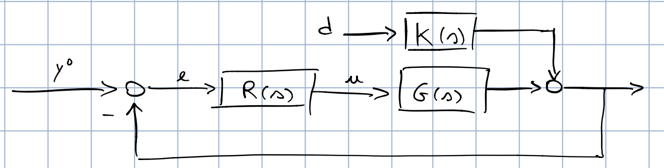
Es 1

Data $G(s) = \dfrac{50}{\left(1+\frac{s}{0{,}1}\right)\left(1+\frac{s}{1}\right)\left(1+\frac{s}{10}\right)}$ III ord. e la FdT $K(s) = \dfrac{5}{1+\frac{s}{100}}$ II ORD



determinare $R(s)$ tale per cui:

a) Risposta allo scalino su $y°$ non oscillante

b) T_a inferiore a 5 udt

c) $|e_\infty| \le 0{,}025$ quando $y°(t) = 10\, sca(t)$ $\quad d(t) = \pm\, sca(t)$

① TRADUZIONE DELLE SPECIFICHE

- NON OSCILLANTE : $\varphi_m \ge 70° \Rightarrow F(s) = \dfrac{Y(s)}{Y°(s)} = \dfrac{L(s)}{1+L(s)} \approx \dfrac{\mu_F}{1+s/\omega_c}$ $\quad \tau = \dfrac{1}{\omega_c}$ $\quad T_a = \dfrac{5}{\omega_c}$

- $T_a \le 5$ udt $\quad T_a = \dfrac{5}{\omega_c} \le 5 \Rightarrow \omega_c \ge 1$ rad/s

- $|e_\infty| \le 0{,}025$

② PROGETTO STATICO : $R(s) = \dfrac{\mu_R}{s^g}$

Vogliamo imporre una condizione su e_∞:

$$e_\infty = \lim_{s\to 0}\Big[s\,E(s) = \frac{s}{1+R(s)G(s)}\Big(1\cdot Y°(s) - K(s)D(s)\Big) = \frac{s}{1+R(s)G(s)}\Big(10 \mp K(s)\Big)\frac{1}{s} = \frac{1}{1+\frac{\mu_R}{s^g}G(s)}\big(10 \mp K(s)\big)$$

Partiamo da $\dfrac{E(s)}{Y°(s)} = \dfrac{1}{1+R(s)G(s)}$ $\quad \dfrac{E(s)}{D(s)} = -\dfrac{K(s)}{1+R(s)G(s)}$ $\quad Y°(s) = \dfrac{10}{s}$ $\quad D(s) = \pm\dfrac{1}{s}$

Sovrapp. effetti

$$= \frac{s^g}{s^g + \mu_R\, G(s)}\big(10 \mp K(s)\big)\Big] \begin{cases} g=0 & \dfrac{1}{1+50\mu_R}(10 \mp 5) \quad \dfrac{15}{1+50\mu_R} \le 0{,}025 \quad \mu_R \ge 12 \\ g\ge 1 & 0 \quad \forall \mu_R \end{cases}$$

prendo il caso massimo

$\mu_R = 20 \Rightarrow R(s) = 20$ per comodità di calcolo

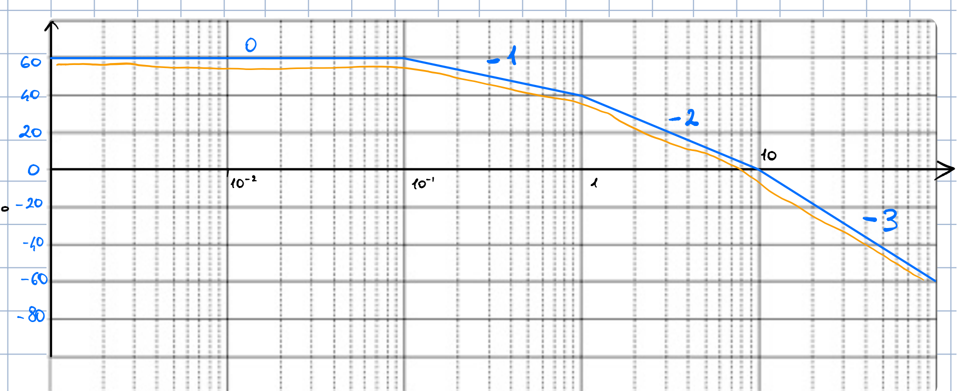
$$L(s) = R(s)G(s) = 1000\,\frac{1}{\left(1+\frac{s}{0{,}1}\right)(1+s)\left(1+\frac{s}{10}\right)}$$

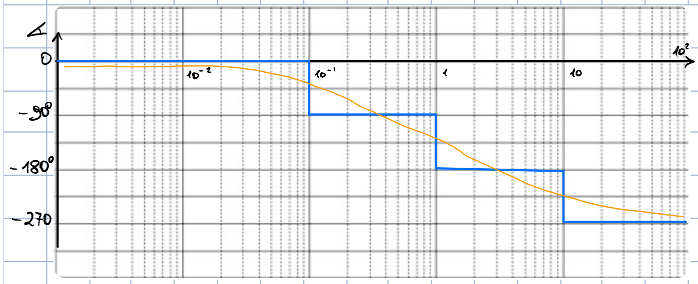
Disegnamo il Diag. di Bode

$\mu_L = 1000$ $\begin{cases} |\mu_L| = 1000 = 60\text{ dB} \\ \mu_L > 0 \end{cases}$

$g = 0$

$\left.\begin{array}{l}\omega_1 = 0{,}1 \text{ rad/s} \\ \omega_2 = 1 \text{ rad/s} \\ \omega_3 = 10 \text{ rad/s}\end{array}\right\}$ POLI SX $\quad \Delta p = -1 \quad \Delta\varphi = -90°$





Già si vede che il margine di fase sarà negativo:

$$\angle L(j\omega_c) = -tg^{-1}\left(\frac{10}{0,1}\right) - tg^{-1}\left(\frac{10}{1}\right) - tg^{-1}\left(\frac{10}{10}\right) = -219°$$

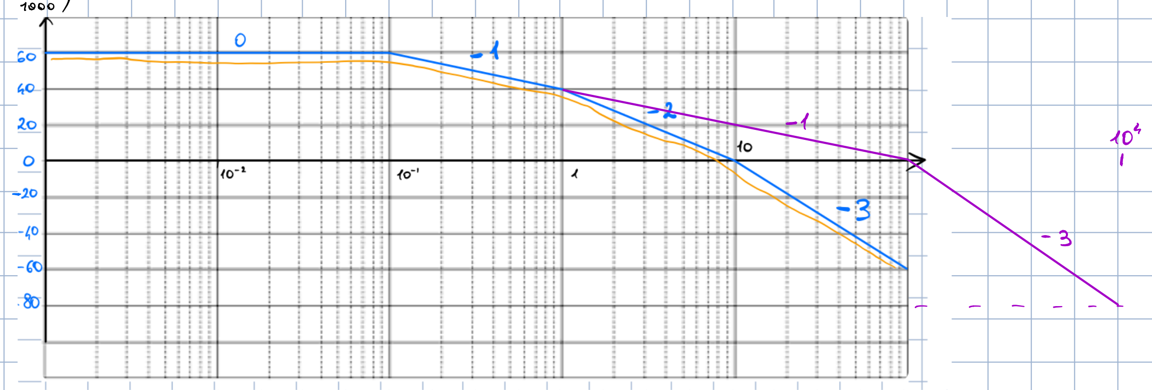
$$\Rightarrow \varphi_m = 180° - |-219°| \approx -39°$$ × Non soddisfa la specifica

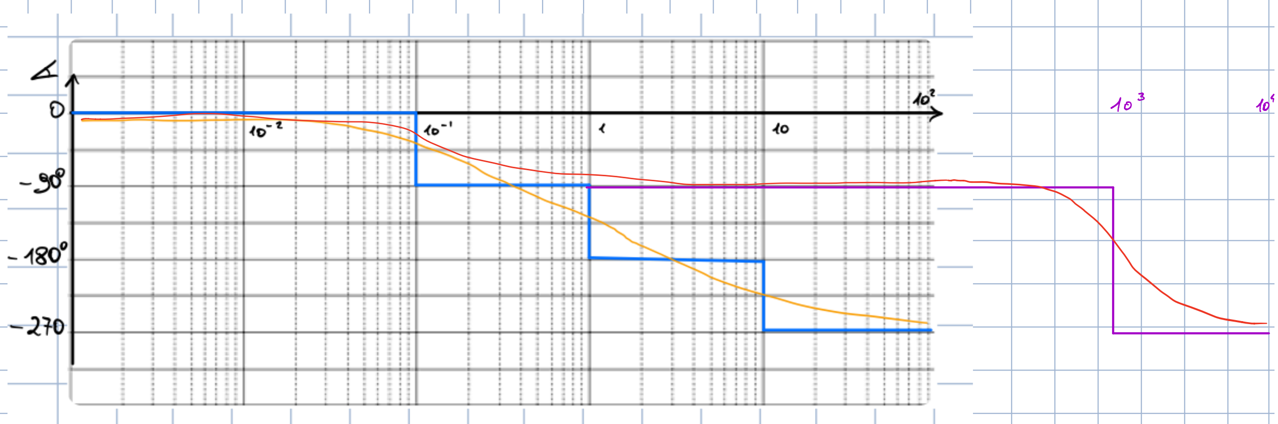
provare x caso $R(s) = \frac{\mu_R}{s}$

③ PROGETTO DINAMICO $R(s) = 20$

Sposto il diagramma di una decade: sul grafico

$$R(s) = 20 \cdot \frac{(1+s)(1+s/10)}{\left(1+\frac{s}{1000}\right)^2}$$

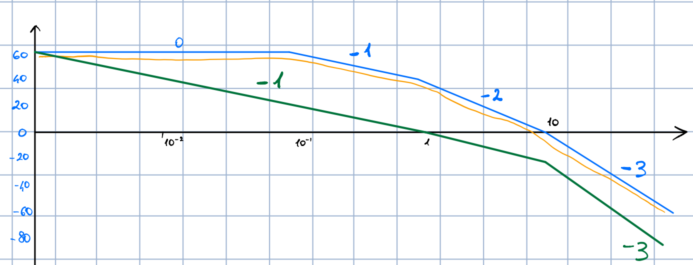


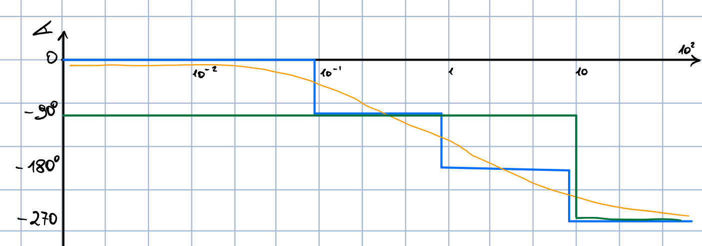


$$L(s) = 1000 \cdot \frac{1}{\left(1+\frac{s}{0,1}\right)\left(1+\frac{s}{10^3}\right)^2}$$ $\omega_c \approx 100$ rad/s (≥ 1 rad/s)

$$\angle L(j\omega_c) = tg^{-1}\left(\frac{100}{0,1}\right) - 2tg^{-1}\left(\frac{100}{1000}\right) \approx -101°$$ $\varphi_m = 180° - |-101°| = 79°$ ($\geq 70°$)

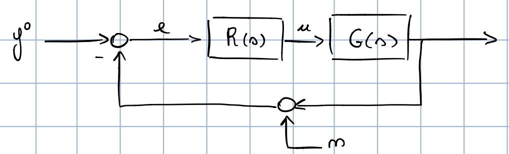
Potrei farlo anche $R(s) = 20 \frac{\left(1+\frac{s}{0,1}\right)\left(1+\frac{s}{1}\right)}{\left(1+\frac{s}{0,001}\right)\left(1+\frac{s}{10}\right)}$ $\omega_c \approx 1$ rad/s $\varphi_m \approx 84°$





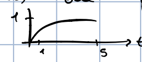
ES 2

Data $G(s) = \dfrac{10}{s(1+s)^2}$ III ORDINE



Determinare $R(s)$ tale per cui:

- $y^o(t) = sca(t)$



- $n(t) = \text{sen}(\omega t)$ $\omega \geq 30$ rad/s ATTENUATO DI ALMENO 40 dB SULL' USCITA

① TRADUZIONE DELLE SPECIFICHE

- Non oscillante $\longrightarrow$ $\varphi_m \geq 70°$
- $e_\infty = 0$
- $T_a = 5$ sdt $\longrightarrow$ $\omega_c \approx 1$ rad/s

$$y(t) = 1 \left| \frac{Y(j\omega)}{N(j\omega)} \right| \text{sen}(\omega t + \measuredangle)$$

$$\frac{Y(s)}{N(s)} = -\frac{L(s)}{1+L(s)} = -F(s) \qquad \left| \frac{Y(j\omega)}{N(j\omega)} \right| = |-F(s)| \approx \begin{cases} 1 & \text{quando } \omega \ll \omega_c \quad |L(j\omega)| \gg 1 \\ \dfrac{1}{|L(j\omega)|} & \omega \gg \omega_c \quad |L(j\omega)| \ll 1 \end{cases}$$

- $\omega_c \ll 30$ rad/s $\qquad |L(j\omega)| \leq -40 \text{ dB} = \dfrac{1}{100} \qquad \forall \omega \geq 30$ rad/s

② PROGETTO STATICO $R(s) = \dfrac{\mu_R}{s^g}$ $\qquad G(s) = \dfrac{10}{s(1+s)^2}$

$$e_\infty = \lim_{s \to 0} \left[sE(s) = s \frac{1}{1+L(s)} \frac{1}{s} = \frac{1}{1 + \frac{\mu_R}{s^{g+1}} \frac{10}{(1+s)^2}} = \frac{s^{g+1}(1+s)^2}{s^{g+1}(1+s)^2 + 10\mu_R} \right. \quad \begin{cases} g=-1 & \dfrac{1}{1+10\mu_R} \\ g \geq 0 & 0 \quad \forall \mu_R \end{cases}$$

$R(s) = \mu_R$ siccome c'è già un integratore nel sistema non occorre aggiungerne un altro

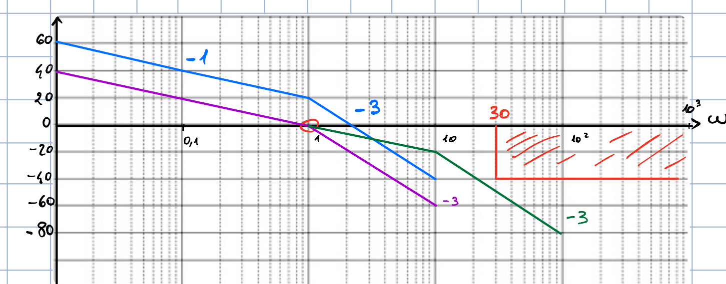
$L(s) = \dfrac{10\mu_R}{s} \cdot \dfrac{1}{(1+s)^2}$ $\qquad \mu_R = 1$

$\mu_L = 10$ $\begin{cases} |\mu_L| = 10 = 20 \text{ dB} \\ \mu_L > 0 \quad +0° \end{cases}$

$g = 1$ $\qquad -90°$ $\qquad -g = -1$

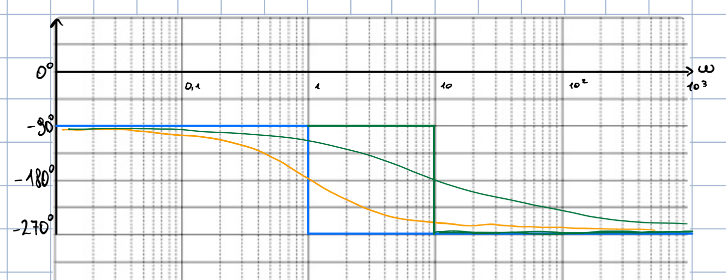
$\omega_1 = 1$ rad/s $\qquad$ 2 POLI SX $\qquad \Delta p = -2$, $\Delta\varphi = -180°$

Se poniamo $\mu_R = 0{,}1$ abbasso il diagramma di 20 dB, la fase non cambia e la specifica è soddisfatta



$\bar{\omega} = 0{,}1$ rad/s

$$|L(j\bar{\omega})| = \frac{10}{0{,}1} \cdot \frac{1}{\left|1 + j\frac{0{,}1}{1}\right|^2} \approx 100 = 40 \text{ dB}$$



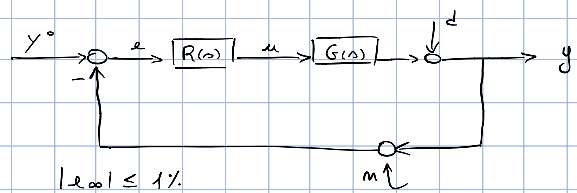
③ PROGETTO DINAMICO $\qquad L(s) = \dfrac{1}{s\left(1+\frac{s}{10}\right)^2}$

$R(s) = \dfrac{0{,}1\,(1+s)^2}{(1+s/10)^2}$ $\qquad \omega_c \approx 1$ rad/s ✓ $\qquad \measuredangle L(j\omega_c) = -90° - 2\,tg^{-1}\left(\frac{1}{10}\right) \approx -101°$ $\qquad \varphi_m \approx 79°$ ✓

$$|L(j\omega)|_{dB} = |L(j10)|_{dB} - 60\,dB\,(\log\omega - \log 10) \approx -48\,dB < -40\,dB \quad ✓$$

$\hookrightarrow \omega = 30$ rad/s

ES 3



$$G(s) = \frac{1}{(1+s)(1+\frac{s}{10})} \qquad \text{II ORDINE}$$

$R(s)$ tale per cui: $- \; y°(t) = A\,\text{sca}(t) \qquad d(t) = 0 \qquad |e_\infty| \le 1\%$

$- \; \varphi_m \ge 80°$

a) PROGETTO STATICO $\quad R(s) = \dfrac{\mu_R}{s^g}$

$$e_\infty = \lim_{s\to 0}\left[s\,E(s) = s\,\frac{1}{1+L(s)}\,\frac{A}{s} = \frac{A}{1+\frac{\mu_R}{s^g}\cdot\frac{1}{(1+s)(1+s/10)}} = \frac{s^g(1+s)(1+\frac{s}{10})\,A}{s^g(1+s)(1+s/10)+\mu_R}\right] = \begin{cases} g=0 & \dfrac{1}{1+\mu_R}\cdot A \\ g\ge 1 & 0 \quad \forall \mu_R \end{cases} \qquad |e_\infty| \le 1\%$$

$$\frac{A}{1+\mu_R} \le \frac{A}{100} \implies \mu_R \ge 99 \qquad R(s) = \mu_R = 100 \qquad L(s) = \frac{100}{(1+\frac{s}{1})(1+\frac{s}{10})}$$

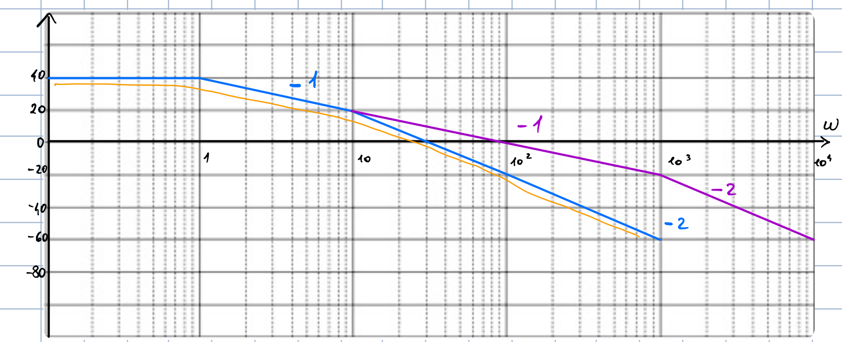
$$\mu_L = 100 \begin{cases} |\mu_L| = 100 = 40dB \\ \mu_L > 0 \quad +0° \end{cases}$$

$g = 0 \qquad +0°$

$-g = 0$

$\omega_1 = 1$ rad/s $\quad$ POLO SX $\rightarrow \begin{cases} \Delta p = -1 \\ \Delta\varphi = -90° \end{cases}$

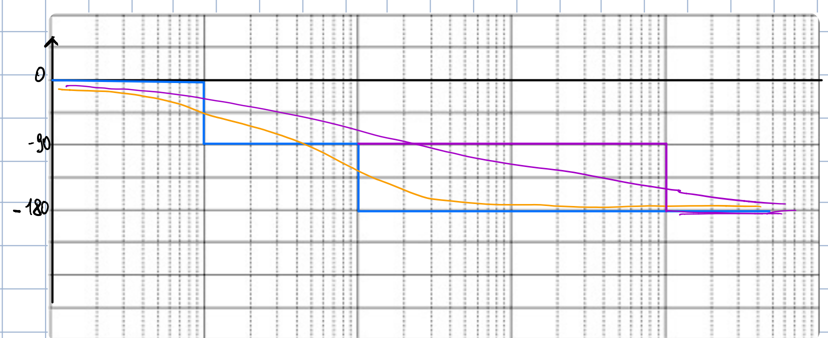
$\omega_2 = 10$ rad/s $\quad$ POLO SX $\rightarrow \begin{cases} \Delta p = -1 \\ \Delta\varphi = -90° \end{cases}$



$\omega_c \approx 30$ rad/s

$$\measuredangle L(j30) = -tg^{-1}(30) - tg^{-1}\left(\frac{30}{10}\right) \approx -160°$$

$$\varphi_m = 180° - |-160°| \approx 20°$$



b) PROGETTO DINAMICO

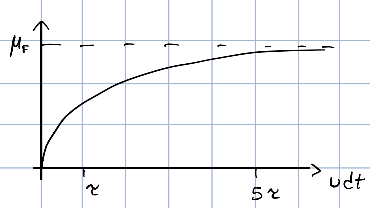
$$R(s) = 100\,\frac{1+s/10}{1+s/10^3} \qquad \omega_c = 100 \text{ rad/s} \qquad \measuredangle L(j\omega_c) = tg^{-1}(100) - tg^{-1}\left(\frac{100}{1000}\right) \approx -95°$$

$\varphi_m \approx 85° \; (> 80°)$

② RISPOSTA $\quad y°(t) = \text{sca}(t)$

$\varphi_m > 70° \qquad F(s) \approx \dfrac{\mu_F}{1+s/\omega_c} \qquad \omega_c \approx 100$ rad/s $\qquad \tau = 0{,}01$ udt

$T_a = 0{,}05$ udt

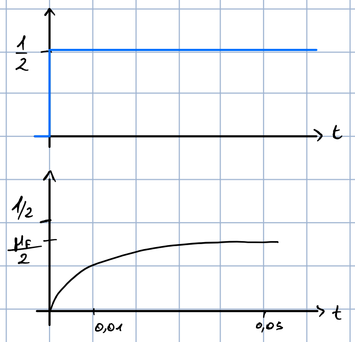


③ RISPOSTA $d(t) = \frac{1}{2}\,\mathrm{sca}(t)$ $\quad \frac{Y(s)}{D(s)} = \frac{1}{1+L(s)} = \frac{1+L(s)-L(s)}{1+L(s)} = 1 - F(s)$

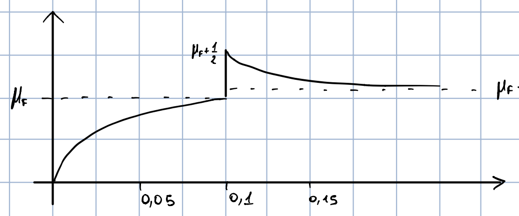
$F(s)$

$Y(s) = D(s) - F(s)D(s)$

$y(t) = d(t) - \tilde{d}(t)$



④ RISPOSTA $y°(t) = \mathrm{sca}(t)$ e $d(t) = \frac{1}{2}\,\mathrm{sca}(t-0{,}1)$



$\mu_F + \frac{1-\mu_F}{2} = \frac{\mu_F+1}{2}$