




















Problem Set 8

Please  your solutions in the \LaTeX template provided. Aim for concise solutions; convoluted and obtuse descriptions might receive low marks, even when they are correct. **There is no coding part to submit.**





Please solve each of the following problems using **dynamic programming**. For each , be sure to define  set of subproblems, relate the subproblems recursively, argue the relation is acyclic, provide base cases, construct  solution from the subproblems, and analyze  . Correct but inefficient dynamic programs will be awarded significant partial credit.





For each  below, please indicate whether the requested   is either:
1 polynomial, **2 pseudopolynomial**, or **3 exponential** in the size of the .
 This categorization will be worth **3 points per** .





Problem 8-1. [25 points] Oil Well that Ends Well



The oil wells of tycoon Ron Jockefeller will produce  oil barrels this . Ron has  list of n orders from potential buyers, where the i th   willingness to  a_i barrels for  total price of p_i (not per barrel), which may be negative.¹ Each  must be filled completely or not at all, and can only be filled once. Ron does not have to sell all of his oil, but he must pay s dollars per unsold barrel in storage costs. Describe an $O(nm)$ -time algorithm to determine which orders to fill so that Ron can maximize his profit (which may be negative).




Problem 8-2. [25 points] Splits Bowling





In Lecture 15, we introduced **Bowling**:  one-player  played on  sequence of n pins, where pin i has integer value  (possibly negative). The player repeatedly knocks down pins in two ways:

- knock down  single pin, providing  points; or
- knock down two adjacent pins i and $i + 1$, providing   points.

Pins may be knocked down at most once, though the player may choose not to knock down some pins.  Bowling variant, **Split Bowling**, adds  third  the player can knock down two pins forming  **split**, specifically:

- knock down two pins i and $j > i + 1$ if all pins in $\{i + 1, \dots, j - 1\}$ between them have already been previously knocked down, providing   points.

Describe an $O(n^3)$ -time algorithm to determine the maximum  possible playing Split Bowling on  given  sequence of n pins.

¹Earlier this year, oil futures contract prices went negative:  were paying  to not  delivery of oil because demand for oil had fallen dramatically and there was  shortage of places to store oil.

Problem 8-3. [25 points] Quarter Partition

Given a set $A = \{a_1, \dots, a_n\}$ containing n distinct positive integers where $M = \sum_{a_i \in A} a_i$, describe an $O(m^3n)$ -time algorithm to partition A into four subsets $A_1, A_2, A_3, A_4 \subseteq A$ (where $A_1 \cup A_2 \cup A_3 \cup A_4 = A$) such that the maximum of their individual sums is as small as possible, i.e., such that $\max \left\{ \sum_{a_i \in A_j} a_i \mid j \in \{1, 2, 3, 4\} \right\}$ is minimized.

Problem 8-4. [25 points] Corrupt Chronicles


Kimmy Jerk is the captain of the USS Exitcost, a starship charged with exploring new worlds. Each day, Capt. Jerk uploads a captain's log to the ship's database: a string of at most M lowercase English letters and spaces, where a word in a log is any maximal substring not containing a space.

One day, Capt. Jerk is abducted, and Communications Officer Uhota Nyura goes to the captain's logs looking for evidence. Unfortunately, the log upload system has malfunctioned, and has corrupted each of the last n logs by dropping all spaces. Officer Nyura wants to restore the spaces based on Capt. Jerk's word patterns in his logs. Given a list L_c of the n corrupted logs, as well as a list L_u of $O(m^2n)$ uncorrupted logs from before the malfunction, Officer Nyura wants to:

- for each word w appearing in any log in L_u , compute $f(w)$: the positive integer number of times word w appears in L_u ; $f(w)$ is 0 for any word w not appearing in L_u ; and
- for each log $\ell_i \in L_c$, find a restoration R_i of ℓ_i (i.e., a sequence of words R_i whose ordered concatenation equals ℓ_i) such that $\sum_{w \in R_i} f(w)$ is maximized over all possible restorations.

Describe an $O(m^3n)$ -time algorithm to restore Capt. Jerk's logs based on the above protocol.

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 6.004 Introduction to Algorithms
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