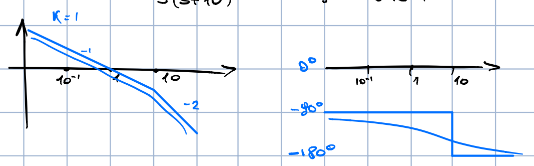
8.1

$P(s) = \frac{10}{s+10} \qquad R(s) = \frac{K}{s} \qquad K > 0$

1) $L(s) = 10K \frac{1}{s(s+10)} = \frac{K}{s} \cdot \frac{1}{0.1s+1} \qquad g = 1 \qquad \mu = K > 0 \qquad p_1 = -10 \quad p_2 = 0$



Bode: no poli Re ≤ 0, no nascosti

ω_c ben definita

- $\mu_L > 0$
- $\varphi_m > 0 \Rightarrow \measuredangle L(j\omega) > -180°$ perchè tende asintoticamente a $-180°$. S è AS $\forall K > 0$

2) $e(t) = y^o(t) - y(t)$

$y^o(t) = A_1 \,\text{sca}(t) \qquad d(t) = A_2 \,\text{sca}(t) \qquad |A_i| < 10$

$$y^o \xrightarrow{+} \bigcirc \xrightarrow{e} \boxed{L(s)} \xrightarrow{+} \bigcirc \xrightarrow{} y$$

(d entra con + nel secondo nodo; retroazione con − al primo nodo)

S è AS $\forall K \Rightarrow$ TVF

$y^o(t) \to e(t) \qquad E(s) = Y^o(s) - E(s)L(s) \to H_y(s) = \frac{1}{1+L(s)} = \frac{1}{1+\frac{K}{s(0.1s+1)}} = \frac{s(0.1s+1)}{s(0.1s+1)+K}$

$d(t) \to e(t) \qquad D(s) + E(s)L(s) = -E(s) \to H_d(s) = -\frac{1}{1+L(s)} = -\frac{1}{1+\frac{K}{s(0.1s+1)}} = -\frac{s(0.1s+1)}{s(0.1s+1)+K}$

$Y^o(s) = \frac{A_1}{s} \qquad D(s) = \frac{A_2}{s}$

$E_{y^o}(s) = \frac{A_1}{s} \cdot \frac{s(0.1s+1)}{s(0.1s+1)+K} \qquad E_d(s) = D(s)\,H_d(s)$

$e_{\infty,y^o} = \lim_{s\to 0} s\frac{A_1}{s} \frac{s(0.1s+1)}{s(0.1s+1)+K} = 0 \qquad \forall A_1 \; \forall K > 0$

$e_{\infty,d} = \lim_{s\to 0} s\frac{A_2}{s} \frac{-(s(0.1s+1))}{s(0.1s+1)+K} = \emptyset \qquad \forall A_2 \; \forall K > 0 \qquad e_\infty = e_{\infty,y^o} + e_{\infty,d} = 0$

Approssimazione a poli dominanti: $F(s) = \frac{\mu_F}{1+\frac{s}{\omega_c}}$ oppure $F(s) = \frac{\mu_F\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

FdT $\quad y^o \to y = \frac{L(s)}{1+L(s)} \qquad \mu_F = \begin{cases} 1 & g_L > 0 \\ \frac{\mu_L}{1+\mu_L} & g_L = 0 \end{cases} \qquad \xi = \sin\left(\frac{\varphi_m}{2}\right)$

$H(s) = 1 - F(s) \qquad T_a = \frac{5}{\omega_c}$ se a singolo polo

$|\measuredangle L(j\bar{\omega}_c)| = 180 - \bar{\varphi}_m = 180° - 60° = 120° \Rightarrow \measuredangle L(j\bar{\omega}_c) = -120°$

$L(s) = \frac{K}{s} \cdot \frac{1}{0.1s+1} \Rightarrow \measuredangle K - \measuredangle j\omega - \measuredangle(0.1j\omega+1) = tg^{-1}(0) - tg^{-1}(\infty) - tg^{-1}(0.1\omega) = 0° - 90° - tg^{-1}(0.1\omega)$

$-90° - tg^{-1}(0.1\bar{\omega}_c) = -120° \Rightarrow tg^{-1}(0.1\bar{\omega}_c) = 30° \Rightarrow 0.1\bar{\omega}_c = \frac{\sqrt{3}}{3} \Rightarrow \bar{\omega}_c = \frac{10}{3}\sqrt{3}$

- $\varphi_m > 60° \Leftrightarrow 0 < K < \bar{K}$
- $\varphi_m \leq 60° \Leftrightarrow K \geq \bar{K}$

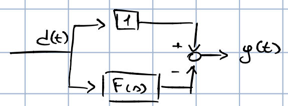
$|L(j\bar{\omega}_c)| = \frac{|K|}{|j\bar{\omega}_c| \cdot |j0.1\bar{\omega}_c+1|} = \frac{K}{\bar{\omega}_c \cdot \sqrt{1+\frac{\bar{\omega}_c^2}{100}}} = \frac{3K}{10\sqrt{3} \cdot \sqrt{1+\frac{100}{100}\cdot\frac{1}{9}\cdot 3}} = \frac{3K}{10\sqrt{3}\cdot\sqrt{\frac{4}{3}}} = \frac{3}{20}K = 1 \Rightarrow K = \frac{20}{3}$

$\varphi_m < 60° \qquad \forall K > \frac{20}{3}$

$\varphi_m > 60° \qquad \forall K < \frac{20}{3}$

$0 < K < \frac{20}{3}$: $\quad F(s) = \frac{1}{1+\frac{s}{\omega_c}}$

FdT $\quad d \to y = \frac{1}{1+L(s)} = 1 - F(s)$



$d(t) = 0.5\, sca(t)$

$$y(t) = d(t) - \mathcal{L}^{-1}\left[F(s) D(s)\right](t)$$

- $K \geq \frac{20}{3}$ $\quad$ approssimazione cc

$$F(s) = \frac{L(s)}{1+L(s)} = \frac{1}{s^2 + 2\xi\omega_c s + \omega_c^2} \qquad \xi = \sin\left(\frac{\varphi_m}{2}\right) \to 0 \text{ se } \varphi_m \to 0 \qquad T_a = \frac{5}{\xi\omega_c}$$

8.2

$$G(s) = \frac{1}{1+s} \qquad n = 1$$

1) $R(s) = K, \quad K \in \mathbb{R}, \, K \neq 0$

$L(s) = \frac{K}{1+s}$ $\quad$ 2 casi : $\cdot\; K > 0$, $\quad \mu_L = K > 0$, no poli $Re > 0$, no AV nascosti

$\varphi_m > 0$ perchè fase asintotica è $-90°$

ω_c ben definita perchè monotona decrescente $|L(j\omega)|$

Criterio di Bode soddisfatto $\Rightarrow$ AS

$\cdot\; K < 0$: $\quad \mu_L = K < 0$ $\quad$ no poli $Re < 0$ nè AV nascosti

ω_c ben definita

Bode: $\quad -1 < K < 0$ è AS, $\; K = 1$ inst., $\; K > 1$ inst.

FdT $\quad y^o \to y$: $\quad F(s) = \frac{L(s)}{1+L(s)} = \frac{\mu_L}{1+\frac{s}{\omega_c}}$

$L(s)$: $\quad p_1 = -1$ $\qquad |L(j\omega_c)| = 1$: $\quad \frac{|K|}{|1+j\omega_c|} = 1 \to |K| = \sqrt{1+\omega_c^2} \Rightarrow \omega_c \simeq |K|$

$\mu_L = K$

$$|F(j\omega)| = \left|\frac{L(j\omega)}{1+L(j\omega)}\right| = \begin{cases} 1 & \omega < \omega_c \\ |L(j\omega)| & \omega > \omega_c \end{cases}$$

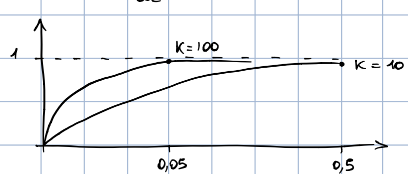
3) $y^o(t) = sca(t) \qquad d(t) = 0$

Approssimazione a polo dominante : $\quad F(s) = \frac{\mu_F}{1+\frac{s}{\omega_c}} \qquad \mu_F = \frac{\mu_L}{1+\mu_L}$

$K = 10 \quad \tau = 0.1 \qquad K = 100 \quad \tau = 0.01$

$T_a^{10} = 0{,}5 \qquad T_a^{100} = 0{,}05$

$\mu_F = \frac{10}{11} \qquad \mu_F = \frac{100}{101}$

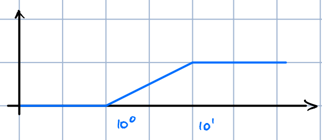


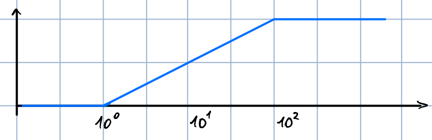
4) $\quad y^o \to u$: $\quad U = E \cdot R \Rightarrow U = Y^o R - UGR$

$E = Y^o - UG \qquad U(1+GR) = Y^o R$

$\frac{U}{Y^o} = \frac{R}{1+GR}$ $\qquad$ FdT $\; y^o \to u$: $\quad \frac{K}{1+\frac{K}{1+s}} = V(s)$

$$|V(j\omega)| = \left|\frac{R(j\omega)}{1+G(j\omega)R(j\omega)}\right| = \left|\frac{F(j\omega)}{G(j\omega)}\right| \quad \begin{cases} \frac{1}{|G(j\omega)|} & \omega < \omega_c \\ |R(j\omega)| & \omega > \omega_c \end{cases} = \begin{cases} -|G(j\omega)|\text{ dB} & \omega < \omega_c \\ |R(j\omega)|\text{ dB} & \omega > \omega_c \end{cases}$$





← ampiezza segnale è maggiore perché c'è più banda passante ma aumenta anche l'energia

5) $K = 10$

a) $d(t) = \frac{1}{2}\,\text{sca}(t)$

b) $d(t) = \frac{1}{2}\sin(0.01t)\,\text{sca}(t)$

c) $d(t) = \frac{1}{2}\sin(100t)\,\text{sca}(t)$

FdT $d(t) \to y(t)$: $Y = D + E(RG)$ $E = -Y$

$Y = D - Y(RG)$

$Y(1+RG) = D$

$\frac{Y}{D} = \frac{1}{1+L(s)} = \frac{1}{1+\frac{10}{1+s}} = \frac{1+s}{11+s}$

a) $d(t) = \frac{1}{2}\,\text{sca}(t)$

$D(s) = \frac{1}{2}\cdot\frac{1}{s}$ TVF: $\lim_{s\to 0} s\,\frac{1}{2}\,\frac{1}{s}\,\frac{1+s}{11+s} = \frac{1}{22} = y_\infty(t)$

b) $d(t) = \frac{1}{2}\sin(0.01t)\,\text{sca}(t)$ $|H(j0.01)| = \frac{|1+j0.01|}{|11+j0.01|} \simeq \frac{1}{11}$ $y_\infty = \frac{1}{22}\sin(0.01t + \angle H(j0.01))$

c) $d(t) = \frac{1}{2}\sin(100t)\,\text{sca}(t)$ $|H(j100)| = \frac{|1+j100|}{|11+j100|} \simeq 1$ $y_\infty = \frac{1}{2}\sin(100t + \angle H(j100))$

(c) degrada maggiormente il sistema

$$|H(j\omega)| \simeq \begin{cases} \frac{1}{|L(j\omega)|} & \omega < \omega_c = 10 \\ 1 & \omega > \omega_c = 10 \end{cases}$$

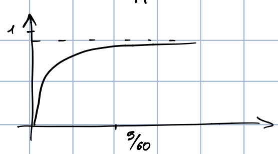
8.3

$\mu = 10$ $z_1 = -1$ $p_1 = -3$ $p_2 = -10$ $p_3 = -1000$

1) $L(s) = \frac{10\,(s+1)}{(s+3)(s+10)(s+1000)}$ Bode: · ω_c ben definita, · $\varphi_m > 0$, · no AR nascosti né poli $Re > 0$

S è A.S. e $\varphi_m > 60° \Rightarrow$ Approssimazione polo dominante $\omega_c \simeq 300$

$F(s) = \frac{10/11}{1+\frac{s}{300}}$



2) $n(t) = A\sin(\bar{\omega}t)$, $\bar{\omega} \in [1,3]$

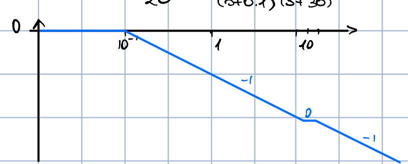
FdT $n(t) \to y(t)$: $Y = -(N+Y)L$, $\frac{Y}{N}(1+L) = -L \Rightarrow H(s) = -\frac{L(s)}{1+L(s)} = \frac{-G(s)}{1+G(s)}$

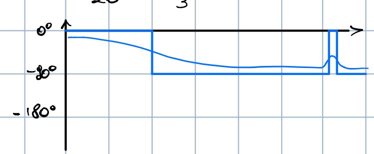
$$|H(j\omega)| = \begin{cases} |G(s)| & \omega > \omega_c \\ 1 & \omega < \omega_c \end{cases}$$

$y_{\infty,n} = A\sin(\omega t + \angle H(j\bar{\omega}))$

8.4

1) $G(s) = \frac{3}{20}\cdot\frac{s+20}{(s+0.1)(s+30)}$ $\mu = G(0) = \frac{3}{20}\cdot\frac{20}{3} = 1$ $z_1 = -20$ $p_1 = -0.1$ $p_2 = -30$





2) a) no perché fase confinata nel IV quadrante, idem (b) ⇒ è (c)

3) $R(s) = K\,\frac{1+sT}{s}$ $\qquad L(s) = \frac{3}{20}\cdot K \cdot \frac{1+sT}{s}\cdot\frac{s+20}{(s+0.1)(s+30)}$

$\mu_F = 1 \Rightarrow g_L > 1$, $T_a = 0.5$ udt , $\varphi_m > 60°$

$\angle L(j\omega) = \angle\left(\frac{3}{20}K\right) + \angle(1+j\omega T) - \angle(j\omega) + \angle(j\omega+20) - \angle(j\omega+0.1) - \angle(j\omega+30)$

$\frac{5}{\omega_c} \leq 0.5 \Rightarrow \omega_c \geq 10$, passo cancellare p_1 con $T = 10$ e traslo verso l' alto il modulo

con $K = 10$ e ottengo $\omega_c = 10$

$L(s) = R(s)G(s) = 10\,\frac{1+10s}{s}\,\frac{1+s/20}{(1+s10)(1+s/30)} = \frac{10}{s}\,\frac{1+s/20}{1+s/30}$

8.5

$\mu = 60\,dB$ $\qquad p_1 = p_2 = 0.1$ c.c. $\qquad p_3 = -100$ $\qquad 0 < \xi < \frac{\sqrt{2}}{2}$

$z_1 = -1$

1) a) Vero, si assesta al valore del guadagno essendo AS per il TVF

b) Vero, due poli sono c.c. e $0<\xi<\frac{\sqrt{2}}{2}$

c) $\tau = \frac{1}{\xi\omega_n} > \frac{1}{\omega_n} = 10$ $\qquad T_a > 50$ dt

d) $u(t) = \sin(\bar{\omega}t)$ $\quad \bar{\omega}\in[100, 1000]$ Vero, sono attenuati di un fattore tra $\frac{1}{1000}$ e $\frac{1}{10}$

2) a) Vero: Criterio di Bode: No poli né a.v. $Re<0$ e ω_c ben definita

✓ $\mu > 0$ ✓ $\varphi_m > 0°$

b) $F(s) = \frac{G(s)}{1+G(s)}$, $\mu_F = \frac{\mu_G}{1+\mu_G} = \frac{1000}{1001} \approx 1$ Falso

c) Vero $\quad F(s)_{\approx} = \frac{1}{1+\frac{s}{\omega_c}} = \frac{1}{1+10s} \Rightarrow T_a = \frac{5}{10}$

d) $y^o(t) = \sin(\bar{\omega}t)$ $\quad \bar{\omega}\in[100, 1000]$ Vero

$|F(j\omega)| = \frac{1}{|1+j\omega 10|}$ $\qquad |F(j100)| = \frac{1}{|1+j1000|} \simeq 10^{-3}$ $\qquad |F(j1000)| \simeq 10^{-4}$

e) $d(t) = \sin(\bar{\omega}t)$ $\quad \bar{\omega}\in[0.01, 0.1]$

FdT $d \to y$: $\frac{1}{1+G(s)} = H(s)$ $\qquad |H(j\omega)| = \begin{cases} 1 & \omega > \omega_c \\ \frac{1}{|G(s)|} & \omega < \omega_c \end{cases}$ Vero

$|H(j\bar{\omega})|$ $\bar{\omega}\in[0.01, 0.1]$ è 10^{-3}

3) a) Vero, non dipende dagli ingressi

b) "

c) "

d) "

e) $Y = (D - Y)G \rightarrow \frac{Y(1+G)}{D} = G$ $\qquad F(s) = \frac{G(s)}{1+G(s)}$

$|F(j\omega)| = \begin{cases} 1 & \omega < \omega_c \\ |G(j\omega)| & \omega > \omega_c \end{cases}$ $\qquad \bar{\omega}\in[0.01, 0.1]$ Falso perchè per questi ω il modulo è >1