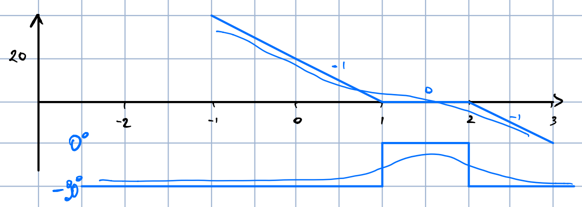
6.1

$$G(s) = \frac{10}{s} \cdot \frac{1+0.1s}{1+0.01s}$$

$g = 1$ $\mu = 10 \rightarrow \mu_{dB} = 20\ dB$

$z_1 = -10$ $p_1 = 0$ $p_2 = -100$ sì a fase minima

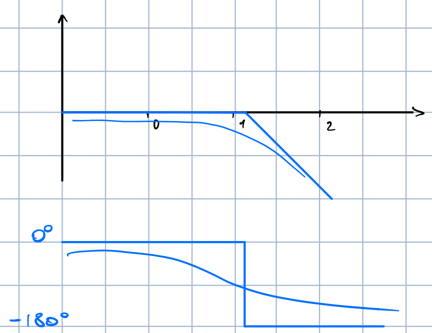


6.2

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$ $\omega_n = 2$ $\xi = 0.8$

1) $g = 0$ $\mu = 1$ $p_{1-2} = -\xi \pm \sqrt{\xi^2 - \omega_n^2} = -0.8 \pm \sqrt{0,64 - 4} = -0.8 \pm i\sqrt{3,36}$

$p_{1-2} \simeq -2$

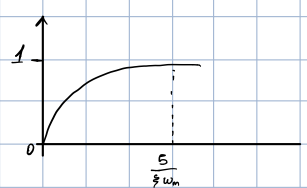


2) picco di risonanza perchè $|\xi| < \frac{\sqrt{2}}{2}$

3) $G(s) \simeq \frac{4}{(s+2)^2}$

$\lim_{s\to\infty} sY(s) = 0$ TVI

$\lim_{s\to 0} sY(s) = 1$ TVF



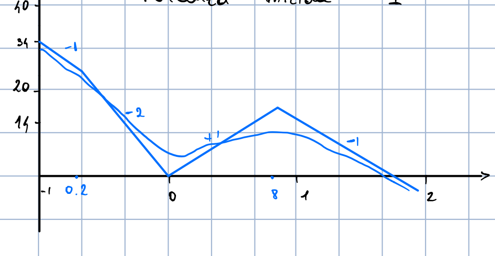
6.3

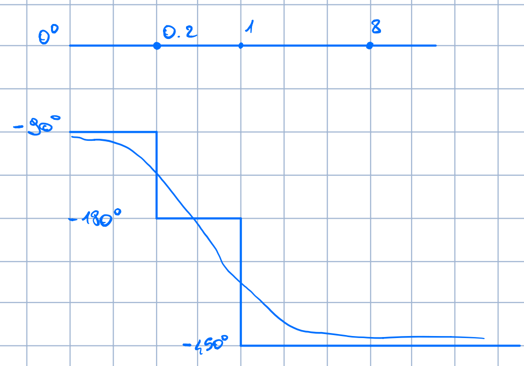
$$G(s) = \frac{5(1-s)^3}{s(1+5s)\left(1+\frac{s}{8}\right)\left(1-\frac{s}{8}\right)}$$

$g = 1$ $\mu = 5$ (13,97 dB) $z_{1-2-3} = +1$

$p_1 = 0$ $p_2 = -\frac{1}{5}$ $p_3 = -8$ $p_4 = +8$

Pendenza iniziale -1





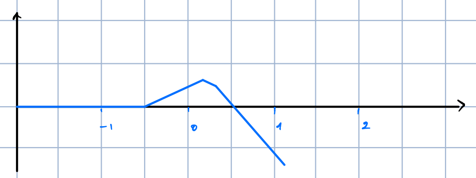
6.4

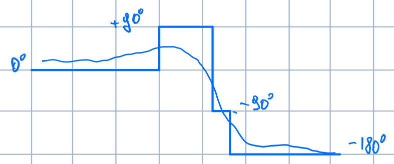
$$G(s) = \frac{1+2s}{\left(1+2\frac{\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}\right)\left(1+\frac{s}{4}\right)}$$

$\xi = 0.6$

$\omega_n = 2$

1) $z_1 = -\frac{1}{2}$ $\quad p_1 = -4$ $\quad \mu = 1$ $\quad p_{2-3} = -2$ $\quad g = 0$



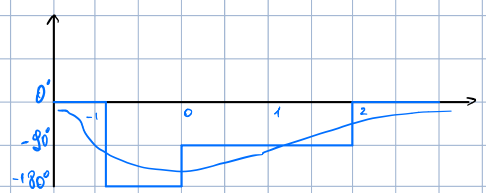


6.5

$g = 0$ $\quad |p_{1-2}| = 0{,}2$ $\quad z_1 = -1$ $\quad z_2 = -100$

$\mu = 26\text{ dB} \simeq 20$

$$G(s) = \frac{20\,(s+1)\,(s+100)}{(s+0{,}2)^2}$$



6.6

$$G(s) = 10\,\frac{s+1}{(s+0.1)\,(s^2+20s+100)}$$

$\mu = 1$

1) poli e zeri a $Re < 0$ e $grD > grN \Rightarrow$ S A.S.

2) $p_{2-3} = -10 \pm \sqrt{100-100} = -10$ $\quad$ Polo dominante 0.1

3) $u(t) = \underbrace{2}_{u_1} + \underbrace{\sin(0.01t)}_{u_2} + \underbrace{\sin(0.1t)}_{u_3} + \underbrace{2\cos(100t)}_{u_4}$ $\qquad U(s) = \frac{2}{s} + \frac{0.01}{(0.01)^2+s^2} + \frac{0.1}{(0.1)^2+s^2} + \frac{2s}{100^2+s^2}$

- $y_{1,\infty}(t) = \lim_{s\to 0} s \cdot \frac{2}{s}\, 10\, \frac{s+1}{(s+0.1)}\, \frac{1}{s^2+20s+100} = 2$

- $y_{2,\infty}(t) = \mu\, |G(j0.01)| \cdot \sin(0.01t + \measuredangle G(j0.01))$ $\qquad G(s) = 10\,\frac{1+s}{0.1\left(1+\frac{s}{0.1}\right)10^2\left(1+\frac{s}{10}\right)^2} = \frac{1+s}{(1+10s)\left(1+0.1s\right)^2}$

$$\measuredangle G(j\omega) = \measuredangle \frac{1+j\omega}{(1+10j\omega)(1+0.1j\omega)^2} = \measuredangle(1+j\omega) - \measuredangle(1+10j\omega) - 2\measuredangle(1+0.1j\omega)$$

$$\operatorname{arctg}(\omega) - \operatorname{arctg}(10\omega) - 2\operatorname{arctg}(0.1\omega)$$

$$\measuredangle G(j0.01) = tg^{-1}(0.01) - tg^{-1}(0.1) - 2tg^{-1}(0.001) = 0{,}001 - 0{,}1 - 2\cdot 0{,}001 = -0{,}1$$

$|G(j0.01)| \simeq 1$ $\qquad y_{\infty,2}(t) = \sin(0.01t)$

- $u_3 = \sin(0.1t)$ $\qquad y_{\infty,3}(t) = |G(j0.1)| \sin(0.1t + \measuredangle G(j0.1))$

$$\measuredangle G(j0.1) = tg^{-1}(0.1) - tg^{-1}(1) - 2tg^{-1}(0.01) \simeq -45°$$

$$|G(j0.1)| = \frac{|1+j0.1|}{|1+j|\cdot|(1+j0{,}01)^2|} = \frac{1}{\sqrt{2}\cdot 1} \simeq \frac{1}{\sqrt{2}}$$

$$y_{3,\infty} = \sin(0{,}1t - 45°)\cdot\frac{1}{\sqrt{2}}$$

- $u_4 = 2\cos(100t) = 2\,\text{sen}(100t + 90°)$

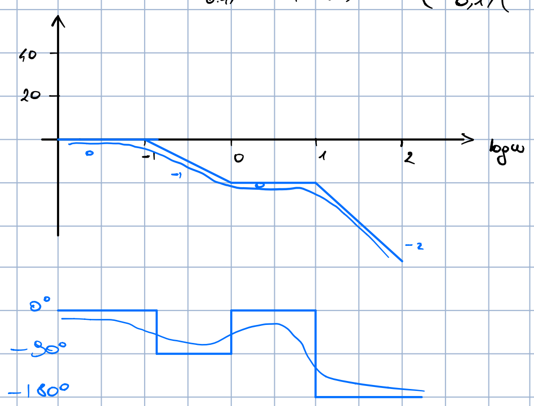
$$\angle G(j100) = tg^{-1}(100) - tg^{-1}(1000) - 2\,tg^{-1}(10) \simeq \frac{\pi}{2} - \frac{\pi}{2} - 2\frac{\pi}{2} = -\pi$$

$$|G(j100)| = \frac{|1+j100|}{|1+j1000|\,|1+j10|^2} = \frac{\cancel{100}}{1000\cdot\cancel{100}} = \frac{1}{1000}$$

$$y_{4,\infty}(t) = \frac{2}{1000}\,\text{sen}(100t + 90° - 180°)$$
$$= \frac{1}{500}\,\text{sen}(100t - 90°)$$

4) $G(s) = \frac{10}{0.1}\,\frac{s+1}{\left(1+\frac{s}{0.1}\right)100\left(1+\frac{s}{10}\right)^2} = \frac{s+1}{\left(1+\frac{s}{0{,}1}\right)\left(1+\frac{s}{10}\right)^2}$ $\mu = 1$ $g = 0$

$z_1 = -1$ $p_1 = -0{,}1$ $p_{2-3} = -10$



5) $\tilde{G}(s) = \frac{\mu}{1+\frac{s}{0.1}}$

6) (d) no (solo III e IV quad)

(c) no (tg orizz.)

(b) no (non è monotòna decr.)

6.7

1) $g = 0$ $\mu = 20dB = 10$ $p_1 = -0{,}05$ $p_{2-3} = -1$

$z_1 = -0{,}1$

È a fase minima con 2 poli cc, 1 polo e 1 zero reale

2) Grafico (c) perchè $T_a = 100$ e oscillazioni

3) (b) no perchè solo III e IV quadrante, idem (c) $\Rightarrow$ (a)

6.8

$G(s) = \frac{1}{(s+5)(s+1)^2}$ $u_3(t) = \sin(t)\,\text{sca}(t)$

$\angle G(j1) = -tg^{-1}\left(\frac{1}{5}\right) - 2\,tg^{-1}(1) = -101°$

$|G(j1)| = \frac{1}{|j+5|\cdot|j+1|^2} = \frac{1}{\sqrt{6}\cdot 2} = \frac{\sqrt{6}}{12}$ $y_3(t) = \frac{\sqrt{6}}{12}\,\text{sen}(t - 101°)$

$u_1(t) \xrightarrow{\mathcal{L}} U_1(s) = 1$ $Y_1(s) = G(s)U(s) = \frac{1}{(s+5)(s+1)^2}\cdot 1$ $y_{1,\infty} = 0 \Rightarrow$ grafico (d)

$u_2(t) = e^t\, sca(t) = \frac{1}{s-1} \Rightarrow Y_2(s) = G(s)\,U_2(s) = \frac{1}{(s-1)(s+5)(s+1)^2} = \frac{\alpha}{s+5} + \frac{\beta}{s+1} + \frac{\gamma}{(s+1)^2} + \frac{\delta}{s-1}$

$$\begin{cases} \alpha(s+1)^2(s-1) = \alpha(s^2+2s+1)(s-1) = \alpha(s^3+2s^2+s-s^2-2s-1) = \\ \beta(s+1)(s+5)(s-1) = \beta(s^2-1)(s+5) = \beta(s^3+5s^2-s-5) = \\ \gamma(s+5)(s-1) = \gamma(s^2+4s-5) \\ \delta(s+5)(s+1)^2 = \delta(s+5)(s^2+2s+1) = \delta(s^3+7s^2+11s+5) \end{cases}$$

$$\begin{cases} -\alpha - 5\beta - 5\gamma + 5\delta = 1 \\ s^3(\alpha+\beta+\delta) = 0 \\ s^2(5\beta \overset{+2\alpha}{+} \gamma + 7\delta) = 0 \\ s(-\beta \overset{-2\alpha}{-} 5\gamma + 5\delta) = 0 \end{cases}$$

$$\begin{cases} \alpha = -\beta - \delta \\ -4\beta - 5\gamma + 6\delta = 1 \\ -2\beta - 2\gamma + 5\beta + \gamma + 7\delta = 0 \\ 2\beta + 2\delta - \beta - 5\gamma + 5\delta = 0 \end{cases}
\qquad
\begin{matrix} \alpha = -\beta - \delta \\ -4\beta - 5\gamma + 6\delta = 1 \\ 3\beta - \gamma + 7\delta = 0 \\ \beta - 5\gamma + 7\delta = 0 \end{matrix}
\qquad
\begin{matrix} \alpha = -\beta - \delta \\ \gamma = 3\beta + 7\delta \\ -4\beta - 15\beta - 35\delta + 6\delta = 1 \\ \beta - 15\beta - 35\delta + 7\delta = 0 \end{matrix}
\qquad
\begin{matrix} \alpha = -\beta - \delta \\ \gamma = 3\beta + 7\delta \\ -19\beta - 29\delta = 1 \\ -14\beta - 28\delta = 0 \end{matrix}$$

$\beta = -2\delta \rightarrow \alpha = \delta$

$\gamma = -6\delta + 7\delta \rightarrow \gamma = -\delta$

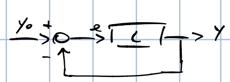
$38\delta - 29\delta = 1 \rightarrow \delta = 1,\ \gamma = -1,\ \alpha = 1,\ \beta = -2$

$Y_2(s) = \frac{1}{s+5} - \frac{2}{s+1} - \frac{1}{(s+1)^2} + \frac{1}{s-1} \rightarrow$ diverge $\Rightarrow$ (a)

$u_3(t) = \sin(t)\, sca(t) \rightarrow$ (b)

6.9

$L(s) = \frac{5}{s(1+0.05s)}$



$(y_0 - y)L = Y$

$y_0 L - yL = y \rightarrow Y(1+L) = y_0 L$

$\frac{Y}{y_0} = \frac{L}{1+L}$

$F(s) = \frac{L(s)}{1+L(s)} = \frac{5}{s(1+\frac{1}{20}s)} \cdot \frac{1}{1+\frac{5}{s(1+\frac{1}{20}s)}} = \frac{5}{s(1+\frac{1}{20}s)} \cdot \frac{s(1+\frac{1}{20}s)}{s(1+\frac{1}{20}s)+5} = \frac{20 \cdot 5}{20 \cdot \left(\frac{s^2}{20} + s + 5\right)} = \frac{100}{s^2+20s+100}$

$= \frac{100}{(s+10)^2} = \frac{1}{\left(\frac{s}{10}+1\right)^2}$

$y_0 = \alpha_1 \sin(0.5t + \beta_1) + \alpha_2 \sin(t + \beta_2) + \alpha_3 \sin(100t + \beta_3)$

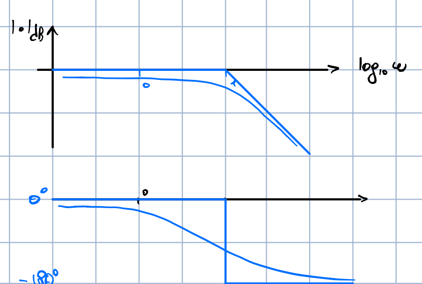
$\angle F(j\omega) = -2\, tg^{-1}\left(\frac{\omega}{10}\right) \qquad |F(j\omega)| = \frac{1}{\left|j\frac{\omega}{10}+1\right|^2} = \frac{1}{\frac{\omega^2}{100}+1} = \frac{100}{\omega^2+100}$

$\omega = 0.5 \qquad \angle F(j0.5) = -5{,}7° \qquad |F(j0.5)| \simeq 1$

$\omega = 1 \qquad \angle F(j1) = -11{,}4° \qquad |F(j1)| \simeq 1$

$\omega = 100 \qquad \angle F(j100) = -168{,}6° \qquad |F(j100)| \simeq 0 \leftarrow$ componente ad alta frequenza filtrato, segnale non riprodotto correttamente

$g = 0 \qquad \mu = 1 \qquad p_{1-2} = -10$

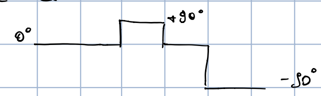


6.10

$\mu = 20\,dB = 10 \qquad z_1 = -0.5 \qquad p_1 = -1 \qquad p_2 = -10$

$$G(s) = \frac{10\,(2s+1)}{(s+1)\left(\frac{s}{10}+1\right)}$$

1) Falso, dal diagramma asintotico il modulo di $G(j60)$ è minore di 10
2) Vero, il modulo è minore di 1
3) Falso, cresce in corrispondenza dello zero negativo



2) è la (b), sovraelongazione da sis. II ord. con 2 poli Re e fase minima, con cost. tempo zero > poli
3) (a) no perché tg è vert., (b) no perché modulo iniz. cresce, è (c)

6.11

$$G(s) = \frac{4s}{(1-s)(1+0.1s)^2} \qquad g = -1 \qquad \mu = 4 \qquad z_1 = 0 \qquad p_1 = 1 \qquad p_{2-3} = -10$$