MATRIX PROJECT

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QUESTION:

The sides of a rhombus ABCD are parallel to the lines

- (1,-1) $\mathbf{X} + 2 = 0$
- (7,-1) $\mathbf{X} + 3 = 0$

If the diagonals of the rhombus intersect at

$\mathbf{P} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and the vertex A (different) from the origin is on the y-axis, then find the ordinate of A.

SOLUTION APPROACH (USING VECTORS) :

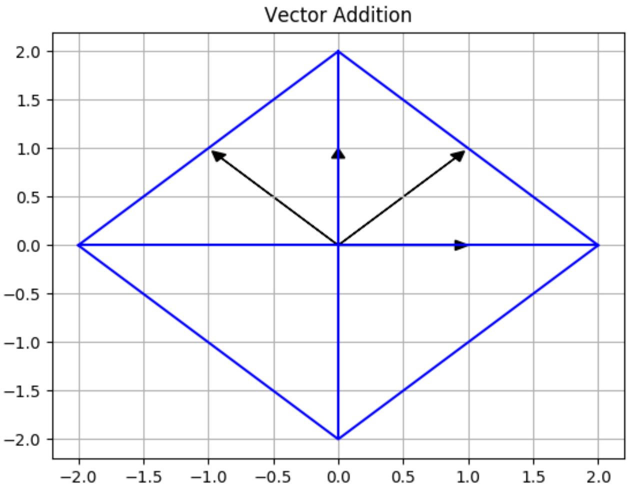
For line: (1,-1) $\mathbf{X} + 2 = 0$

Let $\mathbf{N}_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$ be the **normal unit vector**.

For line: (7,-1) $\mathbf{X} + 3 = 0$

Let $\mathbf{N}_2 = \begin{bmatrix} \frac{7}{5\sqrt{2}} \\ \frac{-1}{5\sqrt{2}} \end{bmatrix}$ be the **normal unit vector**.

Vector Addition



Let D_1 and D_2 be the vectors along diagonals of the Rhombus ABCD and A will be on one of the diagonals.
By parallelogram law of vector addition:

$$\mathbf{D}_1 = \mathbf{N}_1 + \mathbf{N}_2 = \begin{bmatrix} \frac{12}{5\sqrt{2}} \\ \frac{-6}{5\sqrt{2}} \end{bmatrix}$$

$$\mathbf{D}_2 = \mathbf{N}_2 - \mathbf{N}_1 = \begin{bmatrix} \frac{2}{5\sqrt{2}} \\ \frac{4}{5\sqrt{2}} \end{bmatrix}$$

Equation of Diagonals:

Diagonal along $\mathbf{D}_1$ passing through point $\mathbf{P}$:

$$\mathbf{D}_2{}^T\mathbf{X} = \mathbf{D}_2{}^T\mathbf{P} \ (\textit{Because } \mathbf{D}_2 \text{ is normal to } \mathbf{D}_1\ )$$

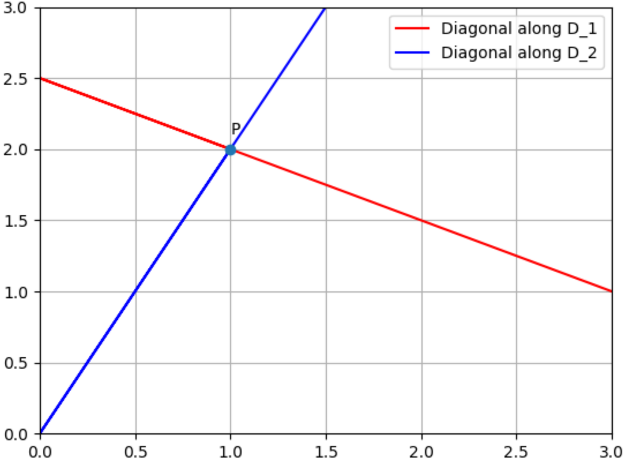
$$\begin{bmatrix} \frac{2}{5\sqrt{2}} & \frac{4}{5\sqrt{2}} \end{bmatrix}\mathbf{X} = \frac{10}{5\sqrt{2}}$$

Diagonal along $\mathbf{D}_2$ passing through point $\mathbf{P}$:

$$\mathbf{D}_1{}^T\mathbf{X} = \mathbf{D}_1{}^T\mathbf{P} \ (\textit{Because } \mathbf{D}_1 \text{ is normal to } \mathbf{D}_2\ )$$

$$\begin{bmatrix} \frac{12}{5\sqrt{2}} & \frac{-6}{5\sqrt{2}} \end{bmatrix}\mathbf{X} = 0$$

Diagonals of Rhombus



Result:

As the Diagonal along $\mathbf{D}_2$ passes through origin so we will neglect it as according to question point A can not be origin.

Hence , A is the point where Diagonal along $\mathbf{D}_1$ intersects y-axis. So:

$$\begin{bmatrix} \frac{2}{5\sqrt{2}} & \frac{4}{5\sqrt{2}} \end{bmatrix} \mathbf{X} = \frac{10}{5\sqrt{2}} \text{ , where } \mathbf{X} = \begin{bmatrix} 0 \\ y \end{bmatrix}$$

Therefore, y $= \frac{5}{2}$ (i.e ordinate of A)

Point **A** is $(0, \frac{5}{2})$.

Rhombus ABCD