Es.1

$$S: \begin{cases} \dot{x}_1 = x_1(x_2-1) + u - 1 \\ \dot{x}_2 = x_2 + x_1 u \\ y = x_1 \end{cases}$$

① Calcolare gli stati di eq. $\quad u(t) = \bar{u} = 1 \quad t \geq 0$

$$\begin{cases} 0 = \bar{x}_1(\bar{x}_2-1) + \bar{u} - 1 \\ 0 = \bar{x}_2 + \bar{x}_1 \bar{u} \\ \bar{y} = \bar{x}_1 \end{cases} \qquad \begin{cases} \bar{x}_1(\bar{x}_2-1) = 0 \\ \bar{x}_1 + \bar{x}_2 = 0 \\ \bar{y} = \bar{x}_1 \end{cases} \qquad \begin{cases} \bar{x}_2 = -\bar{x}_1 \\ \bar{x}_1(-\bar{x}_1 - 1) = 0 \\ y = \bar{x}_1 \end{cases} \qquad \begin{matrix} \bar{x}_2 = -\bar{x}_1 = 0 \\ \bar{x}_1 = -\bar{x}_2 = -1 \end{matrix}$$

eq.1 $\bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad$ eq.2 $\bar{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Diagramma lineare tangente

② Studiare la stabilità di entrambi gli eq. (sistema ordine 2, no metodo grafico)

Basta calcolare A

$\delta x = x - \bar{x} \qquad \delta u = u - \bar{u} \qquad \delta y = y - \bar{y}$

$$\delta S: \begin{cases} \delta \dot{x}_1 = \frac{\partial f_1}{\partial x_1}\Big|_{\bar{x},\bar{u}} \delta x_1 + \left(\frac{\partial}{\partial x_2}\Big|_{\bar{x},\bar{u}} f_1\right) \delta x_2 + \frac{\partial}{\partial u} f_1\Big|_{\bar{x},\bar{u}} \delta u = (\bar{x}_2 - 1)\delta x_1 + \bar{x}_1 \delta x_2 + 1 \\ \delta \dot{x}_2 = \frac{\partial}{\partial x_1} f_2\Big|_{\bar{x},\bar{u}} \delta x_1 + \frac{\partial}{\partial x_2} f_2\Big|_{\bar{x},\bar{u}} \delta x_2 + \frac{\partial}{\partial u} f_2\Big|_{\bar{x},\bar{u}} \delta u = \delta x_1 + \delta x_2 + \bar{x}_1 \delta u \\ \delta y = \frac{\partial g}{\partial x_1}\Big|_{\bar{x},\bar{u}} \delta x_1 + \frac{\partial}{\partial x_2} g\Big|_{\bar{x},\bar{u}} \delta x_2 + \frac{\partial}{\partial u} g\Big|_{\bar{x},\bar{u}} \delta u = \delta x_1 \end{cases}$$

Modello lineare tangente

$$\delta S_1: \begin{cases} \delta \dot{x}_1 = -\delta x_1 + \delta u \\ \delta \dot{x}_2 = \delta x_1 + \delta x_2 \\ \delta y = \delta x_1 \end{cases} \qquad \delta S_2: \begin{cases} \delta \dot{x}_1 = -\delta x_2 + \delta u \\ \delta \dot{x}_2 = \delta x_1 + \delta x_2 - \delta u \\ \delta y = \delta x_1 \end{cases}$$

$$A_1 = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \qquad \lambda I - A_2 = \begin{pmatrix} \lambda & 1 \\ -1 & \lambda - 1 \end{pmatrix}$$

$\lambda_1 = -1 \quad \underline{\lambda_2 = 1} \leftarrow$ instabile

equilibrio instabile

$\det = \lambda(\lambda-1) + 1 = \lambda^2 - \lambda + 1 = 0$

$\lambda = \dfrac{1 \pm \sqrt{1-4}}{2}$ $\qquad$ instabile: c'è almeno una $Re > 0$

Es.2

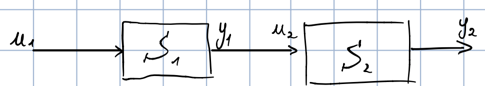
$$S_1: \begin{cases} \dot{x}_1 = u_1 \\ y_1 = x_1 + u_1 \end{cases} \qquad S_2: \begin{cases} \dot{x}_2 = x_2 + u_2 \\ y_2 = 2x_2 \end{cases}$$

② Studiare la stabilità dei sistemi:

$S_1: \ \lambda_1 = 0$ $\qquad$ $S_2: \ \lambda_2 = 1$

Semplicemente stabile $\qquad$ Instabile

② Studiare la stab. di S $\qquad$ (Connessione in cascata o in serie)



$u_2 = y_1$

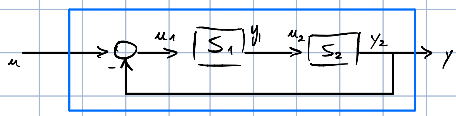
$$S: \begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = x_2 + u_2 = x_2 + y_1 = x_2 + x_1 + u_1 \\ y_1 = x_1 + u_1 \\ y_2 = 2x_1 \\ u_2 = y_1 \end{cases}$$

$$\Rightarrow S: \begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = x_2 + x_1 + u_1 \\ y_2 = 2x_2 \end{cases} \qquad A = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \qquad \lambda_1 = 0 \qquad \lambda_2 = 1$$

Instabile

NOTA: gli autovalori sono gli stessi dei sistemi di partenza sempre quando sono connessi in cascata

③ Stabil. di S



Retroaz. neg.

$u_1 = u - y_2$

$y = y_2$

$u_2 = y_1$

$y_2 = 2x_2$

$\dot{x}_2 = x_2 + u_2$

$\dot{x}_1 = u_1$

$y_1 = x_1 + u_1$

$u_1 = u - 2x_2$

$y_1 = x_1 + u - 2x_2 = u_2$

$y = 2x_2$

$\dot{x}_1 = -2x_2 + u$

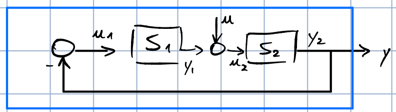
$\dot{x}_2 = x_1 - x_2 + u$

$y = 2x_2$

$A = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix}$ $\quad \det(\lambda I - A) = \lambda(\lambda+1) + 2 = \lambda^2 + \lambda + 2 = 0$

S: AS. STABILE

④

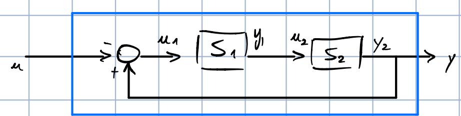


$u_1 = -y_2$

La stabilità è indipendente dall' ingresso, quindi in questo caso le proprietà di stabilità non variano

X CASA studiarlo

⑤



La stab. cambia in questo caso perchè y_2 è cambiato di segno: cambiano le informazioni su come varia il sistema

$\dot{x}_1 = u_1 \longrightarrow \dot{x}_1 = 2x_2 - u$

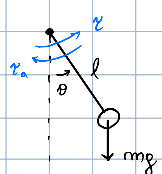
$\dot{x}_2 = x_2 + u_2 \longrightarrow \dot{x}_2 = x_1 + 3x_2 - u$

$u_1 = y_2 - u \longrightarrow y = 2x_2$

$y_2 = 2x_2$

$A = \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}$ $\quad \lambda_1 = 0$, $\lambda_2 = 3$ $\quad S$ instabile

Es 4. Pendolo



① Modello in spazio di stato

$J\ddot{\theta} = \tau - \tau_a - mgl\,\mathrm{sen}(\theta)$

$ml^2\ddot{\theta} = \tau - K\dot{\theta} - mgl\,\mathrm{sen}(\theta)$

$x_1 = \theta$

$x_2 = \dot{\theta}$

$u = \tau$

$y = \theta$

$$P: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{g}{l}\,\mathrm{sen}(x_1) - \frac{K}{ml^2}\,x_2 + \frac{1}{ml^2}\,u \\ y = x_1 \end{cases}$$

② Calcolare stati di eq. per $v(t) = \bar{u} = 0$ $\quad \forall t \geq 0$

$$\begin{cases} 0 = \bar{x}_2 \\ 0 = -\frac{g}{l}\,\mathrm{sen}(\bar{x}_1) - \cancel{\frac{K}{ml^2}\bar{x}_2} + \cancel{\frac{1}{ml^2}\bar{u}} \end{cases}$$

$\bar{x}_1 = 0$ $\quad \bar{x}_2 = \pi$

Eq 1: $\bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad$ Eq 2: $\bar{x} = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$

③ Stabilità di Eq_1, Eq_2

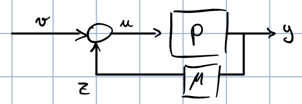
$$\delta P: \begin{cases} \delta\dot{x}_1 = 1 \cdot \delta x_2 \\ \delta\dot{x}_2 = -\frac{g}{l}\cos(\bar{x}_1)\,\delta x_1 - \frac{K}{ml^2}\,\delta x_2 + \frac{1}{ml^2}\,\delta u \\ \delta y = \delta x_1 \end{cases}$$

$\delta P: \begin{pmatrix} 0 & 1 \\ -\frac{g}{l}\cos(\bar{x}_1) & -\frac{K}{ml^2} \end{pmatrix}$

$A_1 = \begin{pmatrix} 0 & 1 \\ -\frac{g}{l} & -\frac{K}{ml^2} \end{pmatrix}$ $\quad A_2 = \begin{pmatrix} 0 & 1 \\ \frac{g}{l} & \frac{K}{ml^2} \end{pmatrix}$

$\det(\lambda I - A_2) = \lambda\left(\lambda - \frac{K}{ml^2}\right) + \frac{g}{l} = \lambda^2 - \lambda\frac{K}{ml^2} - \frac{g}{l}$ $\quad$ INSTABILE

$\det(\lambda I - A_1) = \lambda\left(\lambda + \frac{K}{ml^2}\right) + \frac{g}{l} = \lambda^2 + \overset{>0}{\frac{K}{ml^2}}\lambda + \overset{>0}{\frac{g}{l}} = 0$ $\quad$ A.S. EQ 1

④ 

$\mu, v = ?$ $\quad \bar{x} = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$ sia ancora eq. e che sia A.S.

$z = \mu \cdot y$

$$S: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{g}{l} \operatorname{sen}(x_1) - \frac{K}{ml^2} x_2 + \frac{1}{ml^2} \overbrace{(v + z)}^{u} = -\frac{g}{l} \operatorname{sen}(x_1) - \frac{K}{ml^2} x_2 + \frac{1}{ml_2} v + \frac{1}{ml^2} \mu x_1 \\ y = x_1 \end{cases}$$

Calcoliamo eq. di S e imponiamo che siano uguali a $\begin{pmatrix} \pi \\ 0 \end{pmatrix}$

$$\begin{cases} x_2 = 0 \\ 0 = -\frac{g}{l} \operatorname{sen}(\bar{x}_1) + \frac{1}{ml^2} v + \frac{\mu}{ml^2} \bar{x}_1 \end{cases} \underset{\bar{x}_1 = \pi}{\Longrightarrow} \emptyset + \frac{1}{ml^2} \bar{v} + \frac{\mu}{ml^2} \pi = 0 \rightsquigarrow \bar{v} = -\mu \pi$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cos(\bar{x}_1) + \frac{\mu}{ml^2} & -\frac{K}{ml^2} \end{bmatrix} \qquad \bar{x} = \begin{pmatrix} \pi \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & 1 \\ \frac{g}{l} + \frac{\mu}{ml^2} & -\frac{K}{ml^2} \end{pmatrix}$$

$$\det(\lambda I - A) = \lambda^2 + \frac{K}{ml^2} \lambda \underbrace{- \frac{g}{l} - \frac{\mu}{ml^2}}_{} = 0$$

Per avere $Re < 0$ basta scegliere questo > 0

$-\frac{g}{l} - \frac{\mu}{ml^2} > 0 \qquad -mgl - \mu > 0 \qquad \mu < -mgl$

$\mathcal{L}[a\, v(t)](s) = a\, \mathcal{L}[v(t)](s)$

$\mathcal{L}[v_1(t) + v_2(t)](s) = \mathcal{L}[v_1(t)](s) + \mathcal{L}[v_2(t)](s)$

$e^{i\omega t} = \cos(\omega t) + i \operatorname{sen}(\omega t)$

$\mathcal{L}[e^{pt}](s) = \frac{1}{s-p}$

ti piace?

(La calligrafia di federico haag)

$v(t) = \operatorname{sen}(\omega t) \qquad V(s) = \mathcal{L}[v(t)](s)$

$$v(t) = \operatorname{sen}(\omega t) = \frac{e^{i\omega t} - e^{-i\omega t}}{2i} = \frac{\cos(\omega t) + i \operatorname{sen}(\omega t) - \cos(\omega t) + i \operatorname{sen}(\omega t)}{2i} = \operatorname{sen}(\omega t)$$

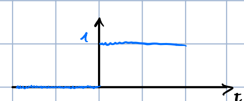
$$\mathcal{L}[\operatorname{sen}(\omega t)](s) = \mathcal{L}\left[\frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)\right] = \frac{1}{2i}\left[\mathcal{L}[e^{i\omega t}](s) - \mathcal{L}[e^{-i\omega t}](s)\right] =$$

$$= \frac{1}{2i}\left[\frac{s + i\omega - (s - i\omega)}{s^2 + \omega^2}\right] = \frac{1}{2i}\left[\frac{2i\omega}{s^2 + \omega^2}\right] = \frac{\omega}{s^2 + \omega^2} = \mathcal{L}[\operatorname{sen}(\omega t)](s)$$

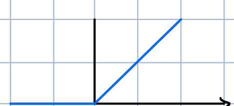
X CASA $\quad \mathcal{L}[\cos(\omega t)]$

④ $\mathcal{L}[\dot{v}(t)](s) = s\,\mathcal{L}[v(t)](s) - v(0)$

⑤ $\mathcal{L}[sca(t)](s) = \frac{1}{s}$



$$w(t) = ram(t) = \begin{cases} 0 & t < 0 \\ t & t > 0 \end{cases}$$



$$\dot{w}(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} = sca(t)$$

$$\mathcal{L}[\dot{w}(t)](s) = s\,\mathcal{L}[w(t)](s) - \overset{0}{\overset{\shortparallel}{w(s)}}$$

$$\mathcal{L}[\dot{w}(t)](s) = \mathcal{L}[sca(t)](s) = \frac{1}{s} \Longrightarrow \frac{1}{s} = s\,\mathcal{L}[w(t)](s) \longrightarrow \mathcal{L}[w(t)] = \frac{1}{s^2}$$

• $z(t) = \begin{cases} 0 & t < 0 \\ t^2/2 & t \geq 0 \end{cases} \qquad \mathcal{L}[z(t)](s) = ? \qquad \dot{z}(t) = w(t) = ram(t) \qquad \mathcal{L}[z(t)] = \frac{1}{s^3}$

- $y(t) = \begin{cases} 0 \\ \dfrac{t^K}{K!} & t \geq 0 \end{cases}$ $\mathcal{L}[y(t)](s) = \dfrac{1}{s^{K+1}}$