CS 188: Artificial Intelligence
Fall 2010

Lecture 8: MEU / Utilities
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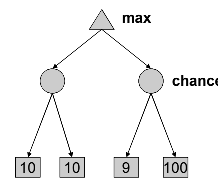
Many slides over the course adapted from either Stuart Russell or Andrew Moore

Expectimax Search Trees

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, next card is unknown
  - In minesweeper, mine locations
  - In pacman, the ghosts act randomly
- Can do **expectimax search** to maximize average score
  - Chance nodes, like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Max nodes as in minimax search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize the underlying problem as a **Markov Decision Process**



[DEMO: minVsExp]

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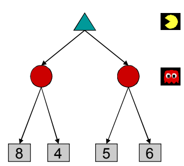
Expectimax Pseudocode

```
def value(s)
    if s is a max node return maxValue(s)
    if s is an exp node return expValue(s)
    if s is a terminal node return evaluation(s)

def maxValue(s)
    values = [value(s') for s' in successors(s)]
    return max(values)

def expValue(s)
    values = [value(s') for s' in successors(s)]
    weights = [probability(s, s') for s' in successors(s)]
    return expectation(values, weights)
```



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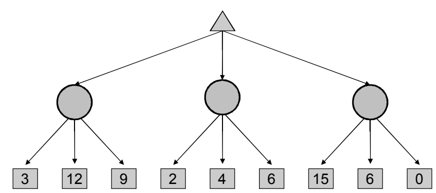
Expectimax Quantities



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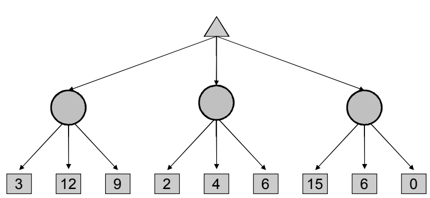
Expectimax Pruning?



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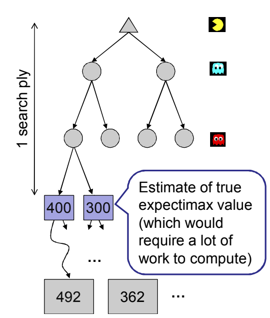
Expectimax Search

- Chance nodes
  - Chance nodes are like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Chance nodes average successor values (weighted)
- Each chance node has a probability distribution over its outcomes (called a model)
  - For now, assume we're given the model
- Utilities for terminal states
  - Static evaluation functions give us limited-depth search



Expectimax for Pacman

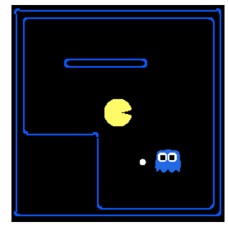
- Notice that we've gotten away from thinking that the ghosts are trying to minimize pacman's score
- Instead, they are now a part of the environment
- Pacman has a belief (distribution) over how they will act
- Quiz: Can we see minimax as a special case of expectimax?
- Quiz: what would pacman's computation look like if we assumed that the ghosts were doing 1-ply minimax and taking the result 80% of the time, otherwise moving randomly?
- If you take this further, you end up calculating belief distributions over your opponents' belief distributions over your belief distributions, etc…
  - Can get unmanageable very quickly!

Expectimax for Pacman

Results from playing 5 games

[demo: world assumptions]

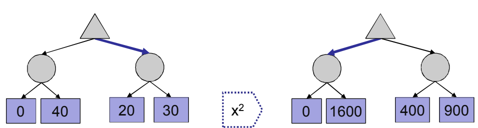
	Minimizing Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 493	Won 5/5 Avg. Score: 483
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503



Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Expectimax Utilities

- For minimax, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For expectimax, we need *magnitudes* to be meaningful

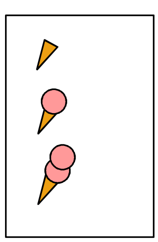


Maximum Expected Utility

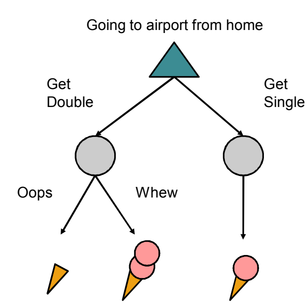
- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
  - A rational agent should chose the action which maximizes its expected utility, given its knowledge
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - Why are we taking expectations of utilities (not, e.g. minimax)?
  - What if our behavior can't be described by utilities?

Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function
- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?



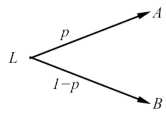
Utilities: Uncertain Outcomes



Preferences

- An agent chooses among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ (1-p), B]$$



- Notation:

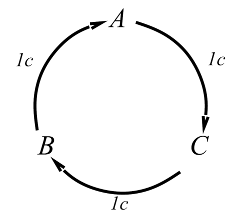
$A \succ B$	A preferred over B
$A \sim B$	indifference between A and B
$A \succeq B$	B not preferred over A

Rational Preferences

- We want some constraints on preferences before we call them rational

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



Rational Preferences

- Preferences of a rational agent must obey constraints.
  - The axioms of rationality:

Orderability
$(A \succ B) \vee (B \succ A) \vee (A \sim B)$

Transitivity
$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

Continuity
$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$

Substitutability
$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$

Monotonicity
$A \succ B \Rightarrow$
$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$

- Theorem: Rational preferences imply behavior describable as maximization of expected utility

MEU Principle

- Theorem:
  - [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \ \Leftrightarrow\ A \succeq B$$
$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \textstyle\sum_i p_i U(S_i)$$

- Maximum expected likelihood (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tictactoe, reflex vacuum cleaner

Utility Scales

- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

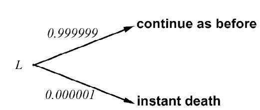
$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Human Utilities

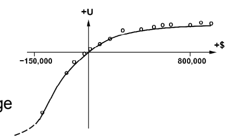
- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment of human utilities:
  - Compare a state A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until A ~ L_p
  - Resulting p is a utility in [0,1]

pay $30 ~



Money

- Money <u>does not</u> behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, \$X; (1-p), \$Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U(\$X) + (1-p)*U(\$Y)
  - Typically, U(L) < U(EMV(L)): why?
  - In this sense, people are risk-averse
  - When deep in debt, we are risk-prone
- Utility curve: for what probability p am I indifferent between:
  - Some sure outcome x
  - A lottery [p,\$M; (1-p),\$0], M large



Example: Insurance

- Consider the lottery [0.5,\$1000; 0.5,\$0]
  - What is its expected monetary value? (\$500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - \$400 for most people
  - Difference of \$100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!

Example: Insurance

- Because people ascribe different utilities to different amounts of money, insurance agreements can increase both parties' expected utility

You own a car. Your lottery:
L_Y = [0.8, \$0 ; 0.2, -\$200]
i.e., 20% chance of crashing

You do not want -\$200!

$U_Y(L_Y) = 0.2 \cdot U_Y(-\$200) = -200$
$U_Y(-\$50) = -150$

Amount	Your Utility U_Y
\$0	0
-\$50	-150
-\$200	-1000

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Insurance company buys risk:
L_I = [0.8, \$50 ; 0.2, -\$150]
i.e., \$50 revenue + your L_Y

Insurer is risk-neutral:
U(L)=U(EMV(L))

$U_I(L_I)$ = U(0.8*50 + 0.2*(-150))
= U(\$10) > U(\$0)

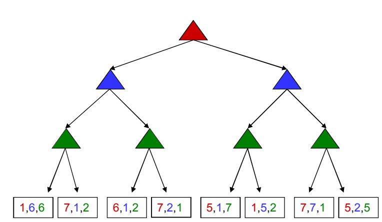
Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8,\$4k; 0.2,\$0]
  - B: [1.0,\$3k; 0.0,\$0]

  - C: [0.2,\$4k; 0.8,\$0]
  - D: [0.25,\$3k; 0.75,\$0]
- Most people prefer B > A, C > D
- But if U(\$0) = 0, then
  - B > A ⇒ U(\$3k) > 0.8 U(\$4k)
  - C > D ⇒ 0.8 U(\$4k) > U(\$3k)

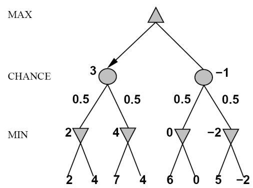
Non-Zero-Sum Utilities

- Similar to minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own utility
  - Can give rise to cooperation and competition dynamically…



Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



ExpectiMinimax-Value(*state*):

```
if state is a MAX node then
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
```

Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = 20 x (21 x 20)^3 = 1.2 x 10^9
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1st AI world champion in any game!