\; 0] \quad D = 0$$

$$x(0) = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \qquad u(t) = 2$$

1)
$$x_3(t) = e^t + \int_0^t e^{(t-\tau)} \cdot 2 \, d\tau = e^t + 2e^t \int_0^t e^{-\tau} d\tau = e^t + 2e^t(1 - e^{-t}) = e^t + 2e^t - 2 = 3e^t - 2$$

$$x_2(t) = 3e^{-3t} + \int_0^t e^{-3(t-\tau)} \cdot 3(3e^\tau - 2)\, d\tau = 3e^{-3t} + 3e^{-3t}\int_0^t e^{3\tau}(3e^\tau - 2)\, d\tau =$$

$$= 3e^{-3t} + 3e^{-3t}\left[\frac{1}{4}e^{4\tau} - \frac{2}{3}e^{3\tau}\right]_0^t = 3e^{-3t} + 3e^{-3t}\left(\frac{1}{4}e^{4t} - \frac{2}{3}e^{3t} - \frac{1}{4} + \frac{2}{3}\right) = 3e^{-3t} + \frac{3}{4}e^t - 2 - \frac{3}{4}e^{-3t} + 2e^{-3t}$$

$$= e^{-3t}\left(3 - \frac{3}{4} + 2\right) + \frac{3}{4}e^t - 2 = \frac{17}{4}e^{-3t} + \frac{3}{4}e^t - 2$$

$$x_1(t) = 2e^{-2t} + \int_0^t e^{-2(t-\tau)} \cdot 2\, d\tau = 2e^{-2t} + 2e^{-2t}\int_0^t e^{2\tau} d\tau = 2e^{-2t} + 2e^{-2t}\left[\frac{1}{2}e^{2\tau}\right]_0^t = 2e^{-2t} + 1 - e^{-2t} =$$
$$= 1 + e^{-2t}$$

$$y(t) = 2 + 2e^{-2t}$$

2)
$$\begin{cases} \dot{x} = f(x(t), u(t)) \\ y = x(t) \end{cases}$$

2.1 $\quad u(t) = 3$

$$f(\bar{x}, 3) = \emptyset \Rightarrow \bar{x}_1 = (-1, 0)$$
$$\bar{x}_2 = (1, 0)$$

3.1

$$G(s) = 2\,\frac{s + 500}{(s+100)(s^2 + 0.2s + 1)}$$

$$g = 0 \qquad \mu = \frac{2 \cdot 500}{100} = 10 \qquad z_1 = -500$$
$$p_1 = -100 \qquad p_{2-3} = \frac{-0.1 \pm \sqrt{10^{-2} - 1}}{} = -0.1 \pm j$$

$$\tau_d = \frac{1}{0.1} = 10 \qquad \omega_n = \sqrt{10^{-2} + 0.99} = 1$$
$$\xi = \frac{0.1}{1} = 0.1$$

(a) e (d) No perchè ci sono oscillazioni poiche $\xi < \frac{\sqrt{2}}{2}$

$$y_{MAX} = y_\infty\left(1 + \frac{S\%}{100}\right)$$

$$y_\infty = \text{TVF}: \lim_{s \to 0} \frac{1}{s} \cdot s \cdot G(s) = 10 \qquad S\% = 100\, e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} = 100\, e^{-} \simeq 73$$

$$y_{MAX} = y_\infty(1 + 0{,}73) = 17{,}3 \Rightarrow \text{è (b)}$$