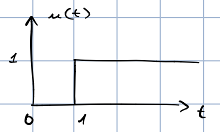
Es 1

Dato $S: \begin{cases} \dot{x}_1 = -2x_1^3 + 2x_2 \\ \dot{x}_2 = x_2 - u \\ y = x_2 \end{cases}$

① Espress. Analitica $y(t)$ con $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ e $u(t) = \text{sca}(t-1)$

Ce ne freghiamo della non linearità di x_1 perchè $y(t)$ dipende da x_2

$\begin{cases} \dot{x}_2(t) = x_2 - u \\ y(t) = x_2 \end{cases}$



a) $0 \le t < 1$
b) $t \ge 1$

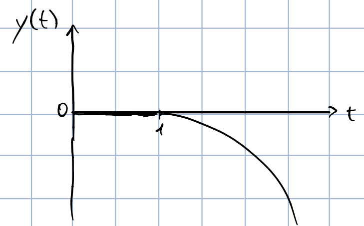
a) $u(t) = 0$ $\quad x_2(0) = 0$ $\quad \dot{x}_2 = 0$

$\Longrightarrow x_2(t) = 0 \qquad y(t) = 0 \qquad \forall t : 0 \le t < 1$

b) $x_2(t) = e^{1(t-1)} \underset{\overset{!}{=}0}{x_2(1)} + \int_1^t e^{1(t-\tau)}(-1)u(\tau)\,d\tau$

$t_0 = 1$

$x_2(t) = \int_1^t -e^{t-\tau}\,d\tau = e^t \int_1^t -e^{-\tau}\,d\tau = e^t \left[e^{-\tau} \right]_1^t = e^t(e^{-t} - e^{-1}) = 1 - e^{t-1} \qquad \forall t \ge 1$



2) Calcolare $\bar{x}$ quando $u(t) = 1 = \bar{u}$

$\begin{cases} 0 = -2\bar{x}_1^3 + 2\bar{x}_2 \\ 0 = \bar{x}_2 - \bar{u} \\ \bar{y} = \bar{x}_2 \end{cases}$ $\quad -2\bar{x}_1^3 + 2 = 0 \longrightarrow \bar{x}_1^3 = 1$

$\longrightarrow \bar{x}_2 - 1 = 0 \longrightarrow \bar{x}_2 = 1$

$\bar{y} = \bar{x}_2 = 1$

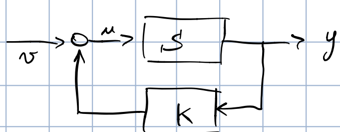
$\bar{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{u} = 1 \qquad \bar{y} = 1$

3) Stab $\bar{x}$ $\quad \delta S : \begin{cases} \delta\dot{x}_1 = -6\bar{x}_1^2\,\delta x_1 + 2\,\delta x_2 \\ \delta\dot{x}_2 = \delta x_2 - \delta u \\ \delta y = \delta x_2 \end{cases}$

$A = \begin{bmatrix} -6\bar{x}_1^2 & +2 \\ 0 & 1 \end{bmatrix}\Bigg|_{\bar{x}} = \begin{bmatrix} -6 & +2 \\ 0 & 1 \end{bmatrix}$

$\lambda_1 = -6 \qquad \lambda_2 = 1 \qquad$ INSTABILE

4) $K \in \mathbb{R}$ $\quad \bar{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



$v \in \mathbb{R}$

t.c. con $\bar{x}$ S' A.S.

$\begin{cases} \dot{x}_1 = -2x_1^3 + 2x_2 \\ \dot{x}_2 = x_2 - (v + Kx_2) = (1-K)x_2 - v \\ y = x_2 \end{cases}$ $\qquad \begin{cases} 0 = -2\bar{x}_1^3 + 2\bar{x}_2 \\ 0 = (1-K)\bar{x}_2 - \bar{v} \\ \bar{y} = \bar{x}_2 \end{cases}$ $\quad ✓ \quad \boxed{1 - K = \bar{v}}$ 1ª condizione

$y = 1$

δ - $\bar{x}_1^2\,\delta x_1 + 2\,\delta x_2$

$\delta\dot{x}_2 = (1-K)\,\delta x_2 - \delta v$

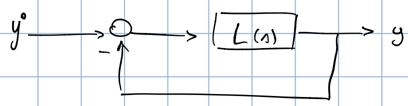
$\delta y = \delta x_2$

$A = \begin{pmatrix} -6\bar{x}_1^2 & 2 \\ 0 & 1-K \end{pmatrix}\Bigg|_{\bar{x}} = \begin{pmatrix} -6 & 2 \\ 0 & 1-K \end{pmatrix} \qquad \lambda_1 = -6 \qquad \lambda_2 = 1-K$

$1 - K < 0 \iff \boxed{K > 1}$ 2ª condiz.

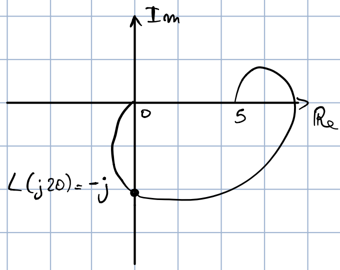
$\begin{cases} K > 1 \\ \bar{v} = 1 - K \end{cases}$ eq. AS con $\bar{x}$

Es. 2



L(s) AS STABILE

con POLI REALI



1) È AS STAB $P=0$ x BODE

AS STAB $\iff$ $\mu_L = 5 > 0$, $\varphi_m > 0$

$\omega_c = 20$ rad/s

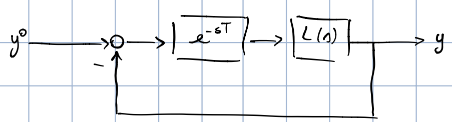
$\measuredangle L(j20) = -90°$

$\varphi_m = 180° - |-90°| = 90° > 0$ per Bode S' è AS

2) Risposta allo scalino (casa)

3) Si introduce un ritardo di $T = 0{,}1$ udt nell' anello

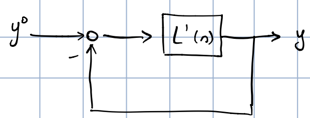
come cambia la stab. del sistema retroazionato?



$L'(s) = e^{-sT} L(s)$

$|L'(j\omega)| = |e^{-j\omega T}| \cdot |L(j\omega)| =$

$= 1 \cdot |L(j\omega)|$



Ci dice che il diagramma del modulo non cambia, e non cambia neanche ω_c $\Rightarrow$ $\omega_c = 20$

$\measuredangle(L'(j20)) = \measuredangle(e^{-j\omega_c T}) + \underbrace{\measuredangle(L(j20))}_{-90°} = -\omega_c T \frac{180°}{\pi} - 90° = -205°$

$\hookrightarrow \omega_c = 20$ rad/s

$T = 0{,}1$ udt

$\varphi_m \simeq -25° < 0$

$\Rightarrow$ INSTABILE

Es. 3

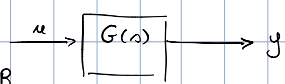
Data $G(s) = 10 \dfrac{1 - s/3}{(1 + s/0{,}1)(1 + \frac{2\xi}{100} s + \frac{s^2}{100^2})}$

III ORDINE

($\Rightarrow$ AS STAB)

$\xi <$ 1, 0,1

1) Tracciare Bode



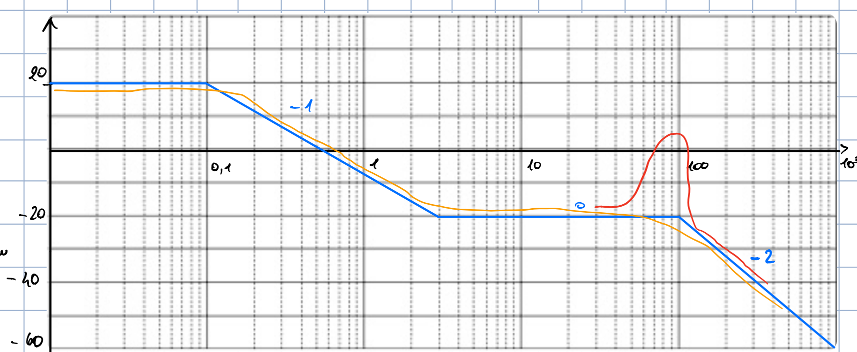
$\mu = 10$ < $|\mu| = 10 = 20$dB ; $\mu > 0$ $+0°$ FASE INIZ.

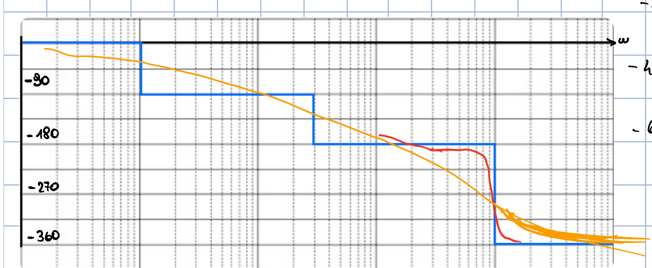
$g = 0$ < $-g = 0$ pendenza iniz ; $-90° g = 0°$ fase iniz.

$\omega_1 = 0{,}1$ rad/s POLO SX

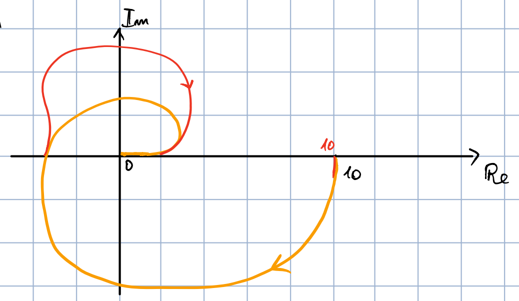
$\omega_2 = 3$ rad/s ZERO DX

$\omega_3 = 100$ rad/s 2 POLI CC SX





② D. POLARI



③ $y_\infty(t)$ $\quad u(t) = \text{sen}(0{,}01t) - \cos(10t)$

$u_1(t) = \text{sen}(0{,}01t)$ $\qquad y_{1,\infty}(t) = 1 \cdot |G(j0{,}01)| \cdot \text{sen}(0{,}01t + \measuredangle G(j0{,}01)) \cong 10\ \text{sen}(0{,}01t)$

$u_2(t) = -\cos(10t)$ $\qquad y_{2,\infty}(t) = -1\, |G(j10)| \cos(10t + \measuredangle G(j10)) \cong -\frac{1}{3} \cos(10t - \pi)$

$y_\infty(t) = 10\,\text{sen}(0{,}01t) - \frac{1}{3}\cos(10t - \pi)$

ES. 4

$G(s) = \dfrac{10}{(1+10s)^2}$ $\quad$ II ORD

$y^0 \to \bigcirc \to \boxed{R(s)} \to \boxed{G(s)} \to \bigcirc \to y$ (con disturbo d e rumore di misura m in retroazione)

1) $R(s)$ t.c.: a) $e_\infty = 0$ $\quad y^0(t) = A\,\text{sca}(t)$

b) $\omega_c \in [0{,}1\ ;\ 10]$ rad/s

c) $\varphi_m \geq 80°$

d) $d(t) = \text{sen}(0{,}01t)$ ATTENUATO DI ALMENO 100 SU $y(t)$

e) $m(t) = \text{sen}(100t)$ " " 1000 "

TRADUZIONE SPECS: $\dfrac{Y(s)}{D(s)} = \dfrac{1}{1+L(s)} = S(s)$ $\qquad |S(j\omega)| \approx \begin{cases} 1 & |L(j\omega)| \ll 1 \quad \omega \gg \omega_c \\ \dfrac{1}{|L(j\omega)|} & |L(j\omega)| \gg 1 \quad \omega \ll \omega_c \end{cases}$

$|S(j0{,}01)| = \dfrac{1}{|L(j0{,}01)|} \leq \dfrac{1}{100}$ $\qquad |L(j0{,}01)| \geq 100$

$\dfrac{Y(s)}{N(s)} = \dfrac{-L(s)}{1+L(s)} = -F(s)$ $\qquad |F(j\omega)| \approx \begin{cases} |L(j\omega)| & |L(j\omega)| \ll \quad \omega \gg \omega_c \\ 1 & |L(j\omega)| \gg 1 \quad \omega \ll \omega_c \end{cases}$

$|F(j100)| \cong |L(j100)| \leq \dfrac{1}{1000} = -60\ dB$

PROG STATICO $\quad R(s) = \dfrac{\mu_R}{s^g}$ $\quad$ a) $e_\infty = 0$ $\quad y^0(t) = A\,\text{sca}(t) \Rightarrow g = 1$ $\qquad R(s) = \dfrac{\mu_R}{s}$

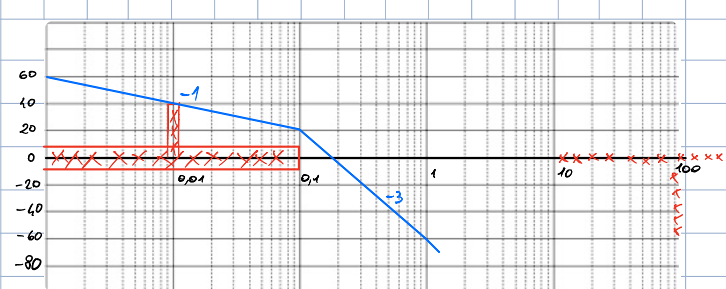
$L(s) = 10 \dfrac{\mu_R}{s} \dfrac{1}{(1+s/0{,}1)^2}$ $\qquad |L(j0{,}01)| = \dfrac{10\mu_R}{0{,}01} \cdot \dfrac{1}{\left|1+j\frac{0{,}01}{0{,}1}\right|^2} \approx 1000\,\mu_R$ $\qquad \mu_R = 0{,}1$

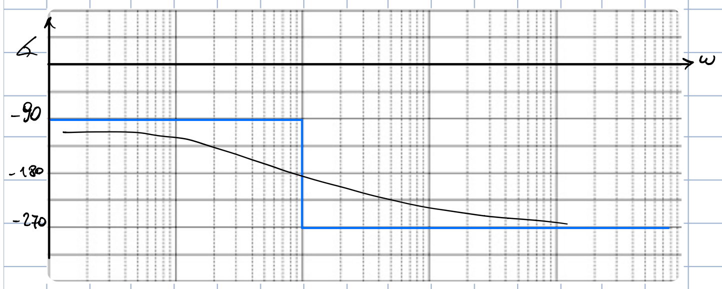
$\approx 40\ dB$

$\mu_L = 1000\,\mu_R = 100$ $\quad |\mu_L| = 40\ dB$ $\quad \mu_L > 0$

$g_L = 1$

$\omega_1 = 0{,}1$ rad/s $\qquad$ 2 POLI SX





$\omega_c = 0{,}2$ rad/s

$\measuredangle L(j0{,}2) = -90° - 2\,tg^{-1}\left(\frac{0{,}2}{0{,}1}\right) \approx -216°$

$\varphi_m \approx -36° < 0$

PROGETTO DIN. $R(s) = \frac{0{,}1}{s}$ $\quad L(s) = \frac{1}{s}\frac{1}{(1+s/0{,}1)^2}$ $\quad \omega_c = 1$ rad/s

$R(s) = \frac{0{,}1}{s}\left(1+s/0{,}1\right)^2$ $\quad \measuredangle L(j1) = -90° - tg^{-1}\left(\frac{1}{10}\right) \approx -96°$ $\quad \varphi_m \approx 84° \;(>80°)$

$L(s) = \frac{1}{s}\frac{1}{1+s/10}$

È realizzabile con un PID reale ?

$R(s) = \frac{0{,}1}{s}\frac{(1+s/0{,}1)^2}{(1+s/10)}$

$$PID_{Re}(s) = K_p\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+s\frac{T_d}{N}}\right) = \frac{K_p}{T_i}\frac{1}{s}\frac{s^2\left(T_iT_d+T_i\frac{T_d}{N}\right)+s\left(T_i+\frac{T_d}{N}\right)+1}{1+s\frac{T_d}{N}}$$

$$= 0{,}1\,\frac{1}{s}\,\frac{1+20s+100s^2}{1+0{,}1s}$$

$\frac{T_d}{N} = 0{,}1 \qquad \frac{K_p}{T_i} = 0{,}1 \qquad T_i + \frac{T_d}{N} = 20$

$T_i = 20 - \frac{T_d}{N} = 19{,}9 \qquad K_p = 0{,}1\,T_i = 1{,}99 \qquad T_iT_d + T_i\frac{T_d}{N} = 100$

$T_d = \frac{1}{T_i}\left(100 - T_i\frac{T_d}{N}\right) = 4{,}925 \qquad N = \frac{T_d}{0{,}1} = 49{,}25$

3) Attuatore è limitato tra [-10, 10] disegnare lo schema di controllo in conf. anti windup e con derivazione uscita