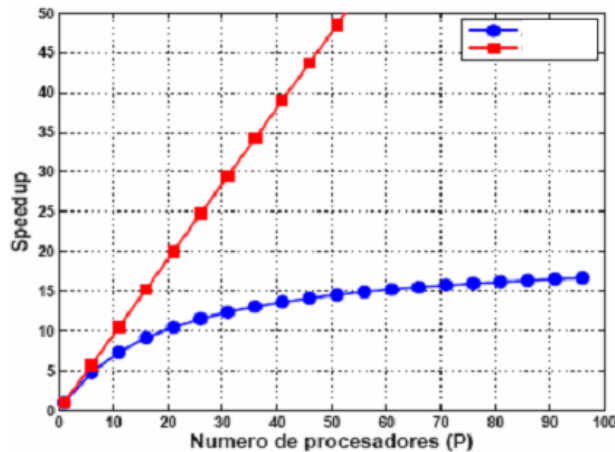




**Exercise 1.-** The figure shows the speedup achieved with P processors when executing a weather forecasting calculation. In one case the execution time is reduced while maintaining the accuracy and in the other case the execution time is maintained while improving the accuracy.



- Explain under which conditions each of the Speedup behaviors represented in the graph occurs.

Squares: fixed time speed-up, workload increases with the number of processors (Gustafson's Law)  
Circles: Speed-up of fixed workload, Execution time decreases with the number of processors but is limited by the serial work (Amdahl's Law).

- Formulate the value of Speedup that corresponds to Amdahl's law and Gustafson's law.  $AmdahlS = \frac{p}{1+fs*(p-1)}$  if  $p$  very large  $S = 1/fs$   
Gustafson  $S = p + (1-p)*fs$
- Identify in the graph which is each case and detail what each parameter represents in both formulations.  
 $S = \text{Speed-up or acceleration} = \text{serial time}/\text{parallel time}$ . Number of processors is  $p$   
The fraction of work done in series is  $fs$   
In Amdahl, the parallel working time on one processor is  $(1-fs)$  and if it is done with  $p$  processors it is  $(1-fs)/p$   
In Gustafson the parallel time with  $p$  processors is  $(1-fs)$  and if done on one processor it is  $p*(1-fs)$ .

From the approximate SpeedUp values obtained from the graph calculate serial fraction,  $fs$ , of the corresponding problem for  $P = 50$  in each case and extrapolate the performance for  $P = 200$ . Calculate the behavior of the serial fraction and the acceleration obtained when  $P$  is very large for each of the cases.

Plot circles(amdahl)  $S$  with large  $p$  is 18  $S = 1/fs$ ,  $fs = 1/18 = 0.056$   $fs = 5.6\%$  for all  $P$  For  $p = 200$ ;  
 $S = 200/(1+199*0.056) = 200/12.1 = 16.5$

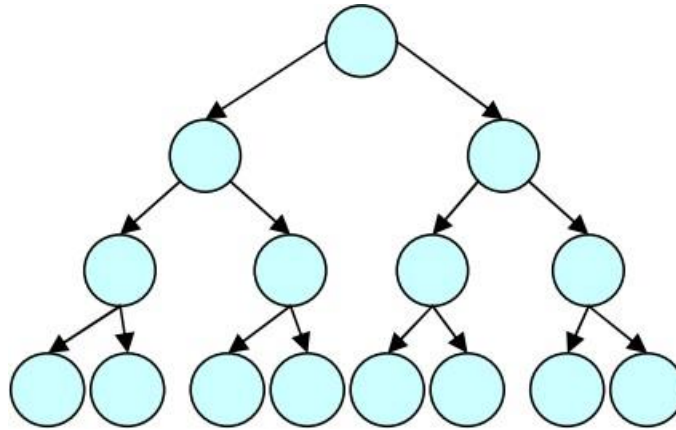
From the square plot  $S=49$  with  $p=50$

$$49 = 50 + (1-50)*fs; 1=49fs, fs = 1/49 = 0.02 \text{ } fs=2\% \text{ for } P=50$$

The behavior of the serial fraction( $fs$ ),

- In fixed workload problems (Amdahl), increasing the number of processors  $fs$  remains constant.
- for fixed time problems (Gustafson) by increasing the number of processors  $fs \Rightarrow 0$ .

The figure represents the dependency network of an application to be executed in a multiprocessor system with  $p$  processors. We want to generalize by calling  $N$  the number of nodes in the network and  $n$  a power of 2, so that there is  $N = \text{summation from } 1 \text{ to } n \text{ of } 2^{(n-1)}$  nodes. In the figure it is represented for  $n=4$ .



It is assumed that there is no penalty for communications and that each node represents a task that takes one unit of time and cannot be divided into subtasks of lower granularity.

- a. Plot the parallelism profile during the execution of the application for the number of nodes represented ( $n=4$ ).



- b. The maximum degree of parallelism and the work performed with maximum degree of parallelism.

$$\text{DOP max} = 8, W = 15$$

- c. Calculate the speedup ( $S$ ) and efficiency ( $E$ ) as a function of the number of processors  $S(p)$  and compare for this case the values  $S()$ ,  $S(p=4)$  and  $S(p=8)$ .

$$S(\infty) = S(8) = (1+2+4+8) / 1+1+1+1+1 = 15/4 = 4.5 \quad | \quad E(8) = 15/4 * 8 = 0.46$$

$$S(4) = (1+2+4+8) / 1+1+1+2 = 15/5 = 3 \quad | \quad E(4) = 3/4 = 0.75$$

- d. Express the maximum degree of parallelism and the maximum Speedup as a function of  $n$ .

$$\text{DOP max} = 2^{(n-1)} \quad | \quad S(\infty) = \text{ceil}((n+1)/2)$$

Given the following loop, indicate how to execute it in parallel in the most efficient way possible. Write the resulting code.

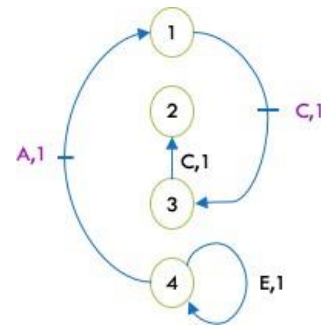
```
do i = 2, 1001
  (1) A(i) = C(i) - 10
  (2) B(i+1) = C(i-2) + alpha
  (3) C(i-1) = D(i+1) * 3
  (4) E(i) = E(i-1) + A(i+1)
enddo
```

Note: alpha is a constant value

- a. We are asked to analyze the dependencies between the instructions and generate the dependency graph and the iteration space.

```
A(2) = C(2) - 10
B(3) = C(0) +
alpha C(1) = D(3)
* 3 E(2) = E(1) +
A(3)

A(3) = C(3) - 10
B(4) = C(1) +
alpha C(2) = D(4)
* 3 E(3) = E(2) +
A(4)
```



- b. Write the code (pseudocode) resulting from the most efficient parallelization possible.

```
do i = 2, 1001
  (4) E(i) = E(i-1) + A(i+1)
enddo

C(1) = D(3) * 3
B(3) = C(0) + alpha
B(4) = C(1) + alpha

do i = 2, 999
  (1) A(i) = C(i) - 10
  (3) C(i) = D(i+2) * 3
  (2) B(i+3) = C(i) + alpha
enddo

A(1000) = C(1000) - 10
A(1001) = C(1001) - 10
C(1001) = D(1002) * 3
```

- c. If the execution time of each instruction is  $T$ , make an estimate of the speedup factor and efficiency to be achieved by using 100 processors.

Time series. 1000 iterations of 4 instructions  $T_s = 4000 T$

Parallel time, sum: 1000 iterations of 1 instruction in series,  $1000T$   
 $6 T$  of prologue and epilogue + 998 iterations of 3 instructions

$$T_p = 1000T + 6T + 998 \times 3T/100 = 1006T + 2994T/100 = 1036T$$

$$S = 4000T/1036T = 3.86$$

- d. What is the maximum achievable acceleration with as many processors as needed? Justify the results.

$$\text{As } f_s = 0.25 = 1/4S \quad (\text{infinity}) = 1/f_s = 4 \text{ it is also worth } 4000T/1009T = 3.96$$