CENTRAL WASHINGTON UNIVERSITY, DEPARTMENT OF COMPUTER SCIENCE

# CS471 Project 5

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## 1 INTRODUCTION

The purpose of this lab was to code an optimization algorithms that use job Scheduling, the NEH algorithm, The NEH Flow shop optimization algorithm uses a two-dimensional array which holds the completion time of each job on each machine. Each row in the array represents a separate machine that each column (job) must process on. To begin the Algorithm each job must be sorted in order of largest total completion time first with the last value being the shortest job. Then each job is inserted into the make span function starting with the longest job first. Then the next largest job is inserted into the schedule at each available slot and tested for best the shortest make span. Once all positions have been tested the schedule with the shortest make span is selected, locking in the order of that schedule, upon which time the process is repeated with the next longest job until all jobs have been placed in the schedule. Once all jobs have been placed into the schedule the resulting schedule should be close to the optimal schedule for the given flow shop.

#### 1.0.1 FLOW SHOPS

A general flowshop scheduling problem is defined as follows.

A set of n jobs, 1,2,...,n available at time zero has to be processed in a shop with m (ordered) machines $M_1$,$M_2$,...$M_m$

Each job is processed first on $M_1$, next on $M_2$, and so on, and lastly on $M_m$.
No machine can process more than one job at a time, no preemption is allowed, all setup times are included into the job processing times, and there is unlimited storage between the machines.

# 2 RESULTS

### 2.0.1 NEH ALGORITHM

The NEH Algorithm is delivered the known optimal result for FSS consistently when it came to the first functions, however when compared to other published results using the same function there are noticeable differences between those results and the results of this experiment. After analyzing the functions where the results were varied between experiments a pattern emerged, when jobs have the same total completion time as each other but varying completion time and different machines it can result in the NEH algorithm generating different results when using the same data from experiment to experiment. This is because when the algorithm uses the sorted jobs based on total completion time and there is a tie between two or more jobs the order of the jobs will be different between sorts. Since the algorithm does not check for completion time on each machine individually the algorithm might not generate the globally optimal value but instead just an optimal value for that particular sorting. In cases that use fewer than 50 jobs and 10 machines this results in a margin of error of less than 1 percent, but as the number of machines and jobs grow this margin of error grows to the point where in the case of function 120 which has 20 machines and 500 jobs it can result in a margin of error of up to 3 percent.

### 2.0.2 RESULTS(FLOW SHOP CALLS)

The main drawback of the NEH Algorithm is that the number of calls to the flow shop function grows exponentially with the growth of the the number of jobs and machines. Depending on the implementation of the matrix that holds the data for the jobs and machines this can inherently result in much longer execution times.

### 2.0.3 RESULTS(EXECUTION TIME)

The execution time of the NEH Algorithm is cubic with the number of calls to the flow shop function. However in the data shown in tables 3.1 through 3.4 this is not always the case, with the first 3 tables this is the case, however in the 4th table (3.4) the execution time grows more than cubic. This may be because the data structure used for the matrix to hold the schedule in this experiment was a vector rather than an array. The resulting time change is likely because the vector is required to resize and copy multiple times as the schedule is built as the vector would have to be resized roughly 9 times for every job which could result in a longer execution time due to the frequent resizing and copying of the vector.

# 3 CONCLUSION

The NEH Algorithm is a very powerful algorithm for determining the make span of a flow shop, however it has some short-falls. The main short-fall of the algorithm is that it does not account for jobs that have the same total completion time such as the case in function one where jobs 2 and 4 have the same total execution time, resulting in the flow shop with no waiting having different results every time the experiment is run. What the algorithm does do well is producing consistent accurate results, the largest difference generated between this experiment and the known optimum was for function 120 where the result for flow shop with now wait enabled generated a result 2.4 percent deviation for the known minimums found in other published works.

Table 3.1: Computation NEH Problems 1-30

| Problem | FFS | | | FFSB | | | FFSNW | | |
|---|---|---|---|---|---|---|---|---|---|
| | makespan | time | Evaluations | makespan | time | Evaluations | makespan | time | Evaluations |
| $f_1$ | 1286 | 0.012 | 209 | 1435 | 0.01 | 209 | 16394 | 0.01 | 209 |
| $f_2$ | 1365 | 0.01 | 209 | 1477 | 0.01 | 209 | 18018 | 0.01 | 209 |
| $f_3$ | 1159 | 0.01 | 209 | 1353 | 0.011 | 209 | 16701 | 0.012 | 209 |
| $f_4$ | 1325 | 0.01 | 209 | 1552 | 0.01 | 209 | 18963 | 0.01 | 209 |
| $f_5$ | 1305 | 0.01 | 209 | 1398 | 0.01 | 209 | 16368 | 0.011 | 209 |
| $f_6$ | 1228 | 0.011 | 209 | 1464 | 0.011 | 209 | 16145 | 0.011 | 209 |
| $f_7$ | 1278 | 0.011 | 209 | 1458 | 0.01 | 209 | 16728 | 0.011 | 209 |
| $f_8$ | 1223 | 0.009 | 209 | 1449 | 0.01 | 209 | 16675 | 0.009 | 209 |
| $f_9$ | 1291 | 0.01 | 209 | 1457 | 0.012 | 209 | 17219 | 0.011 | 209 |
| $f_{10}$ | 1151 | 0.011 | 209 | 1349 | 0.011 | 209 | 16159 | 0.011 | 209 |
| $f_{11}$ | 1680 | 0.012 | 209 | 1767 | 0.012 | 209 | 25968 | 0.012 | 209 |
| $f_{12}$ | 1729 | 0.011 | 209 | 1903 | 0.011 | 209 | 28022 | 0.016 | 209 |
| $f_{13}$ | 1557 | 0.012 | 209 | 1772 | 0.013 | 209 | 24133 | 0.012 | 209 |
| $f_{14}$ | 1439 | 0.011 | 209 | 1622 | 0.012 | 209 | 23044 | 0.012 | 209 |
| $f_{15}$ | 1502 | 0.012 | 209 | 1722 | 0.012 | 209 | 24591 | 0.012 | 209 |
| $f_{16}$ | 1453 | 0.011 | 209 | 1679 | 0.012 | 209 | 24047 | 0.013 | 209 |
| $f_{17}$ | 1562 | 0.011 | 209 | 1738 | 0.013 | 209 | 23609 | 0.012 | 209 |
| $f_{18}$ | 1609 | 0.011 | 209 | 1814 | 0.014 | 209 | 25187 | 0.012 | 209 |
| $f_{19}$ | 1647 | 0.011 | 209 | 1832 | 0.014 | 209 | 25497 | 0.013 | 209 |
| $f_{20}$ | 1653 | 0.011 | 209 | 1854 | 0.014 | 209 | 25699 | 0.012 | 209 |
| $f_{21}$ | 2410 | 0.017 | 209 | 2530 | 0.016 | 209 | 40401 | 0.016 | 209 |
| $f_{22}$ | 2150 | 0.016 | 209 | 2285 | 0.018 | 209 | 39688 | 0.016 | 209 |
| $f_{23}$ | 2411 | 0.015 | 209 | 2564 | 0.017 | 209 | 39373 | 0.018 | 209 |
| $f_{24}$ | 2262 | 0.015 | 209 | 2399 | 0.017 | 209 | 40068 | 0.016 | 209 |
| $f_{25}$ | 2397 | 0.015 | 209 | 2538 | 0.017 | 209 | 40127 | 0.017 | 209 |
| $f_{26}$ | 2349 | 0.016 | 209 | 2472 | 0.017 | 209 | 41575 | 0.018 | 209 |
| $f_{27}$ | 2362 | 0.016 | 209 | 2498 | 0.019 | 209 | 41826 | 0.02 | 209 |
| $f_{28}$ | 2249 | 0.016 | 209 | 2411 | 0.017 | 209 | 39515 | 0.016 | 209 |
| $f_{29}$ | 2320 | 0.015 | 209 | 2421 | 0.017 | 209 | 39825 | 0.017 | 209 |
| $f_{30}$ | 2277 | 0.016 | 209 | 2425 | 0.016 | 209 | 40589 | 0.016 | 209 |

[1] , 3.4GHz Intel core i5-3570K, 8 GB RAM

Table 3.2: Computation NEH Problems 31-60

| Problem | FFS | | | FFSB | | | FFSNW | | |
|---|---|---|---|---|---|---|---|---|---|
| | makespan | time | Evaluations | makespan | time | Evaluations | makespan | time | Evaluations |
| $f_{31}$ | 2733 | 0.034 | 1274 | 3321 | 0.033 | 1274 | 86186 | 0.033 | 1274 |
| $f_{32}$ | 2843 | 0.034 | 1274 | 3550 | 0.034 | 1274 | 90841 | 0.034 | 1274 |
| $f_{33}$ | 2643 | 0.033 | 1274 | 3223 | 0.035 | 1274 | 85678 | 0.033 | 1274 |
| $f_{34}$ | 2783 | 0.034 | 1274 | 3382 | 0.034 | 1274 | 90543 | 0.036 | 1274 |
| $f_{35}$ | 2868 | 0.033 | 1274 | 3431 | 0.035 | 1274 | 91410 | 0.034 | 1274 |
| $f_{36}$ | 2850 | 0.032 | 1274 | 3413 | 0.033 | 1274 | 90676 | 0.033 | 1274 |
| $f_{37}$ | 2752 | 0.032 | 1274 | 3266 | 0.032 | 1274 | 85490 | 0.033 | 1274 |
| $f_{38}$ | 2694 | 0.031 | 1274 | 3326 | 0.032 | 1274 | 86989 | 0.032 | 1274 |
| $f_{39}$ | 2576 | 0.031 | 1274 | 3118 | 0.033 | 1274 | 80209 | 0.032 | 1274 |
| $f_{40}$ | 2787 | 0.031 | 1274 | 3446 | 0.032 | 1274 | 91517 | 0.033 | 1274 |
| $f_{41}$ | 3135 | 0.042 | 1274 | 3947 | 0.042 | 1274 | 123660 | 0.043 | 1274 |
| $f_{42}$ | 3032 | 0.043 | 1274 | 3795 | 0.041 | 1274 | 124042 | 0.041 | 1274 |
| $f_{43}$ | 3016 | 0.04 | 1274 | 3819 | 0.044 | 1274 | 116443 | 0.042 | 1274 |
| $f_{44}$ | 3198 | 0.041 | 1274 | 3938 | 0.043 | 1274 | 123369 | 0.042 | 1274 |
| $f_{45}$ | 3128 | 0.04 | 1274 | 3897 | 0.041 | 1274 | 125226 | 0.044 | 1274 |
| $f_{46}$ | 3178 | 0.041 | 1274 | 3828 | 0.042 | 1274 | 122324 | 0.042 | 1274 |
| $f_{47}$ | 3277 | 0.043 | 1274 | 3975 | 0.042 | 1274 | 123145 | 0.042 | 1274 |
| $f_{48}$ | 3123 | 0.042 | 1274 | 3794 | 0.044 | 1274 | 123066 | 0.041 | 1274 |
| $f_{49}$ | 3015 | 0.042 | 1274 | 3878 | 0.043 | 1274 | 123067 | 0.042 | 1274 |
| $f_{50}$ | 3257 | 0.04 | 1274 | 3943 | 0.042 | 1274 | 123685 | 0.045 | 1274 |
| $f_{51}$ | 4013 | 0.06 | 1274 | 4889 | 0.062 | 1274 | 183151 | 0.064 | 1274 |
| $f_{52}$ | 3921 | 0.06 | 1274 | 4597 | 0.061 | 1274 | 174576 | 0.062 | 1274 |
| $f_{53}$ | 3923 | 0.063 | 1274 | 4653 | 0.064 | 1274 | 178800 | 0.062 | 1274 |
| $f_{54}$ | 3987 | 0.062 | 1274 | 4714 | 0.065 | 1274 | 176460 | 0.062 | 1274 |
| $f_{55}$ | 3835 | 0.06 | 1274 | 4456 | 0.065 | 1274 | 178973 | 0.062 | 1274 |
| $f_{56}$ | 3914 | 0.062 | 1274 | 4566 | 0.063 | 1274 | 172641 | 0.061 | 1274 |
| $f_{57}$ | 3952 | 0.063 | 1274 | 4575 | 0.062 | 1274 | 180844 | 0.063 | 1274 |
| $f_{58}$ | 3974 | 0.063 | 1274 | 4719 | 0.061 | 1274 | 179363 | 0.061 | 1274 |
| $f_{59}$ | 3952 | 0.063 | 1274 | 4575 | 0.061 | 1274 | 174485 | 0.062 | 1274 |
| $f_{60}$ | 4016 | 0.06 | 1274 | 4802 | 0.059 | 1274 | 181715 | 0.063 | 1274 |

[1] , 3.4GHz Intel core i5-3570K, 8 GB RAM

Table 3.3: Computation NEH Problems 61-90

| Problem | FFS | | | FFSB | | | FFSNW | | |
|---|---|---|---|---|---|---|---|---|---|
| | makespan | time | Evaluations | makespan | time | Evaluations | makespan | time | Evaluations |
| $f_{61}$ | 5519 | 0.094 | 5049 | 6677 | 0.096 | 5049 | 346551 | 0.1 | 5049 |
| $f_{62}$ | 5284 | 0.098 | 5049 | 6591 | 0.091 | 5049 | 337898 | 0.098 | 5049 |
| $f_{63}$ | 5248 | 0.094 | 5049 | 6341 | 0.095 | 5049 | 321373 | 0.096 | 5049 |
| $f_{64}$ | 5023 | 0.094 | 5049 | 6185 | 0.093 | 5049 | 310277 | 0.098 | 5049 |
| $f_{65}$ | 5266 | 0.093 | 5049 | 6433 | 0.093 | 5049 | 314738 | 0.097 | 5049 |
| $f_{66}$ | 5139 | 0.092 | 5049 | 6382 | 0.094 | 5049 | 325058 | 0.094 | 5049 |
| $f_{67}$ | 5281 | 0.095 | 5049 | 6458 | 0.092 | 5049 | 340579 | 0.098 | 5049 |
| $f_{68}$ | 5129 | 0.094 | 5049 | 6277 | 0.099 | 5049 | 320482 | 0.101 | 5049 |
| $f_{69}$ | 5489 | 0.096 | 5049 | 6672 | 0.094 | 5049 | 335976 | 0.099 | 5049 |
| $f_{70}$ | 5354 | 0.094 | 5049 | 6767 | 0.095 | 5049 | 318738 | 0.097 | 5049 |
| $f_{71}$ | 5835 | 0.134 | 5049 | 7627 | 0.137 | 5049 | 460166 | 0.137 | 5049 |
| $f_{72}$ | 5500 | 0.131 | 5049 | 7304 | 0.13 | 5049 | 432785 | 0.137 | 5049 |
| $f_{73}$ | 5802 | 0.134 | 5049 | 7362 | 0.13 | 5049 | 451055 | 0.139 | 5049 |
| $f_{74}$ | 5966 | 0.133 | 5049 | 7599 | 0.133 | 5049 | 462018 | 0.136 | 5049 |
| $f_{75}$ | 5637 | 0.135 | 5049 | 7373 | 0.132 | 5049 | 450673 | 0.133 | 5049 |
| $f_{76}$ | 5391 | 0.134 | 5049 | 7199 | 0.132 | 5049 | 423947 | 0.136 | 5049 |
| $f_{77}$ | 5735 | 0.129 | 5049 | 7294 | 0.134 | 5049 | 443647 | 0.137 | 5049 |
| $f_{78}$ | 5723 | 0.131 | 5049 | 7437 | 0.134 | 5049 | 443329 | 0.136 | 5049 |
| $f_{79}$ | 6051 | 0.134 | 5049 | 7632 | 0.132 | 5049 | 451817 | 0.135 | 5049 |
| $f_{80}$ | 5915 | 0.132 | 5049 | 7508 | 0.129 | 5049 | 467223 | 0.145 | 5049 |
| $f_{81}$ | 6613 | 0.207 | 5049 | 8378 | 0.211 | 5049 | 607717 | 0.22 | 5049 |
| $f_{82}$ | 6565 | 0.212 | 5049 | 8327 | 0.212 | 5049 | 604330 | 0.217 | 5049 |
| $f_{83}$ | 6595 | 0.211 | 5049 | 8276 | 0.212 | 5049 | 612867 | 0.218 | 5049 |
| $f_{84}$ | 6649 | 0.207 | 5049 | 8345 | 0.21 | 5049 | 611044 | 0.224 | 5049 |
| $f_{85}$ | 6653 | 0.216 | 5049 | 8303 | 0.212 | 5049 | 595347 | 0.215 | 5049 |
| $f_{86}$ | 6713 | 0.21 | 5049 | 8331 | 0.211 | 5049 | 602323 | 0.219 | 5049 |
| $f_{87}$ | 6595 | 0.207 | 5049 | 8519 | 0.211 | 5049 | 616825 | 0.219 | 5049 |
| $f_{88}$ | 6785 | 0.213 | 5049 | 8567 | 0.215 | 5049 | 621720 | 0.217 | 5049 |
| $f_{89}$ | 6668 | 0.212 | 5049 | 8411 | 0.209 | 5049 | 604596 | 0.217 | 5049 |
| $f_{90}$ | 6728 | 0.208 | 5049 | 8512 | 0.211 | 5049 | 630701 | 0.218 | 5049 |

[1] , 3.4GHz Intel core i5-3570K, 8 GB RAM

Table 3.4: Computation NEH Problems 91-120

| Problem | FFS | | | FFSB | | | FFSNW | | |
|---|---|---|---|---|---|---|---|---|---|
| | makespan | time | Evaluations | makespan | time | Evaluations | makespan | time | Evaluations |
| $f_{91}$ | 10942 | 0.514 | 20099 | 14425 | 0.515 | 20099 | 1.63079e+06 | 0.538 | 20099 |
| $f_{92}$ | 10651 | 0.511 | 20099 | 14429 | 0.504 | 20099 | 1.62148e+06 | 0.53 | 20099 |
| $f_{93}$ | 11061 | 0.503 | 20099 | 14568 | 0.5 | 20099 | 1.62579e+06 | 0.537 | 20099 |
| $f_{94}$ | 11057 | 0.506 | 20099 | 14496 | 0.494 | 20099 | 1.63502e+06 | 0.539 | 20099 |
| $f_{95}$ | 10615 | 0.506 | 20099 | 14402 | 0.504 | 20099 | 1.63788e+06 | 0.526 | 20099 |
| $f_{96}$ | 10491 | 0.51 | 20099 | 14255 | 0.501 | 20099 | 1.65436e+06 | 0.522 | 20099 |
| $f_{97}$ | 11001 | 0.508 | 20099 | 14633 | 0.497 | 20099 | 1.72748e+06 | 0.528 | 20099 |
| $f_{98}$ | 10823 | 0.502 | 20099 | 14642 | 0.497 | 20099 | 1.6848e+06 | 0.532 | 20099 |
| $f_{99}$ | 10617 | 0.503 | 20099 | 14232 | 0.495 | 20099 | 1.64083e+06 | 0.528 | 20099 |
| $f_{100}$ | 10791 | 0.511 | 20099 | 14387 | 0.501 | 20099 | 1.63722e+06 | 0.522 | 20099 |
| $f_{101}$ | 11588 | 0.861 | 20099 | 15649 | 0.924 | 20099 | 2.18707e+06 | 0.941 | 20099 |
| $f_{102}$ | 11740 | 0.858 | 20099 | 15838 | 0.924 | 20099 | 2.23527e+06 | 0.915 | 20099 |
| $f_{103}$ | 11825 | 0.866 | 20099 | 15840 | 0.927 | 20099 | 2.24087e+06 | 0.919 | 20099 |
| $f_{104}$ | 11804 | 0.857 | 20099 | 15931 | 0.929 | 20099 | 2.23525e+06 | 0.923 | 20099 |
| $f_{105}$ | 11657 | 0.864 | 20099 | 15710 | 0.935 | 20099 | 2.17681e+06 | 0.912 | 20099 |
| $f_{106}$ | 11689 | 0.859 | 20099 | 16010 | 0.921 | 20099 | 2.19759e+06 | 0.944 | 20099 |
| $f_{107}$ | 11894 | 0.899 | 20099 | 16003 | 0.956 | 20099 | 2.21672e+06 | 0.954 | 20099 |
| $f_{108}$ | 11870 | 0.86 | 20099 | 15906 | 0.951 | 20099 | 2.23052e+06 | 0.91 | 20099 |
| $f_{109}$ | 11631 | 0.87 | 20099 | 15991 | 0.923 | 20099 | 2.18026e+06 | 0.926 | 20099 |
| $f_{110}$ | 11800 | 0.853 | 20099 | 15943 | 0.93 | 20099 | 2.20513e+06 | 0.912 | 20099 |
| $f_{111}$ | 26693 | 7.259 | 125249 | 38247 | 9.555 | 125249 | 1.27043e+07 | 8.411 | 125249 |
| $f_{112}$ | 27197 | 7.242 | 125249 | 38699 | 9.519 | 125249 | 1.29825e+07 | 8.455 | 125249 |
| $f_{113}$ | 26875 | 7.362 | 125249 | 38081 | 9.721 | 125249 | 1.27635e+07 | 8.455 | 125249 |
| $f_{114}$ | 27125 | 7.26 | 125249 | 38669 | 9.487 | 125249 | 1.29629e+07 | 8.41 | 125249 |
| $f_{115}$ | 26837 | 7.235 | 125249 | 38311 | 9.576 | 125249 | 1.2766e+07 | 8.609 | 125249 |
| $f_{116}$ | 27010 | 7.201 | 125249 | 38528 | 9.593 | 125249 | 1.27253e+07 | 8.437 | 125249 |
| $f_{117}$ | 26806 | 7.258 | 125249 | 38000 | 9.611 | 125249 | 1.27854e+07 | 8.427 | 125249 |
| $f_{118}$ | 27144 | 7.245 | 125249 | 38254 | 9.829 | 125249 | 1.27445e+07 | 8.329 | 125249 |
| $f_{119}$ | 26653 | 7.219 | 125249 | 38252 | 9.609 | 125249 | 1.27319e+07 | 8.464 | 125249 |
| $f_{120}$ | 27054 | 7.228 | 125249 | 38623 | 9.625 | 125249 | 1.287e+07 | 8.394 | 125249 |

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