Artificial Intelligence- Games

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Outline

1. Games
2. Game playing strategies
   - minimax decisions
   - α-β pruning
3. Games that includes chance
4. Games of imperfect information

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   3. There are two agents whose actions must alternate.
   4. Utility values at end of game are equal and opposite.

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3. Actions' effects are defined by precise rules.
4. Games are (too) hard toy problem.
5. Games usually have a huge state space (chess: 35^{100})

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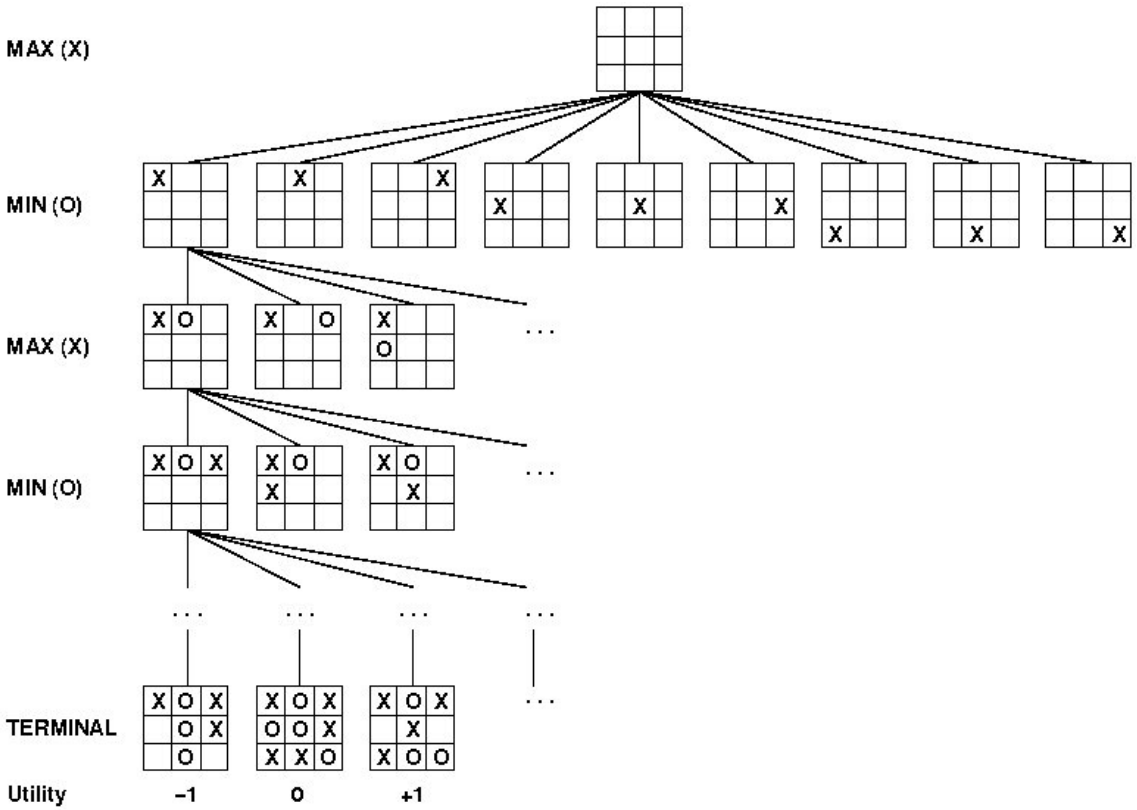
3. The minmax strategy is an optimal strategy for deterministic games with complete information.

Optimal Strategy, Cont



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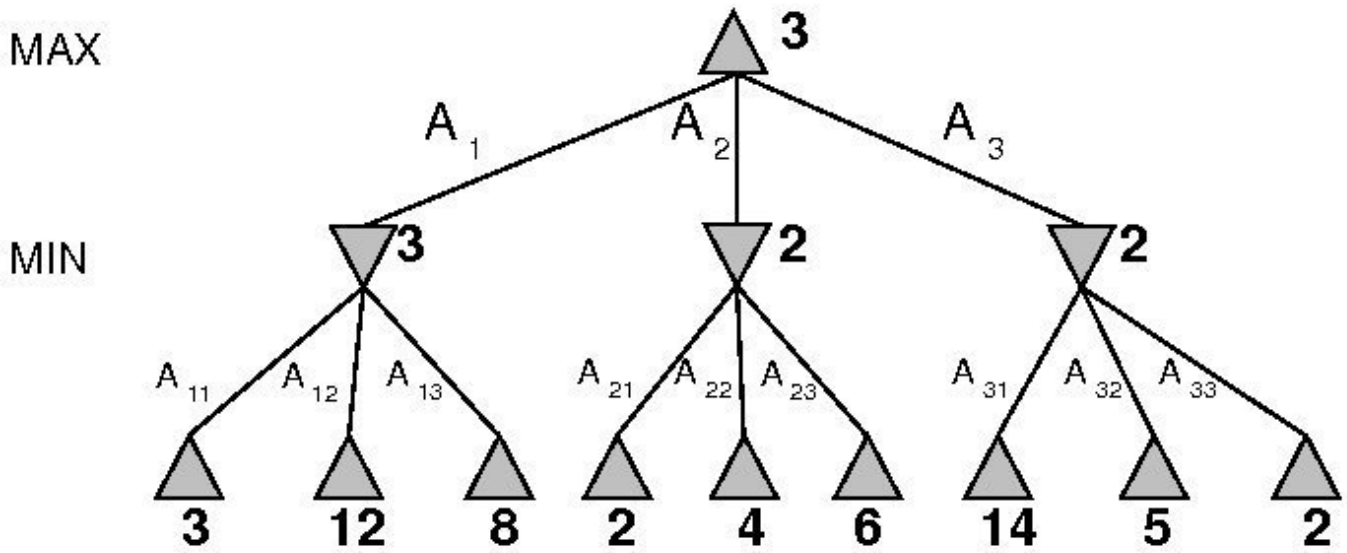

Optimal Strategy, Cont

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function ALPHA-BETA-DECISION (state) returns an action
  return the a in ACTIONS(state) maximizing MIN-VALUE(RESU

function MAX-VALUE (state) returns a utility value
  if TERMINAL-TEST (state) then return UTILITY (state)
  v= -Infty
  for a, s in SUCCESSORS(state) do v= MAX(v, MIN-VALUE(s))
  return v

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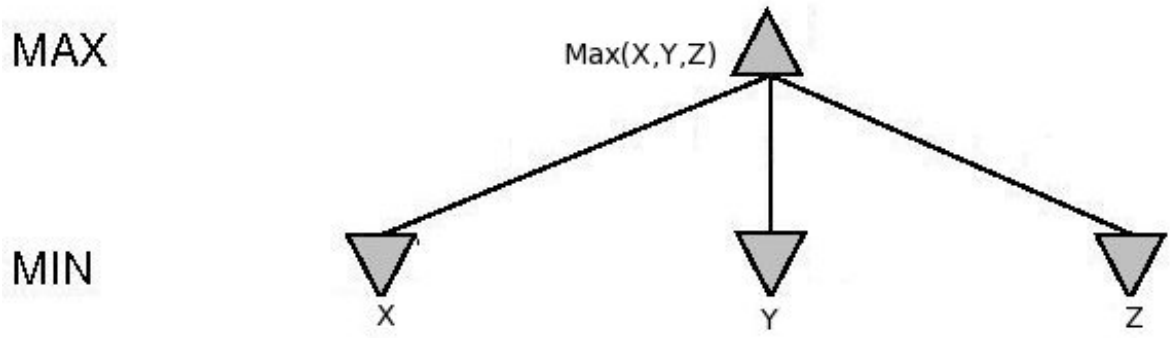
2. Minmax is an optimal strategy but needs exponential time.
3. Is it necessary to visit all nodes in tree?
4. Can the tree be pruned?

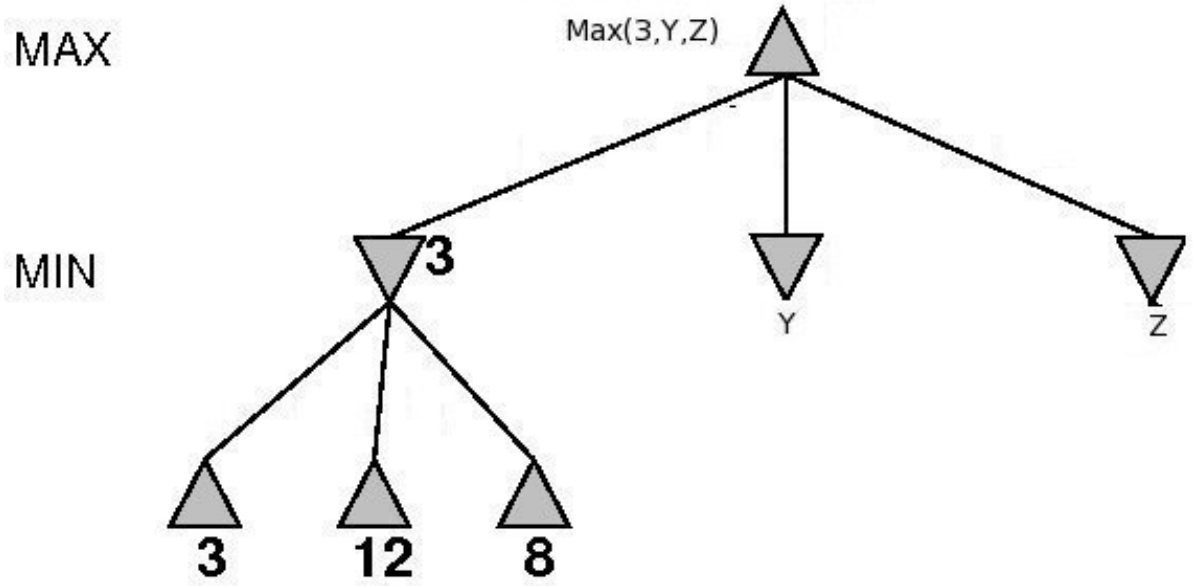
Alpha-Beta Pruning

To calculate minmax root we should exand all its children.



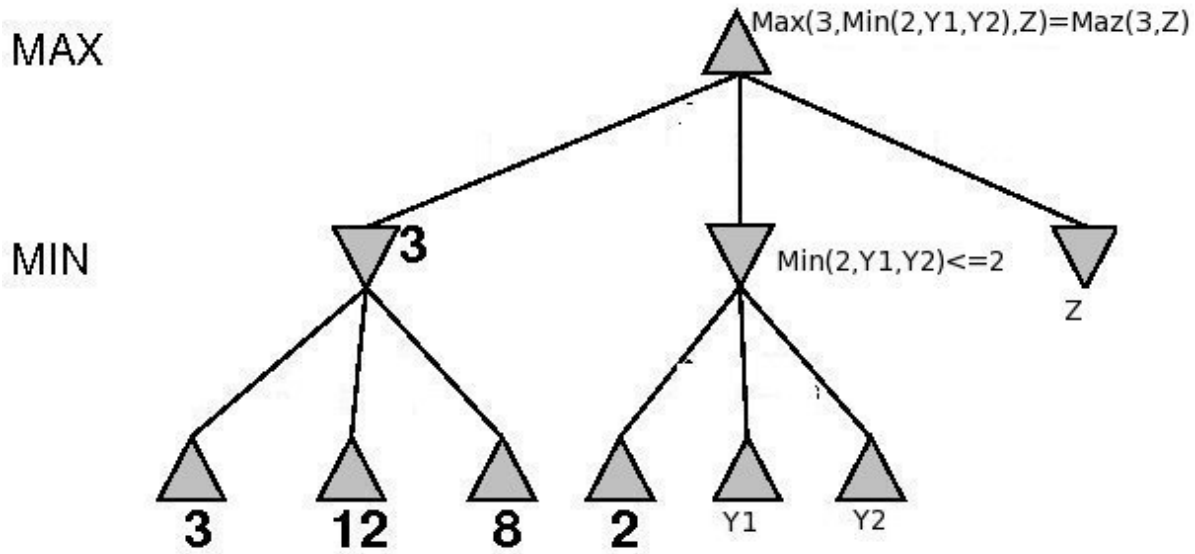
Alpha-Beta Pruning, Cont

To calculate minmax(X) we should exand all its children.



Alpha-Beta Pruning, Cont

To calculate minmax(Y), knowing minmax(X), only the first child is visited.



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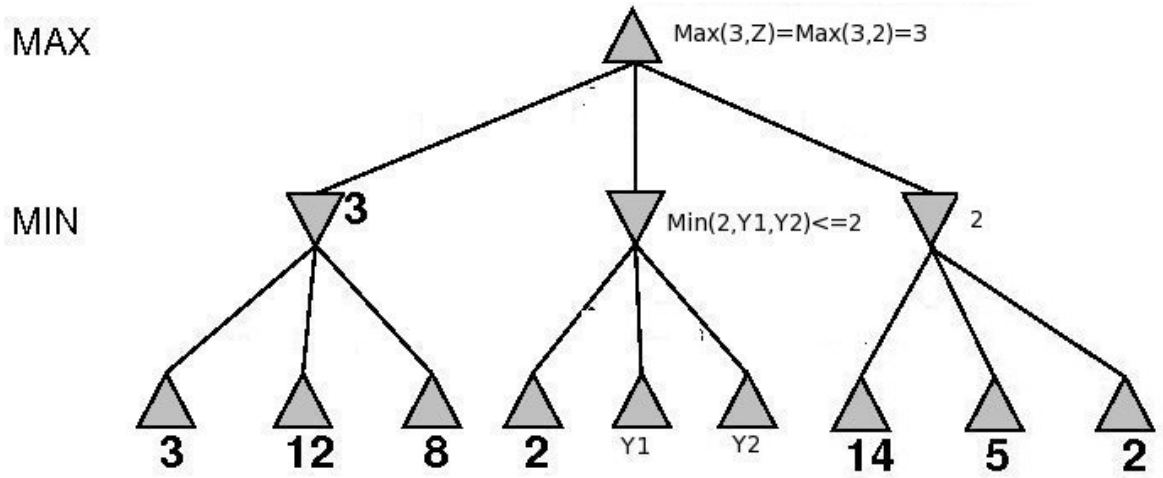

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5. Alpha-Beta updates the value of α and β as it goes along.

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2. In best case, alpha-beta pruning needs to explore $b^{\frac{m}{2}}$ nodes instead of b^m nodes.
3. The effective branching factor becomes $\sqrt{b}$ instead of b.
4. For chess, 6 instead of 35.

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  for a, s in SUCCESSORS(state) do
    v= MAX(v, MIN-VALUE(s,a,b))
    if b<= v then return v
    a=MAX(a,v)
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4. In chess, 35^{50} is still too big!.

Real-Time Decision Making

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   - Terminal test is replaced by **cutoff test** that decides when to apply EVAL.

Evaluation functions

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2. can be in form of $Eval(s) = w_1 f_1(s) + w_1 f_1(s) + \cdots + w_n f_n(s)$.
   e.g. $w_1 = 5$ and
   $f_1(s)$=(number of white rooks)- (number of black rooks)

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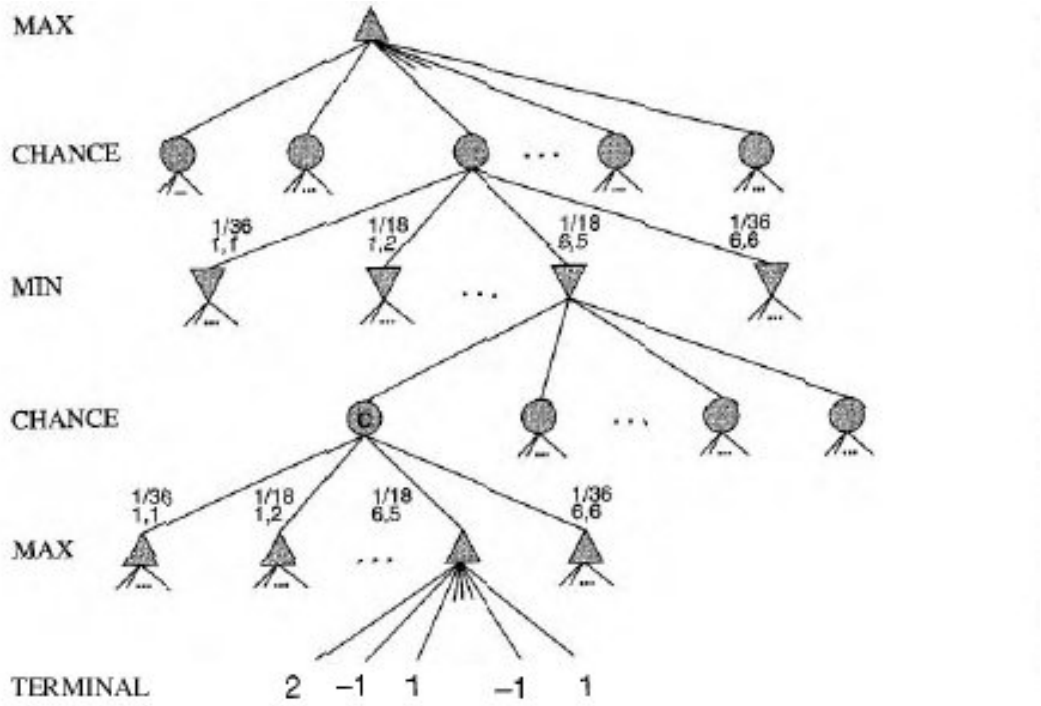
2. The game tree for this games are:



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3. Alpha-beta pruning can be used.

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4. Eval function can be used here.

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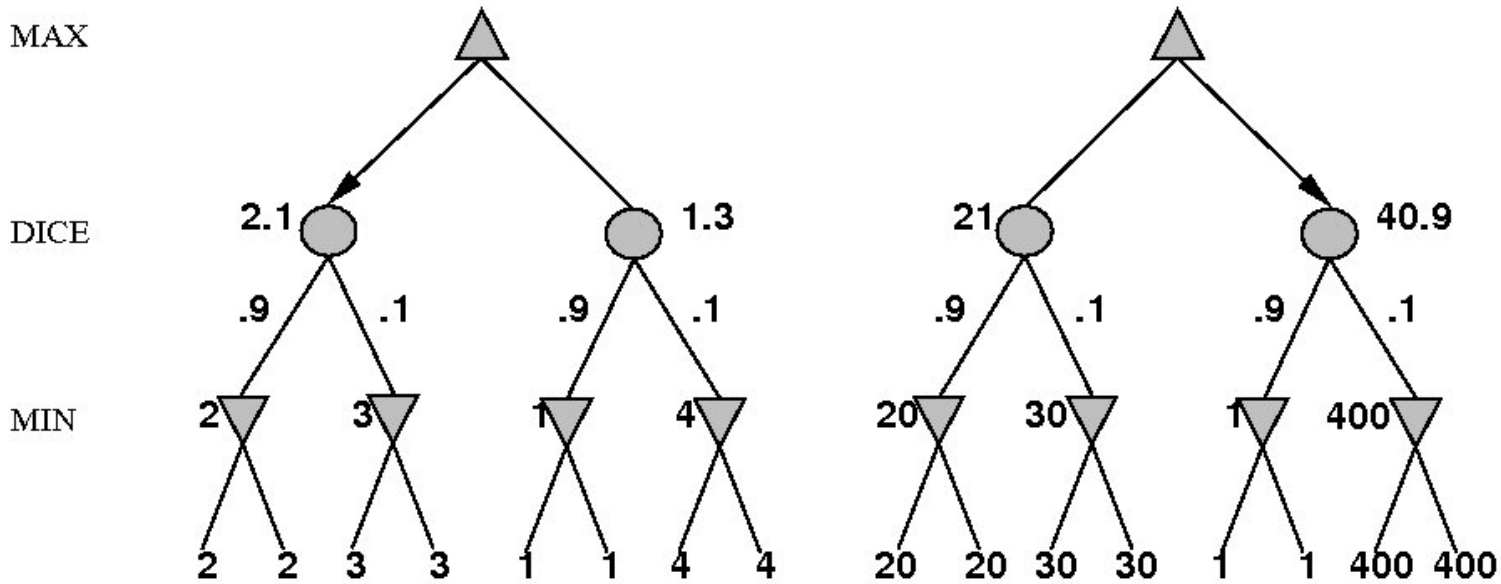

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4. TDGammon uses depth-2 search+ very good Eval $\approx$ world-champion level.

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3. Then, compute the minmax value of each action in each deal.