Homework 9

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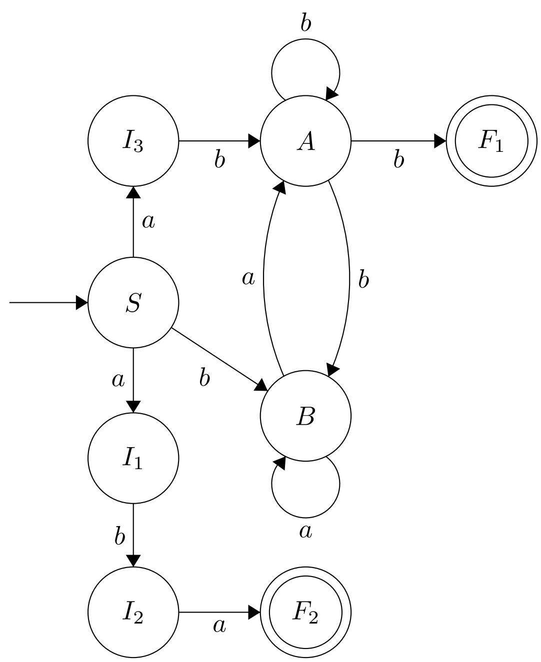
4.28. Draw an NFA for the following grammar:
$S \to abA \mid bB \mid aba$
$A \to b \mid aB \mid bA$
$B \to aB \mid aA$



4.29.

d. $S \to AB \quad A \to aAa \mid bAb \mid a \mid b \quad B \to aB \mid bB \mid \lambda$
Odd length palindrones are being generated. A regular grammar that generates odd length palindrones is:
$S \to aS \mid bS \mid aA \mid bA$
$A \to \lambda$

e. $S \rightarrow AA \mid B \quad A \rightarrow AAA \mid Ab \mid bA \mid a \quad B \rightarrow bB \mid \lambda$ Clearly this grammar describes the language with an even number of a's. A regular grammar for this is:
$S \rightarrow aA \mid bS \mid \lambda$
$A \rightarrow aS \mid bA$

55. Let $L \subset \Sigma_1^*$ be a context free language. There must be a CFG to represent this language call it: $G = (V, \Sigma_1, S, P_1)$ s.t. $L = L(G_1)$. Let $G_2 = (V, \Sigma_2, S, P_2)$ and let the function $f : \Sigma_1^* \rightarrow \Sigma_2^*$ be homomorphic. Extend f to $g : (\Sigma_1 \cup V)^* \rightarrow (\Sigma_2 \cup V)^*$ where $F(A) = A \ \ !\forall A \in V$ Finally P_2 is defined as $\forall$ productions $A \rightarrow \alpha \in P_1$, there is a corresponding production $A \rightarrow f(\alpha) \in P_2$ Now we must show that $f(L) = L(G_2)$

$\Rightarrow$ if $y \in f(L)$ then $\Rightarrow y \in L(G_2)$
Assume that $y \in f(L)$ then $\exists x \in L$ s.t. $y = f(x)$. But since $x \in L = L(G_1)$ there is a derivation $S \rightarrow_{G_1}^* x$. Now g can be applied to every derivation in G_2 hence $y \in L(G_2)$.

$\Leftarrow$ If $y \in L(G_2)$ then there is a derviation $S \rightarrow_{G_2}^* y$ of the form $A \rightarrow f(x)$ and $x \in L$ therefore y must be an element of $F(L)$.

5.1 a.

String: ab $(q_0, ab, Z_0) \vdash (q_1, b, aZ_0) \vdash (q_2, \lambda, Z_0) \vdash (q_3, \lambda, Z_0)$. Hence ab is accepted.

String: aab $(q_0, aab, Z_0) \vdash (q_1, ab, aZ_0) \vdash (q_1, b, aaZ_0) \vdash (q_2, \lambda, aZ_0)$. No transition exists for the given inputs, hence the string is not accepted.

String: abb $(q_0, abb, Z_0) \vdash (q_1, bb, aZ_0) \vdash (q_2, b, Z_0)$. No transition exists for the given inputs, hence the string is not accepted.

b.

String: bacab $(q_0, bacab, Z_0) \vdash (q_0, acab, bZ_0) \vdash (q_0, cab, abZ_0) \vdash (q_1, ab, abZ_0) \vdash (q_1, b, bZ_0) \vdash (q_1, \lambda, Z_0) \vdash (q_2, \lambda, Z_0)$. Hence bacab is accepted.

String: baca $(q_0, baca, Z_0) \vdash (q_0, aca, bZ_0) \vdash (q_0, ca, abZ_0) \vdash (q_1, a, abZ_0) \vdash (q_1, \lambda, bZ_0)$. No transition exists for the given inputs, hence the string is not accepted.

4. a. The language of even-length palindromes.

Move	State	Input	Stack	Move(s)
1	q_0	a	Z_0	(q_0, aZ_0)
2	q_0	a	a	(q_0, aa)
3	q_0	a	b	(q_0, ab)
4	q_0	b	Z_0	(q_0, bZ_0)
5	q_0	b	a	(q_0, ba)
6	q_0	b	b	(q_0, bb)
7	q_0	λ	Z_0	(q_1, Z_0)
8	q_0	λ	a	(q_1, a)
9	q_0	λ	b	(q_1, b)
10	q_1	a	a	(q_1, λ)
11	q_1	b	b	(q_1, λ)
12	q_1	λ	Z_0	(q_2, Z_0)

5.5 a. The language of all odd-length strings over a, b with middle symbol a.

Move	State	Input	Stack	Move(s)
1	q_0	a	Z_0	$(q_0, aZ_0), (q_1, Z_0)$
2	q_0	a	a	$(q_0, aa), (q_1, a)$
3	q_0	a	b	$(q_0, ab), (q_1, b)$
4	q_0	b	Z_0	(q_0, bZ_0)
5	q_0	b	a	(q_0, ba)
6	q_0	b	b	(q_0, bb)
7	q_1	a	a	(q_1, λ)
8	q_1	a	b	(q_1, λ)
9	q_1	b	a	(q_1, λ)
0	q_1	b	b	(q_1, λ)
10	q_1	λ	Z_0	(q_2, Z_0)

b. $\{a^n x | n \geq 0, x \in \{a, b\}^*$ and $|x| \leq n\}$.

Move	State	Input	Stack	Move(s)
1	q_0	λ	Z_0	(q_1, Z_0)
2	q_0	a	Z_0	(q_1, aZ_0)
3	q_0	a	a	$(q_0, aa), (q_1, a)$
4	q_1	a	a	(q_1, λ)
5	q_1	b	a	(q_1, λ)

c. $\{a^i b^j c^k | i, j, k \geq 0$ and $j = i$ or $j = k\}$.

Move	State	Input	Stack	Move(s)
1	q_0	a	Z_0	$(q_0, aZ_0), (q_0, Z_0)$
2	q_0	a	a	(q_0, aa)
3	q_0	b	Z_0	(q_1, bZ_0)
4	q_1	b	a	(q_1, λ)
5	q_1	b	b	(q_1, bb)
6	q_1	c	Z_0	(q_3, Z_0)
7	q_1	c	b	(q_2, λ)
8	q_2	c	b	(q_2, λ)
9	q_2	λ	Z_0	(q_3, Z_0)
10	q_3	c	Z_0	(q_3, Z_0)
11	q_3	λ	Z_0	(q_4, Z_0)

5.7 All non-palindromes in $\{a, b\}^*$