## Prove the union of CFLs is a CFL

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## December 8, 2013

Define the  $G_u$  to be the union of the grammars  $G_1$ ,  $G_2$ . More concretly  $G_u=(V_u,\Sigma,S_u,P_u),\ G_1=(V_1,\Sigma,S_1,P_1)$  and  $S_2=(V_2,\Sigma,S_2,P_2)$ . Assume that  $V_1\cap V_2=\phi$  since the elements can be arbitrarily renamed.

Let 
$$V_u = V_1 \cup V_2 \cup \{S_u\}$$
 where  $S_u$  is not in  $V_1$  or  $V_2$ .  
Let  $P_u = P_1 \cup P_2 \cup \{S \to S_1 \mid S_2\}$ 

We must show that  $L(G_u) = L_1 \cup L_2$ Firstly  $L(G_u) \subseteq L_1 \cup L_2$ 

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Secondly  $L_1 \cup L_2 \subseteq L(G_u)$