HyperZEXE

Recursive HyperPlonk for fully function-private smart contract

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Zero knowledge proofs

Attest a statement while hidding some inputs of the statement

- Zcash: prove the validity of UTXO without leaking the ID info
- zkRollups: prove the soundness of a list of transactions
- zkBridge: prove the soundness of a list of transactions from another chain
- zkDID: prove who you are without revealing who you are
- zkOracle: attest historical data

Function Privacy

Execute a function with additional guarantees:

- The inputs and outputs to the function remain hidden.
  - Zcash: execute a Layer 1 transaction where sender/receiver and amount are hidden
  - Tornado cash: execute a smart contract transaction where sender/receiver and amount are hidden
- Above, and the function itself is also secret.
  - Aleo (earlier version of testnet): smart contract 1 is IND from SC 2

Function-private smart contract

Applications

- Distributed private computations.
- Miner-extractable values (MEV).
  - all info. w.r.t the smart contract are hidden, no MEV to extract.
- Plausible deniability.
  - Miners do not see if a smart contract is sanctioned.

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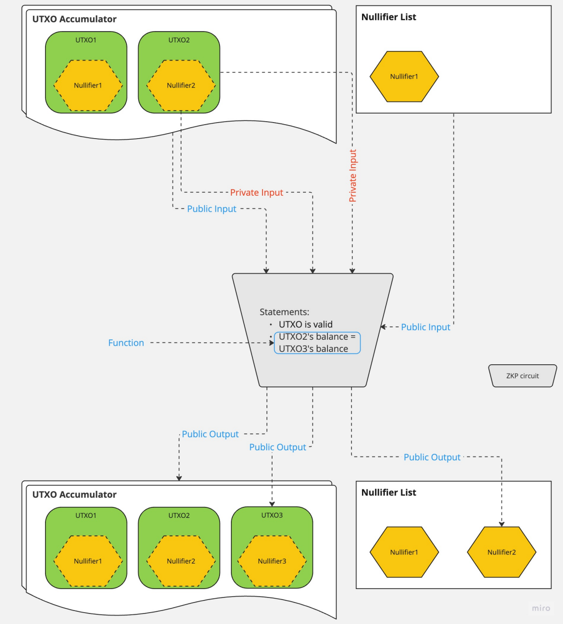
Bonus

- An efficient recursive prover.



UTXO Accumulator
UTXO1
Nullifier1
UTXO2
Nullifier2
Nullifier List
Nullifier1
Private Input
Public Input
Private Input
Statements:
UTXO is valid
UTXO2's balance = UTXO3's balance
Function
Public Input
ZKP circuit
Public Output
Public Output
Public Output
UTXO Accumulator
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Nullifier1
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Nullifier2
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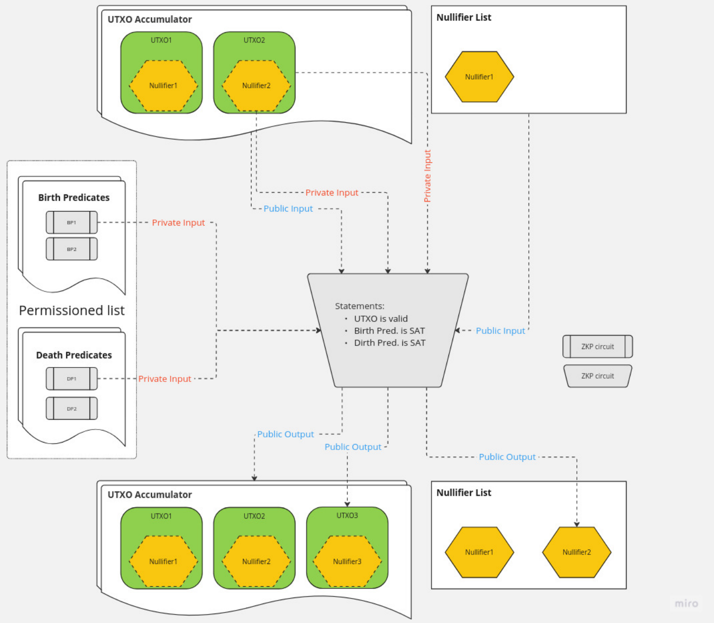
miro



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Recursive proof

- Input a proof π_1 that is valid w.r.t. verification key vk
- Generate a new proof π_2 asserting $\mathtt{verify}(\pi_1, vk) == 1$

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EC1: Two chain proofs

- Two curves `CurveA` and `CurveB`
- `CurveA::BaseField` = `CurveB::ScalarField`
- e.g.: `ZEXE`

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EC2: Cyclic curves

- Two curves `CurveA` and `CurveB`
- `CurveA::BaseField` = `CurveB::ScalarField` and `CurveB::BaseField` = `CurveA::ScalarField`
- e.g.: `Halo2-Pasta`, `Nova`, etc.

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EC3: Non-native arithmetics

- Single Curve `BN254`
- Use $\mathbb{F}_{|\mathbb{G}|}$ to emulate $\mathbb{F}_q$
- Penalty: 30× larger circuit (`Halo2-lib`)
- e.g.: zkEVM via `Halo2-KZG`

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Code based provers

- Relation and proof uses a same field
- FRI, Breakdown, etc...

ZEXE paradigm

	Inner Prover		Outer Prover	
	Scheme	Curve	Scheme	Curve
`ZEXE`	Groth16	BLS12-377	Groth16	CP6-782
`SnarkVM`	Marlin	BLS12-377	Groth16	BW6-761
`VeriZEXE`	TurboPlonk	BLS12-377	UltraPlonk	BW6-761

Table: 2-Chain recursive proof systems in ZEXE

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Table: 2-Chain recursive proof systems in ZEXE

- Plonk arithmetization is $10 \sim 30\times$ more expressive than R1CS

ZEXE on-chain

Challenges 1

- The outer proof has to be on BN254 curve
- Ethereum does not support other popular ZK-friendly curves or fields
  - BN254 curve group mul: 6K Gas
  - Pasta curves group mul: 3M Gas
  - BW6-761 curve group mul: ??? Gas

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Solution 1

- Use `Grumpkin` $\Longleftrightarrow$ BN254 cyclic curves

ZEXE on-chain

Challenges 2

- `Grumpkin` does not support FFT
- Groth16 and Plonk require FFT

ZEXE on-chain

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- `Grumpkin` does not support FFT
- Groth16 and Plonk require FFT

Solution 2

- Use an FFT-free prover
- Candidates:
  - Nova (R1CS)
  - HyperPlonk

ZEXE on-chain

Challenges 3

- `Grumpkin` does not support pairing
- ML-KZG commitment requires pairing
- Other commitment schemes are less verifier friendly

ZEXE on-chain

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Solution 3

- Use Hyrax, verifier does $2\sqrt{n}$ group muls
- Or IPA, verifier does n group muls (deferred and aggregated)

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`HyperZEXE`	HyperPlonk	Grumpkin	UltraPlonk	BN254

Table: 2-Chain recursive proof systems in ZEXE

Concrete efficiency

- vs non-native `Halo2-KZG`: saves $30\times$ in # constraints
- vs (`Veri`)ZEXE: saves $5\times$ due to smaller field (254 bits vs 761 bits)
- and more ...

Native ECAdd: $(x_3, y_3) := (x_1, y_1) + (x_2, y_2)$

(Veri)ZEXE: prove Short Weierstrass formula directly

- $x_3 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2 - x_1 - x_2$
- $y_3 = \frac{(2x_1 + x_2)(y_2 - y_1)}{x_2 - x_1} - \left(\frac{y_2 - y_1}{x_2 - x_1}\right)^3 - y_1$

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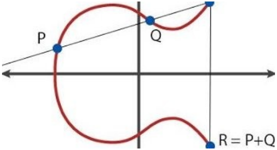
HyperZEXE

- (x_3, y_3) is on curve: $y_3^2 = x_3^3 + b$
- (x_1, y_1), (x_2, y_2) and $(x_3, -y_3)$ are on the same line: $(x_1 - x_3)(y_2 + y_3) = (x_2 - x_3)(y_1 + y_3)$
- $(x_1, y_1)! = (x_3, -y_3)$ and $(x_2, y_2)! = (x_3, -y_3)$



Native Double: $(x_2, y_2) := (x_1, y_1) + (x_1, y_1)$

`(Veri)ZEXE`: prove Short Weierstrass formula directly

- $x_2 = \left(\frac{3x_1}{2y_1}\right)^2 - 2x_1$
- $y_2 = \frac{9x_1^3}{2y_1} - \left(\frac{3x^2}{2y_1}\right)^3 - y_1$

Native Double: $(x_2, y_2) := (x_1, y_1) + (x_1, y_1)$

(Veri)ZEXE: prove Short Weierstrass formula directly

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HyperZEXE

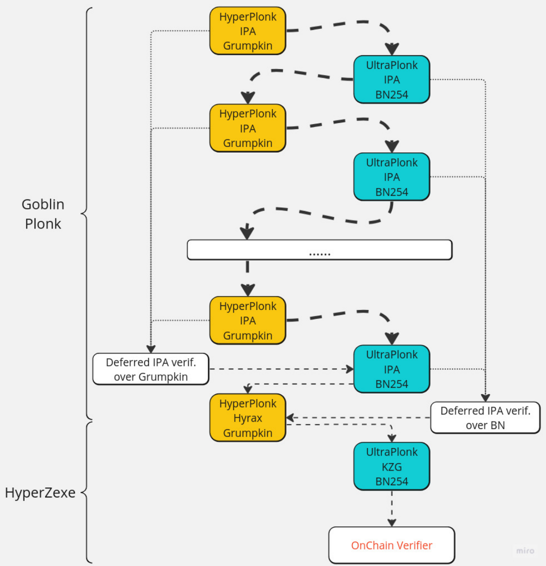
- (x_2, y_2) is on curve: $y_2^2 = x_2^3 + b$
- (x_1, y_1) and $(x_2, -y_2)$ are on a tangential line of the curve $\frac{x_1 - x_2}{y_1 + y2} = \frac{3x_1^2}{2y_1}$
- $(x_1, y_1)! = (x_2, -y_2)$

HyperZEXE's custom gate

OpCodes	Advices		Selectors			
	w_1	w_2	q_{ecc}	q_1	q_2	q_3
On Curve	a_0	b_0	1	0	0	1
EC double	a_1 a_2	b_1 b_2	1	0	1	0
Conditional EC Add	a_3 a_4 cond a_5	b_3 b_4 _ b_5	1	1	0	0

- Custom gate degree: 5 (c.f. 6 for `VeriZEXE` on SW curve)
- Total witness cells per EC mul: 2442 cells
  - c.f., 9325 cells in `VeriZEXE`
  - Further reduced by 5$\times$ via Pippenger and lookups

Beyond ZEXE: infinity recursion for Ethereum applications


Goblin
Plonk
HyperZexe
HyperPlonk
IPA
Grumpkin
UltraPlonk
IPA
BN254
HyperPlonk
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Grumpkin
UltraPlonk
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HyperPlonk
IPA
Grumpkin
Deferred IPA verif.
over Grumpkin
UltraPlonk
IPA
BN254
HyperPlonk
Hyrax
Grumpkin
Deferred IPA verif.
over BN
UltraPlonk
KZG
BN254
OnChain Verifier
miro

Use HyperZEXE as a recursive prover

Target: generate a recursive proof for zkEVM

- Typical circuit size: 2^{20} rows and ≈ 500 columns

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baseline

- Single layer, non-native halo2-KZG: BN254 $\rightarrow$ BN254
- Cost: $2^{25} \times 20 \approx 640M$ cells

Use HyperZEXE as a recursive prover

Target: genererate a recursive proof for zkEVM

- Typical circuit size: 2^{20} rows and ≈ 500 columns, or 2^{29} witness cells

HyperZEXE first layer: BN254 $\rightarrow$ Grumpkin

- verify π_1 dominated by batch verifying 500 KZG openings
- requires roughly 1000 ECMULs, or $\approx 2^{19}$ witness cells
- generate a proof π_2 with Hyrax commitment

HyperZEXE second layer: Grumpkin $\rightarrow$ BN254

- verify π_2 dominated by batch verifying 2 hyrax openings
- requires roughly $2 \times \sqrt{2^{19}} = 2^{11}$ ECMULs, or $\approx 2^{20}$ witness cells

	Inner Prover		Outer Prover		Prover time	OnChain Verifier
	Scheme	Setup	Scheme	Setup		
ZEXE	Groth16	Trusted	Groth16	Trusted		
SnarkVM	Marlin	Universal	Groth16	Trusted	150 s	N/A
VeriZEXE	TurboPlonk	Universal	UltraPlonk	Universal	13 s	N/A
HyperZEXE	HyperPlonk	Transparent	UltraPlonk	Universal	$<$ 1s	450K Gas

Progress

- ✓HyperPlonk with Hyrax commitment
- ✓Optimized native-ECC custom gate
- ✓Solidity onchain verifier
- × UltraPlonk verifier circuit
- × HyperPlonk verifier circuit