1.2

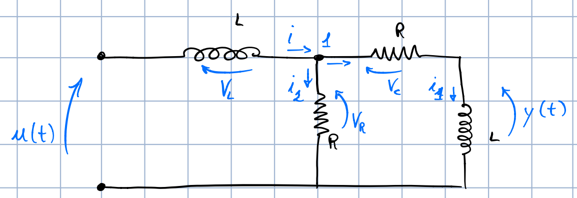
$$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 3x_2(t)\,(1+\alpha x_2(t)) + u(t) \\ \dot{x}_2(t) = -x_1(t) + x_2(t) \\ y(t) = x_1(t) + 3u(t) \end{cases} \qquad \alpha \in \mathbb{R}$$

1) SISO: ingresso $u(t)$, uscita $y(t)$
2) Dinamico ordine 2
3) Proprio (compare $u(t)$ in $y(t)$)
4) Non lineare ($(x_2)^2$) se $\alpha \neq 0$

Per $\alpha = 0$:

$$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 3x_2(t) + u(t) \\ \dot{x}_2(t) = -x_1(t) + x_2(t) \\ y(t) = x_1(t) + 3u(t) \end{cases} \Rightarrow \begin{aligned} \dot{X}(t) &= \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) \\ Y(t) &= \begin{pmatrix} 1 & 0 \end{pmatrix} x(t) + (3)\,u(t) \end{aligned}$$

1.2



KCL #1: $i(t) - i_1(t) - i_2(t) = 0$

$y(t) = L\dfrac{di_1(t)}{dt}$ $\qquad V_L = L\dfrac{di(t)}{d}$

KVL 1: $L\dfrac{di}{dt} = u(t) - R i_2$ $\qquad i_2 = i - i_1$

KVL 2: $L\dfrac{di_1}{dt} = Ri_2 - Ri_1 \longrightarrow \begin{cases} L\dfrac{di_1}{dt} = Ri - 2Ri_1 \\ L\dfrac{di}{dt} = u(t) + Ri_1 - Ri \end{cases}$ $\qquad x_1(t) = i_1(t) \qquad x_2(t) = i(t)$

$$\begin{cases} \dot{x}_1 = R/L\; x_2(t) - 2R/L\; x_1(t) \\ \dot{x}_2 = R/L\; x_1(t) - R/L\; x_2(t) + 1/L\; u(t) \end{cases}$$

$y = L\dot{x}_1(t)$

$A = \begin{bmatrix} -\frac{2R}{L} & \frac{R}{L} \\ R/L & -\frac{R}{L} \end{bmatrix} \qquad B = \begin{pmatrix} 0 \\ 1/L \end{pmatrix}$

$C = \begin{pmatrix} -2R & R \end{pmatrix} \qquad D = 0$

- Strett. proprio
- lineare

3) $L = 1 \quad R = 1 \quad u(t) = 1 \quad t \geq 0$

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + x_2(t) \\ \dot{x}_2(t) = x_1(t) - x_2(t) + u(t) \end{cases}$$

$y = -2x_1(t) + x_2(t)$

$$\begin{cases} -2x_1(t) + x_2(t) = 0 \\ x_1(t) - x_2(t) + 1 = 0 \end{cases} \quad \begin{cases} x_2(t) = 2x_1(t) \\ x_1(t) - 2x_1(t) + 1 = 0 \end{cases} \quad \begin{cases} x_1(t) = 1 \\ x_2(t) = 2 \end{cases}$$

$y(t) = -2 + 2 = 0$

1.3

$$\begin{cases} 1° \text{ carrello}: & m_1 \ddot{p}_1 = F_1 - K_1 p_1(t) + K_2 (p_2(t) - p_1(t)) \\ 2° \quad '' \quad : & m_2 \ddot{p}_2 = F_2 - K_2 (p_2(t) - p_1(t)) \end{cases}$$

1) Sistema MIMO: 2 ing: F_1, F_2 Dinamico ordine 4

2 usc: p_1, p_2

$$\begin{cases} F_1(t) = u_1(t) \\ F_2(t) = u_2(t) \end{cases} \qquad \begin{cases} y_1 = p_1(t) \\ y_2 = p_2(t) \end{cases} \qquad \begin{aligned} x_1 &= p_1(t) \\ x_2 &= \dot{p}_1(t) \\ x_3 &= p_2(t) \\ x_4 &= \dot{p}_2(t) \end{aligned}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_3 = x_4 \\ m_1 \dot{x}_2 = u_1(t) - K_1 x_1(t) + K_2 x_3(t) - K_2 x_1(t) \\ m_2 \dot{x}_4 = u_2(t) - K_2 (x_3(t) - x_1(t)) \end{cases}$$

$$\begin{cases} \dot{x}_2 = -\frac{1}{m_1} x_1(t) (K_1 + K_2) + \frac{K_2}{m_1} x_3(t) + \frac{1}{m_1} u_1(t) \\ \dot{x}_4 = +\frac{K_2}{m_2} x_1(t) - \frac{K_2}{m_2} x_3(t) + \frac{1}{m_2} u_2(t) \end{cases}$$

$$\begin{cases} y_1 = x_1(t) \\ y_2 = x_3(t) \end{cases}$$ Strettamente proprio

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(K_1+K_2)}{m_1} & 0 & \frac{K_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{m_2} & 0 & -\frac{K_2}{m_2} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3) $m_1 = 1$ $m_2 = 2$ $K_1 = K_2 = 1$ $F_1(t) = 0$ $F_2(t) = 1$

$$\begin{cases} \dot{x}_2 = -2x_1(t) + x_3(t) + u_1(t) \\ \dot{x}_4 = \frac{1}{2} x_1(t) - \frac{1}{2} x_3(t) + \frac{1}{2} u_2(t) \end{cases} \qquad \begin{cases} u_1 = 0 \\ u_2 = 1 \end{cases}$$

$$\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{pmatrix} \qquad \begin{cases} -2\bar{x}_1 + \bar{x}_3 = 0 \\ \frac{1}{2}\bar{x}_1 - \frac{1}{2}\bar{x}_3 + \frac{1}{2} = 0 \end{cases} \qquad \begin{cases} 2\bar{x}_1 - 2\bar{x}_3 + 2 = 0 \\ 2\bar{x}_1 = \bar{x}_3 \end{cases} \rightarrow -\bar{x}_3 + 2 = 0 \rightarrow \bar{x}_3 = 2$$

$$\bar{x}_1 = 1$$

$$\bar{x}_2 = \bar{x}_4 = 0$$

2.1

$u(t) = F(t)\ \hat{x}$ $\qquad u(t) = M\ddot{s}(t)$

$y(t) = s(t)$ $\qquad \dot{s} = \dot{x}_1$ $\qquad$ II ordine

$F_{att}(t) = -\alpha \dot{s}(t)\ \hat{x} =$ $\qquad \dot{x}_2 = \ddot{x}_1$

$$\begin{cases} M\ddot{s} = F(t) - F_{att} \\ y(t) = s(t) \end{cases} \qquad \begin{cases} F(t) = u(t) \\ F_{att} = \alpha \dot{x}_1 \\ y(t) = x_1 \end{cases}$$

$x_2 = \dot{x}_1$

$M\dot{x}_2 = u(t) - \alpha\dot{x}_1$

$$\begin{cases} \dot{x}_2 = -\frac{\alpha}{M}x_2 + \frac{u(t)}{M} \\ \dot{x}_1 = \dot{s} = x_2(t) \\ y(t) = x_1 \end{cases} \qquad A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\alpha}{M} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/M \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0 \qquad \underline{\text{strett. proprio}}$$

3) Strett. proprio, lineare, II ordine, dinamico

4) $\alpha = 3$, $M = 1$ $\qquad x_L = ?$ $\qquad x(t_0) = x_0$

$x_L = e^{A(t-t_0)}x(t_0)$ $\qquad A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$ $\qquad \lambda_1 = 0$, $\lambda_2 = -3$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) \longrightarrow \begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = -3x_2(t) + u(t) \end{cases}$$

$$y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$u(t) \rightarrow \boxed{\dot{x}_2} \rightarrow \boxed{\int} \xrightarrow{x_2(t)} \boxed{\dot{x}_1} \rightarrow \boxed{\int} \xrightarrow{x_1(t)} \boxed{y} \rightarrow y(t)$

$$x_2(t) = \int -3x_2(t) + u(t)\ dt \Big|_{u(t)=0,\ x_2(t_0)=x_{2,0}} = e^{-3t}x_{2,0}, \quad t \geq 0$$

$$\dot{x}_1(t) = x_2(t) \implies x_1(t) = e^{0t}x_{1,0} + \int_{t_0}^{t} e^{0(t-\tau)}x_2(\tau)\ d\tau$$

$x_1(t_0) = x_{1,0}$

$$= x_{1,0} + \int_0^t e^{-3\tau}x_{2,0}\ d\tau = x_{1,0} + \left[-\frac{1}{3}e^{-3\tau}\right]_0^t x_{2,0} = x_{1,0}\left(-\frac{1}{3}e^{-3t} + \frac{1}{3}\right)x_{2,0} \checkmark$$

Mov. libero:

$$\begin{cases} x_1(t) = x_{1,0} + \frac{1}{3}\left(1 - e^{-3t}\right)x_{2,0} \\ x_2(t) = e^{-3t}x_{2,0} \end{cases} \longrightarrow x(t) = \begin{bmatrix} 1 & -\frac{1}{3}(e^{-3t} - 1) \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} x_{1,0} \\ x_{2,0} \end{bmatrix}, \ t \geq 0$$

$$y_L = x_1(t) = x_{1,0} + \frac{1}{3}x_{2,0} - \frac{1}{3}e^{-3t}x_{2,0} \quad t \geq 0$$

5) $x_F = \int_{t_0}^{t} e^{A(t-\tau)}B\bar{u}(\tau)\ d\tau$ $\qquad u(t) = \bar{u} \quad \forall t \geq 0$

$$x_{2,F}(t) = \int_0^t e^{-3(t-\tau)}\bar{u}\ d\tau = e^{-3t}\bar{u}\int_0^t e^{3\tau}d\tau = \frac{1}{3}e^{-3t}\bar{u}\left[e^{3\tau}\right]_0^t = \frac{1}{3}e^{-3t}\bar{u}\left(e^{3t} - 1\right) = \frac{1}{3}\bar{u}\left(1 - e^{-3t}\right), \quad t \geq 0$$

$$x_{1,F}(t) = \int_0^t \frac{1}{3}\bar{u}\left(1 - e^{-3\tau}\right)d\tau = \frac{1}{3}\bar{u}\int_0^t 1 - e^{-3\tau}\ d\tau = \frac{1}{3}\bar{u}\left[\tau + \frac{1}{3}e^{-3\tau}\right]_0^t = \frac{1}{3}\bar{u}\left(t + \frac{1}{3}e^{-3t} - \frac{1}{3}\right), \quad t \geq 0$$

$$\begin{cases} x_{1,F} = \frac{1}{3}\bar{u}\left(t + \frac{1}{3}e^{-3t} - \frac{1}{3}\right), \ t \geq 0 \\ x_{2,F} = \frac{1}{3}\bar{u}\left(1 - e^{-3t}\right), \ t \geq 0 \end{cases} \qquad y_F = x_{1,F} = \frac{1}{3}\bar{u}\left(t + \frac{1}{3}e^{-3t} - \frac{1}{3}\right)$$

6) $x(t) = x_L(t) + x_F(t)$, $\quad t \geq 0$

$y(t) = y_L(t) + y_F(t)$, $\quad t \geq 0$

2.2

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + x_2(t) + u(t) \\ \dot{x}_2(t) = -3x_2(t) + u(t) \\ y(t) = x_2(t) \end{cases}$$

1) $x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $u(t) = 0 \quad t \geq 0$ $\qquad x(t) = ?$

$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = [0 \quad 1] \quad D = \emptyset$

$x_2(t) = e^{-3t}$

$x_1(t) = e^{-2t} \cdot \emptyset + \int_0^t e^{-2(t-\tau)} \cdot e^{-3\tau} \, d\tau = e^{-2t} \int_0^t e^{-\tau} \, d\tau = e^{-2t} \left[-e^{-\tau} \right]_0^t = e^{-2t} (-e^{-t} + 1) = e^{-2t} - e^{-3t}$

Modo dominante: e^{-2t}

$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad T^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \quad TAT^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix}$

$e^{At} = T^{-1} e^{A_d t} T = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{-3t} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} e^{-2t} & e^{-3t} \\ 0 & -e^{-3t} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} e^{-2t} & e^{-2t} - e^{-3t} \\ 0 & e^{-3t} \end{pmatrix}$

$x(t) = e^{At} x(0) = \begin{pmatrix} e^{-2t} & e^{-2t} - e^{-3t} \\ 0 & e^{-3t} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} e^{-2t} - e^{-3t} \\ e^{-3t} \end{pmatrix}, \quad t \geq 0$

2.3

KVL: $v(t) - v_R(t) - v_L(t) - v_C(t) = \emptyset \qquad v_C(t) = y(t) \qquad v(t) = u(t)$

$v_R(t) = R i(t) \qquad v_L = L \frac{di(t)}{dt}$

$\begin{cases} u(t) - R i(t) - L \frac{di(t)}{dt} - y(t) = \emptyset \\ i(t) = C \frac{dv_C}{dt} \end{cases} \qquad \begin{cases} \frac{dv_C}{dt} = \frac{1}{C} i(t) \\ \frac{di(t)}{dt} = \frac{1}{L} v_L \end{cases} \qquad v_L(t) = v(t) - v_R(t) - v_C(t) \overset{!}{=} v(t) - R i(t) - v_C(t)$

$\frac{di(t)}{dt} = \frac{v(t)}{L} - \frac{R}{L} i(t) - \frac{1}{L} v_C(t) \longrightarrow \begin{matrix} x_1(t) = i(t) \\ x_2(t) = v_C(t) \end{matrix} \qquad \begin{cases} \dot{x}_1 = \frac{1}{L} (u(t) - R x_1(t) - x_2(t)) \\ \dot{x}_2 = \frac{1}{C} x_1(t) \\ y(t) = x_2(t) \end{cases}$ Lineare II ordine

2. $R = 4 \quad C = 1 \quad L = 3$

$y(t)\big|_{u(t)=0} \qquad \left. \begin{matrix} v_C(0) = 2 \\ i(0) = 0 \end{matrix} \right\} \; x(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$x_{2,L} = e^{A(t-t_0)} x(0) \qquad A = \begin{pmatrix} -\frac{4}{3} & -\frac{1}{3} \\ 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad C = (0 \quad 1) \quad D = \emptyset$

$\lambda I - A = \begin{pmatrix} \lambda + \frac{4}{3} & \frac{1}{3} \\ -1 & \lambda \end{pmatrix} \xrightarrow[\det]{} = \lambda \left(\lambda + \frac{4}{3} \right) + \frac{1}{3} = \lambda^2 + \frac{4}{3} \lambda + \frac{1}{3} = 0 \qquad \lambda_{1,2} = \frac{-\frac{4}{3} \pm \sqrt{\frac{16}{9} - \frac{4}{3}}}{2} \rightsquigarrow \frac{-\frac{4}{3} \pm \sqrt{\frac{16-12}{9}}}{2} = \frac{-\frac{4}{3} \pm \frac{2}{3}}{2} = \begin{cases} -1 \\ -\frac{1}{3} \end{cases}$

$\lambda_1 = -1 \longrightarrow Av = -v \qquad \begin{cases} -\frac{4}{3} \alpha - \frac{1}{3} \beta = -\alpha \\ \alpha = -\beta \end{cases} \qquad \begin{cases} \alpha = -\beta \\ \frac{4}{3} \beta - \frac{1}{3} \beta = \beta \end{cases} \qquad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lambda_2 = -\frac{1}{3} \qquad \begin{cases} -\frac{4}{3} \alpha - \frac{1}{3} \beta = -\frac{1}{3} \alpha \\ \alpha = -\frac{1}{3} \beta \end{cases} \qquad \begin{cases} -\alpha = \frac{1}{3} \beta \\ \frac{4}{9} \beta - \frac{1}{3} \beta = \frac{1}{9} \beta \end{cases} \qquad \begin{cases} 4\beta - 3\beta = \beta \\ \alpha = -\frac{1}{3} \beta \end{cases} \qquad \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$T^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & -3 \end{pmatrix} \qquad T = \frac{1}{-2} \begin{pmatrix} -3 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix} \qquad e^{At} = T^{-1} e^{A_d t} T = T^{-1} \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-\frac{1}{3}t} \end{pmatrix} T$

$e^{At} = \begin{pmatrix} e^{-t} & e^{-\frac{1}{3}t} \\ -e^{-t} & -3 e^{-\frac{1}{3}t} \end{pmatrix} \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} e^{-t} + \frac{1}{2} e^{-\frac{1}{3}t} & \frac{1}{2} e^{-t} - \frac{1}{2} e^{-\frac{1}{3}t} \\ -\frac{3}{2} e^{-t} - \frac{3}{2} e^{-\frac{1}{3}t} & -\frac{1}{2} e^{-t} + \frac{3}{2} e^{-\frac{1}{3}t} \end{pmatrix}$

$y_L = (0 \quad 1) \, e^{At} x(0) = \begin{pmatrix} -\frac{3}{2} e^{-t} - \frac{3}{2} e^{-\frac{1}{3}t} & -\frac{1}{2} e^{-t} + \frac{3}{2} e^{-\frac{1}{3}t} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = -e^{-t} + 3 e^{-\frac{1}{3}t}$

2.4

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$A = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda+1 & -2 \\ 2 & \lambda+1 \end{bmatrix} \qquad \chi_A = 0 \Rightarrow (\lambda+1)^2 + 4 = \lambda^2 + 1 + 2\lambda + 4 =$$

$$= \lambda^2 + 2\lambda + 5 \qquad \lambda_{1,2} = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2j$$

$$A \cdot v = (-1+2j)v$$

$$\begin{cases} -\alpha + 2\beta = (-1+2j)\alpha \\ -2\alpha - \beta = (-1+2j)\beta \end{cases} \longrightarrow \begin{cases} \cancel{-\alpha} + 2\beta = \cancel{-\alpha} + 2\alpha j \\ -2\alpha \cancel{-\beta} = \cancel{-\beta} + 2\beta j \end{cases} \begin{matrix} \rightarrow \beta = \alpha j \\ \rightarrow \alpha = -\beta j \end{matrix} \qquad \begin{pmatrix} 1 \\ j \end{pmatrix}$$

$$A \cdot v = (-1-2j)v$$

$$\begin{cases} -\alpha + 2\beta = (-1-2j)\alpha \\ -2\alpha - \beta = (-1-2j)\beta \end{cases} \qquad \begin{cases} \cancel{-\alpha} + 2\beta = \cancel{-\alpha} - 2\alpha j \\ -\cancel{2}\alpha \cancel{-\beta} = \cancel{-\beta} - \cancel{2}\beta j \end{cases} \qquad \begin{cases} \beta = -\alpha j \\ \alpha = \beta j \end{cases} \qquad \begin{pmatrix} j \\ 1 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix} \qquad \det T^{-1} = 1 + 1 = 2 \qquad T = \frac{1}{2}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & -j/2 \\ -j/2 & 1/2 \end{pmatrix}$$

$$e^{At} = T^{-1}\begin{pmatrix} e^{(-1+2j)t} & 0 \\ 0 & e^{(-1-2j)t} \end{pmatrix} T = \begin{pmatrix} e^{(-1+2j)t} & j e^{(-1-2j)t} \\ j e^{(-1+2j)t} & e^{(-1-2j)t} \end{pmatrix}\begin{pmatrix} 1/2 & -j/2 \\ -j/2 & 1/2 \end{pmatrix}$$

$$x_l = e^{At}x(0) = \begin{pmatrix} e^{(-1+2j)t} & j e^{(-1-2j)t} \\ j e^{(-1+2j)t} & e^{(-1-2j)t} \end{pmatrix}\begin{pmatrix} 1/2 & -j/2 \\ -j/2 & 1/2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} e^{(-1+2j)t} & j e^{(-1-2j)t} \\ j e^{(-1+2j)t} & e^{(-1-2j)t} \end{pmatrix}\begin{pmatrix} 1 - j/2 \\ -j + 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} (1-j/2)e^{(-1+2j)t} + (-j+1/2)je^{(-1-2j)t} \\ (1-j/2)je^{(-1+2j)t} + (-j+1/2)e^{(-1-2j)t} \end{pmatrix} = \begin{pmatrix} e^{(-1+2j)t} - j/2\, e^{(-1+2j)t} + e^{(-1-2j)t} + j/2\, e^{(-1-2j)t} \\ je^{(-1+2j)t} + \frac{1}{2}e^{(-1+2j)t} + \frac{1}{2}e^{(-1-2j)t} - je^{(-1-2j)t} \end{pmatrix}$$

$$= \begin{pmatrix} (1-j/2)e^{-t}(\cos(2t) + j\,\mathrm{sen}(2t)) + (-j+1/2)e^{-t}j(\cos(2t) - j\,\mathrm{sen}(2t)) \\ (1-j/2)je^{-t}(\cos(2t) + j\,\mathrm{sen}(2t)) + (-j+1/2)e^{-t}(\cos(2t) - j\,\mathrm{sen}(2t)) \end{pmatrix}$$

$$= \begin{pmatrix} 2e^{-t}\cos(2t) + e^{-t}\mathrm{sen}(2t) \\ -2e^{-t}\mathrm{sen}(2t) + e^{-t}\cos(2t) \end{pmatrix}, \quad t \geq 0$$

2.5

$$\begin{cases} \dot{x}_1(t) = \alpha x_1(t) + 2x_3(t) \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) + 2u(t) \\ \dot{x}_3(t) = -x_1(t) \\ y(t) = x_2(t) \end{cases} \qquad \alpha \in \mathbb{R}$$

1) α | Sistema A.S. $\Rightarrow$ λ_i con $Re < 0$

$$A = \begin{bmatrix} \alpha & 0 & 2 \\ 1 & -1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \qquad \lambda I - A = \begin{pmatrix} \lambda - \alpha & 0 & -2 \\ -1 & \lambda+1 & -1 \\ 1 & 0 & \lambda \end{pmatrix} = (\lambda-\alpha)(\lambda+1)\lambda + 2(\lambda+1) = (\lambda^2 - \alpha\lambda)(\lambda+1) + 2(\lambda+1) =$$

$$= (\lambda+1)(\lambda^2 - \alpha\lambda + 2) \qquad \lambda_1 = -1$$

$$\lambda_{2-3} < 0 \iff \alpha < 0$$

2) $\alpha = -1$ $\qquad x(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$y(t) = ?$ $\qquad u(t) = 1, \quad t \geq 0$

Perchè è A.S. $\Rightarrow$ x_l tende a 0, calcolo solo x_F che è dato dallo stato di equilibrio ($\dot{x}(t) = 0$ perchè costante)

$$\begin{cases} -x_1(t) + 2x_3(t) = 0 \\ x_1(t) - x_2(t) + x_3(t) + 2 = 0 \\ -x_1(t) = 0 \end{cases} \qquad \begin{cases} x_1(t) = 0 \\ x_3(t) = 0 \\ x_2(t) = 2 = y(t) \end{cases}$$

2.6

$$\begin{cases} \dot{x}_1(t) = -5x_1(t) - 2x_2(t) + u(t) \\ \dot{x}_2(t) = -x_2(t) + u(t) \\ y(t) = 3x_1(t) \end{cases}$$

1) $A = \begin{bmatrix} -5 & -2 \\ 0 & -1 \end{bmatrix}$ $\lambda_1 = -5$, $\lambda_2 = -1$ $\Rightarrow$ A.S.

2) $x(0) = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$ $u(t) = 10,\ t \geq 0$

$$x_2(t) = 10e^{-t} + \int_0^t e^{-(t-\tau)} \cdot 10\, d\tau = 10e^{-t} + 10e^{-t}\int_0^t e^{\tau}\, d\tau = 10e^{-t} + 10e^{-t}\left[e^{\tau}\right]_0^t =$$

$$= \cancel{10e^{-t}} + 10 - \cancel{10e^{-t}} = 10$$

$$x_1(t) = e^{-5t} + \int_0^t e^{-5(t-\tau)}(-10)\, d\tau = e^{-5t} - 10e^{-5t}\int_0^t e^{5\tau}\, d\tau = e^{-5t} - 10e^{-5t}\left[\frac{1}{5}e^{5\tau}\right]_0^t = e^{-5t} - 10e^{-5t}\left(\frac{1}{5}e^{5t} - \frac{1}{5}\right) =$$

$$= e^{-5t} - 2 + 2e^{-5t} = 3e^{-5t} - 2$$

$$y(t) = 9e^{-5t} - 6, \quad t \geq 0$$

3) $u(t) = 20,\ t \geq 0$ $x(0) = \begin{bmatrix} 2 \\ 20 \end{bmatrix}$ $x_2(t) = 20e^{-t} + \int_0^t e^{-(t-\tau)} \cdot 20\, d\tau = 20e^{-t} + 20e^{-t}\int_0^t e^{\tau}\, d\tau = 20$

$$x_1(t) = 2e^{-5t} + \int_0^t e^{-5(t-\tau)}(-20)\, d\tau = 2e^{-5t} - 20e^{-5t}\int_0^t e^{5\tau}\, d\tau = 2e^{-5t} - 20e^{-5t}\left[\frac{1}{5}e^{5\tau}\right]_0^t = 2e^{-5t} - 4 + 4e^{-5t} = 6e^{-5t} - 4$$

$$y(t) = 18e^{-5t} - 12, \quad t \geq 0$$

2.7

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + u(t) \\ \dot{x}_2(t) = -3x_2(t) + 3x_3(t) \\ \dot{x}_3(t) = x_3(t) + u(t) \\ y(t) = 2x_1(t) \end{cases}$$

$x(0) = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ $u(t) = 2$

1)

$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $C = [2\ 0\ 0]$ $D = 0$

$\lambda I - A = \begin{bmatrix} \lambda+2 & 0 & 0 \\ 0 & \lambda+3 & -3 \\ 0 & 0 & \lambda-1 \end{bmatrix}$ $\chi = (\lambda+2)(\lambda+3)(\lambda-1)$ $\lambda_1 = -2$ $\lambda_2 = -3$ $\lambda_3 = 1$ INSTABILE

$$x_3(t) = e^t + \int_0^t e^{(t-\tau)} \cdot 2\, d\tau = e^t + 2e^t\int_0^t e^{-\tau} d\tau = e^t + 2e^t\left[-e^{-\tau}\right]_0^t = e^t + 2e^t(-e^{-t} + 1) = e^t - 2 + 2e^t = 3e^t - 2$$

$$x_1(t) = 2e^{-2t} + \int_0^t e^{-2(t-\tau)} \cdot 2\, d\tau = 2e^{-2t} + 2e^{-2t}\int_0^t e^{2\tau}\, d\tau = 2e^{-2t} + 2e^{-2t}\left(\frac{1}{2}e^{2t} - \frac{1}{2}\right) = 2e^{-2t} + 1 - e^{-2t} = e^{-2t} + 1$$

$$x_2(t) = 3e^{-3t} + \int_0^t e^{-3(t-\tau)}\, 3(3e^{\tau} - 2)\, d\tau = 3e^{-3t} + \int_0^t 3e^{-3t} \cdot e^{3\tau} \cdot 3e^{\tau} - 6 \cdot e^{-3t} \cdot e^{3\tau}\, d\tau$$

$$= 3e^{-3t} + 9e^{-3t}\int_0^t e^{4\tau}\, d\tau - 6e^{-3t}\int_0^t e^{3\tau} d\tau = 3e^{-3t} + 9e^{-3t}\left[\frac{1}{4}e^{4\tau}\right]_0^t - 6e^{-3t}\left[\frac{1}{3}e^{3\tau}\right]_0^t =$$

$$= 3e^{-3t} + 9e^{-3t}\left(\frac{1}{4}e^{4t} - \frac{1}{4}\right) - 6e^{-3t}\left(\frac{1}{3}e^{3t} - \frac{1}{3}\right) =$$

$$= \underline{3e^{-3t}} + \frac{9}{4}e^{t} - \underline{\frac{9}{4}e^{-3t}} - 2 + \underline{2e^{-3t}} = e^{-3t}\underbrace{\left(3 - \frac{9}{4} + 2\right)}_{\frac{20-9}{4} = \frac{11}{4}} + \frac{9}{4}e^{t} - 2$$

$$y(t) = 2e^{-2t} + 2$$

$y_L = 2x_{1,L}(t) \longrightarrow x_{1,L} = 0 \Rightarrow e^{-2t}x_0 = 0 \iff x_0 = 0$

$X(0) = \begin{pmatrix} 0 & \alpha & \beta \end{pmatrix}^T, \quad \alpha, \beta \in \mathbb{R}$

2.8

$$\begin{cases} \dot{x}_1(t) = -4x_1(t) - 4\alpha x_2(t) + u(t) \\ \dot{x}_2(t) = \alpha x_1(t) - 4x_2(t) + u(t) \\ y(t) = x_1(t) + x_2(t) \end{cases} \qquad \alpha \in \mathbb{R}$$

1) $\alpha \mid$ Sis. A.S. $\quad A = \begin{pmatrix} -4 & -4\alpha \\ \alpha & -4 \end{pmatrix} \quad \lambda I - A = \begin{pmatrix} \lambda+4 & 4\alpha \\ -\alpha & \lambda+4 \end{pmatrix} \longrightarrow \det \lambda I - A = (\lambda+4)^2 + 4\alpha^2 = \lambda^2 + 8\lambda + (16 + 4\alpha^2)$

$16 + 4\alpha^2 > 0 \rightarrow \forall \alpha \in \mathbb{R}$

2) $\alpha = 0 \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad u(t) = 3 \quad t \geq 0$

$A = \begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix} \quad x(t) = \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-4t} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \int_0^t e^{A(t-\tau)} 3\, d\tau & 0 \\ 0 & \int_0^t e^{A(t-\tau)} 3\, d\tau \end{bmatrix} = \begin{cases} e^{-4t} + 3e^{-4t}\left[\frac{1}{4}e^{4\tau}\right]_0^t = e^{-4t} + \frac{3}{4} - \frac{3}{4}e^{-4t} = \frac{3}{4} - \frac{1}{4}e^{-4t} \\ e^{-4t} + 3e^{-4t}\left[\frac{1}{4}e^{4\tau}\right]_0^t = e^{-4t} + \frac{3}{4} - \frac{3}{4}e^{-4t} = \frac{3}{4} - \frac{1}{4}e^{-4t} \end{cases}$

$y(t) = x_1(t) + x_2(t) = \frac{3}{2} - \frac{1}{2}e^{-4t}$

3) $\alpha = 0$

$u(t) = \bar{u}, \quad t \geq 0 \quad x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x_{eq} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$$\begin{cases} \dot{x}_1 = -4x_1 + \bar{u} = 0 \longrightarrow -\frac{\bar{u}}{4} = 4 \\ \dot{x}_2 = -4x_2 + \bar{u} = 0 \longrightarrow -\frac{\bar{u}}{5} = 5 \end{cases} \quad \nexists\, \bar{u}$$

2.9

$$\begin{cases} \dot{x}_1(t) = -2x_2(t) + u(t) \\ \dot{x}_2(t) = x_1(t) - 3x_2(t) + 2u(t) \\ y(t) = x_1(t) + x_2(t) \end{cases}$$

1) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \quad \det(\lambda I - A) = \det\begin{bmatrix} \lambda & 2 \\ -1 & \lambda+3 \end{bmatrix} = \lambda(\lambda+3) + 2 = \lambda^2 + 3\lambda + 2$ ✓ A.S.

2) $y(t)\big|_{x(0)} \quad x(0) = (1 \quad 1)^T \quad y(t) = y_L(t) = x_{1,L}(t) + x_{2,L}(t)$

$u(t) = 0 \quad t \geq 0$

$x_L = e^{At}x(0) \quad \lambda_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} = \begin{cases} -1 \\ -2 \end{cases}$

$Av = -v \quad \begin{cases} -2\beta = -\alpha \rightarrow \alpha = 2\beta \\ \alpha - 3\beta = -\beta \end{cases} \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad Av = -2v \quad \begin{cases} -2\beta = -2\alpha \\ \alpha - 3\beta = -2\beta \end{cases} \quad \{\alpha = \beta \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$T^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \quad e^{At} = T^{-1}e^{A_d t}T = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2e^{-t} & e^{-2t} \\ e^{-t} & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} & -2e^{-t} + 2e^{-2t} \\ e^{-t} - e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$

$x_L(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & -2e^{-t} + 2e^{-2t} \\ e^{-t} - e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{cases} x_{1,L} = 2e^{-t} - e^{-2t} - 2e^{-t} + 2e^{-2t} = e^{-2t} \\ x_{2,L} = e^{-t} - e^{-2t} - e^{-t} + 2e^{-2t} = e^{-2t} \end{cases}$

$y_L(t) = 2e^{-2t}$

3.1

$$\begin{cases} \dot{x}_1(t) = x_1(t)\,(x_2(t)-1) + u(t) - 1 \\ \dot{x}_2(t) = x_2(t) + x_1(t)\,u(t) \\ y(t) = x_1(t) \end{cases}$$

1) $u(t) = \bar{u} = 1$ $\quad \begin{cases} \bar{x}_1(\bar{x}_2-1)+1-1=0 \\ \bar{x}_2+\bar{x}_1=0 \\ y(t)=\bar{x}_1 \end{cases} \quad \begin{cases} \bar{x}_1\bar{x}_2-\bar{x}_1=0 \\ \bar{x}_2=-\bar{x}_1 \end{cases} \quad \begin{cases} -\bar{x}_1^2-\bar{x}_1=0 \\ \bar{x}_2=-\bar{x}_1 \end{cases} \to \bar{x}_1(-\bar{x}_1-1)=0$

$\bar{x}_1' = 0 \to \bar{x}_2 = 0$

$\bar{x}_1'' = -1 \to \bar{x}_2 = 1$

$x_a = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad x_b = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

2) $A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_1} \end{bmatrix} = \begin{bmatrix} x_2-1 & x_1 \\ 1 & 1 \end{bmatrix}$ $\quad \det(\lambda I - A) = \det\begin{vmatrix} \lambda+1-x_2 & -x_1 \\ -1 & \lambda-1 \end{vmatrix} = (\lambda-1)(\lambda+1-x_2) - x_1 = \lambda^2 + \cancel{\lambda} - \lambda x_2 - \cancel{\lambda} - 1 + x_2 - x_1$

$\chi = \lambda^2 + \lambda x_2 - 1 + x_2 - x_1$

$\chi_a = \lambda^2 + 0 - 1$ Non AS

$A_b = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ $\quad \det(\lambda I - A_b) = \det\begin{pmatrix} \lambda & 1 \\ -1 & \lambda-1 \end{pmatrix} = \lambda(\lambda-1)+1 = \lambda^2-\lambda+1$ Non AS

3.2

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + x_2^2(t) + x_2(t)\,u(t) \\ \dot{x}_2(t) = 3x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

1) $\bar{u} \mid \bar{x} = (0 \;\; 0)^T$ è equilibrio

$\begin{cases} -\bar{x}_1 + \bar{x}_2^2 + \bar{x}_2\bar{u} = 0 \\ 3\bar{x}_2 + \bar{u} = 0 \end{cases} \quad \begin{cases} \bar{u} = -3\bar{x}_2 \\ -\bar{x}_1 + \bar{x}_2^2 - 3\bar{x}_2^2 = 0 \end{cases} \quad \begin{cases} \bar{x}_1 = -2\bar{x}_2^2 \\ \bar{u} = -3\bar{x}_2 \end{cases} \qquad \bar{u} = 0$

2) $x(0) = \begin{bmatrix} 0 \\ \delta \end{bmatrix}$, $u(t) = 0,\ t \geq 0$

$\begin{cases} \dot{x}_1 = -x_1(t) + x_2^2(t) \\ \dot{x}_2 = 3x_2(t) \end{cases}$ $\qquad x_2(t) = e^{3t}\delta + \int 0\, d\tau = e^{3t}\cdot\delta$

$\Rightarrow \dot{x}_1(t) = -x_1(t) + e^{6t}\delta^2$

$x_1(t) = e^{-t}\cdot 0 + \int_0^t e^{-(t-\tau)} e^{6\tau}\delta^2 = \delta^2 e^{-t}\left[\frac{1}{7}e^{7\tau}\right]_0^t = \delta^2\left(\frac{1}{7}e^{6t} - \frac{e^{-t}}{7}\right)$

3) $\bar{x} = [0 \;\; 0]^T \qquad \bar{u} = 0$

Linearizzazione attorno a un equilibrio

$A = \begin{bmatrix} -1 & 2\bar{x}_2 + \bar{u}(t) \\ 0 & 3 \end{bmatrix}_{\bar{x},\bar{u}} \longrightarrow \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$ $\qquad \lambda_1 = -1$, $\lambda_2 = 3$ instabile

3.3

$$\begin{cases} \dot{x}_1(t) = -x_1^3(t) + 27\,u(t) \\ \dot{x}_2(t) = -x_1^2(t) - x_2(t) + u(t) - 1 \\ y(t) = x_1(t) + x_2(t) \end{cases}$$

1) $u(t) = 1$

$\begin{cases} -x_1^3 + 27 = 0 \\ -x_1^2 - x_2 = 0 \end{cases} \quad \begin{cases} \bar{x}_1 = 3 \\ -\bar{x}_2 = 9 \end{cases} \quad \begin{cases} \bar{x}_1 = 3 \\ \bar{x}_2 = -9 \end{cases} \quad \begin{pmatrix} 3 \\ -9 \end{pmatrix}$

2) $A = \begin{pmatrix} -3x_1^2 & 0 \\ -2x_1 & -1 \end{pmatrix}_{\bar{x},\bar{u}} = \begin{pmatrix} -27 & 0 \\ -6 & -1 \end{pmatrix}$ $\qquad \chi_A = \det\begin{vmatrix} \lambda+27 & 0 \\ 6 & \lambda+1 \end{vmatrix} = (\lambda+1)(\lambda+27)$ $\qquad$ A.S. $\quad Re(\lambda_i) < 0$

3) $x(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \qquad u(t) = 1$

$x_1(t) = x_1(0) = 3, \quad t \geq 0$

$\dot{x}_2(t) = -9 - x_2(t)$

$x_2(0) = 0 \qquad x_2(t) = 0 + \int_0^t e^{-(t-\tau)}(-9)\,d\tau = -9e^{-t}\left[e^{\tau}\right]_0^t = -9e^{-t}(e^t - 1) = -9 + 9e^{-t}$

$y(t) = 3 - 9 + 9e^{-t} = 9e^{-t} - 6$

3.4 $\begin{cases} \dot{x}_1(t) = x_2^2(t) + x_1(t) \\ \dot{x}_2(t) = -2x_2(t) + 2u(t) \\ y(t) = x_1(t) \end{cases}$

1) $u(t) = 1, \ t \geq 0 \qquad x(0) = \begin{pmatrix} -1+\delta \\ 1 \end{pmatrix} \qquad \delta \in \mathbb{R}$

$\begin{cases} \Delta\dot{x}_1(t) = 1 + 2x_2(t) \\ \Delta\dot{x}_2(t) = -2 + 2 \\ \Delta y(t) = \Delta x_1(t) + \Delta x_2(t) \end{cases}$

$x_2(t) = e^{-2t}(1) + \int_0^t e^{-2(t-\tau)} \cdot 2\,d\tau = e^{-2t} + 2e^{-2t}\int_0^t e^{2\tau}d\tau = e^{-2t} + 2e^{-2t}\left[\frac{1}{2}e^{2\tau}\right]_0^t =$

$= e^{-2t} + 2e^{-2t}\left(\frac{1}{2}e^{2t} - \frac{1}{2}\right) = e^{-2t} + 1 - e^{-2t} = 1$

$\dot{x}_1(t) = x_1(t) + 1 \rightsquigarrow x_1(t) = e^t(-1+\delta) + \int_0^t e^{(t-\tau)} \cdot 1\,d\tau = e^t(-1+\delta) + e^t\left[-e^{-\tau}\right]_0^t = e^t(-1+\delta) + e^t\left(-e^{-t} + 1\right) =$

$= e^t(-1+\delta) - 1 + e^t = -e^t + e^t - 1 + \delta e^t = \delta e^t - 1$

2) $\bar{x} = [\bar{x}_1 \ \ \bar{x}_2]^T \qquad u(t) = 1$

$\begin{cases} 0 = \bar{x}_2^2 + \bar{x}_1 \\ 0 = -2\bar{x}_2 + 2 \end{cases} \qquad \begin{cases} \bar{x}_2 = 1 \\ \bar{x}_1 = -1 \end{cases} \longrightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

3) $x_1(t) = \delta e^t - 1 \qquad x_1(0) = \delta - 1 \qquad \begin{cases} \Delta\dot{x}_1(t) = \tilde{x}_1(t) - \bar{x}_1 = \delta e^t \\ \Delta\dot{x}_2(t) = \tilde{x}_2(t) - \bar{x}_2 = 0 \end{cases} \quad t \geq 0$ diverge $\Rightarrow$ eq. instabile

$x_2(t) = 1 \qquad x_2(0) = 1$

3.5 $\begin{cases} \dot{x}(t) = x(t)^2 - 4|x(t)| + u(t) \\ y(t) = x(t) \end{cases}$

1) $u(t) = 4 \qquad 0 = \bar{x}^2 - 4|\bar{x}| + 4$

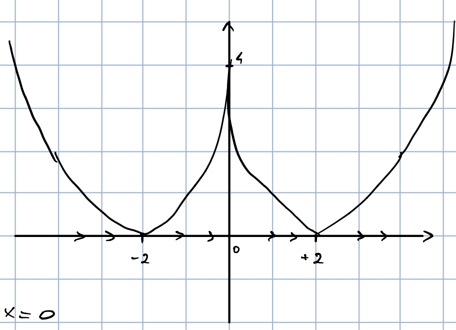
$\bar{x} > 0 \Rightarrow \bar{x} = \frac{4 \pm \sqrt{16-16}}{2} = 2$

$\bar{x} < 0 \Rightarrow \bar{x}^2 + 4\bar{x} + 4 \rightarrow \bar{x} = \frac{-4 \pm \sqrt{0}}{2} = -2$

1° eq $\begin{cases} x(t) = 2 \\ y(t) = 2 \end{cases} \qquad$ 2° eq $\begin{cases} x(t) = -2 \\ y(t) = -2 \end{cases}$

2) $h(x) = x(t)^2 - 4|x(t)| + 4$

$\begin{cases} x = 0 \\ y = 4 \end{cases} \qquad \begin{cases} y = 0 \\ x = \pm 2 \end{cases}$



$h'(x) = 2x - 4\frac{x}{|x|} + 4$

$x > 0: \quad 2x - 4 + 4 \rightarrow 2x = 0 \Leftrightarrow x = 0$

$x < 0: \quad 2x + 8 = 0 \rightarrow x = -4$

3.6

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = x(t) \end{cases}$$

1) Stati di equilibrio $-1 \le \bar{x} \le 1$

2) $-1 < x < 1$ semp. stabile

$x = -1$ semp. stabile, $x = 1$ instabile

3) Non cambia nulla

3.7

$$\begin{cases} \dot{x}_1(t) = 2\alpha^3 x_1^3(t) + 2x_2(t)u(t) \\ \dot{x}_2(t) = -x_2(t) + u(t) \\ y(t) = x_2(t) \end{cases} \qquad \alpha \in \mathbb{R} \setminus \{0\}$$

1) $u(t) = 1 \quad \forall t \ge 0$

$$\begin{cases} 0 = 2\alpha^3\bar{x}_1^3 + 2\bar{x}_2 \\ 0 = -\bar{x}_2 + 1 \end{cases} \quad \begin{cases} \bar{x}_2 = 1 \\ \alpha^3\bar{x}_1^3 + 1 = 0 \end{cases} \longrightarrow \bar{x}_1^3 = -\frac{1}{\alpha^3} \longrightarrow \bar{x}_1 = -\frac{1}{\alpha} \qquad \bar{x} = \begin{bmatrix} -\frac{1}{\alpha} \\ 1 \end{bmatrix}$$

$y(t) = 1$

2)

$$\begin{cases} \Delta\dot{x}_1(t) = 6\alpha^3\bar{x}_1^2\Delta x_1(t) + 2\bar{u}\,\Delta x_2(t) + 2\bar{x}_2\,\Delta u(t) \\ \Delta\dot{x}_2(t) = -\Delta x_2(t) + \Delta u(t) \\ \Delta y(t) = \Delta x_2(t) \end{cases} \qquad \begin{cases} \Delta\dot{x}_1(t) = 6\alpha\Delta x_1(t) + 2\Delta x_2(t) + 2\Delta u(t) \\ \Delta\dot{x}_2(t) = -\Delta x_2(t) + u(t) \\ \Delta y(t) = \Delta x_2(t) \end{cases}$$

3) $A|_{\bar{x}} = \begin{bmatrix} 6\alpha & 2 \\ 0 & -1 \end{bmatrix}$ $\quad \alpha < 0$ AS, $\quad \alpha > 0$ INSTAB.

3.8

$$\begin{cases} \dot{x}_1(t) = -x_1^3(t) - x_1(t) + 3x_2(t) + u(t) \\ \dot{x}_2(t) = -x_2^2(t) + x_1(t) \\ y(t) = x_1(t) \end{cases}$$

1) $x(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $u(t) = -1$

$$\begin{cases} -\bar{x}_1^3 - \bar{x}_1 + 3\bar{x}_2 - 1 = 0 \\ -\bar{x}_2^2 + \bar{x}_1 = 0 \end{cases} \quad \begin{cases} \bar{x}_1 = \bar{x}_2^2 \\ -\bar{x}_2^6 - \bar{x}_2^2 + 3\bar{x}_2 - 1 = 0 \end{cases} \Rightarrow \begin{cases} 1 = 1^2 \ \checkmark \\ -1 - 1 + 3 - 1 = 0 \ \checkmark \end{cases}$$

$$\begin{cases} \Delta\dot{x}_1(t) = -4\Delta x_1(t) + 3\Delta x_2(t) + \Delta u(t) \\ \Delta\dot{x}_2(t) = -2\Delta x_2(t) + \Delta x_1(t) \\ \Delta y(t) = \Delta x_1(t) \end{cases}$$

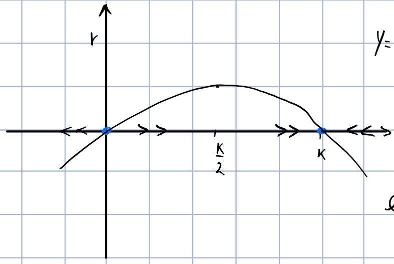
2) $A = \begin{bmatrix} -4 & 3 \\ 1 & -2 \end{bmatrix} \xrightarrow[\chi_A]{} \lambda I - A = 0 \Rightarrow \begin{pmatrix} \lambda+4 & -3 \\ -1 & \lambda+2 \end{pmatrix} \xrightarrow[\chi_a]{} (\lambda+4)(\lambda+2) - 3 = \lambda^2 + 4\lambda + 8 + 2\lambda - 3$ ✓ AS.

3) Grafico a) perchè autov. $\in \mathbb{R}$ e sis. A.S

3.9

$$\dot{x}(t) = r\,x(t)\left(1 - \frac{x(t)}{K}\right) \qquad r, K \in \mathbb{R}^+$$

- $r\bar{x}\left(1 - \frac{\bar{x}}{K}\right) = 0 \longrightarrow \bar{x} = 0 \ \vee \ \bar{x} = K$



$y = rx - \frac{r}{K}x^2 = 0 \rightarrow y' = r - 2\frac{r}{K}x > 0 \iff x < \frac{K}{2}$

$y\left(\frac{K}{2}\right) = \frac{rK}{2} - \frac{rK}{4} = \frac{rK}{4}$

eq instabile

$\bar{x} = 0$ instabile
$\bar{x} = K$ stabile

4.1

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + u(t) \\ \dot{x}_2(t) = -x_2(t) + 9u(t) \\ y(t) = x_1(t) + x_2(t) \end{cases}$$

1) $$\begin{cases} sX_1(s) = -X_1(s) + U(s) \rightarrow X_1(s+1) = U(s) \\ sX_2(s) = -X_2(s) + 9U(s) \rightarrow X_2(s+1) = 9U(s) \\ Y(s) = X_1(s) + X_2(s) \end{cases}$$

$\rightarrow Y(s) = \frac{10}{s+1} U(s)$

$G(s) = \frac{10}{s+1}$ autovalore nascosto $= -1$

2) $u(t) = 2e^{-3t} \longleftrightarrow U(s) = \frac{2}{s+3}$

$Y(s) = \frac{20}{(s+3)(s+1)} = \frac{\alpha}{s+3} + \frac{\beta}{s+1}$ $\begin{cases} s(\alpha+\beta) = 0 \\ 3\beta + \alpha = 20 \end{cases}$ $\begin{cases} \alpha = -\beta \\ \beta = 10 \end{cases}$

$Y(s) = \frac{-10}{s+3} + \frac{10}{s+1} \longleftrightarrow y(t) = -10e^{-3t} + 10e^{-t}$

3) TVI: $\lim_{s\to\infty} sY(s) = \lim_{s\to\infty} s \cdot \frac{20}{(s+3)(s+1)} = 0 = y(0)$

TVF: $\lim_{s\to 0} sY(s) = \lim_{s\to 0} s \cdot \frac{20}{(s+3)(s+1)} = 0 = y_\infty$

4) $x(0) = [1 \ \ 2]^T$ $u(t) = e^{-3t}$

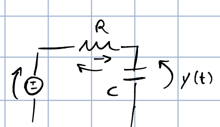
$y(t) = y_L(t) + y_F(t)$ $y_F(t) = -5e^{-3t} + 5e^{-t}$

$y_L = e^{At}x(0)$ $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = e^{-t} + 2e^{-t} = 3e^{-t}$

$y(t) = 3e^{-t} - 5e^{-3t} + 5e^{-t} = 8e^{-t} - 5e^{-3t}$

4.2

1) 

$y(t) = V_C(t)$

$u(t) - Ri(t) - V_C(t) = 0$

$i(t) = C\frac{dV_C(t)}{dt} \rightsquigarrow \dot{x}_1(t) = -\frac{x_1(t)}{RC} + \frac{u(t)}{RC}$

$y(t) = x_1(t)$

$i(t) = \frac{u(t)}{R} - \frac{V_C(t)}{R}$

$n = 1$

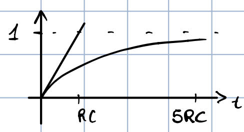
2) $sX(s) = -\frac{1}{RC}X(s) + \frac{1}{RC}U(s) \rightarrow X(s) = U(s) \cdot \frac{1}{RC\left(s + \frac{1}{RC}\right)}$

$G(s) = \frac{1}{RC\left(s + \frac{1}{RC}\right)}$

3) $u(t) = sca(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$ $\qquad \int_0^{+\infty} 1\, e^{-st}\, dt = -\frac{1}{s} e^{-st}\Big|_0^{+\infty} = \frac{1}{s}$

$Y(s) = \frac{1}{s} \cdot \frac{1}{RC\left(s+\frac{1}{RC}\right)}$

TVI : $\lim_{s\to\infty} G(s) = 0$



TVF : $\lim_{s\to 0} G(s) = 1$

TVI : $\lim_{s\to\infty} \left[s\dot{Y}(s) = sG(s) \right] = \lim_{s\to\infty} s \frac{1/RC}{(s+1/RC)} = \frac{1}{RC} > 0$

4) $Y(s) = \frac{1}{RC} \cdot \frac{1}{s} \cdot \frac{1}{s+\frac{1}{RC}} = \frac{1}{RC}\left(\frac{\alpha}{s} + \frac{\beta}{s+\frac{1}{RC}} \right)$ $\qquad \begin{cases} s(\alpha+\beta) = 0 \longrightarrow \beta = -RC \\ \frac{\alpha}{RC} = 1 \longrightarrow \alpha = RC \end{cases}$

$Y(s) = \frac{1}{RC}\left(\frac{RC}{s} - \frac{RC}{s+\frac{1}{RC}} \right) \longleftrightarrow y(t) = \underbrace{1}_{sca(t)} - e^{-\frac{t}{RC}}, \; t \geq 0$

4.3

$G(s) = \frac{1}{(s+1)(s+10)}$

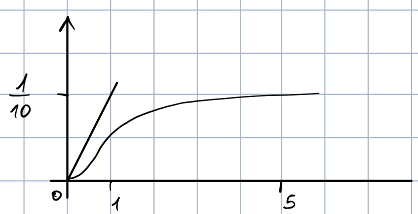
1) $U(s) = \frac{1}{s}$ $\qquad Y(s) = \frac{1}{s(s+1)(s+10)}$

TVI: $y(0) = \lim_{s\to\infty} G(s) = 0 \longrightarrow \dot{y}(0) = \lim_{s\to\infty} s\mathcal{L}[\dot{y}(t)](s) = \lim_{s\to\infty} s[sY(s) - y(0)] = \lim_{s\to\infty} sG(s) = 0$ pendenza nulla

TVF: $y_\infty = \lim_{s\to 0} G(s) = \frac{1}{10}$ $\qquad \ddot{y}(0) = \lim_{s\to\infty} s\mathcal{L}[\ddot{y}(t)](s) = \lim_{s\to\infty} s[s^2 Y(s) - \dot{y}(0)] = \lim_{s\to\infty} s^2 G(s) = 1$

concavità verso l'alto

$p_1 = -1$ $\qquad p_2 = -10$ $\qquad \tau_d = \tau_1$ $\qquad T_a = 5\tau_1 = 5$ udt



2) $\tilde{G}(s) = \frac{\tilde{\mu}}{1+s\tilde{\tau}}$ tc $\tilde{Y} \approx Y$ con $\tilde{U}(s) = \frac{1}{s}$

$\tilde{Y}(s) = \frac{1}{s} \cdot \frac{\tilde{\mu}}{1+s\tilde{\tau}}$

TVI : $\lim_{s\to\infty} G(s) = 0$, $\qquad \dot{y}(0) = \lim_{s\to\infty} s[\mathcal{L}(\dot{y})](s) = \lim_{s\to\infty} s[sY(s) - y(0)] = \lim_{s\to\infty} sG(s) = \lim_{s\to\infty} \frac{s\tilde{\mu}}{1+s\tilde{\tau}} = 1$

$\ddot{y}(0) = \lim_{s\to 0} s[s^2 Y(s) - 1] = \lim_{s\to 0} s^2 G(s) - s = \lim_{s\to 0} \frac{s^2\tilde{\mu}}{1+s\tilde{\tau}} - s = 0$

TVF : $G(0) = \tilde{\mu} = \frac{1}{10}$; $p = -\frac{1}{\tilde{\tau}} = -1 \Rightarrow \tilde{\tau} = 1$

$\Rightarrow \tilde{G}(s) = \frac{0.1}{1+s}$

4.4

1) $\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -10x_1(t) - 11x_2(t) + u(t) \\ y(t) = x_1(t) - x_2(t) \end{cases} \longrightarrow \begin{cases} sX_1(s) = X_2(s) \longrightarrow X_1(s) = \frac{1}{s}(X_2(s)) \\ sX_2(s) = -10X_1(s) - 11X_2(s) + U(s) \\ X_2(s)(s+11) = -10X_1(s) + U(s) \longrightarrow X_2(s)\left(s + 11 + \frac{10}{s}\right) = U(s) \end{cases}$

$X_2(s) = U(s) \cdot \frac{s}{s^2+11s+10}$

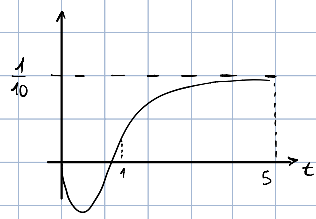
$Y(s) = X_1(s) - X_2(s) = \frac{X_2(s)}{s} - X_2(s) = \frac{U(s)}{s^2+11s+10} - U(s)\frac{s}{s^2+11s+10} = U(s)\left[\frac{1}{s^2+11s+10} - \frac{s}{s^2+11s+10} \right] = U(s)\frac{1-s}{s^2+11s+10}$

$$G(s) = \frac{1-s}{s^2+11s+10} \longrightarrow (s+1)(s+10)$$

2) $Y(s) = \frac{1}{s}\, G(s)$

TVI : $\lim_{s\to\infty} G(s) = 0 = y(0)$

$$\dot y(0) = \lim_{s\to\infty} s\,G(s) = -1$$



TVF : $y_\infty = \mu = G(0) = \frac{1}{10}$; $p_1 = -1$ $p_2 = -10$ $\tau_d = 1$ $T_a = 5$ sec

3) $$Y(s) = \frac{1}{s}\cdot\frac{1-s}{(s+1)(s+10)} = \frac{1-s}{s(s+1)(s+10)} = \frac{\alpha}{s} + \frac{\beta}{s+1} + \frac{\gamma}{s+10} = \frac{\alpha s^2 + 10\alpha s + \alpha s + 10\alpha + \beta s^2 + 10\beta s + \gamma s^2 + \gamma s}{s(s+1)(s+10)}$$

$$\begin{cases} (\alpha+\beta+\gamma) = 0 \\ (11\alpha + 10\beta + \gamma) = -1 \\ 10\alpha = 1 \end{cases} \quad \begin{cases} 1/10 + \beta + \gamma = 0 \\ 11/10 + 10\beta + \gamma = -1 \\ \alpha = \frac{1}{10} \end{cases} \quad \begin{cases} \frac{11}{10} + 10\beta - \frac{1}{10} - \beta = -1 \longrightarrow 9\beta = -2 \longrightarrow \beta = -\frac{2}{9} \\ \alpha = \frac{1}{10} \longrightarrow \alpha = \frac{1}{10} \\ \gamma = -\frac{1}{10} - \beta \longrightarrow \gamma = -\frac{1}{10} + \frac{2}{9} = \frac{-9+20}{90} = \frac{11}{90} \end{cases}$$

$$Y(s) = \frac{1}{10}\cdot\frac{1}{s} - \frac{1}{9}\cdot\frac{1}{s+1} + \frac{1}{90}\cdot\frac{1}{s+10} \xrightarrow[\mathcal{L}^{-1}]{} y(t) = \frac{1}{10}\,sca(t) - \frac{2}{9}e^{-t} + \frac{11}{90}e^{-10t}$$

4.5

$$\ddot y = -10\dot y(t) + u(t) \qquad m = 1$$
$$F_a(t) = 10\dot y(t)$$

1) $\mathcal{L}[\ddot y(t)](s) = s\,\mathcal{L}[\dot y(t)](s) = s^2 Y(s) = -10s\,Y(s) + U(t) \implies Y(s)(s^2+10s) = U(s)$

$$Y(s) = \frac{1}{s(s+10)}\,U(s), \quad G(s) = \frac{1}{s(s+10)} \qquad p_1 = 0,\; p_2 = -10 \quad \text{stabile semp.}$$

2) $u(t) = 1 + 10t$, $t \geq 0$

$$U(s) = \frac{1}{s} + \frac{10}{s^2}, \quad Y(s) = \frac{1}{s}\cdot\frac{1}{s+10}\left(\frac{1}{s} + \frac{10}{s^2}\right) = \frac{1}{s(s+\cancel{10})}\left(\frac{s+10}{s^2}\right) = \frac{1}{s^3}$$

$$y(t) = \frac{t^2}{2}, \quad t \geq 0$$

4.6

$$\begin{cases} \dot x_1(t) = -x_1(t) + u(t) \\ \dot x_2(t) = x_1(t) - 10x_2(t) \\ y(t) = x_2(t) \end{cases}$$

1) $$\begin{cases} sX_1(s) = -X_1(s) + U(s) \\ sX_2(s) = X_1(s) - 10X_2(s) \\ Y(s) = X_2(s) \end{cases} \quad \left.\begin{cases} X_1(s) = \frac{1}{s+1}U(s) \\ X_2(s)(s+10) = \frac{1}{s+1}U(s) \\ Y(s) = X_2(s) \end{cases}\right\} \longrightarrow X_2(s) = \frac{1}{(s+1)(s+10)}U(s) = Y(s)$$

$$G(s) = \frac{1}{(s+1)(s+10)}$$

2) $u(t) = 2$, $U(s) = \frac{2}{s} \longrightarrow Y(s) = \frac{2}{s(s+1)(s+10)} \implies \frac{\alpha}{s} + \frac{\beta}{s+1} + \frac{\gamma}{s+10} = \frac{(\alpha s+\alpha)(s+10) + \beta s(s+10) + \gamma s(s+1)}{s(s+1)(s+10)} =$

$$\frac{\alpha s^2 + 10\alpha s + \alpha s + 10\alpha + \beta s^2 + 10\beta s + \gamma s^2 + \gamma s}{s(s+10)(s+1)}$$

$$\begin{cases} \alpha+\beta+\gamma = 0 \\ 11\alpha s + 10\beta s + \gamma s = 0 \\ 10\alpha = 2 \end{cases} \quad \begin{cases} \alpha = 1/5 \\ 11/5\, s + 10\beta s + \gamma s = 0 \\ \frac{1}{5} + \beta + \gamma = 0 \end{cases} \quad \begin{cases} \gamma = -\beta - \frac{1}{5} \\ \frac{11}{5}s + 10\beta s - \beta s - \frac{1}{5}s = 0 \longrightarrow 2s + 9\beta s = 0 \end{cases}$$

$\beta = -\frac{2}{9}$, $\alpha = \frac{1}{5}$, $\gamma = \frac{2}{9} - \frac{1}{5} = \frac{10-9}{45} = \frac{1}{45}$

$y(t) = \frac{1}{5} - \frac{2}{9} e^{-t} + \frac{1}{45} e^{-10t}$

3) $p_1 = -1$ $\quad p_2 = -10$ $\quad$ A.S.

$\tau_1 = 1$ $\quad \tau_2 = \frac{1}{10}$ $\longrightarrow \tau_d = \tau_1$, $T_a = 5$ udt

4) $\exists\, u(t) : \ y_{|u(t)} \longrightarrow \infty$?

TVF: $\lim_{t\to 0} s\,Y(s) = +\infty$? $\qquad \lim_{t\to 0} s \frac{1}{(s+1)(s+10)} \cdot U(s) = \infty \iff U(s) = \frac{1}{s^K}$, $K \geq 2$

$u(t) = \frac{t^K}{K!}$ $\quad K \geq 1$

4.7

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + u(t) \\ \dot{x}_2(t) = 2x_1(t) - 2x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

1) $x(0) = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$ $\quad u(t) = 2$ $\quad t \geq 0$

$$x_1(t) = 2e^{-t} + \int_0^t e^{-(t-\tau)} \cdot 2\, d\tau = 2e^{-t} + 2e^{-t} \int_0^t e^{\tau} d\tau = 2e^{-t} + 2e^{-t}\left(e^{t} - 1\right) = 2$$

$$x_2(t) = 10e^{-2t} + \int_0^t e^{-2(t-\tau)} \cdot 6\, d\tau = 10e^{-2t} + 6e^{-2t} \int_0^t e^{2\tau} d\tau = 10e^{-2t} + 6e^{-2t}\left[\frac{1}{2} e^{2\tau}\right]_0^t = 10e^{-2t} + 6e^{-2t}\left(\frac{1}{2}e^{2t} - 1\right)$$

$$= 10e^{-2t} + 3 - 6e^{-2t} = 4e^{-2t} + 3$$

$y(t) = x_1(t) = 2$

2) $x(0) = \begin{bmatrix} 0 \\ \alpha \end{bmatrix}$ $\quad \alpha \in \mathbb{R}$

3) $G(s)$?

$$\begin{cases} sX_1(s) = -X_1(s) + U(s) \\ sX_2(s) = 2X_1(s) - 2X_2(s) + U(s) \\ Y(s) = X_1(s) \end{cases} \longrightarrow X_1(s) = \frac{U(s)}{s+1}$$

$G(s) = \frac{1}{s+1}$ Non è possibile valutare le proprietà di stabilità del sistema perchè c'è un autovalore nascosto

4.8

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + x_2(t) \\ \dot{x}_2(t) = -5x_2(t) + u(t) \\ y(t) = 2x_1(t) \end{cases} \qquad n = 2$$

1) $$\begin{cases} sX_1(s) = -X_1(s) + X_2(s) \\ sX_2(s) = -5X_2(s) + U(s) \\ Y(s) = 2X_1(s) \end{cases} \quad \begin{cases} X_2(s) = U(s) \cdot \frac{1}{s+5} \\ X_1(s) = \frac{U(s)}{(s+5)(s+1)} \\ Y(s) = 2X_1(s) \end{cases} \qquad G(s) = \frac{2}{(s+5)(s+1)}$$ AS. STABILE

2) (a) e (b) No perchè non ci sono sovraelongazioni nè oscillazioni smorzate

(d) No perchè T_a non è $5\tau_d$

è (c) perchè $\tau_d = 1$ e $T_a = 5$ udt

3) $x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $u(t) = 5$

$$x_2(t) = e^{-5t} + \int_0^t e^{-5(t-\tau)} \cdot 5\, d\tau = e^{-5t} + e^{-5t}\int_0^t 5e^{5\tau}\, d\tau = e^{-5t} + e^{-5t}\left(e^{5t} - 1\right) = e^{-5t} + 1 - e^{-5t} = 1$$

$$x_1(t) = 2e^{-t} + \int_0^t e^{-(t-\tau)}\, d\tau = 2e^{-t} + e^{-t}\int_0^t e^{\tau}\, d\tau = 2e^{-t} + 1 - e^{-t} = 1 + e^{-t}$$

$$y(t) = 2 + 2e^{-t}$$

4) $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $u(t) = 5$

$x_{2,F}(t) = 1 - e^{-5t}$

$x_{1,F}(t) = 1 - e^{-t}$

$y_F(t) = 2 - 2e^{-t}$ la differenza tende a zero

4.9

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + 2x_2(t) + u(t) \\ \dot{x}_2(t) = -3x_2(t) \\ y(t) = x_1(t) \end{cases}$$

$A = \begin{bmatrix} -2 & 2 \\ 0 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = [1 \ 0]$ $D = 0$

1) $G(s) = C\left[(sI - A)^{-1}\right]B + D = [1 \ 0]\begin{bmatrix} s+2 & -2 \\ 0 & s+3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Calcolo dell'inversa: $\frac{1}{(s+2)(s+3)}\begin{bmatrix} s+3 & 0 \\ 2 & s+2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+2} & 0 \\ \frac{2}{(s+2)(s+3)} & \frac{1}{s+3} \end{bmatrix}$

$$[1 \ 0]\begin{bmatrix} \frac{1}{s+2} & 0 \\ \frac{2}{(s+2)(s+3)} & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [1 \ 0]\begin{bmatrix} \frac{1}{s+2} \\ \frac{2}{(s+2)(s+3)} \end{bmatrix} = \frac{1}{s+2}$$

$$G(s) = \frac{1}{s+2}$$

$U(s) = \frac{1}{s+1}$ $Y(s) = \frac{1}{(s+2)(s+1)} \rightarrow \frac{\alpha}{s+2} + \frac{\beta}{s+1}$ $\begin{cases} \alpha + \beta = 0 \\ \alpha + 2\beta = 1 \end{cases}$ $\begin{cases} \alpha = -\beta \\ \alpha - 2\alpha = 1 \end{cases}$ $\begin{cases} -\alpha = 1 \rightarrow \alpha = -1 \\ \beta = 1 \end{cases}$

$$Y(s) = \frac{-1}{s+2} + \frac{1}{s+1} \xrightarrow[\mathcal{L}^{-1}]{} y(t) = e^{-t} - e^{-2t}$$

3) TVI: $\lim_{s \to \infty}\left[sY(s) = \frac{s}{(s+2)(s+1)}\right] = 0$

TVF: $\lim_{s \to 0}\left[sY(s) = \frac{s}{(s+2)(s+1)}\right] = 0$

4.10

$$\begin{cases} \dot{x}_1(t) = \alpha x_1(t) - (\alpha+2)^2 x_2(t) + 2u(t) \\ \dot{x}_2(t) = x_1(t) - 4x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

1) $A = \begin{bmatrix} \alpha & -(\alpha+2)^2 \\ 1 & -4 \end{bmatrix}$ $\det(\lambda I - A)$: $\det\begin{bmatrix} \lambda - \alpha & (\alpha+2)^2 \\ -1 & \lambda + 4 \end{bmatrix} = (\lambda - \alpha)(\lambda + 4) + (\alpha+2)^2 = \lambda^2 + 4\lambda - \alpha\lambda \cancel{- 4\alpha} + \alpha^2 + 4 \cancel{+ 4\alpha}$

$= \lambda^2 + \lambda(4 - \alpha) + 4 + \alpha^2$

$$\begin{cases} 4 + \alpha^2 > 0 & \forall \alpha \\ 4 - \alpha > 0 & \alpha < 4 \end{cases} \quad \boxed{\forall \alpha < 4}$$

2) $\alpha = -2$ $\quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad \begin{cases} \dot{x}_1(t) = -2x_1(t) + 2u(t) \\ \dot{x}_2(t) = x_1(t) - 4x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$

$x_{1,L} = e^{-2t}$

$y_L = e^{-2t}$

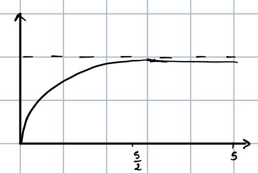
$$\dot{x}_2(t) = e^{-2t} - 4x_2(t) \longrightarrow x_2(t) = e^{-4t}\cdot x_2(0) + \int_0^t e^{-4(t-\tau)}\cdot e^{-2\tau}\,d\tau = e^{-4t}\int_0^t e^{2\tau}\,d\tau = e^{-4t}\left[\frac{1}{2}e^{2\tau}\right]_0^t = e^{-4t}\left(\frac{1}{2}e^{2t} - \frac{1}{2}\right) = \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}$$

$$\begin{cases} x_1(t) = e^{-2t} \\ x_2(t) = \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t} \end{cases}$$

3) $sX_1(s) = -2X_1(s) + 2U(s) \longrightarrow G(s) = \dfrac{2}{s+2}$

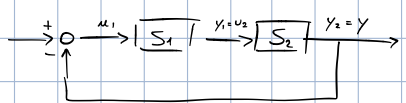
4) $A = \begin{bmatrix} -2 & 0 \\ 1 & -4 \end{bmatrix}$ $\quad \lambda_1 = -2$, $\lambda_2 = -4$ $\quad \tau_d = \frac{1}{2} \Rightarrow T_a = \frac{5}{2}$ udt

$y(0) = 0$ $\quad y_\infty = 1$



5.1

$S_1: \begin{cases} \dot{x}_1(t) = u_1(t) \\ y_1(t) = x_1(t) + u_1(t) \end{cases}$ $\qquad S_2: \begin{cases} \dot{x}_2(t) = x_2(t) + u_2(t) \\ y_2(t) = 2x_2(t) \end{cases}$



1) S_1 semplicemente stabile ($\lambda = 0$)
S_2 instabile ($\lambda = 1$)

2) $S_1 S_2$ instabile

3) $u_1 = u - y_2$ $\qquad \begin{aligned} \dot{x}_1(t) &= u(t) - 2x_2(t) \\ \dot{x}_2(t) &= -x_2(t) + x_1(t) + u(t) \\ y_2(t) &= 2x_2(t) \end{aligned}$

4) $A = \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix}$ $\quad \det\begin{bmatrix} \lambda & 2 \\ -1 & \lambda+1 \end{bmatrix} = \lambda(\lambda+1) + 2 = \lambda^2 + \lambda + 2$ ✓ A.S.

5) fig 5.8 non cambiano

fig 5.9 $\begin{cases} \dot{x}_1(t) = u_1(t) \\ y_1(t) = x_1(t) + u_1(t) \\ \dot{x}_2(t) = x_2(t) + u_2(t) \\ y_2(t) = 2x_2(t) \\ u_1(t) = y_2(t) - u(t) \end{cases}$ $\quad \begin{cases} \dot{x}_1 = y_2(t) - u(t) = 2x_2(t) - u(t) \\ \dot{x}_2 = x_2(t) + x_1(t) + 2x_2(t) - u(t) \end{cases}$ $\quad A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ $\quad \det(\lambda I - A) = \det\begin{bmatrix} \lambda & -2 \\ -1 & \lambda-3 \end{bmatrix}$

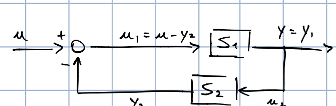
$= \lambda(\lambda-3) - 2 =$

$= \lambda^2 - 3\lambda - 2$

instabile

5.2

$S_1: \begin{cases} \dot{x}_1(t) = x_1(t) + u_1(t) \\ y_1(t) = x_1(t) \end{cases}$ $\qquad S_2: \begin{cases} \dot{x}_2(t) = -6x_2(t) + 6u_2(t) \\ y_2(t) = 2x_2(t) \end{cases}$



1) S_1 è instabile, S_2 è asintoticamente stabile

2) $u_1 = u(t) - 2x_2(t)$, $u_2 = x_1(t)$

$$\begin{cases} \dot{x}_1(t) = x_1(t) - 2x_2(t) + u(t) \\ \dot{x}_2(t) = -6x_2(t) + 6x_1(t) \end{cases}$$

3) $A = \begin{bmatrix} 1 & -2 \\ 6 & -6 \end{bmatrix}$ $\quad \det(\lambda I - A) = \det\begin{bmatrix} \lambda - 1 & 2 \\ -6 & \lambda+6 \end{bmatrix} = (\lambda+6)(\lambda-1) + 12 = \lambda^2 - \lambda + 6\lambda - 6 + 12 = \lambda^2 + 5\lambda + 6$ ✓ A.S.

5.3

$\alpha > 0$, $M = 1$, $\quad u(t) = M \cdot a(t)$, $\quad y(t) = s(t)$

1) $$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\alpha x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

2) $A = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix}$ $\quad \det\begin{bmatrix} \lambda & -1 \\ 0 & \lambda+\alpha \end{bmatrix} = \lambda(\lambda+\alpha)$ $\quad \lambda_1 = 0$

$\lambda_2 = -\alpha$ $\quad \neg\exists\, \alpha :$ A.S.

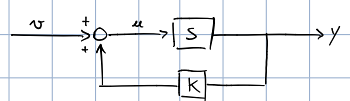
3) S_{a} : $\quad v \to \boxed{K} \xrightarrow{u} \boxed{S} \xrightarrow{y}$

$u(t) = K v(t)$, $\quad K \in \mathbb{R}$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\alpha x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

$\neg\exists\, \alpha :$ A.S.

4) S_{cl} :



$u(t) = v(t) + K y(t)$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2 = -\alpha x_2(t) + v(t) + K x_1(t) \\ y(t) = x_1(t) \end{cases}$$

$A = \begin{bmatrix} 0 & 1 \\ K & -\alpha \end{bmatrix}$, $\quad \det\begin{bmatrix} \lambda & -1 \\ -K & \lambda+\alpha \end{bmatrix} = \lambda(\lambda+\alpha) - K = \lambda^2 + \alpha\lambda - K$

per $K < 0$ sistema A.S.

5) $\alpha = 3$, $K = -2$

$v(t) = \bar{v}$, $t \geq 0$ $\qquad \bar{y} = 2$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -3x_2(t) + v(t) - 2x_1(t) \\ y(t) = x_1(t) \end{cases}$$

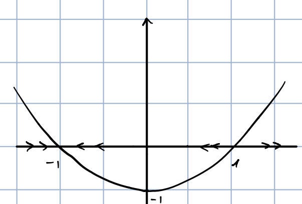
$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ $\qquad \bar{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$$\begin{cases} \bar{x}_2 = 0 \\ -3\bar{x}_2 + \bar{v} - 2\bar{x}_1 = 0 \\ \bar{y} = \bar{x}_1 = 2 \end{cases} \qquad \begin{cases} \bar{v} - 4 = 0 \\ \bar{x}_1 = 2 \end{cases} \longrightarrow \bar{v} = 4$$ ✓ A.S.

5.4

$$S : \begin{cases} \dot{x}(t) = x^2(t) + u(t) \\ y(t) = x(t) \end{cases}$$

1) $u(t) = \bar{u} = -1$, $t \geq 0$ $\longrightarrow$ $\dot{x}(t) = x^2(t) - 1$



$\bar{x} = -1$ eq. as. stabile

$\bar{x} = 1$ eq. instabile

2) $v(t) = \bar{v}$ $\quad K \in \mathbb{R}$: $\bar{x} > 0$ e S. A.S.

$$\begin{cases} \dot{x}(t) = x^2(t) + Kx(t) + v(t) \\ y(t) = x(t) \end{cases} \longrightarrow 0 = 1 + K + \bar{v} \longrightarrow \bar{v} = -1 - K$$

$u(t) = v(t) + Kx(t)$ $\qquad A = \frac{\partial f}{\partial x} \longrightarrow A = 2\bar{x} + K \big|_{\bar{x}=1} = 2 + K$ $\qquad 2 + K < 0$, $K < -2$

5.5

$J = m\ell^2, \quad \tau_a = K\dot{\theta}(t), \quad K>0$

1) $y(t) = \theta(t)$ $\qquad J\ddot{\theta}(t) = \tau(t) - \tau_a(t) - mg\ell\,\sin(\theta(t))$

$u(t) = \tau(t)$ $\qquad \Rightarrow m\ell^2\ddot{\theta}(t) = \tau(t) - K\dot{\theta}(t) - mg\ell\,\sin(\theta(t))$

$x_1(t) = \theta(t)$

$x_2(t) = \dot{\theta}(t)$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\frac{g}{\ell}\sin(x_1(t)) - \frac{K}{m\ell^2}x_2(t) + \frac{u(t)}{m\ell^2} \\ y(t) = x_1(t) \end{cases}$$

2) $$\begin{cases} \bar{x}_2 = 0 \\ -\frac{g}{\ell}\sin(\bar{x}_1) = 0 \rightarrow x_1 = k\pi \\ y(t) = \bar{x}_1 \end{cases} \qquad \bar{x} = \begin{bmatrix} \pi \\ 0 \end{bmatrix}$$

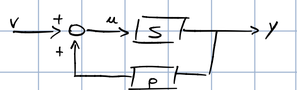
3) $$\begin{cases} \Delta\dot{x}_1(t) = \Delta x_2(t) \\ \Delta\dot{x}_2(t) = -\frac{g}{\ell}\cos(\bar{x}_1)\Delta x_1(t) - \frac{K}{m\ell^2}\Delta x_2(t) + \frac{1}{m\ell^2}\Delta u(t) \\ \Delta y(t) = \Delta x_1(t) \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{g}{\ell}\cos(\bar{x}_1) & -\frac{K}{m\ell^2} \end{bmatrix} \qquad \det(\lambda I - A) = \lambda\left(\lambda + \frac{K}{m\ell^2}\right) + \frac{g}{\ell}\cos(\bar{x}_1)$$

$$\lambda\left(\lambda + \frac{K}{m\ell^2}\right) + \frac{g}{\ell}\cos(\bar{x}_1) = \lambda^2 + \frac{K}{m\ell^2}\lambda + \frac{g}{\ell}\cos(\bar{x}_1), \quad \bar{x}_1 = 0 \Rightarrow \text{A.S.}$$

$\bar{x}_1 = \pi \Rightarrow$ instabile

4)



$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\frac{g}{\ell}\sin(x_1(t)) - \frac{K}{m\ell^2}x_2(t) + \frac{v(t) + p x_1(t)}{m\ell^2} \\ y(t) = x_1(t) \end{cases}$$

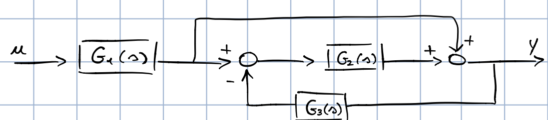
$0 = \bar{x}_2$

$$0 = -\frac{g}{\ell}\sin(\bar{x}_1) + \frac{\bar{v}}{m\ell^2} + \frac{p\bar{x}_1}{m\ell^2} \xrightarrow[\bar{x}_1 = \pi]{} \frac{\bar{v}}{m\ell^2} + \frac{\pi p}{m\ell^2} = 0 \rightarrow \bar{v} = -\pi p$$

$y = \bar{x}_1$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{p}{m\ell^2} - \frac{g}{\ell}\cos(\bar{x}_1) & -\frac{K}{m\ell^2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{p}{m\ell^2} + \frac{g}{\ell} & -\frac{K}{m\ell^2} \end{bmatrix} \Rightarrow p < -m\ell g$$

5.6



1) $G_1(s) = \frac{1}{s-2}$ Non può essere AS perchè $G_1(s)$ è in cascata e l'autovalore +2 si preserva

2) $G_2(s) = \frac{1}{s-3}$ S può essere A.S. perchè nella retroazione non si preservano gli autovalori

3) $Y = W + Y(-G_2G_3) \rightarrow Y = \frac{1}{1+G_2G_3}W$

$Y = G_2(W - G_3Y) \rightarrow Y = -G_2G_3Y + G_2W$

$Y = \frac{G_2}{1+G_2G_3}W$

parallelo : $G_R(s) = \left(\frac{1}{1+G_2G_3} + \frac{G_2}{1+G_2G_3}\right)$

$$G_R(s) = \frac{1 + G_2(s)}{1 + G_2(s)G_3(s)}, \quad G_1 \text{ è in serie a } G_R \Rightarrow G(s) = G_1 \cdot \frac{1 + G_2(s)}{1 + G_2(s)G_3(s)}$$

5.7

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + x_2(t) + u(t) \\ \dot{x}_2(t) = -3x_2(t) + 3u(t) \\ y(t) = x_2(t) \end{cases}$$

1) $u(t) = 2\,\mathrm{sca}(t) \longrightarrow U(s) = \frac{2}{s}$

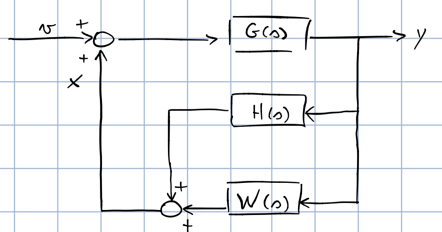
$$\begin{cases} sX_2(s) = -3X_2(s) + 3U(s) \\ Y(s) = \frac{3}{s+3}U(s) \end{cases} \longrightarrow Y(s) = \frac{6}{s+3}\frac{1}{s} = \frac{\alpha}{s-3} + \frac{\beta}{s}$$

$$\begin{cases} \alpha + \beta = 0 & \alpha = -2 \\ +3\beta = 6 \longrightarrow & \beta = 2 \end{cases}$$

$$Y(s) = \frac{-2}{s+3} + \frac{2}{s} \xrightarrow[\mathcal{L}^{-1}]{} y(t) = 2 - 2e^{-3t}$$

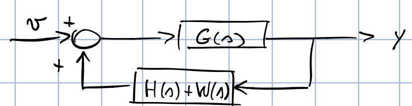
2) $G(s) = \frac{3}{s+3}$

3)



$$H(s) = -\frac{K}{s+2} \qquad W(s) = -\frac{K}{s+4}$$

FDT $v(t) \longrightarrow y(t)$



$$Y = G(s)\left(v + Y\left(H(s)+W(s)\right)\right)$$

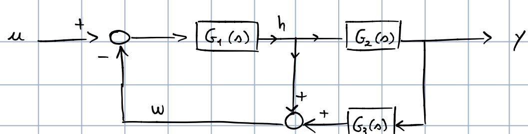
$$Y\left(1 - G(H+W)\right) = G\,v$$

FDT: $$\frac{G(s)}{1 - G(s)\left(H(s)+W(s)\right)} = \frac{\frac{3}{s+3}}{1 - \frac{3}{s+3}\left(\frac{-Ks-4K-Ks-2K}{(s+2)(s+4)}\right)} =$$

$$= \frac{\frac{3}{s+3}}{1 - \frac{3}{s+3}\left(\frac{-6K-2Ks}{(s+2)(s+4)}\right)} = \frac{\frac{3}{s+3}}{1 - \frac{3}{\cancel{s+3}}\cdot(-2K)\left(\frac{\cancel{3+s}}{(s+2)(s+4)}\right)} = \frac{\frac{3}{s+3}}{1 + \frac{6K}{(s+2)(s+4)}} = \frac{\frac{3}{s+3}}{\frac{(s+2)(s+4)+6K}{(s+2)(s+4)}} = \frac{3(s+2)(s+4)}{s+3\left[(s+2)(s+4)+6K\right]}$$

4) K : S è AS $\quad S(s) = \frac{3(s+2)(s+4)}{(s+3)\left(s^2+6s+6K+8\right)}$, $6K+8>0 \longrightarrow K > -\frac{8}{6} = -\frac{4}{3}$

5.8



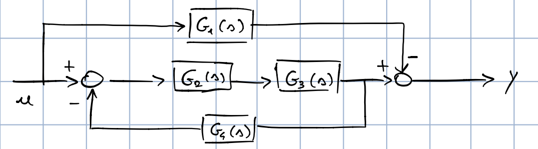
$$H = G_1(U - W) \qquad \longrightarrow H = G_1(U - YG_3 - H)$$
$$Y = G_2 H \qquad H(1+G_1) = G_1(U - YG_3)$$
$$W = YG_3 + H \qquad H = \frac{G_1}{1+G_1}(U - YG_3)$$

$$Y(s) = G_2(s)\frac{G_1(s)}{1+G_1(s)}\left(U(s) - Y(s)G_3(s)\right)$$

$$Y(s) = \frac{G_1G_2}{1+G_1}U(s) - \frac{G_1G_2G_3}{1+G_1}Y(s) \longrightarrow Y(s)\left(1 + \frac{G_1G_2G_3}{1+G_1}\right) = \frac{G_1G_2}{1+G_1}U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{G_1G_2}{\cancel{1+G_1}}\left(\frac{\cancel{1+G_1}}{1+G_1+G_1G_2G_3}\right) = \frac{G_1(s)G_2(s)}{1+G_1(s)+G_1(s)G_2(s)G_3(s)}$$

5.9



1) G_2 e G_3 in cascata: $G_2 G_3$
G_4 retroazione neg.: $\frac{G_2 G_3}{1+G_2 G_3 G_4}$, in parallelo a G_1: $Y(s) = U\left(\frac{G_2 G_3}{1+G_2 G_3 G_4} - G_1\right)$

$$H(s) = \frac{G_2 G_3 - G_1 G_2 G_3 G_4 - G_1}{1+G_2 G_3 G_4}$$

2) $G_1 = \frac{1}{s+10}$ $\quad G_2 = \frac{s-1}{s+2}$ $\quad G_3 = \frac{1}{s-1}$ $\quad G_4 = -\frac{8}{s+9}$

$$H(s) = \frac{\frac{(s+10)(s+9) + 8 - (s+2)(s+9)}{(s+10)\cancel{(s+2)(s+9)}}}{\frac{(s+2)(s+9) - 8}{\cancel{(s+2)(s+9)}}} = \frac{(s+10)(s+9) - s^2 - 11s - 10}{(s+10)(s^2+11s+10)} = \frac{s^2+19s+90 - s^2 - 11s - 10}{(s+10)(s^2+11s+10)} = \frac{8s+80}{(s+10)(s^2+11s+10)}$$

$$= \frac{8}{(s+1)(s+10)}$$

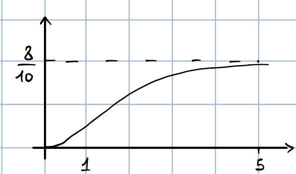
3) $G_2 G_3 \Rightarrow$ autov. nascosto $\lambda = 1 \Rightarrow$ Sistema instabile

4) Guadagno $\mu = \frac{8}{10}$, poli $p_1 = -1$ $p_2 = -10$, No zeri

Tipo: $g = 0$

5) $U(s) = \frac{1}{s}$ $\quad Y(s) = H(s)\, U(s) = \frac{8}{(s+1)(s+10)} \cdot \frac{1}{s}$

a) TVI: $\lim_{s\to\infty} s\, Y(s) = 0$ $\quad T_a = 5$ udt

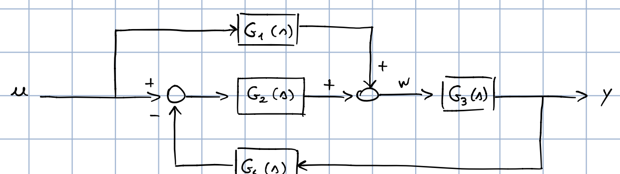


TVF: $\lim_{s\to 0} s\, Y(s) = \frac{8}{10}$

$\dot{y}(t) = \lim_{s\to\infty} s^2\, Y(s) = 0$

$\ddot{y}(t) = \lim_{s\to\infty} s^2\, H(s) = 8 > 0$

5.10



1) $W = G_1 U + G_2 (U - G_4 Y) = G_1 U + G_2 U - G_2 G_4 Y$
$Y = G_3 W \longrightarrow Y = G_3 U (G_1 + G_2) - G_2 G_3 G_4 Y \longrightarrow Y(s) = \frac{G_3 (G_1 + G_2)}{1+G_2 G_3 G_4} U(s)$

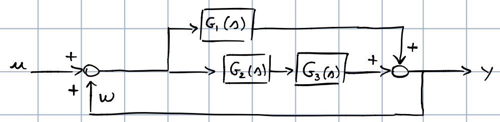
$$H(s) = \frac{G_3(s)\left(G_1(s) + G_2(s)\right)}{1+G_2(s) G_3(s) G_4(s)}$$

2) L'A.S. è necessaria ma non sufficiente

$$H(s) = \frac{\frac{N_3}{D_3}\left(\frac{N_1}{D_1} + \frac{N_2}{D_2}\right)}{1+\frac{N_2}{D_2}\frac{N_3}{D_3}\frac{N_4}{D_4}} = \frac{\frac{N_3}{\cancel{D_3}}\left(\frac{N_1 D_2 + N_2 D_1}{D_1 \cancel{D_2}}\right)}{\frac{D_2 D_3 D_4 + N_2 N_3 N_4}{\cancel{D_2}\cancel{D_3} D_4}} = \frac{D_4 N_3 (N_1 D_2 + N_2 D_1)}{D_1 (D_2 D_3 D_4 + N_2 N_3 N_4)}$$

$\Rightarrow$ $G_1(s)$ deve essere necessariamente AS

5.11



1) $W = (U+W)G_1 + (U+W)(G_2G_3) = Y$

$(U+Y)(G_1+G_2G_3) = Y \rightarrow \frac{Y}{U} = \frac{G_1+G_2G_3}{1-(G_1+G_2G_3)}$

$$H(s) = \frac{\frac{4}{s+10} + \frac{s+2}{(s+2)(s+10)}}{1-\left(\frac{4}{s+10}+\frac{s+2}{(s+2)(s+10)}\right)} = \frac{\frac{4s+8+s+2}{(s+10)(s+2)}}{\frac{(s+10)(s+2)-4s-8-s-2}{(s+10)(s+2)}} = \frac{5s+10}{(s+10)(s+2)-5s-10} = \frac{5s+10}{s^2+12s+20-5s-10} = \frac{5s+10}{s^2+7s+10}$$

$$= \frac{5(s+2)}{(s+2)(s+5)} = \frac{5}{s+5}$$

2) Il sistema è AS perchè i poli nascosti sono $p_1 = -2$, $p_2 = -10$

3) $u(t) = e^{-2t} + sca(t) \longrightarrow U(s) = \frac{1}{s+2} + \frac{1}{s}$

$Y(s) = H(s)\cdot U(s)$

$$= \frac{5}{s+5}\left(\frac{1}{s+2}+\frac{1}{s}\right) = \frac{5}{s(s+5)} + \frac{5}{(s+5)(s+2)} = \left(\frac{\alpha}{s}+\frac{\beta}{s+5}\right)+\left(\frac{\gamma}{s+5}+\frac{\Delta}{s+2}\right)$$

$$\begin{cases} \alpha+\beta = 0 & \rightarrow \beta = -1 \\ 5\alpha = 5 & \rightarrow \alpha = 1 \end{cases} \qquad \begin{cases} \gamma+\Delta = 0 & \rightarrow \gamma = -\Delta \\ 2\gamma+5\Delta = 5 & \rightarrow -2\Delta+5\Delta = 5 \end{cases} \rightarrow \Delta = \frac{5}{3}, \quad \gamma = -\frac{5}{3}$$

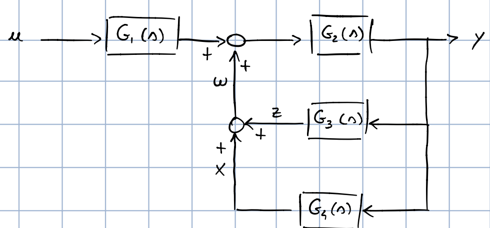
$$= \frac{1}{s} - \frac{1}{s+5} - \frac{5}{3}\cdot\frac{1}{s+5} + \frac{5}{3}\cdot\frac{1}{s+2}$$

$$y(t) = 1 - e^{-5t} - \frac{5}{3}e^{-5t} + \frac{5}{3}e^{-2t}$$

TVI: $\lim_{s\to\infty}\left[s\,Y(s) = \frac{5}{s+5} + \frac{5s}{(s+5)(s+2)}\right] = 0$

TVF: $\lim_{s\to 0} s\,Y(s) = 1$

5.12



1) $Y = (G_1U + W)G_2$

$W = Z + X$

$Z = YG_3 \longrightarrow W = Y(G_3+G_4)$

$X = YG_4$

$\rightarrow Y = (G_1U + YG_3 + YG_4)G_2$

$= G_1G_2U + G_2G_3Y + G_2G_4Y$

$Y(1-G_2G_3-G_2G_4) = G_1G_2U$

$\frac{Y}{U} = \frac{G_1G_2}{1-G_2G_3-G_2G_4}$

$$H(s) = \frac{G_1(s)G_2(s)}{1-G_2(s)G_3(s)-G_2(s)G_4(s)}$$

2) $$H(s) = \frac{\frac{s-2}{(s-2)(s+7)}}{1-\frac{2(s-2)}{(s+7)(s+2)}+\frac{s-2}{(s+7)(s+2)}} = \frac{\frac{s-2}{(s-2)(s+7)}}{\frac{(s+2)(s+7)-2(s-2)+(s-2)}{(s+2)(s+7)}} = \frac{\frac{s-2}{(s-2)\cancel{(s+7)}}}{\frac{(s+2)(s+7)+(2-s)}{(s+2)\cancel{(s+7)}}} = \frac{s^2-4}{(s^2-4)(s+7)-(s-2)^2} = \frac{s^2-4}{s^3+7s^2-\cancel{4s}-28-s^2-4+\cancel{4s}}$$

$$= \frac{s^2-4}{s^3+6s^2-32}$$

$$\begin{array}{r|rrr|r} & 1 & 6 & 0 & -32 \\ -4 & & -4 & -8 & +32 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

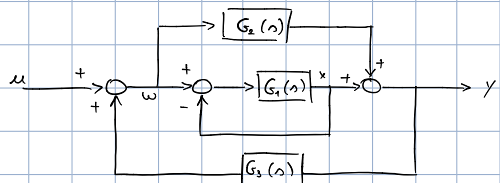
$$= \frac{s^2-4}{(s+4)^2(s-2)} = \frac{s+2}{(s+4)^2}$$

b) È instabile perchè il polo nascosto è $p = 2$

3) $u(t) = e^{-2t} \longrightarrow U(s) = \frac{1}{s+2}$

$$Y(s) = \frac{s+2}{(s+4)^2} \cdot \frac{1}{s+2} = \frac{1}{(s+4)^2} = t e^{-4t}$$

5.13



1) $W = U + G_3 Y$

$$X = (W - X)G_1 \longrightarrow X = \frac{G_1 W}{1+G_1}$$

$$Y = G_2 W + X = G_2 W + \frac{G_1 W}{1+G_1}$$

$$= W\left(G_2 + \frac{G_1}{1+G_1}\right) = W\left(\frac{G_2 + G_1G_2 + G_1}{1+G_1}\right)$$

$$Y = (U + G_3 Y)\left(\frac{G_1 + G_2 + G_1G_2}{1+G_1}\right)$$

$$Y\left(1 - G_3\left(\frac{G_1 + G_2 + G_1G_2}{1+G_1}\right)\right) = U\left(\frac{G_1 + G_2 + G_1G_2}{1+G_1}\right)$$

$$Y\left(\frac{1+G_1 - G_1G_3 - G_2G_3 - G_1G_2G_3}{\cancel{1+G_1}}\right) = U\left(\frac{G_1 + G_2 + G_1G_2}{\cancel{1+G_1}}\right)$$

$$\frac{Y}{U} = \frac{G_1 + G_2 + G_1G_2}{1 + G_1 - G_1G_3 - G_2G_3 - G_1G_2G_3} = \frac{\frac{1}{s+1} + \frac{1}{s+2} + \frac{1}{(s+1)(s+2)}}{1 + \frac{1}{s+1} - \frac{4}{(s+1)(s+9)} - \frac{4}{(s+2)(s+9)} - \frac{4}{(s+2)(s+9)(s+1)}} = \frac{\frac{s+2+s+1+1}{(s+1)(s+2)}}{\frac{(s+2)(s+9)(s+1) + (s+2)(s+9) - 4(s+2) - 4(s+1) - 4}{(s+2)(s+9)(s+1)}} =$$

$$= \frac{\frac{2\cancel{(s+2)}}{\cancel{(s+1)}\cancel{(s+2)}}}{\frac{s^3+12s^2+29s+18+s^2+11s+18-8s-16}{(s+2)(s+9)\cancel{(s+1)}}} = \frac{2}{\frac{s^3+13s^2+32s+20}{(s+2)(s+9)}} = \frac{2(s+2)(s+9)}{s^3+13s^2+32s+20}$$

$$(s^2+11s+18)(s+1) = s^3 + 12s^2 + 29s + 18$$

$$= \frac{2\cancel{(s+2)}(s+9)}{\cancel{(s+2)}(s^2+11s+10)} = \frac{2(s+9)}{(s+1)(s+10)}$$

	1	13	32	20
-2		-2	-22	-20
	1	11	+10	0

$$s = \frac{-11 \pm \sqrt{121-40}}{2} = \frac{-11 \pm \sqrt{81}}{2} \begin{cases} -1 \\ -10 \end{cases}$$

2) A.S. con polo nascosto $p_1 = -2$

3) $\mu = \frac{9}{5} \approx 1{,}8$

(a) no (no oscillaz)

(b) No (sottoelongazione)

(d) NO (No sovraelong.)

TVF: $\lim_{s\to 0}\left[s Y(s) = \frac{2(s+9)}{(s+1)(s+10)}\right] = \frac{18}{10}$

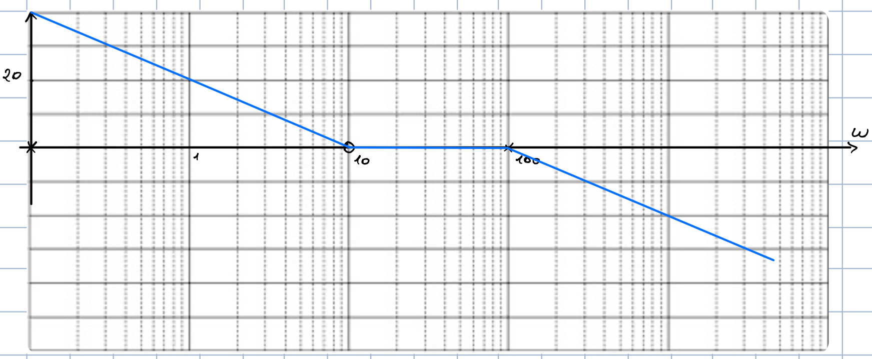
CAP. 6

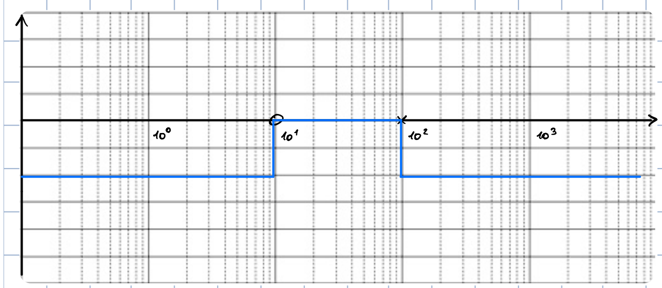
6.1

$$G(s) = \frac{10}{s}\,\frac{1+0{,}1s}{1+0{,}01s}$$

$p_1 = \dfrac{1}{0{,}01} = -10^2$ $\qquad z_1 = -10$

$p_2 = 0$ $\qquad g = 1$





6.2

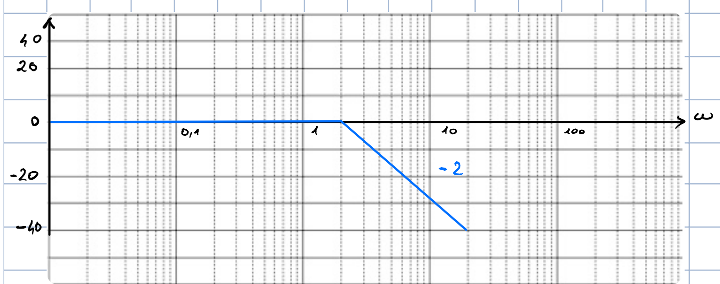
$$G(s) = \frac{\omega_n^2}{s^2 + 2\,\xi\,\omega_n s + \omega_n^2}\ ,\quad \omega_n = 2,\quad \xi = 0.8$$

1) $\mu = G(0) = 1$

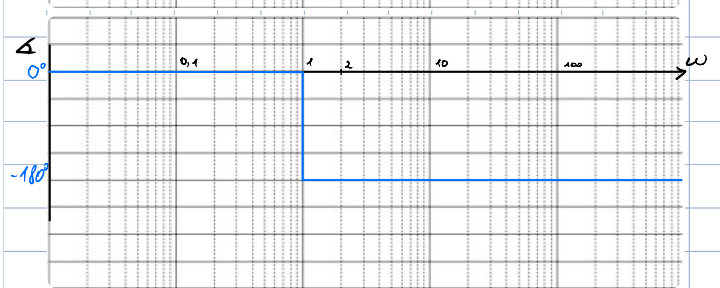
$$G(s) = \frac{4}{s^2 + \frac{16}{5}s + 4} \Rightarrow \text{poli c.c.} \left(-\frac{8}{5} \pm 12i\right)$$

Sostituisco con $\bar{p}_1 = \bar{p}_2 = \text{sign}(\text{Re}(p_1)) \cdot \omega_n = -2$

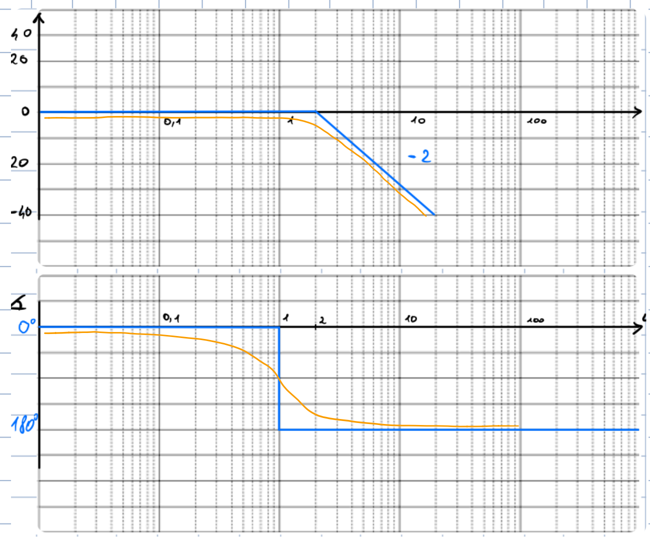
Diagramma del modulo: pendenza iniziale $-20g = 0$ dB e $|\mu| = 0$ dB
In $\omega = 2$ i contributi dei due poli $-2 \cdot 20 = -40$ dB/decade

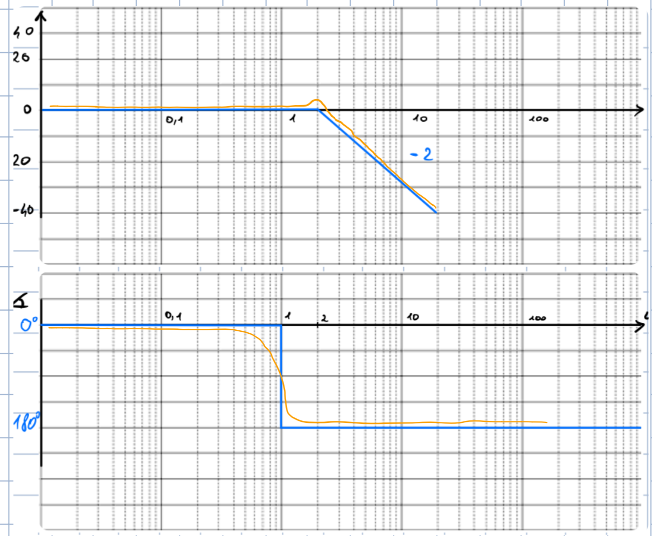


Fase : 2 zeri reali coincidenti
$-180°$

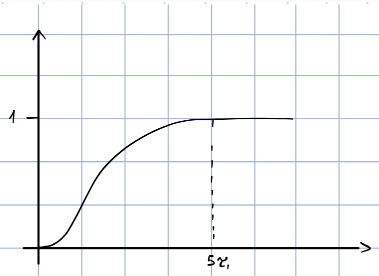


2) $\xi = 0.1$ non cambia i diagrammi asintotici





3)



6.3

$$G(s) = \frac{5(1-s)^3}{s(1+s/0.2)(1+s/8)(1-s/8)}$$

$g = 1$

$-20\,dB/decade$

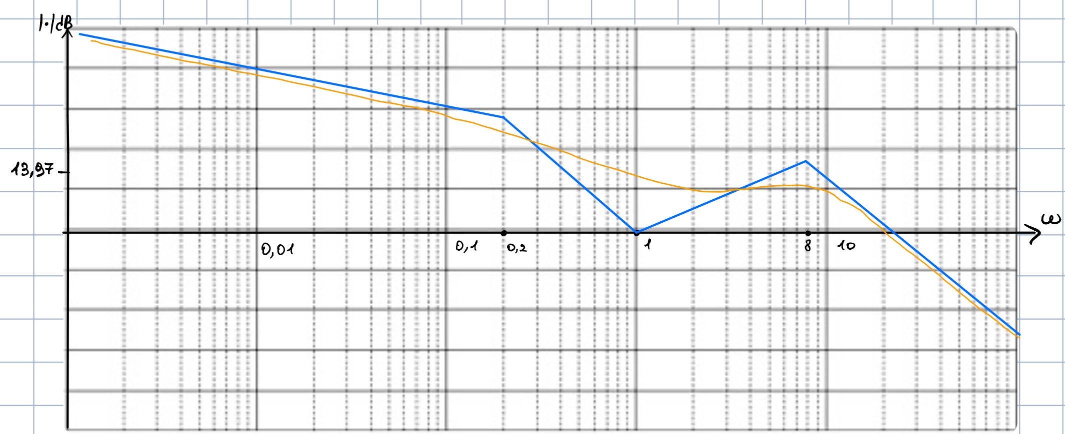
$\mu = sG(0) = 5 \simeq 13,97\,dB$

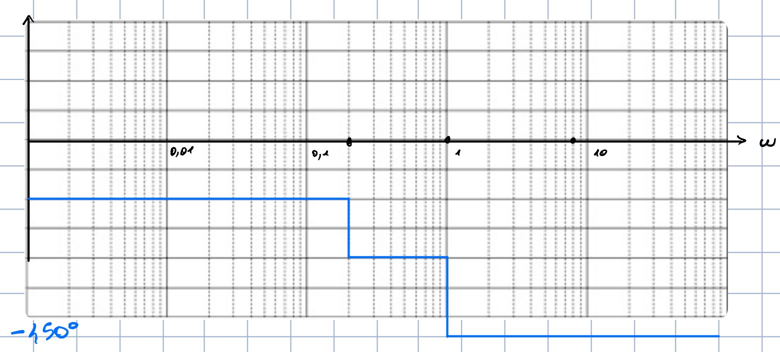
3 zeri in $s = 1$ ⟶ $+60\,dB/decade$

1 polo in $-0,2$ ⟶ $-20\,dB/dec$

1 polo in -8 ⟶ $-20\,dB/dec$

1 polo in $+8$ ⟶ $-20\,dB/dec$





6.4

$$G(s) = \frac{1 + s/0.5}{\left(1 + 2\frac{\xi}{\omega_n}s + \frac{s^2}{\omega_n^2}\right)\left(1 + s/4\right)}$$

$\xi = 0.8\ ,\quad \omega_n = 2$

1) $g = 0 \rightarrow$ pendenza iniziale 0 $\qquad \mu = 1$

$z_1 = -0{,}5 \longrightarrow +20\,dB$

$p_1 = p_2 = -2 \longrightarrow -40\,dB$

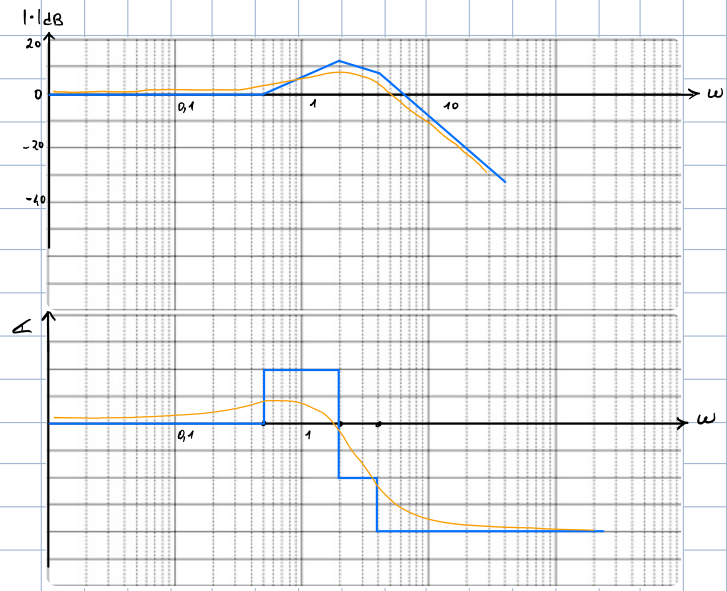
$p_3 = -4 \longrightarrow -20\,dB$

Fase : $-90g \Rightarrow$ fase iniziale $0°$

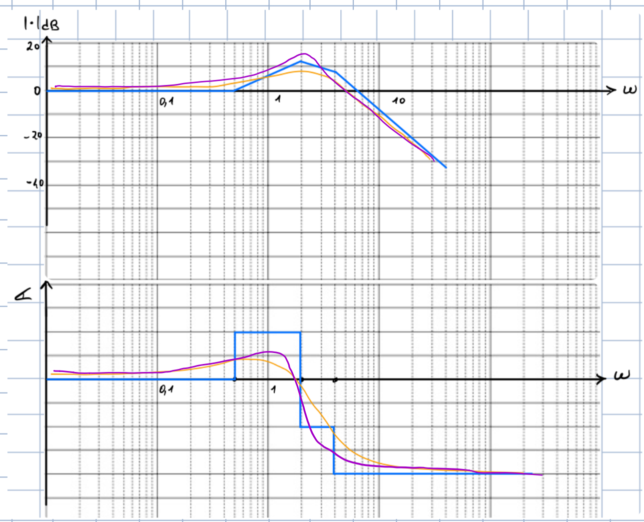
$z_1 \longrightarrow +90°$

$p_1 + p_2 \Rightarrow -180°$

$p_3 \rightarrow -90°$



2) Con smorzamento pari a 0.1 il modulo ha un picco in $\omega = \omega_n\sqrt{1-2\xi^2} = 2\sqrt{1-2\cdot} = 1{,}98$ e il diagramma della fase tende all' asintotico.



6.5

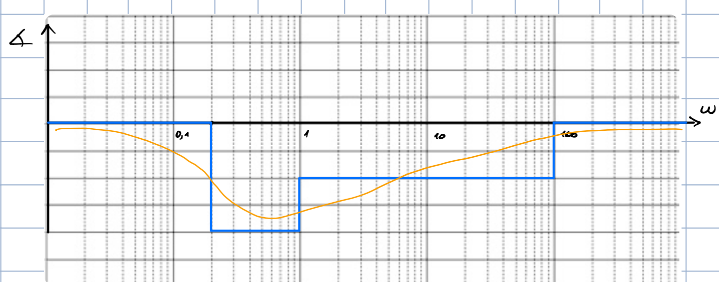
2 poli in $-0{,}2$ $\qquad g = 0$ perchè parte da retta orizzontale

1 zero in -1 $\qquad \mu = 26\ dB$

1 zero in -100

$26 = 20\log_{10} x \longrightarrow \qquad \mu = 20$

$$G(s) = \frac{20\left(1 + \frac{s}{100}\right)(1+s)}{\left(1 + \frac{s}{0{,}2}\right)^2}$$



6.6

$$G(s) = \frac{10\,(s+1)}{(s+0.1)\,(s^2+20s+100)} \qquad n = 3$$

1) # poli = 3 $\Rightarrow$ S A.S.

$p_1 = -0.1 \qquad p_{2-3} = \frac{-20 \pm \sqrt{20^2-400}}{2} = -10$

2) polo dominante $p_1 = -0,1$

3) $y_\infty(t)$ quando $u(t) = \underbrace{2}_{u_1} + \underbrace{\sin(0,01\,t)}_{u_2} + \underbrace{\sin(0,1\,t)}_{u_3} + \underbrace{2\cos(100t)}_{u_4}$

$y_{1,\infty}(t) = 2 \cdot G(0) = 2$

$y_{2,\infty}(t) = |G(j\,0,01)| \cdot \sin(0,01\,t + \measuredangle G(j0,01))$

$$G(s) = \frac{1+s}{(1+10s)\,(1+0,1s)^2} \longrightarrow \measuredangle G(j\omega) = \measuredangle \frac{1+j\omega}{(1+10j\omega)\,(1+0,1j\,\omega)^2} = \measuredangle(1+j\omega) - \measuredangle(1+10j\omega) - 2\measuredangle(1+0,1j\,\omega) = tg^{-1}\omega - tg^{-1}10\omega - 2tg^{-1}0,1\,\omega$$

$\measuredangle G(j\omega) = \arctan\omega - \arctan(10\omega) - 2\arctan(0,1\omega)$

$\measuredangle G(j0,01) = tg^{-1}0,01 - tg^{-1}0,1 - 2tg^{-1}0,001 \simeq 0$ rad

$$|G(j0,01)| = \left|\frac{1+j0,01}{(1+j0,1)(1+j0,001)^2}\right| = \frac{\sqrt{1+10^{-4}}}{\sqrt{1+10^{-2}}\cdot\sqrt{1+10^{-6}}} = \simeq 1$$

$\rightarrow y_{2,\infty}(t) = \sin(0,01\,t)$

$y_{3,\infty}(t) = |G(j0,1)|\sin(0,1t + \measuredangle G(j0,1))$

$\measuredangle G(j0,1) = tg^{-1}0,1 - tg^{-1}1 - 2tg^{-1}0,01 = -\frac{\pi}{4}$

$|G(j0,1)| = \frac{\sqrt{1+10^{-2}}}{\sqrt{2}\;\sqrt{1+10^{-4}}} \simeq \frac{1}{\sqrt{2}}$

$\Big\}\; y_{3,\infty}(t) = \frac{1}{\sqrt{2}}\operatorname{sen}(0,1t - \frac{\pi}{4})$

$y_{4,\infty}(t) = 2|G(j100)|\cos(100t + \measuredangle G(j100))$

$\measuredangle G(j100) = tg^{-1}100 - tg^{-1}1000 - 2tg^{-1}10 \cong -169°$

$$|G(j100)| = \frac{\sqrt{1+10^4}}{\sqrt{1+10^6}\left(\sqrt{1+10^2}\right)^2} = \frac{10^2}{10^2\cdot 10^3} = 10^{-3}$$

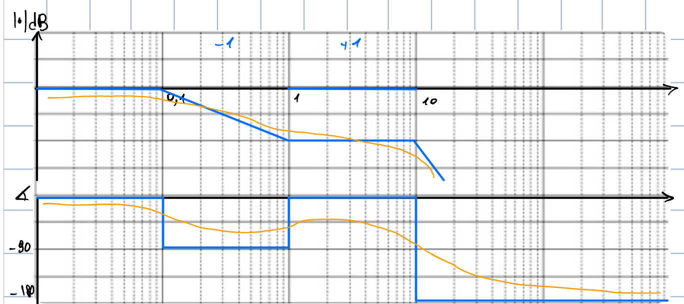
$y_{4,\infty}(t) = 2\cdot 10^{-3}\cdot\cos(100t - 169°)$

$y_\infty(t) = y_{1,\infty}(t) + y_{2,\infty}(t) + y_{3,\infty}(t) + y_{4,\infty}(t)$

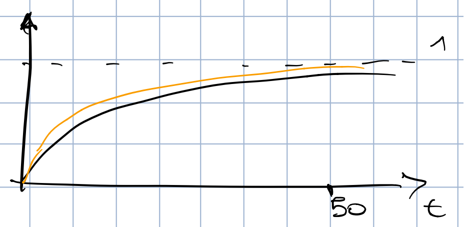
4) $G(s) = 10\,\frac{s+1}{(s+0,1)(s^2+20s+100)}$ $\qquad \mu = 1 = 0$dB

$p_1 = -0,1 \qquad p_2 = p_3 = -10$ $\qquad$ S a fase minima

$z_1 = -1$



5) Approssimazione : sistema a 1 polo $\tilde{G}(s) = \frac{1}{1+\frac{s}{0,1}}$



6) d) No, non passa per II quadrante

$G(j\omega) = 10 \dfrac{j\omega + 1}{(j\omega+0,1)(-\omega^2+20j\omega+100)}$ $\quad |G(j\omega)| = 10 \dfrac{\sqrt{1+\omega^2}}{\sqrt{10^{-2}+\omega^2} \cdot \sqrt{(100-\omega^2)^2+400\omega^2}}$

$\angle G(j\omega) = \angle \omega - \angle 10\omega - \angle \dfrac{100-\omega^2}{20\omega}$

$|G(j1)| = \dfrac{10\sqrt{2}}{\sqrt{10^{-2}+1}\sqrt{(99)^2+400}} = \dfrac{10\sqrt{2}}{101} \simeq \dfrac{\sqrt{2}}{10}$

c) no perche $\omega \to -180°$ e non $-90°$

b) no perche $\angle$ non è monotona decrescente

È a)

6.7

1) $g = 0$ $\quad \mu = 20\ dB = 10$

$p_1 = -0,05$ $\quad z_1 = -0,1$

$p_2 = p_3 = -1$

Poli complessi e sistema a fase minima, asintoticamente stabile

2) Andamento (c) perchè c'è il picco e perchè $\tau_d = 20 \Rightarrow T_a = 100$ udt

3) Diagramma (a)

6.8

$G(s) = \dfrac{1}{(s+5)(s+1)^2}$ $\quad \mu = \dfrac{1}{5}$ $\quad \tau_1 = 0,2$ $\quad \tau_2 = 1$ $\quad T_a \simeq 5$ udt

1) $u_1(t) = imp(t) \longrightarrow$ (d)

2) $u_2(t) = e^t sca(t) \longrightarrow \dfrac{1}{(s-1)}$ $\quad Y(s) = \dfrac{1}{(s-5)(s+1)^2(s-1)} = \dfrac{\alpha}{s-5} + \dfrac{\beta}{s+1} + \dfrac{\gamma}{(s+1)^2} + \dfrac{\delta}{s-1}$

diverge $\to +\infty \Rightarrow$ (a)

3) $u_3 = sin(t) sca(t) \longrightarrow y(t) = |G(j1)| \sin(t + \angle G(j1))$ $\quad |G(j1)| = \dfrac{1}{|j+5|\,|j+1|^2} = \dfrac{1}{\sqrt{26} \cdot 2} = \dfrac{\sqrt{26}}{52}$

$\angle G(j1) = -tg^{-1}\frac{1}{5} - 2tg^{-1}1 = -11,3 - 90 = -101,3°$

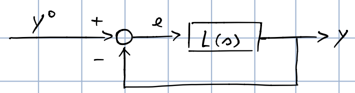
$\to$ (b)

4) $u_4(t) = \sin(100t) sca(t) \to \dfrac{1}{100^2 + s^2}$

$|G(j100)| = \dfrac{1}{|j100+5|\,|j100+1|^2} = \dfrac{1}{\sqrt{100^2+25} + 100^2+1} \simeq \dfrac{1}{10^4} \longrightarrow$ (c)

$\angle G(j100) = -tg^{-1} 20 - 2 tg^{-1} 100 = -87,1° - 178,8° = -265,9°$

6.9



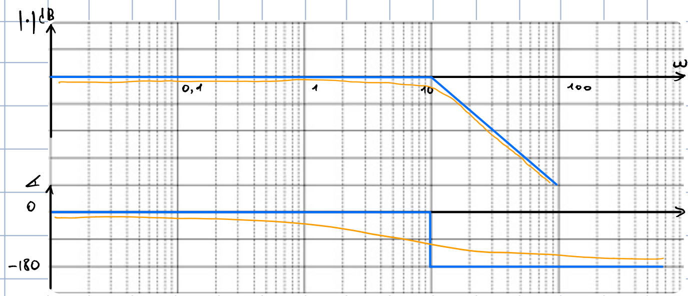
$$L(s) = \frac{5}{s(1+0{,}05s)}$$

$$y^\circ(t) = a_1 \sin(0{,}5t+\beta_1) + a_2 \sin(t+\beta_2) + a_3 \sin(100t+\beta_3)$$

FdT $y^\circ \to y(t)$: $F(s) = \dfrac{L(s)}{1+L(s)} = \dfrac{\frac{5}{s(1+0{,}05s)}}{1+\frac{5}{s(1+0{,}05s)}} = \dfrac{5}{s(1+0{,}05s)+5} = \dfrac{5}{0{,}05s^2+s+5} = \dfrac{100}{s^2+20s+100} =$

$$= \frac{100}{(s+10)^2} = \frac{1}{(1+s/10)^2}$$

$\mu = 1 = 0dB$ $\quad$ $p_1 = p_2 = -10$ $\qquad$ $-40dB = 10^{-2} = 0{,}01$



$u_1(t) = a_1 \sin(0.5t+\beta_1) \longrightarrow y_1(t) \simeq a_1 \sin(0.5t+\beta_1)$

$u_2(t) = a_2 \sin(t+\beta_2) \longrightarrow y_2(t) \simeq a_2 \sin(0{,}5t+\beta_2)$

$u_3(t) = a_3 \sin(100t+\beta_3) \longrightarrow y_3(t) \simeq 10^{-2} a_2 \sin(0{,}5t+\beta_2-180^\circ)$ $\quad$ NON riproduce correttamente u_3

6.10

1. a) No, in $\omega = 60$ il modulo di $G(s)$ è < 10
   b) Vero, modulo decrescente
   c) Falso
2. $z_1 = 0{,}5$ $\quad$ $p_1 = 1$ $\quad$ $p_2 = 10$

   $\tau_d = 1$ $\quad$ $T_a = 5$ udt

   è la (b) perchè si vede la presenza di una sovraelongazione
3. È la (c) => tg. verticale in -90°

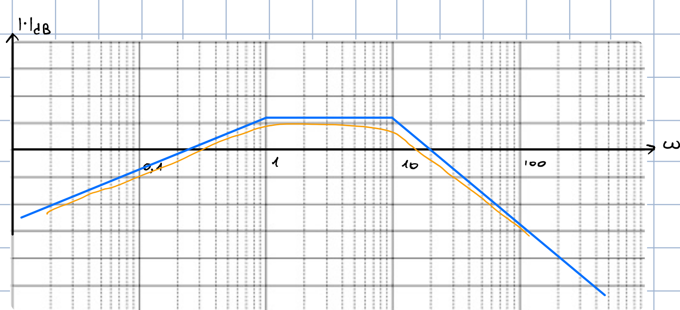
6.11

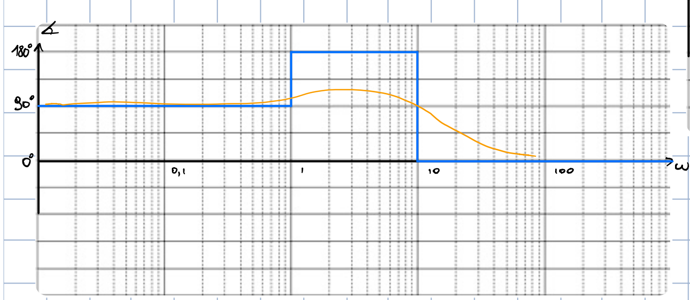
$$G(s) = \frac{4s}{(1-s)(1+0{,}1s)^2}$$

1) $g = -1$ $\qquad$ $\mu = 4 \simeq 12dB$

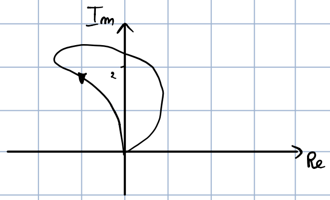
$p_1 = 1$ $\quad$ $p_2 = p_3 = -10$

$\tau_d = 1$

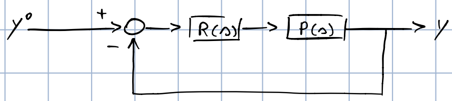




2)



7.1



$P(s) = \frac{1}{(s+1)^3}$ $\quad n = 3 \quad$ $R(s) = K,\ K \in \mathbb{R}^+$

1) Per quali K > 1 è AS?

$L(s) = \frac{K}{(s+1)^3} \qquad F(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{K}{(s+1)^3}}{1+\frac{K}{(s+1)^3}} = \frac{\frac{K}{\cancel{(s+1)^3}}}{\frac{(s+1)^3+K}{\cancel{(s+1)^3}}} = \frac{K}{(s+1)^3+K}$

$p_1 = p_2 = p_3 = -1$

Per AS: $\mu_L > 0$ ✓ $\forall K > 1$

$\varphi_m > 0$

$\varphi_m > 0$: $|\varphi_c| < 180 \qquad \cancel{\angle K}^{\emptyset} - 3\angle(j\omega+1) = -180°$

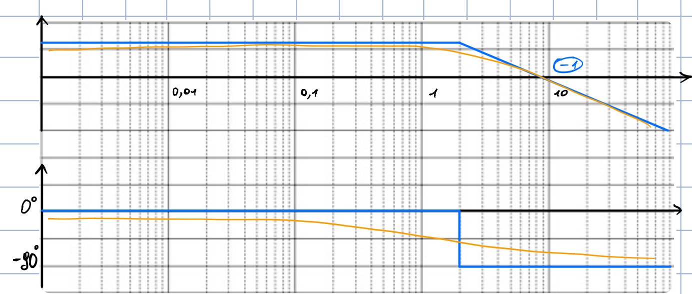
$3\,tg^{-1}\omega = 180 \Rightarrow tg^{-1}\omega = 60° \Rightarrow \bar{\omega} = \sqrt{3}$

$\bar{K}$: $|L(j\bar{\omega})| = 1$: $\left|\frac{\bar{K}}{(j\sqrt{3}+1)^3}\right| = 1 \Rightarrow |\bar{K}| = 8 \Rightarrow K < 8$

7.2

$G(s) = \frac{3K}{s+2} \qquad K \in \mathbb{R}^+$

∃ K > 1 : S è instabile?

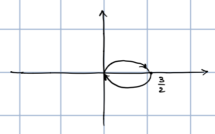


$p_1 = -2 \qquad \mu = \frac{3}{2}K$

↑ -1 ↑ -90°

$\varphi_m > 0$ ✓

$|\mu| > 0 \Rightarrow \frac{|3K|}{|s+2|}$



N=0

P=0 ✓ A.S.

Non esiste K > 1 tc S inst.

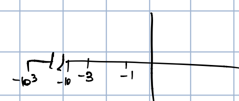
7.3

a) Sì perchè poli e zeri non sono tutti a Re > 0

b) Sì perchè ci sono 3 poli

c) Sì, pendenza mod e fase cambia solo di ± 20dB/dec

d) $G(s) = 10\,\frac{(1+s)}{(1+\frac{s}{3})(1+\frac{s}{10})(1+\frac{s}{10^3})}$ TVF: $y_\infty = \lim_{s\to 0} s\,Y_s = 10$ ✗ $\neq \frac{10}{11}$



e) Sì perchè lo zero è negativo e più vicino a Im dei poli

f) $\tau_d = 3 \rightarrow T_a = \frac{5}{3}$

g) Vero, si vede dal diagramma

2) (c) perchè il modulo cresce fino a 30 e poi decresce, infine fase asintotica $-180°$

3) $R(s) = \alpha, \quad \alpha \in \mathbb{R}^+$

$$L(s) = 10\alpha \frac{1+s}{\left(1+\frac{s}{3}\right)\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)} \qquad |L(s)|_{dB} > 0 \longrightarrow |L(s)| > 1$$

$P = 0 \quad N = 0 \quad$ per $\alpha > 1$

7.4

1. $g = 0, \quad \mu = 100 \quad p_1 = -1 \quad z_1 = -20 \quad p_2 = -100$

a) Sì, i poli sono a $Re < 0$

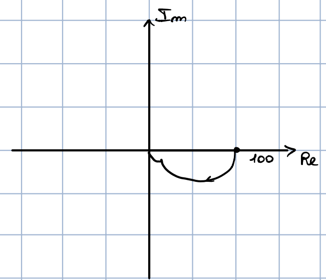
b) Sì perchè poli e zeri sono a $Re < 0$

c) Sì sono tutti reali $\Rightarrow$ le uniche variazioni di pendenza sono di ± 20dB/dec

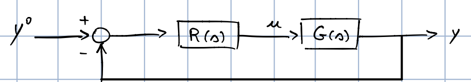
d) Falso il sistema è di ordine 2

2. (b) perche $T_a = 5$ e valore finale 100

3.



4.



$R(s) = \frac{1}{10}$

$L(s) = \frac{1}{10} G(s) \Rightarrow$ traslazione verso il basso del diagramma del modulo, la fase non cambia e $\mu = 10 = 20$ dB > 0 ✓ è A.S.

7.5

1. $g = 0 \quad p_1 = -0,2 \quad z_1 = -2 \quad p_2 = -20 \quad p_3 = -200 \quad \mu = 26$ dB

a) No è di ordine 3

b) Sì è AS perchè $Re(\text{poli}) < 0$

c) No perchè il guadagno è ~ 20

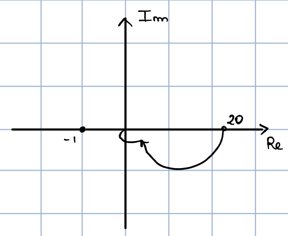
d) Sì perchè $\tau_d = 0,2 \Rightarrow T_a = 5 \cdot 5 = 25$

e) Non ci sono oscillazioni perchè i poli sono reali

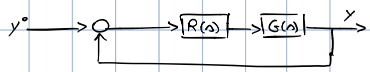
f) Vero, come si nota dal diagramma

g) No perchè non compromette la fase

2.



3.



$R(s) = K > 0$

$L(s) = K\,G(s)$ amplificazione o contrazione che non modifica φ_m => A.S.

4. a) No, è di ordine 3 perchè $R(s)$ non ha poli

b) Sì è A.S.

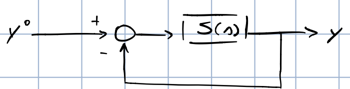
c) $H(s) = e^{-s}$ $L(s) = e^{-s}\,G(s)$

Modulo inalterato, sfasamento in ω_c pari a:

$$-\frac{180}{\pi}\,\omega_c\,\tau = -\frac{5400}{\pi} \simeq -1718{,}87° \quad e \quad \varphi_c = \measuredangle G(j\omega_c) - \frac{180}{\pi}\,\omega_c\,\tau < -180°$$

=> $\varphi_m < 0$ e il $\mathcal{S}$ non è A.S.

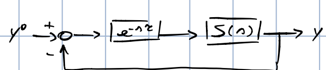
7.6



1) $\mu > 0$ ✓ => è A.S. per Bode

$\varphi_c \approx -80°$ => $\varphi_m > 0$ ✓

2) $\tau > 0$



sfasamento: $-\frac{180}{\pi}\,\omega_c\,\tau = -114$

$$\varphi_c = \measuredangle G(j\omega_c) - \frac{180}{\pi}\,\omega_c\,\tau = -204°$$

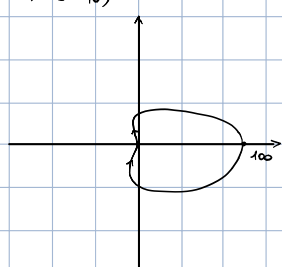
=> $\varphi_m < 0$ $\mathcal{S}$ è instabile

7.7

1) $n = 3$ $g = 0$ $\mu = 100$ $p_1 = p_2 = -1$ $p_3 = 10$

$$G(s) = \frac{100}{(s+1)^2\left(1 - \frac{s}{10}\right)}$$

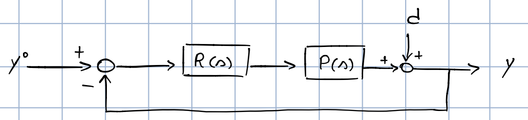
2) $P = 1$ $N = \emptyset$



3) $R(s) = h$, $L(s) = h\,G(s)$ modulo traslato in alto, fase non cambia $\omega_c = 10$ $\varphi_m > 0$ $\mu > 0$

però $P \neq N$ => NON è AS

8.1



$P(s) = \dfrac{10}{s+10}$ AS

$R(s) = \dfrac{K}{s}$, $K > 0$

1) $L(s) = \dfrac{10}{s(s+10)} K$ Bode: $\mu > 0$ ✓, ω_c ben definita ✓, poli $\mathrm{Re} \leq 0$ ✓, $\varphi_m > 0$ $\forall K > 0$ $\Rightarrow$ AS $\forall K > 0$

2) $e(t) = y^\circ(t) - y(t)$

e_∞ quando $y^\circ(t) = A_1 \, \mathrm{sca}(t)$ $d(t) = A_2 \, \mathrm{sca}(t)$ $|A_i| < 10$

FdT $y^\circ \to e$: $H(s) = \dfrac{1}{1+L(s)}$

FdT $d \to e$: $-H(s)$

$$H_{y^\circ}(s) = \frac{1}{1+R(s)P(s)} = \frac{1}{1+\frac{10K}{s(s+10)}} = \frac{s(s+10)}{s(s+10)+10K} = \frac{s(0.1s+1)}{s(0.1s+1)+K}$$

$$H_d(s) = -\frac{s(0.1s+1)}{s(0.1s+1)+K}$$

$Y^\circ(s) = \dfrac{A_1}{s}$ $D(s) = \dfrac{A_2}{s}$

$E_{y^\circ}(s) = \dfrac{A_1(0.1s+1)}{s(0.1s+1)+K}$ $E_d(s) = \dfrac{-A_2(0.1s+1)}{s(0.1s+1)+K}$

$$e_{\infty,y^\circ} = \lim_{s\to 0}\left[sE_{y^\circ}(s) = \frac{A_1}{1+K} s\right] = \emptyset \quad \forall A_1, K>0 \quad \Longrightarrow \quad e_\infty = \emptyset$$

$$e_{\infty,d} = \lim_{s\to 0}\left[sE_d(s) = \frac{-A_2}{1+K} s\right] = \emptyset \quad \forall A_2, K>0$$

3) $y^\circ(t) = \mathrm{sca}(t)$ $d(t) = 0,5 \, \mathrm{sca}(t)$

$F(s) = \dfrac{L(s)}{1+L(s)}$ $H(s) = 1 - F(s)$ $T_a = \dfrac{5}{\omega_c}$

Valore limite di ω_c tc $\bar{\varphi}_m = 60°$

$$|\angle L(j\bar{\omega}_c)| = 180° - \bar{\varphi}_m = 120° \Rightarrow \angle L(j\omega_c) = -120°$$

$$\angle L(j\bar{\omega}_c) = \angle K - \angle j\bar{\omega}_c - \angle(1+0.1j\bar{\omega}_c) = tg^{-1}(0) - 90° - tg^{-1}(0.1\bar{\omega}_c) = 0° - 90° - tg^{-1}(0.1\bar{\omega}_c)$$

$$-90° - tg^{-1}(0,1\bar{\omega}_c) = -120° \Rightarrow tg^{-1}(0,1\bar{\omega}_c) = 30° \Rightarrow 0,1\,\bar{\omega}_c = \frac{\sqrt{3}}{3} \to \bar{\omega}_c = \frac{10\sqrt{3}}{3}$$

$\varphi_m > 60° \iff 0 < K < \bar{K}$

$\varphi_m < 60° \iff K \geq \bar{K}$

$$|L(j\omega_c)| = |K| \cdot \frac{1}{|j\bar{\omega}_c|\left|1+j\frac{\bar{\omega}_c}{10}\right|} = \frac{K}{\bar{\omega}_c\sqrt{\frac{\bar{\omega}_c^2}{100}+1}} = \frac{3K}{10\sqrt{3}\sqrt{\frac{4}{3}}} = \frac{3K}{20}$$

$$\frac{3}{20}\bar{K} = 1 \iff \bar{K} = \frac{20}{3}$$

Se $0 < K < \dfrac{20}{3}$ FdT approssimabile a singolo polo: $F(s) = \dfrac{L(s)}{1+L(s)} \simeq \dfrac{1}{1+\frac{s}{\omega_c}}$

FdT da $d(t)$ a $y(t)$: $H(s) = \dfrac{1}{1+L(s)} = 1 - F(s)$

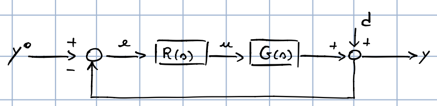
$$y(t) = d(t) - \mathcal{L}^{-1}[F(s)D(s)](t)$$

Se $K > \frac{20}{3}$ $\quad F(s) = \frac{L(s)}{1+L(s)}$ $\quad L(s) = \frac{\mu_F \omega_c^2}{s^2 + 2\xi\omega_c s + \omega_c^2}$ $\quad \mu_F = \frac{\mu_L}{1+\mu_L} = \frac{10K}{1+10K} \approx 1$

$\xi = \sin\left(\frac{\varphi_m}{2}\right) \to 0$ se $\varphi_m \to 0$

$T_a \simeq \frac{5}{\xi\omega_c}$

8.2



$G(s) = \frac{1}{1+s}$

1) $R(s) = K$ $\quad K \in \mathbb{R},\ K \neq \emptyset$

$L(s) = \frac{K}{1+s}$ $\quad p_1 = -1$

Per $K > 0$ $\quad P = \emptyset \implies$ AS.

$N = \emptyset$

Per $K < 0$, se $-1 < K < 0 \implies$ AS

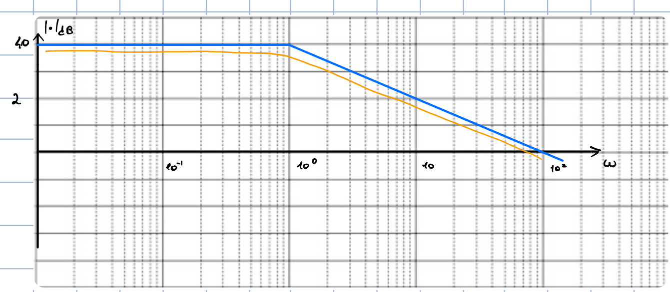
se $K < -1 \implies$ Instabile

se $K = -1 \implies$ NON AS

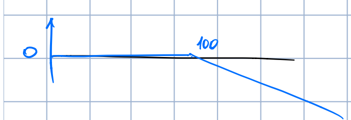
2) FdT $y° \to y$

es. $K = 100$

Diagramma $L(s)$:



$F(j\omega)$: $\quad |F(j\omega)| = \frac{|L(j\omega)|}{|1+L(j\omega)|} \implies |F(j\omega)| \simeq |F(j\omega)|_\sim = \begin{cases} 1 & \omega < \omega_c \\ |L(j\omega)| & \omega > \omega_c \end{cases}$



3) $y°(t) = sca(t)$

$d(t) = \emptyset$

$K = 10,\ K = 100$

$F(s) \simeq \frac{\mu_F}{1 + \frac{s}{\omega_c}}$ $\quad$ con $\quad \mu_F = \frac{\mu_L}{1+\mu_L} = \frac{K}{1+K}$

$\tau_{10} = \frac{1}{10}$ $\quad \tau_{100} = \frac{1}{\omega_c} = \frac{1}{100}$

$\mu_{10} = \frac{10}{11}$ $\quad \mu_{100} = \frac{100}{101}$

4) FdT $y° \to u(t)$ $\quad V(s) = \frac{R(s)}{1+L(s)}$

$$|V(j\omega)| = \left| \frac{R(j\omega)}{1+R(j\omega)G(j\omega)} \right| = \frac{|F(j\omega)|}{|G(j\omega)|}$$

8.3

$$R(s) = 1$$

1) 8.23 è AS ? $\quad y(t)\Big|_{y^{\circ}(t) = sca(t)}$

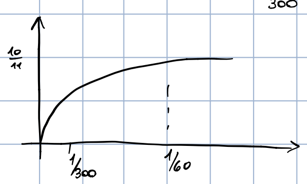
$\mu = 10 \qquad z_1 = -1 \qquad p_1 = -3 \qquad p_2 = -10 \qquad p_3 = -10^3 \qquad G(s) = \dfrac{10\,(s+1)}{\left(\frac{s}{3}+1\right)\left(\frac{s}{10}+1\right)\left(\frac{s}{10^3}+1\right)}$

$$L(s) = \frac{G(s)}{1+G(s)} = \frac{10(s+1)}{\left(\frac{s}{3}+1\right)\left(\frac{s}{10}+1\right)\left(\frac{s}{10^3}+1\right) + 10\,(s+1)}$$

È AS $\Rightarrow$ poli a $Re < 0$

$\varphi_m > 60° \Rightarrow$ polo dominante ω_c : $\quad F(s) = \dfrac{10}{1+\frac{s}{300}}$

$T_a = \dfrac{\cancel{5}}{\cancel{300}} \quad \dfrac{1}{60} \qquad \mu = \dfrac{\mu_L}{1+\mu_L} = \dfrac{10}{11}$



2) $n(t)$ con $\omega \in [1, 3]$

$n(t) = A \sin(\omega t)$

$y_\infty(t) = |H(j\omega)|\, A \sin(\omega t + \measuredangle H(j\omega)) \qquad H(s) = -\dfrac{L(s)}{1+L(s)} = \dfrac{-G(s)}{1+G(s)}$

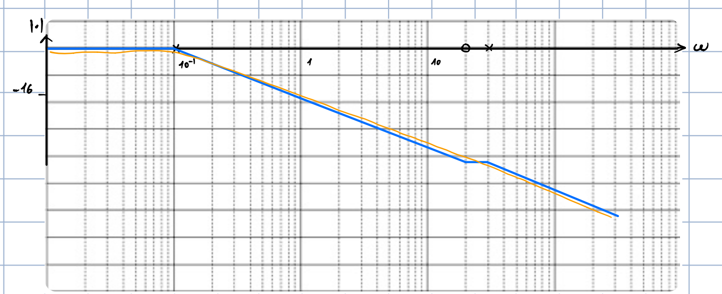
$$|H(j\omega)| = \frac{|G(j\omega)|}{|1+G(j\omega)|} \simeq \begin{cases} 1 & |G(j\omega)| > 1 \Rightarrow 1 \qquad \omega < \omega_c = 300 \\ |G(j\omega)| & |G(j\omega)| < 1 \Rightarrow |G(j\omega)| \qquad \omega > \omega_c = 300 \end{cases}$$

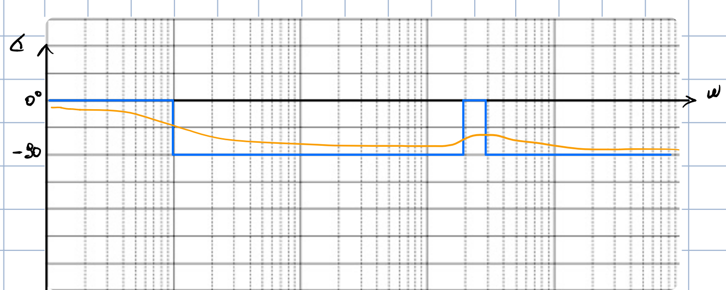
I disturbi sinusoidali non sono attenuati ma passano con ampiezza inalterata

8.4

$$G(s) = \frac{3}{20} \cdot \frac{s+20}{(s+0{,}1)(s+30)} = \frac{3}{20} \; \frac{\frac{s}{20}+1}{(10s+1)\left(\frac{s}{30}+1\right)}$$

1) $\mu = \dfrac{3}{20} \simeq -16\,dB$





2) (b) non è perchè dalla fase si vede che occupa solo IV quadrante
(a) non è perchè il modulo non cresce
è (c)

3) $R(s) = K \dfrac{1+sT}{s}$ $\qquad T_a = \frac{1}{2}$ udt $\qquad y^\circ(t) = A\,sca(t),\ t \geq 0$

$L(s) = G(s)R(s) = \dfrac{3}{20} K \dfrac{(1+sT)(s+20)}{s(10s+1)\left(\frac{s}{30}+1\right)}$ $\qquad \dfrac{5}{\omega_c} \leq \dfrac{1}{2} \longrightarrow \omega_c \leq 10$

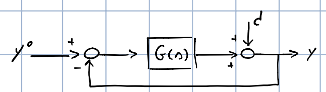
$g_c = 1$ ✓ $\qquad \varphi_m > 60°$

$\varphi_m = 180° - |\measuredangle L(j\omega_c)| > 60°$ $\Rightarrow$ elimino polo 0.1 con $T = 10$
e $K = 10$ per traslare verso l'alto il modulo e avere $\omega_c = 10$

8.5

1) $G(s)$ $\qquad \mu = 10^3$ $\qquad p_1 = p_2 = -10^{-1}$ $\qquad z_1 = -1$ $\qquad p_3 = -100$ $\qquad g = 0$

(a) vero, $\mu = 1000$
(b) vero
(c) falso, $T_a \simeq 50$ udt
(d) vero, attenuati tra 0,1 e 0,001

2)  $\qquad L(s) = G(s)$

a) Bode: $\mu > 0$ ✓ poli $Re < 0$ ✓ $\qquad \varphi_m > 0$ ✓ $\qquad \omega_c$ ben definita ✓
b) $y^\circ(t) = sca(t)$ $\qquad y(t) = 1000$?
$d(t) = 0$

$$F(s) = \frac{G(s)}{1+G(s)} = \frac{10^3 \dfrac{s+1}{\cancel{(0{,}01s+1)(s^2+2\xi 0{,}1+0{,}01)}}}{(0{,}01s+1)(s^2+2s\xi 0{,}1+0{,}01)+10^3(s+1)}, \quad \lim_{s\to 0} F(s) = \frac{10^3}{0{,}01+10^3} \simeq 1 = \mu_F$$

c) $\varphi_m > 60°$ $\Rightarrow$ $F(s) = \dfrac{10^3}{1+\frac{s}{10}}$ $\qquad p = -10$
$T_a = 0{,}5$ udt

d) $y^\circ = \sin(\omega t)$ $\qquad |F(j\omega)| = \dfrac{|G(j\omega)|}{|1+G(j\omega)|} \simeq \begin{cases} 1 & \omega < \omega_c \\ |G(j\omega)| & \omega > \omega_c \end{cases}$ $\qquad \omega_c = 10$
$\omega \in [100, 1000]$

$y_\infty(t) = |F(j\omega)| \sin(\omega t + \measuredangle F(j\omega)) \simeq |G(j\omega)| \sin(\omega t + \measuredangle F(j\omega))$ e $|G(j\omega)| < \frac{1}{10}$ per $\omega \in [100, 1000]$

e) $d(t) = \sin(\omega t)$ $\qquad \omega \in [0{,}01; 0{,}1]$

FdT $d \to y$: $H(s) = \dfrac{1}{1+G(s)}$ $\qquad |H(j\omega)| = \dfrac{1}{|1+G(j\omega)|} \simeq \begin{cases} 1 & \omega > \omega_c \\ \dfrac{1}{|G(j\omega)|} & \omega < \omega_c \end{cases}$

poichè $\omega \in [0{,}01; 0{,}1]$
$y_\infty = |H(j\omega)| \sin(\omega t + \measuredangle H(j\omega))$ $\qquad |H(j\omega)| \simeq \dfrac{1}{1000}$

3) $d(t) \to y(t)$: $F(s) = \dfrac{G(s)}{1+G(s)} \simeq \begin{cases} 1 & \omega < \omega_c \\ |G(s)| & \omega > \omega_c \end{cases}$

9.1

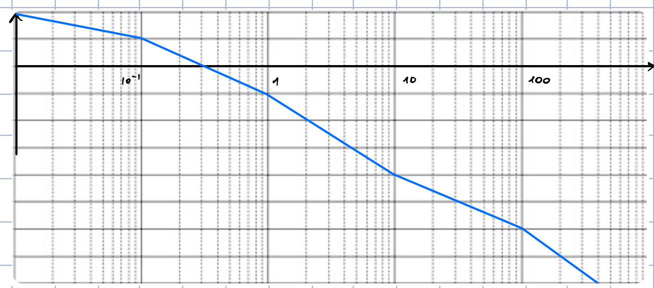
$$G(s) = \frac{1+0.1s}{(1+10s)(1+s)(1+0{,}01s)}$$

1) $y^o(t) = sca(t) \wedge d(t) = 0 \Rightarrow y_\infty = 1 \wedge T_a = 5 \text{ sdt}$
2) $d(t) = \sin(0.05)$ attenuato di almeno $10 \Rightarrow |H(j0{,}05)| \leq \frac{1}{10}$
3) Ordine minimo di R

$L(s) = R(s)\, G(s)$

- $e_\infty = 0 \Rightarrow$ polo nell' origine $g_R = 0$
- $\varphi_m > 60° \Rightarrow$ FdT polo dominante $F_s = \frac{\mu_c}{1+\frac{s}{-\omega_c}}$

- $5/\omega_c \simeq 5 \Rightarrow \omega_c \simeq 1$
- $|L(j0{,}05)| \geq 10$



Provo $R_1(s) = \frac{1}{s} \Rightarrow L_1(s) = \frac{1}{s}\,\frac{1+\frac{s}{10}}{(1+10s)(1+s)\left(1+\frac{s}{100}\right)}$

ω_c non soddisfatto $\Rightarrow$ aumento μ_R

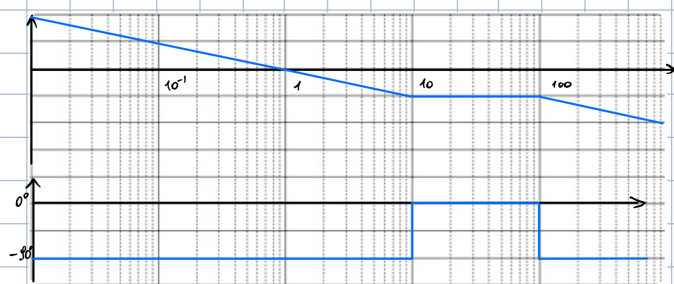
In $\omega = 1$ $|L_1(j\omega)|$ $\varphi_m < 0$

Per $\varphi_m > 60°$ inserisco due zeri in $-0{,}1$ e -1 di $R(s)$

Provo $R_2(s) = \frac{(1+10s)(1+s)}{s}$

$$L_2(s) = R_2(s) \cdot G(s) = \frac{1}{s}\,\frac{1+\frac{s}{10}}{1+\frac{s}{100}}$$

$\varphi_m > 60°$



Introduco polo in $R_2(s)$ in -100
così $R(s)$ raz fratta

9.2

$$G(s) = \frac{1-\frac{1}{10}s}{\left(1+\frac{1}{10}s\right)(1+s)(1+10s)}$$

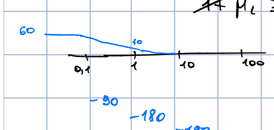
$g = 0$ $z_1 = 10$ $p_1 = -10$ $p_2 = -1$ $p_3 = -\frac{1}{10}$

- $|e_\infty| \leq 0{,}001$ con $y(t) = sca(t)$ e $d(t) = 0$
- $\varphi_m \geq 50°$
- $\omega_c \geq 3$

FdT $y^o \to e$: $H(s) = \frac{1}{1+L(s)}$ $L(s) = G(s)\,R(s)$

$$e_\infty = \lim_{s\to 0} \frac{1}{s}\cdot s \cdot H(s) = \lim_{s\to 0} \frac{1}{1+L(s)} = \lim_{s\to 0} \frac{1}{1+\mu_L} = \mu_H \leq 10^{-3}$$

$\cancel{1+}\,\mu_L \geq 10^3 \longrightarrow R_1(s) = 1000$



$\omega_c = 3$? cancello p_2 e p_3

$\varphi_m < 0$