

5.1. Introduction to Graph Transformation

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Index

- **Introduction.**



- **Motivation.**

- **Types of Transformation.**

- **Types of Languages.**

- Overview of Graph Transformation.
- Exercises.
- Control Languages.
- Meta-modelling and Graph Transformation.
- Applications.
- Tools.
- Conclusions.

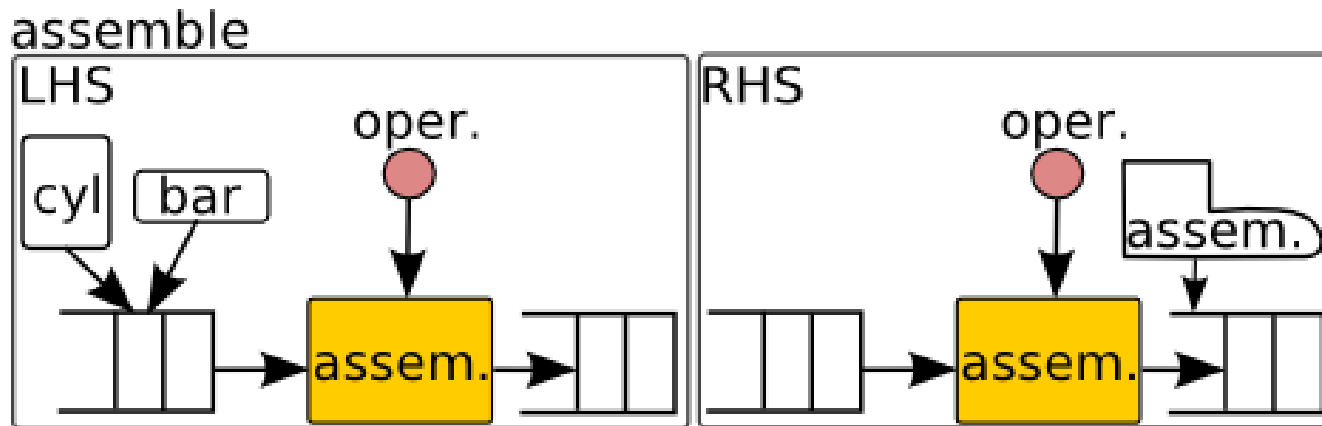


Motivation

- In Model-Driven Engineering (MDE), model transformation is a fundamental activity:
 - Transformation into other languages (e.g. for analysis).
 - In-place transformations:
 - simulations or animations.
 - optimisations and refactorings.
 - Code generation.
 - ...
- Need of a suitable (domain-specific) language for model transformation.
 - Intuitive.
 - Analysable.



- In MDE, it is common to manipulate Domain-Specific (Visual) models.
- Why not using the (Visual) concrete syntax of the models in the transformation language?

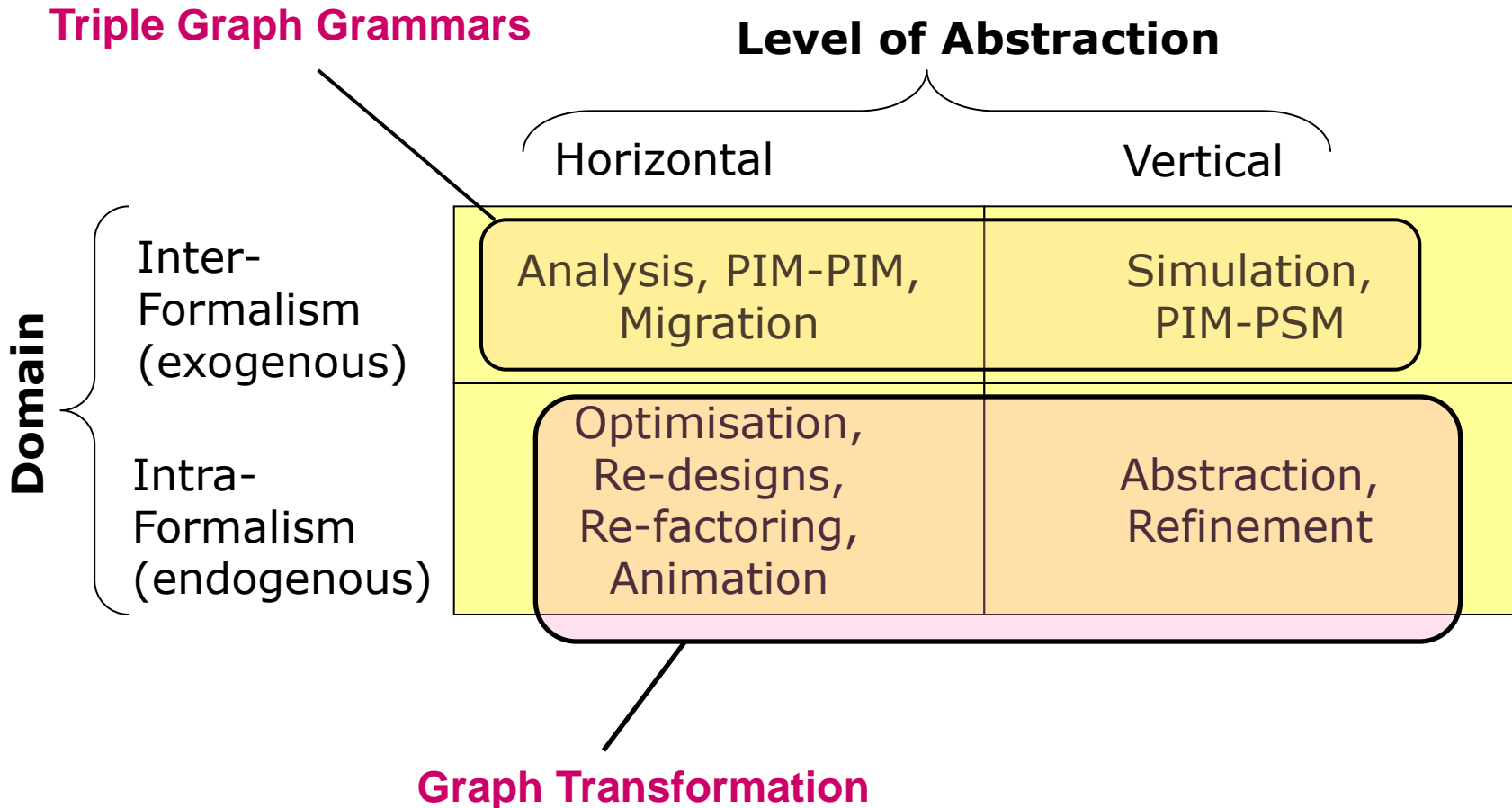


- Similar to programming by demonstration.
- No need to learn a new language!



- How do we know if our transformation is correct?
 - Are there conflicting rules?
 - What are the rule dependencies?
 - Can they be executed in any order?
 - ...?
- Language founded on mathematics that allows answering these questions.

Types of Transformations



Index

- Introduction.
- **Overview of Graph Transformation.**
 - Rule.
 - Match and Derivations.
 - Application Conditions.
 - Grammar.
 - Attributes.
 - Dangling Edges and non-injectivity.
- Exercises.
- Control Languages.
- Meta-modelling and Graph Transformation.
- Applications.
- Tools.
- Conclusions.



What is Graph Transformation?

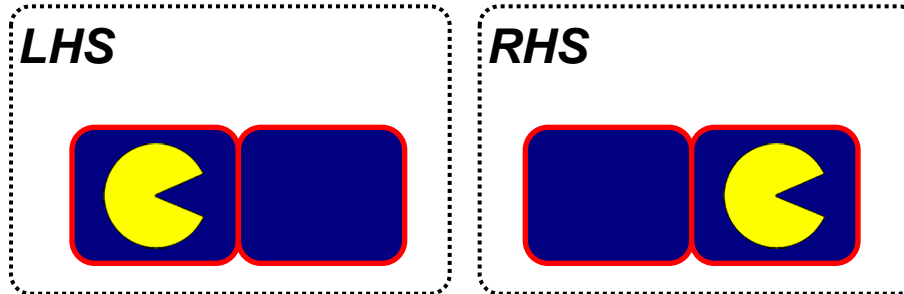
- Graph Transformation (GT) is a declarative, graphical, formal way of manipulating graphs.
 - A graph has nodes and edges....
 - ... nodes and edges can have attributes...
 - ... we can also add types to nodes and edges...
 - ... so we can encode models as graphs.
- Formal technique, many formalisations, the most popular ones based on category theory.
- Hartmut Ehrig and others at TU Berlin in the 70s.

What is GT useful for?

- GT has been used for:
 - Generation of (graph) languages.
 - An alternative to meta-models.
 - Language recognition (parsing).
 - Definition of the semantics of Domain Specific Visual Languages.
 - Specification of simulators or animators.
 - Definition of refactorings, redesigns.
 - Model Transformations.
 - Mutation operators for search-based software engineering (e.g., genetic programming)
 - Computations on graphs.
 - Google's Pregel, Apache Giraph (used at Facebook)

Rules

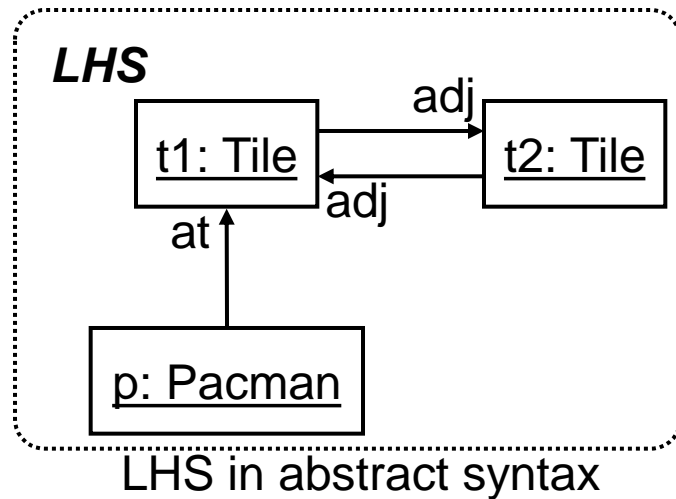
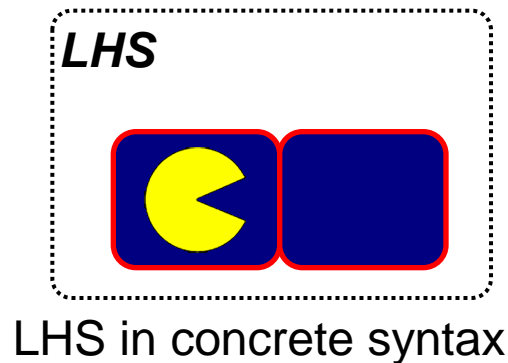
Rule: Pac-Man moves



- A rule is made of a *Left Hand Side* (LHS) and a *Right Hand Side* (RHS).
- Both are graphs.
- They describe an action declaratively through pre- and post-conditions.

Rules

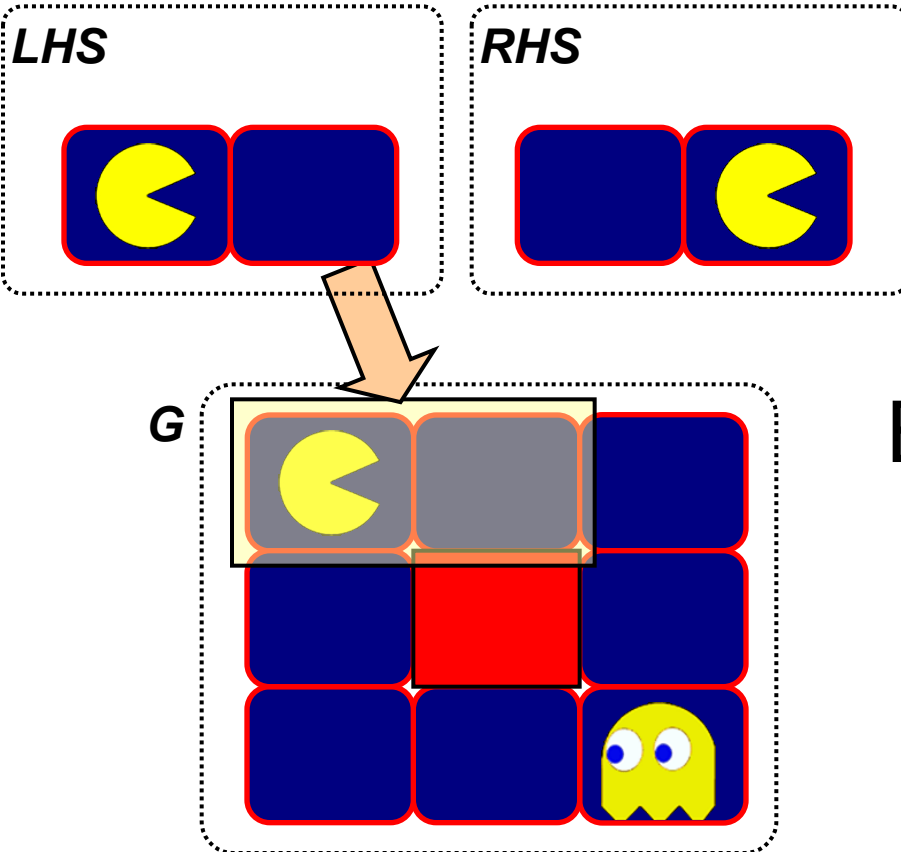
- Are they really graphs?



- Yes, we can use the *concrete syntax* of the language we are manipulating.
- We can also use the *abstract syntax*.

Direct Derivation

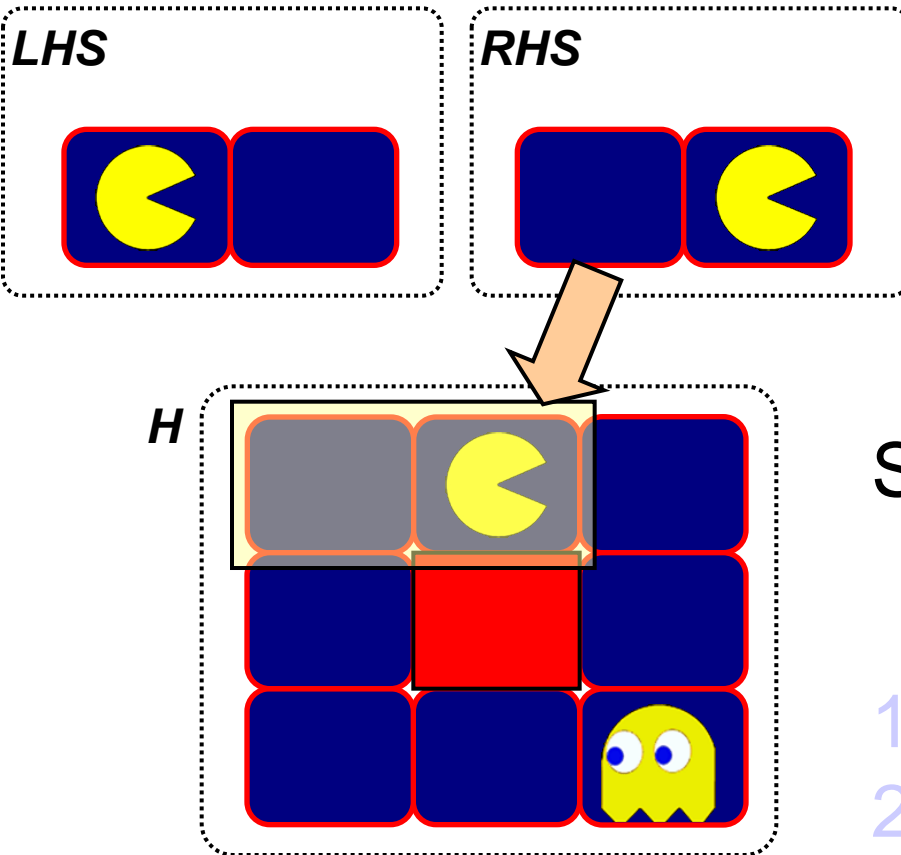
Rule: Pac-Man moves



By pattern matching:
Step 1: Find an
occurrence of the LHS
in the *host graph* G.

Direct Derivation

Rule: Pac-Man moves



Step 2: Do the substitution,
which means:

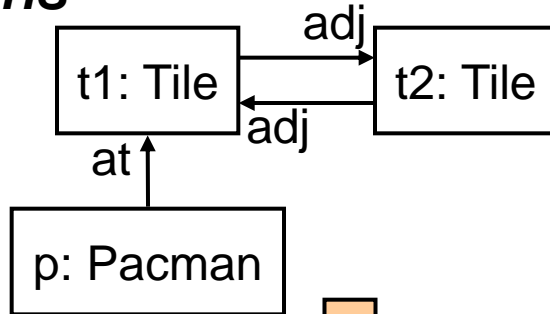
1. Delete $LHS - (LHS \cap RHS)$
2. Create $RHS - (LHS \cap RHS)$
3. Preserve $LHS \cap RHS$.

We write $G \Rightarrow^r H$

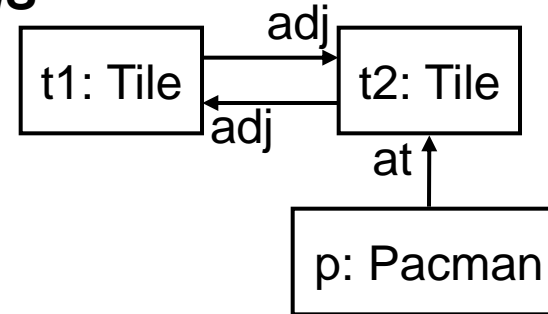
Derivation

in abstract syntax

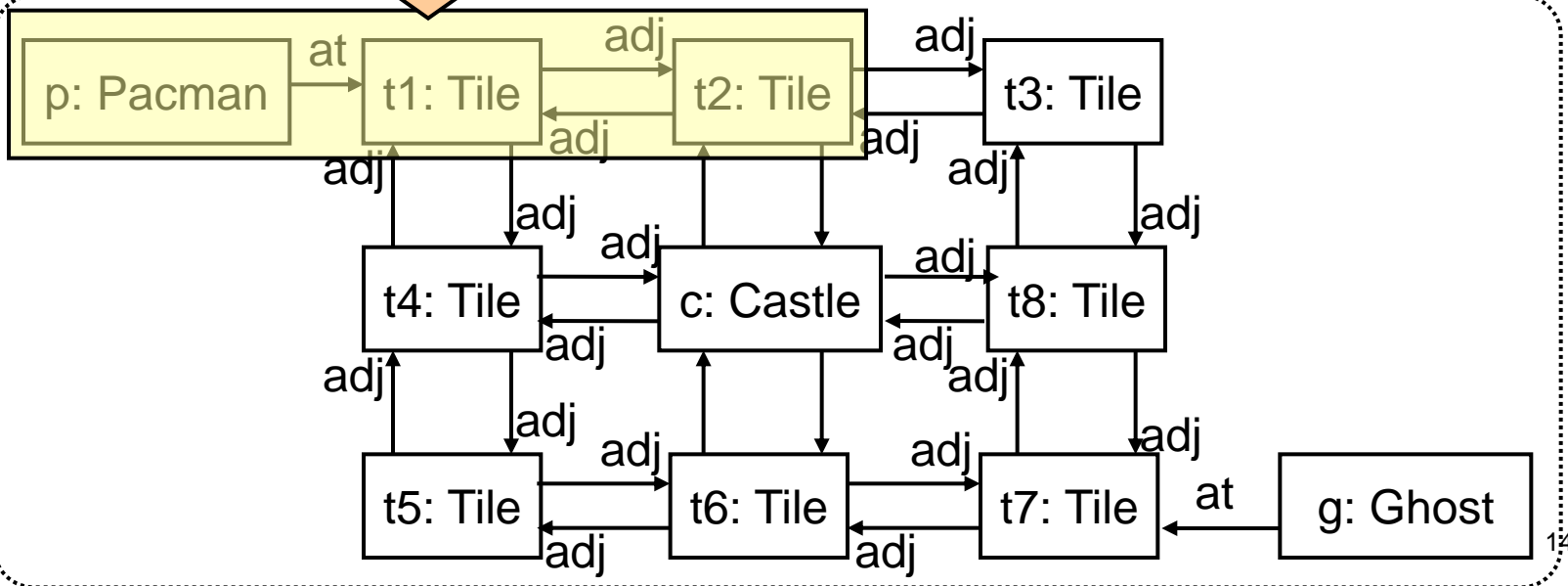
LHS



RHS



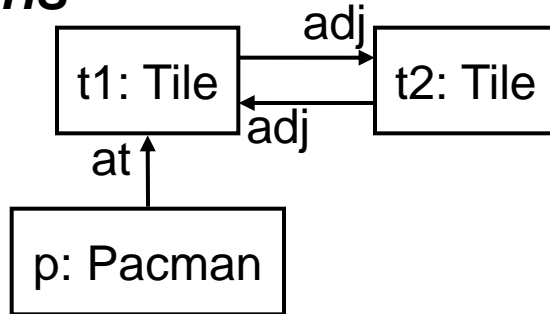
G



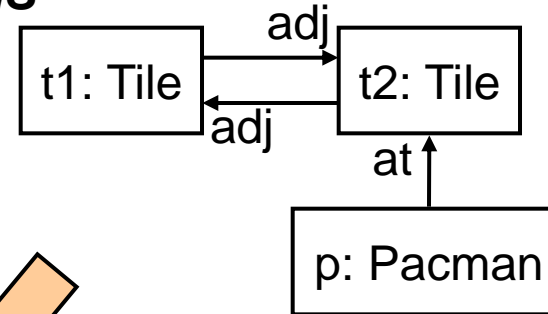
Derivation

in abstract syntax

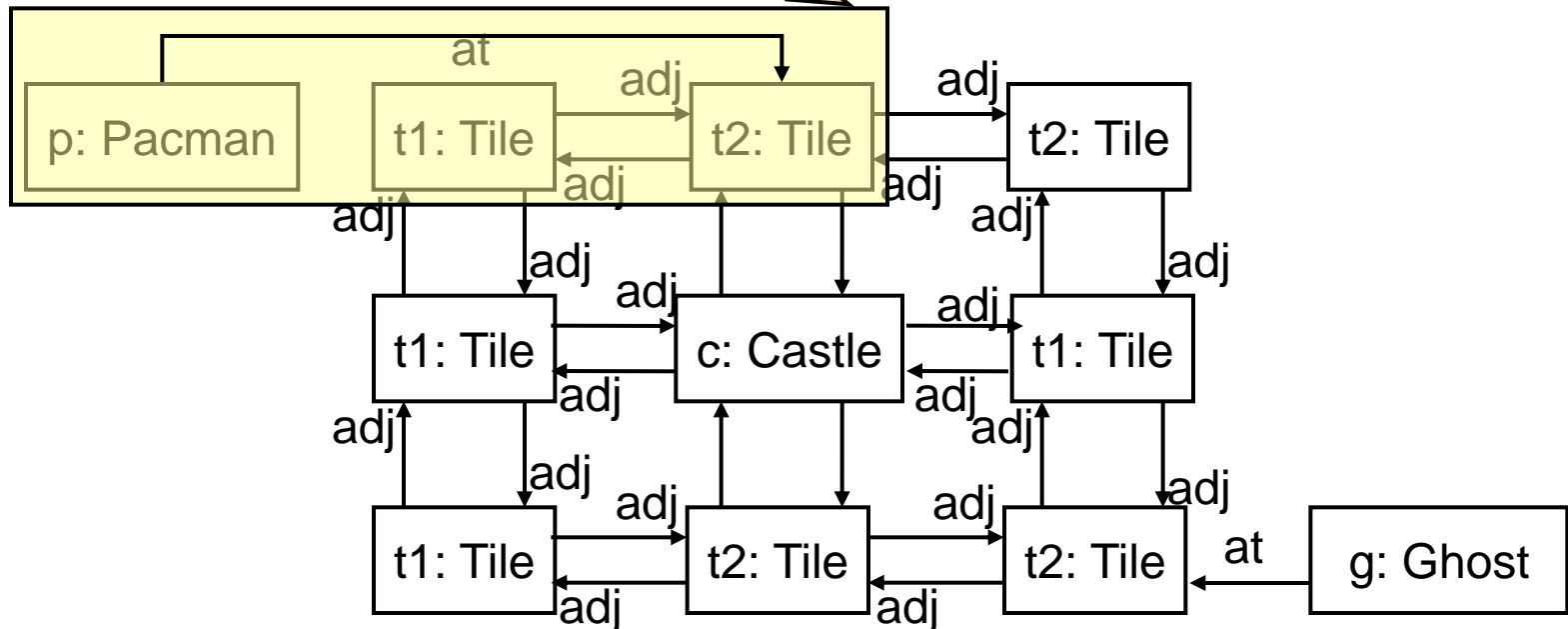
LHS



RHS

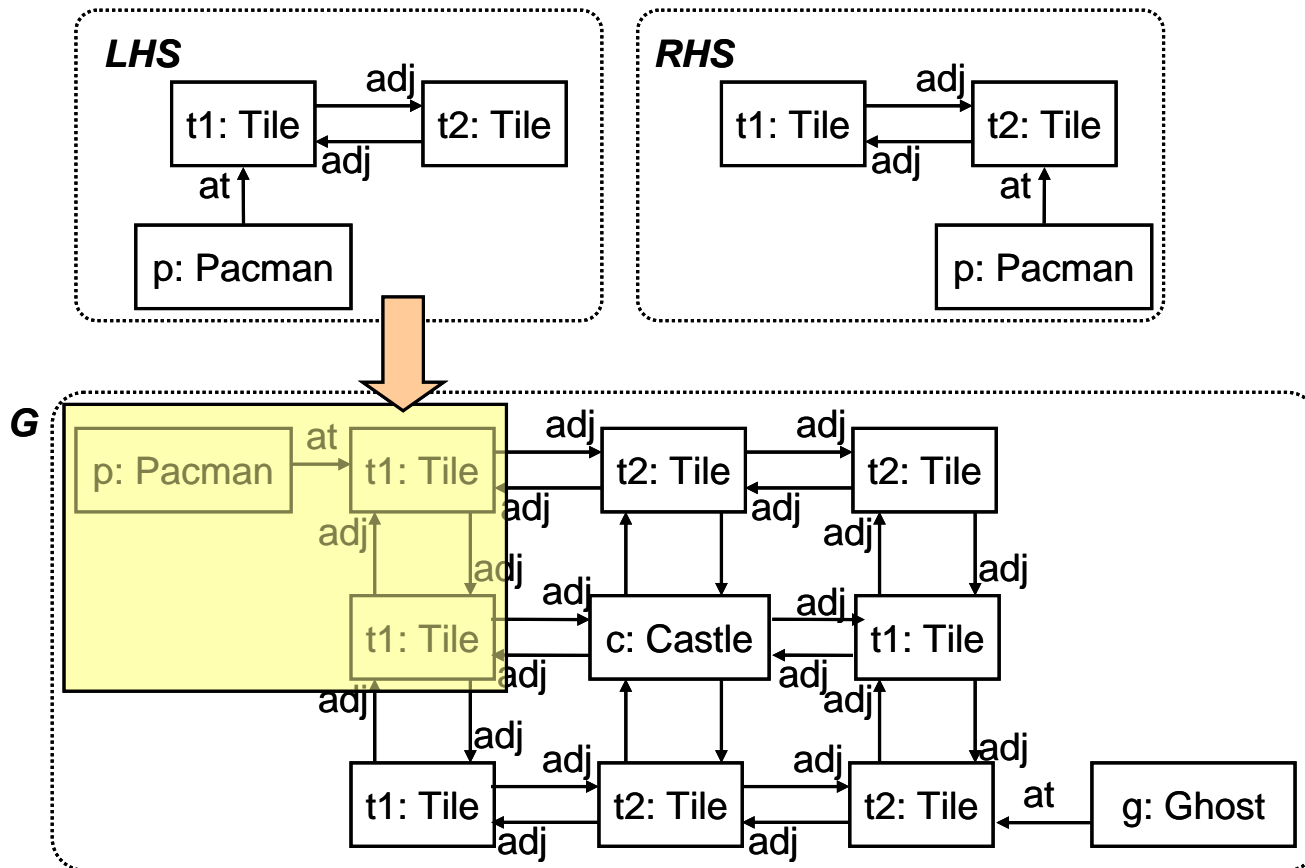


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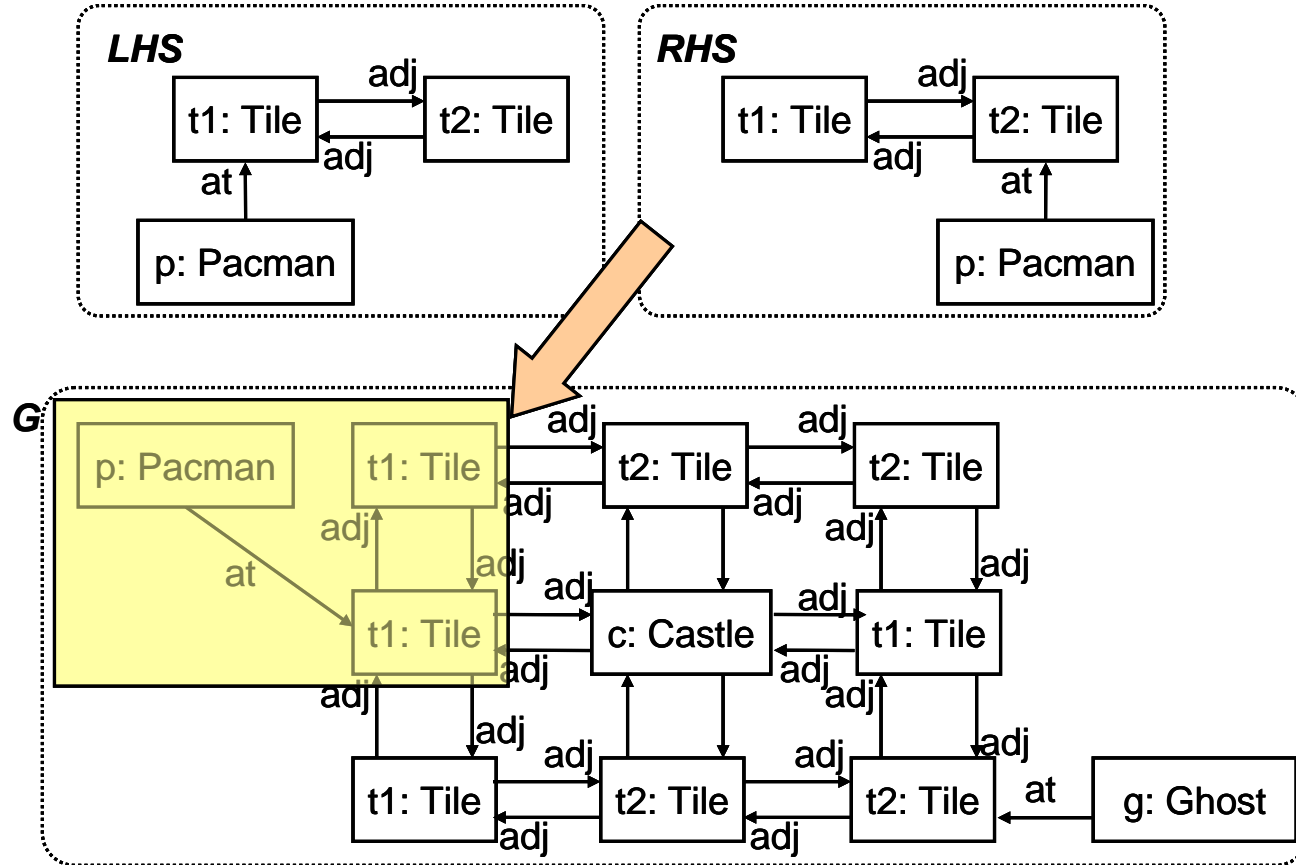
Match

- The place in the graph where the rule is applied.
- More than one valid match may exist for the rule, one has to be selected.



Match

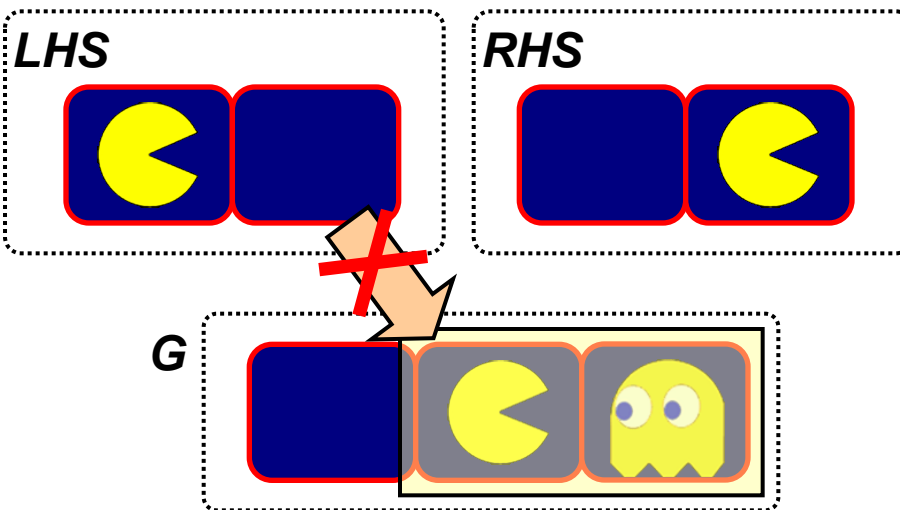
- The place in the graph where the rule is applied.
- More than one valid match can exist for the rule, one has to be selected.



Application Conditions

- The LHS of the rule expresses positive conditions for the rule to be applied.
- We can incorporate more complex application conditions into the rules, in particular to test negative conditions.
 - These are called NACs (Negative Application Conditions).

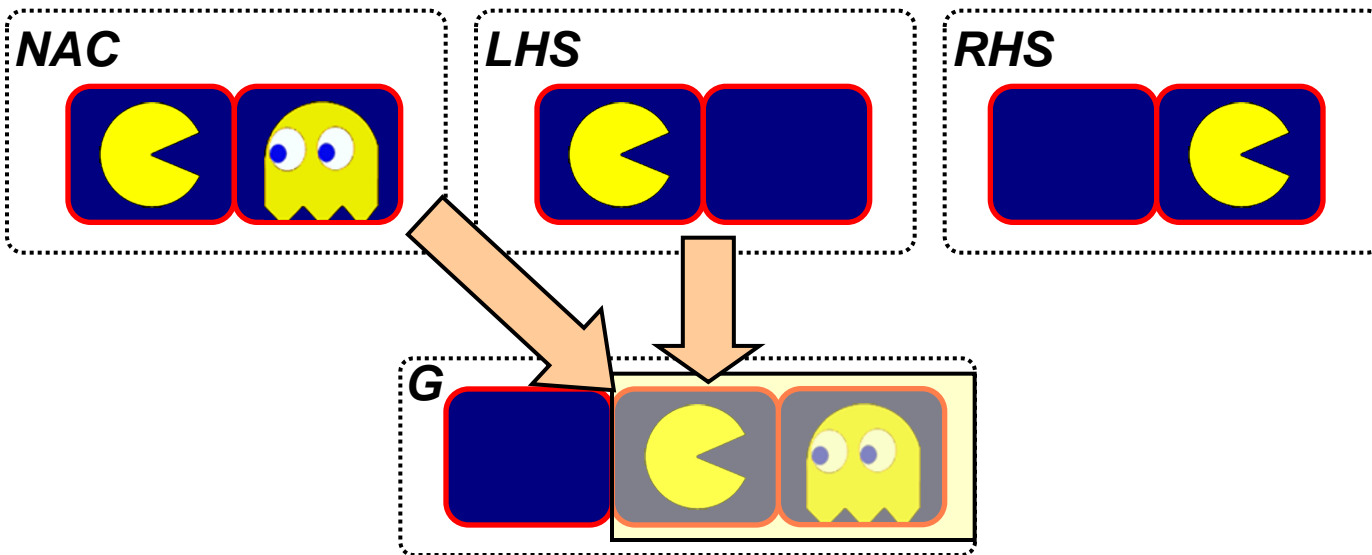
Rule: Pac-Man moves



- Avoid certain matches. A smarter PAC-MAN!

Application Conditions

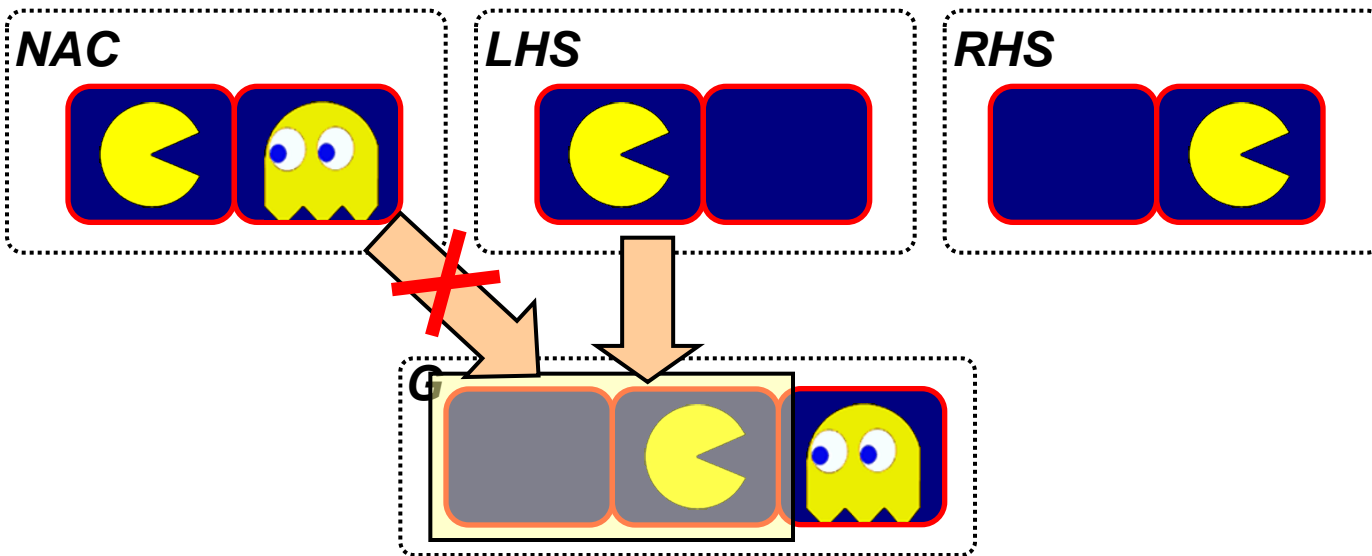
Rule: Pac-Man moves



- This match is invalid, because there is an occurrence of the NAC.

Application Conditions

Rule: Pac-Man moves

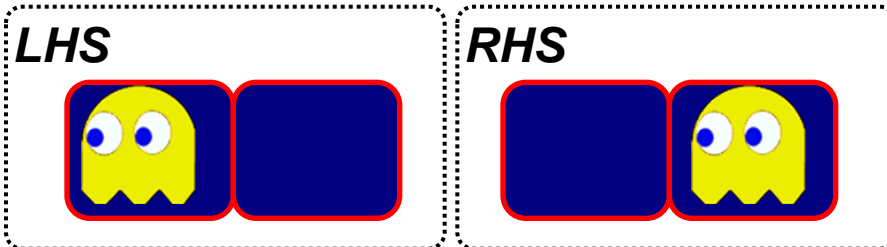


- This match is valid, because there is NO occurrence of the NAC.

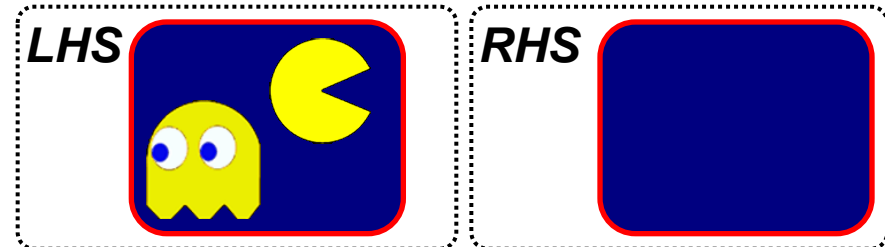
Grammars

- A grammar is a set of rules, and an initial host graph, to which the rules are applied.
- The rules are applied in arbitrary order, until none is applicable.

Rule: Ghost moves



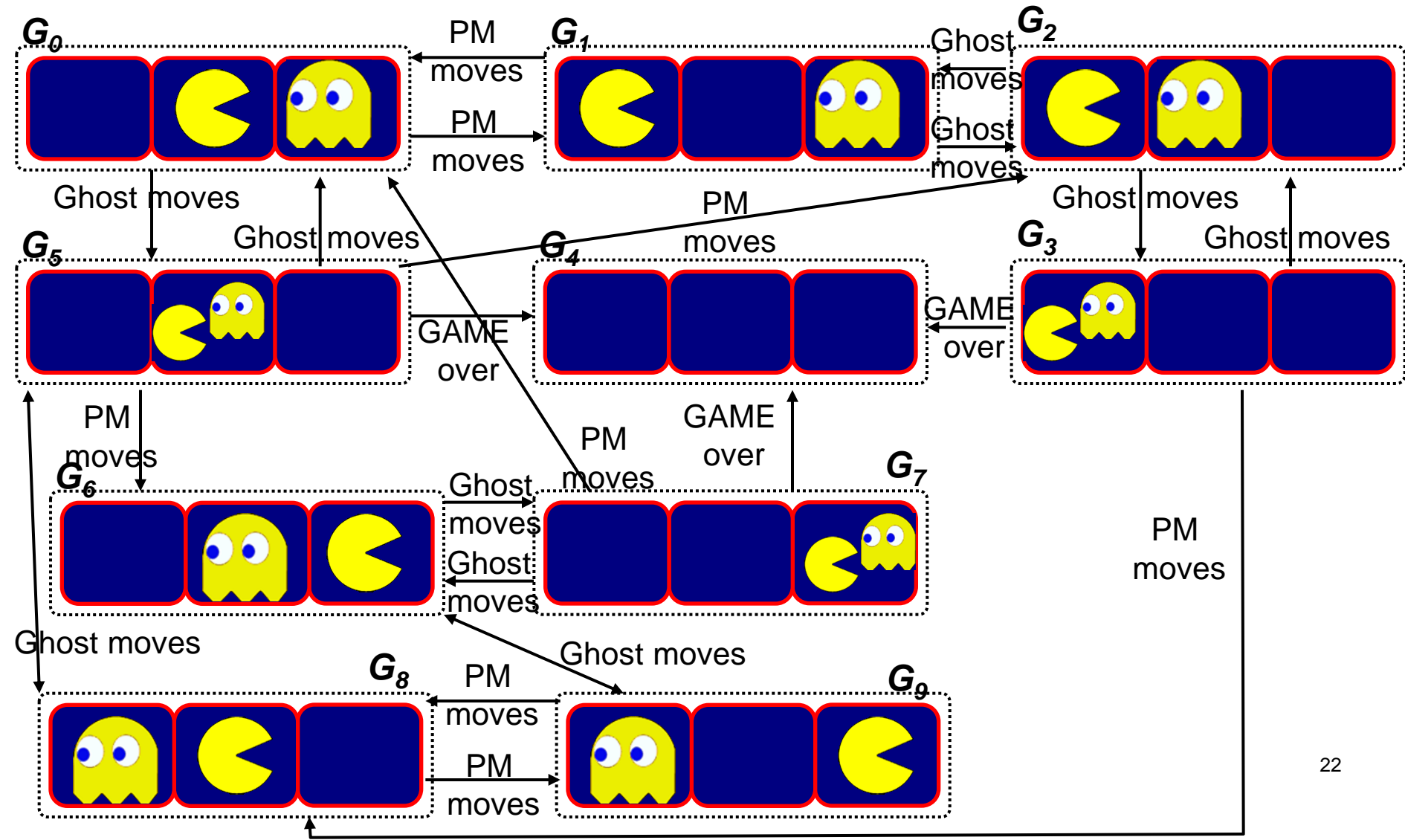
Rule: GAME Over



- $\text{Game} = \{\{\text{PACMAN moves}, \text{Ghost moves}, \text{GAME Over}\}, G\}$.
- The semantics of the grammar are all reachable graphs.
 - $\text{SEM}(\text{Game}) = \{ H \mid G \Rightarrow^* H \}$
 - In this case the set is finite (if the graphs are finite), but there are derivations of infinite length.

PAC-MAN Grammar

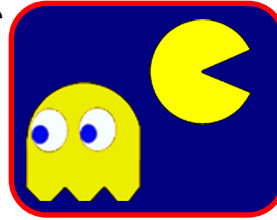
What is wrong? And how can we fix it?



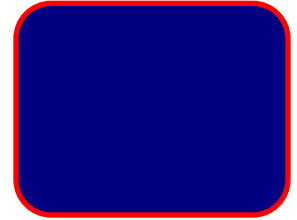
PAC-MAN Grammar

Rule: GAME Over

LHS

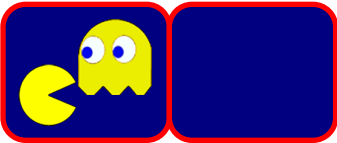


RHS

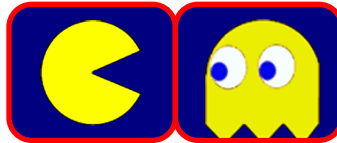


Rule: Pac-Man moves

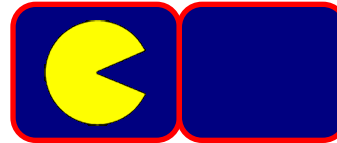
NAC



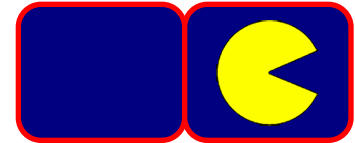
NAC



LHS

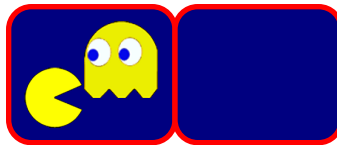


RHS



Rule: Ghost moves

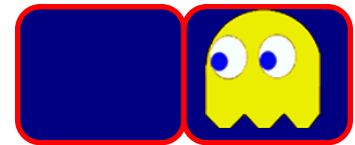
NAC



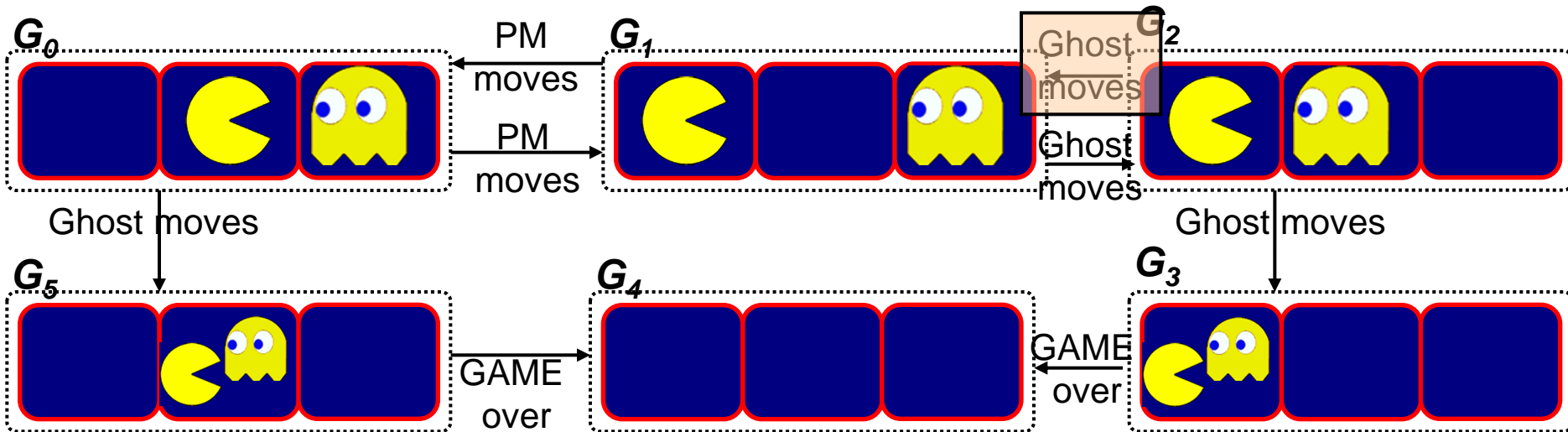
LHS



RHS



PAC-MAN Grammar



Question: How do we make the Ghost Smarter?

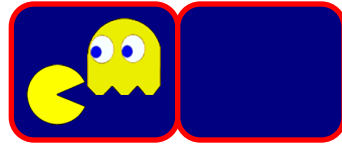
A Smarter Ghost

Rule: Ghost moves

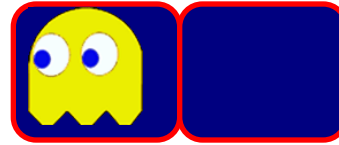
NAC



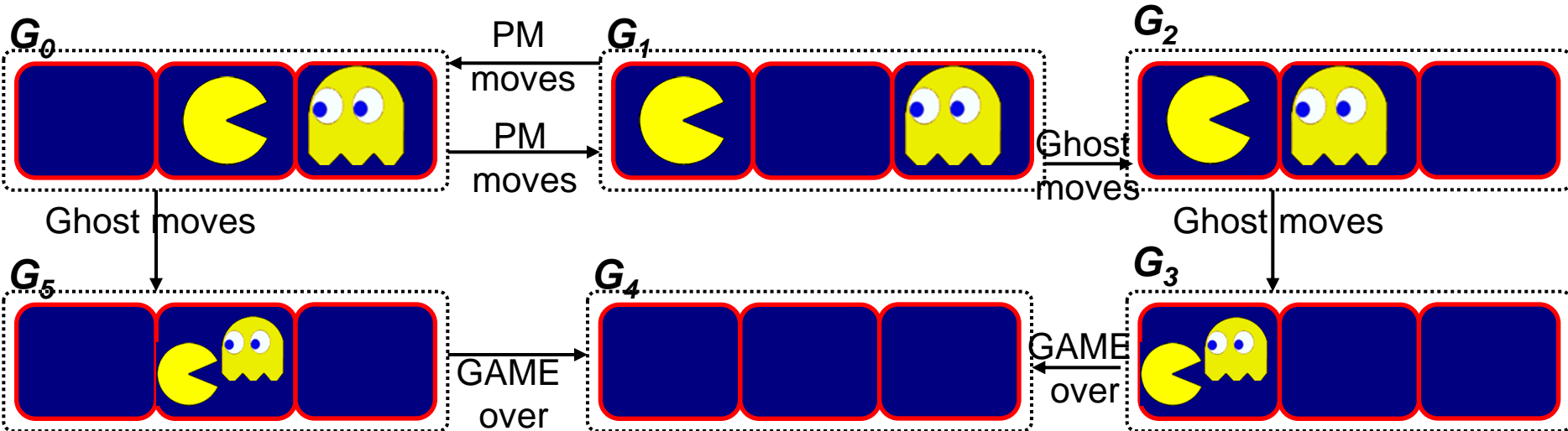
NAC



LHS



RHS



Attributes

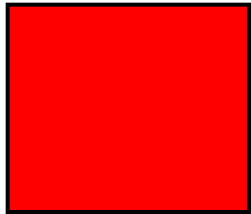


- Nodes and edges in graphs can be assigned attributes of some sort.
- Nodes and edges in rules can be assigned attributes, whose values are variables.
- The possible values that the variables can take are controlled by formulae: ***Attribute conditions.***
- We can assign new values to the attributes in the RHS: ***Attribute computations.***

Attributes

Rule: PACMAN hurted

LHS



Attribute Condition
 $X > 0$

RHS



Rule: GAME over

LHS



Attribute Condition
 $X == 0$

RHS

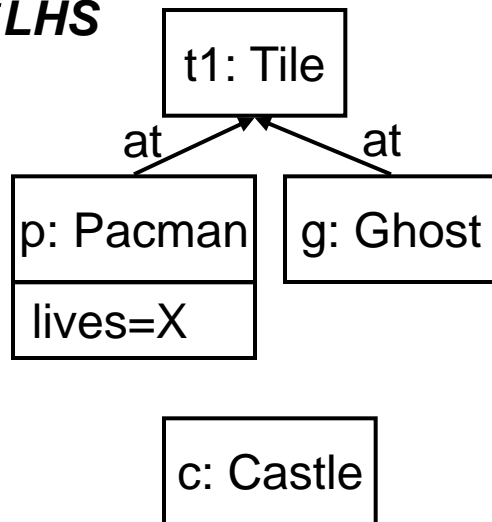


Attributes

In abstract syntax

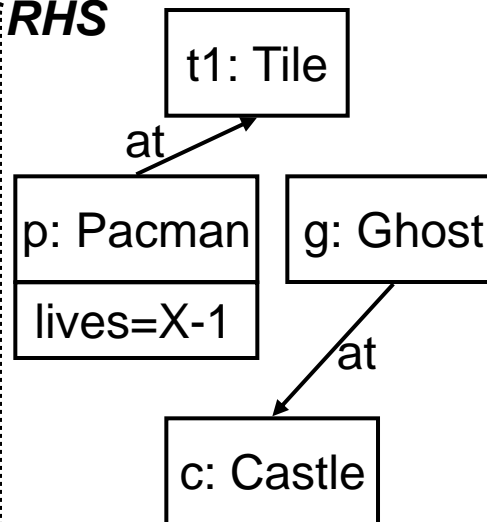
Rule: PACMAN hurted

LHS



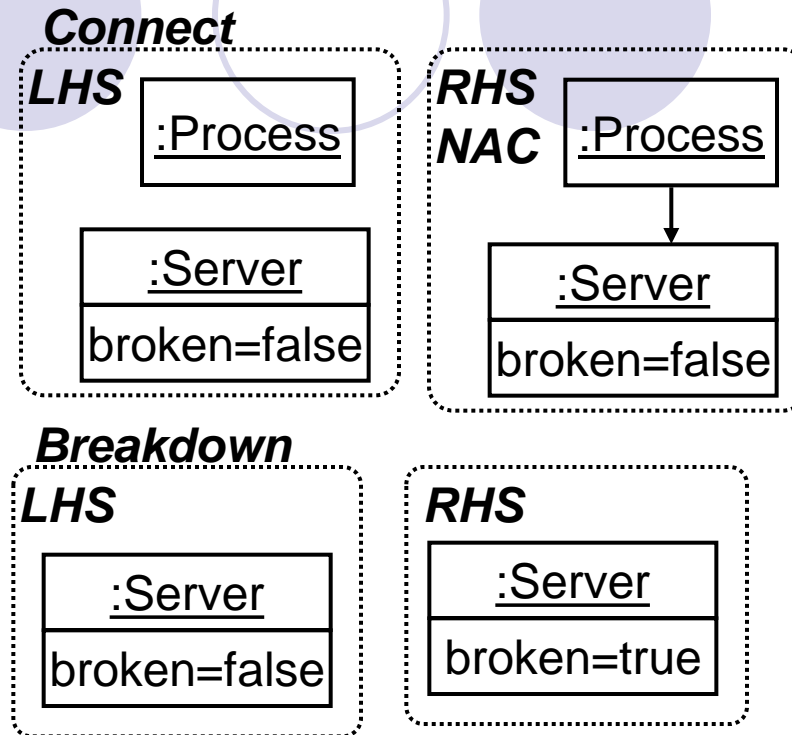
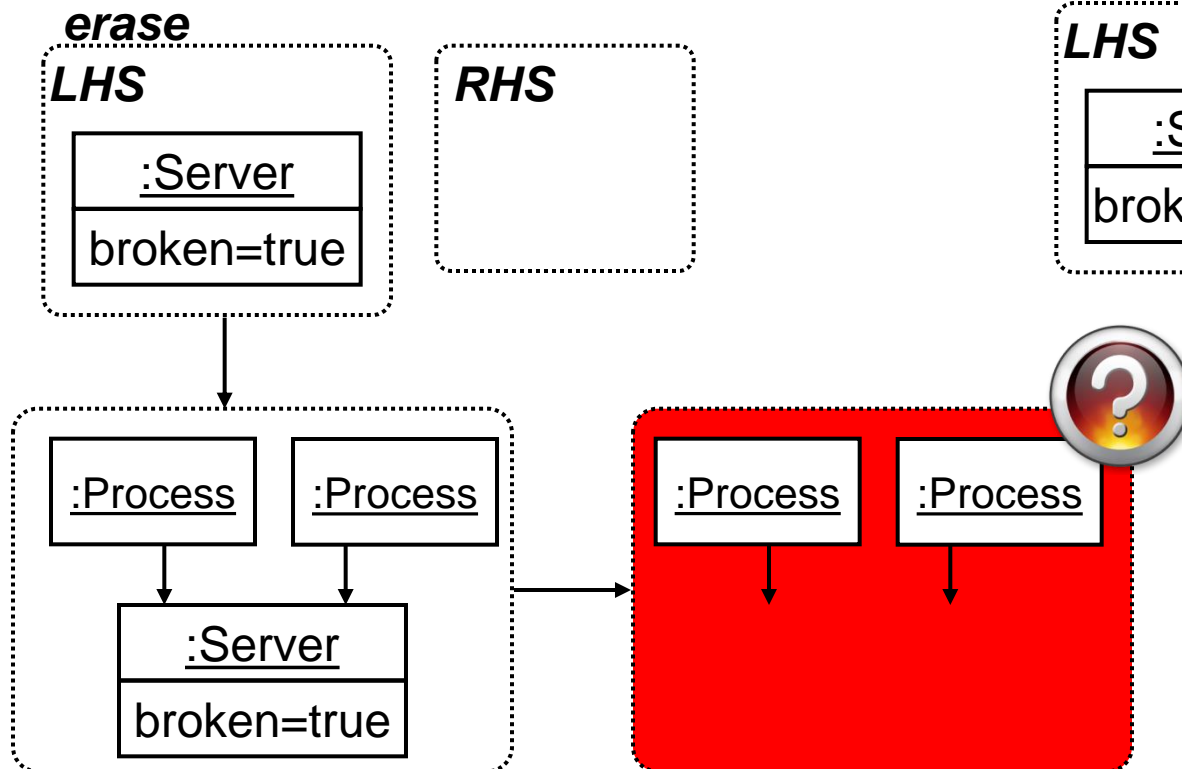
Attribute Condition
 $X > 0$

RHS

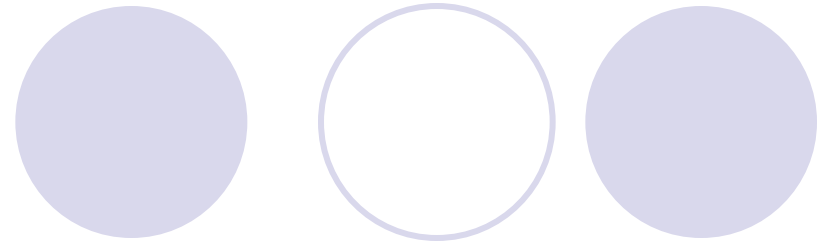


Dangling Edges

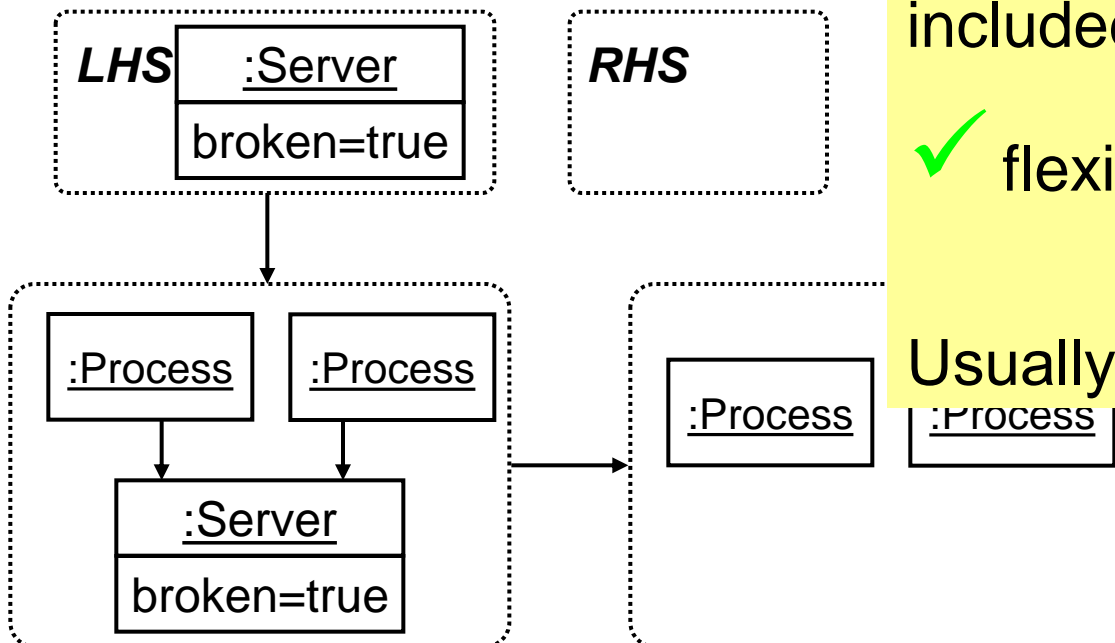
- What happens if a rule states that a node should be deleted, and such node receives arbitrary edges in the host graph?



Dangling Edges



- We no longer have a graph, as several dangling edges appear.
- Two options:
 1. Delete the dangling edges.

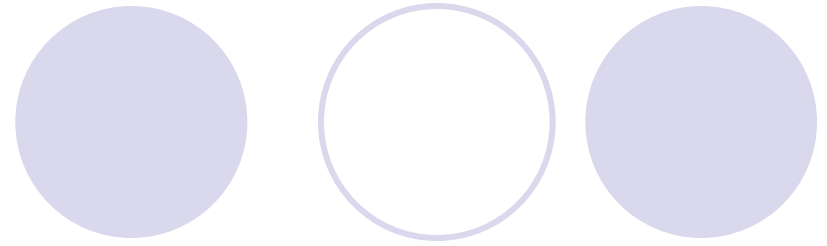


✗ the rule may have secondary effects, not included in its definition.

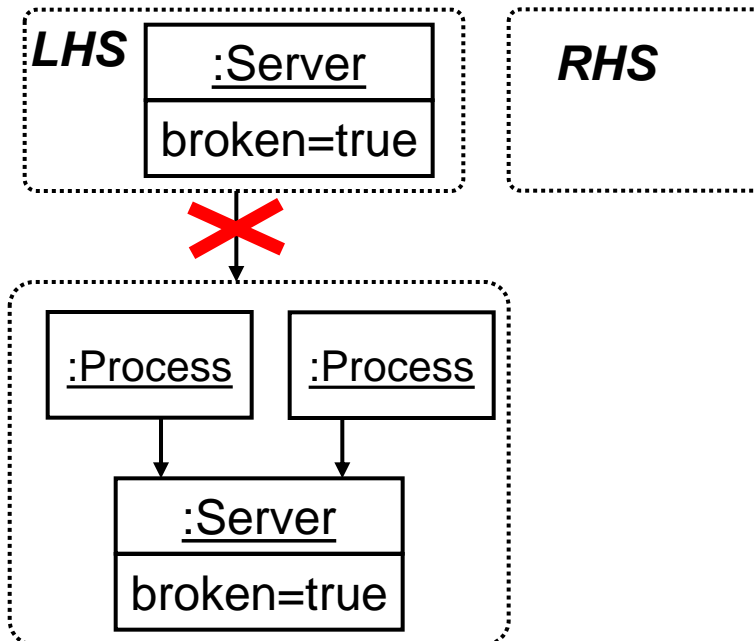
✓ flexibility.

Usually called 'SPO'

Dangling Edges



- We no longer have a graph, as several dangling edges appear.
- Two options:
 1. Forbid rule application.



✗ We need additional rules to explicitly delete the node's context.

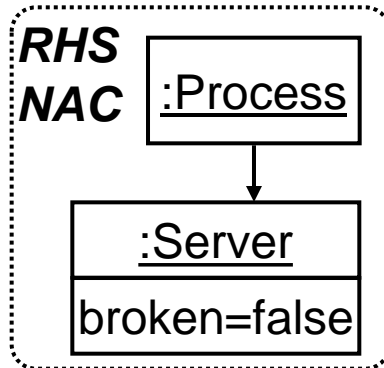
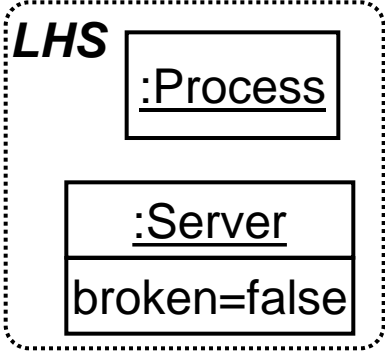
✓ rules easier to analyse.

Usually called 'DPO'

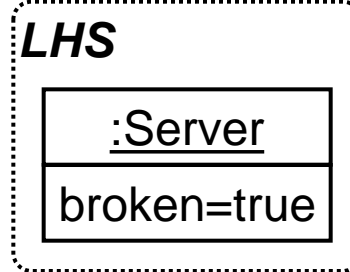
Dangling Edges

- Exercise: Design the missing rule(s) for the DPO case.

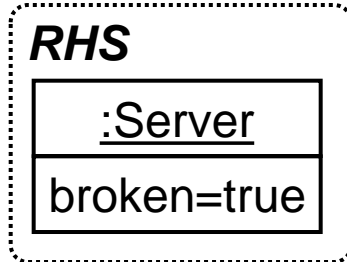
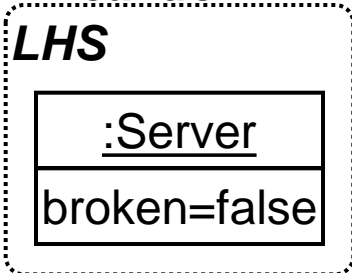
Connect



erase



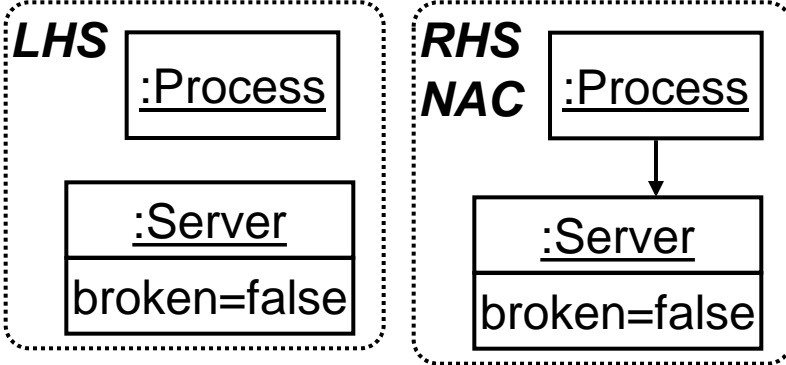
Breakdown



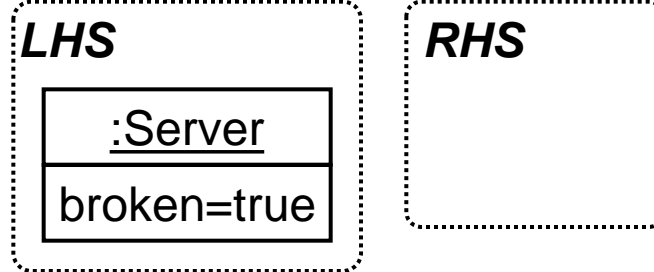
Dangling Edges

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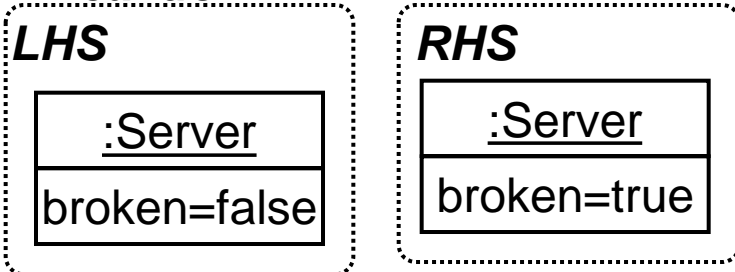
Connect



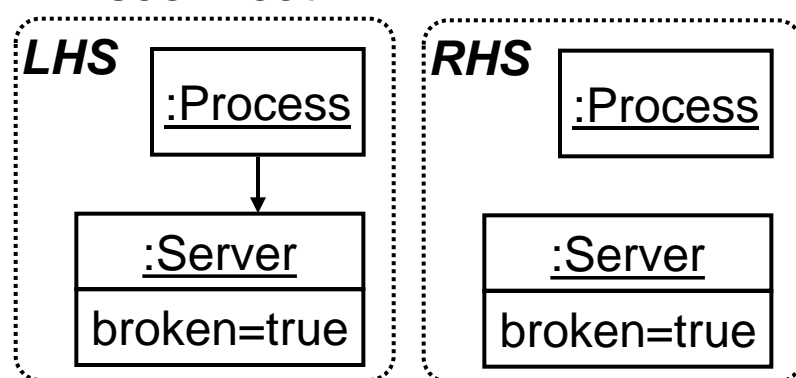
erase



Breakdown

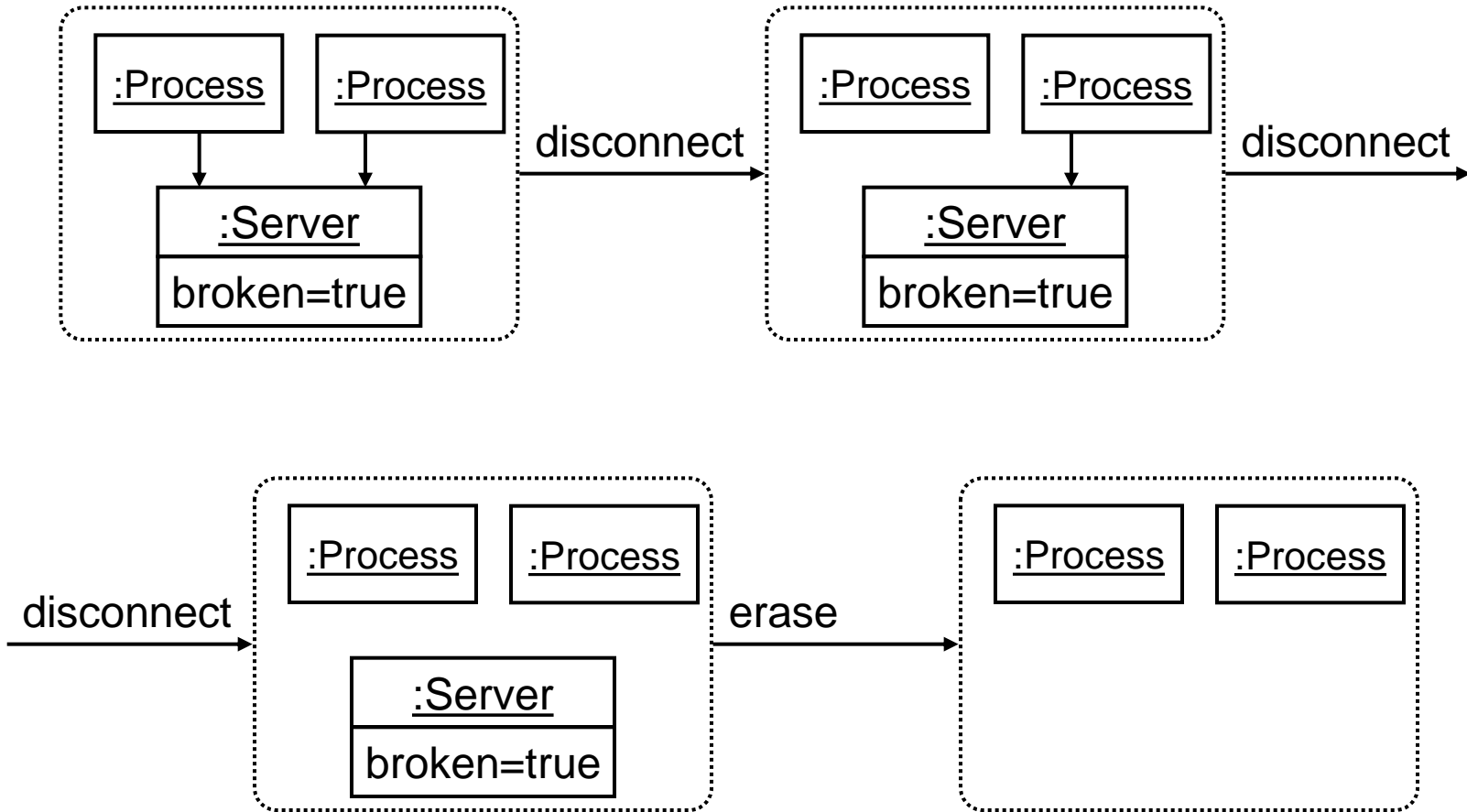


Disconnect



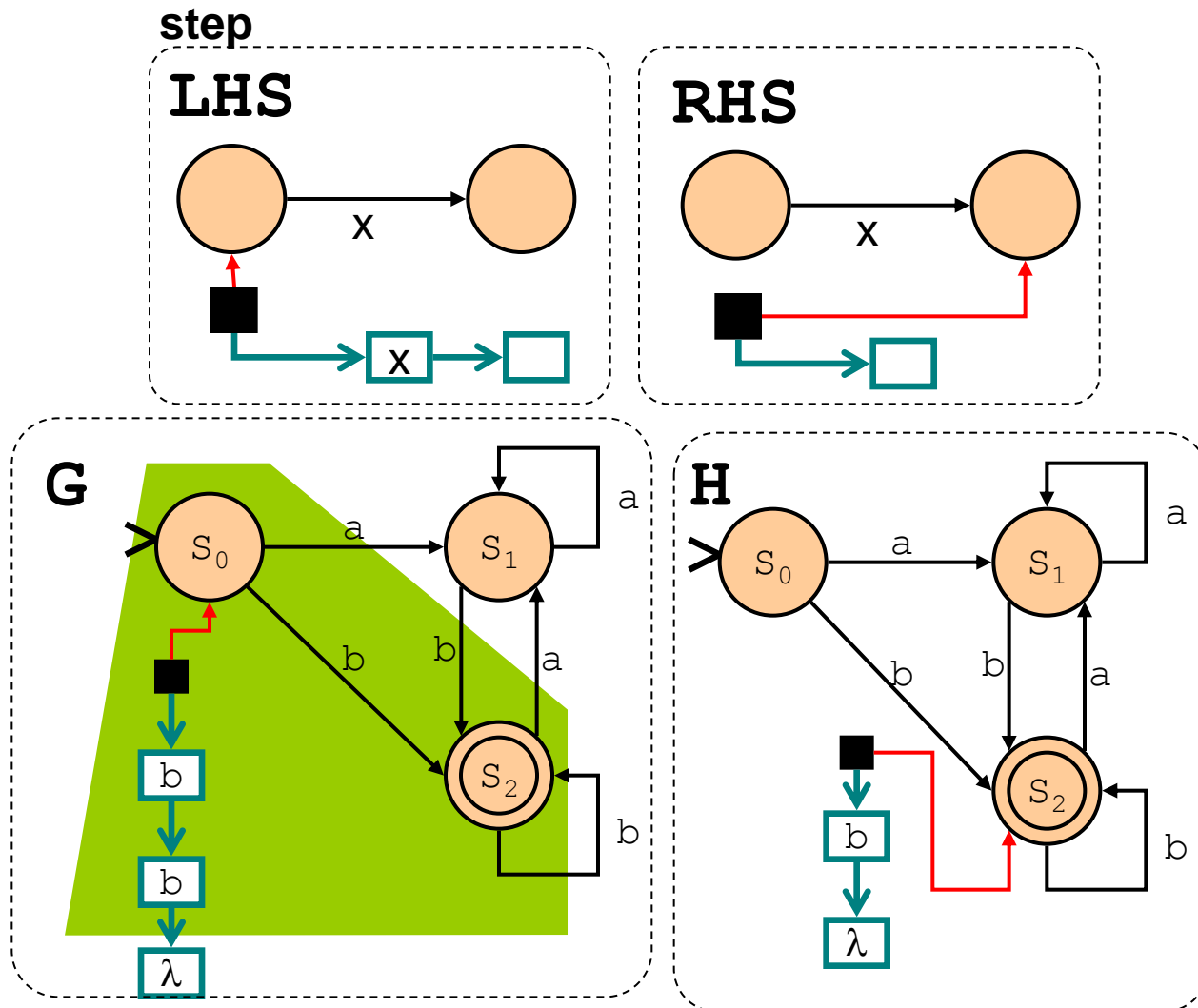
Dangling Edges

- DPO:



Non-Injective Match

Example: Simulating a State Automaton

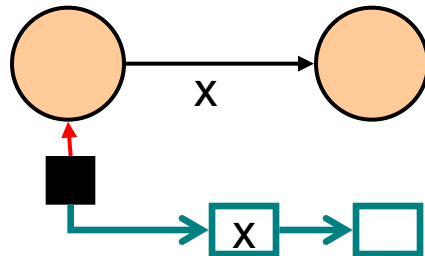


Non-Injective Match

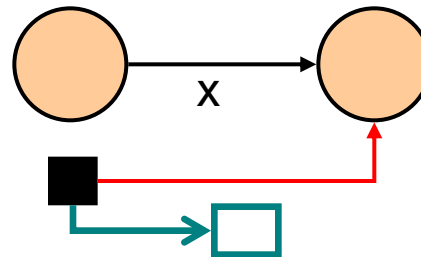
Example: Simulating a State Automaton

step

LHS

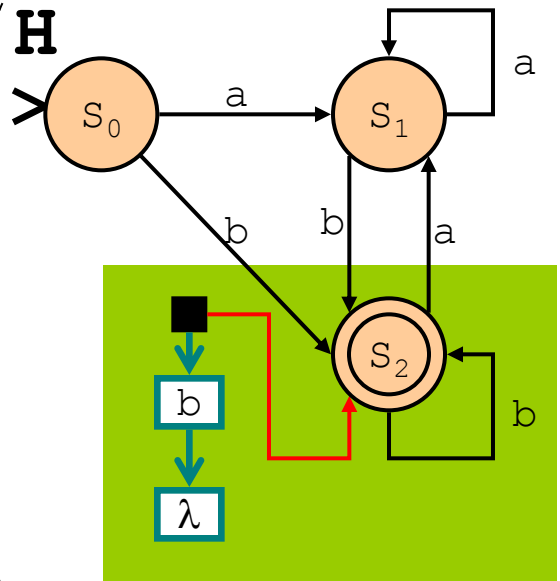


RHS

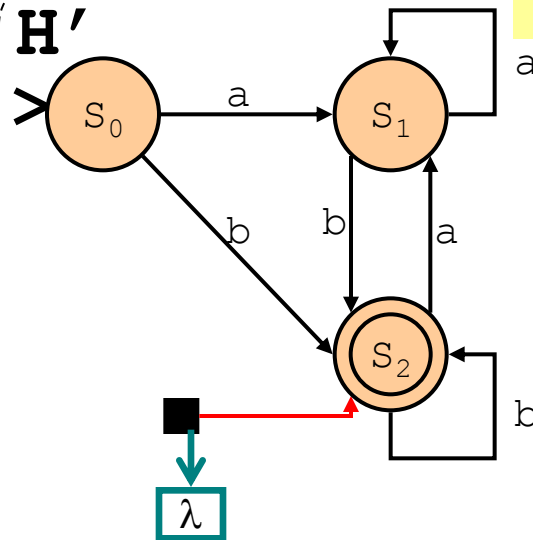


● Non-Injective match: Several elements of the rule can be identified to the same element in the host graph.

H

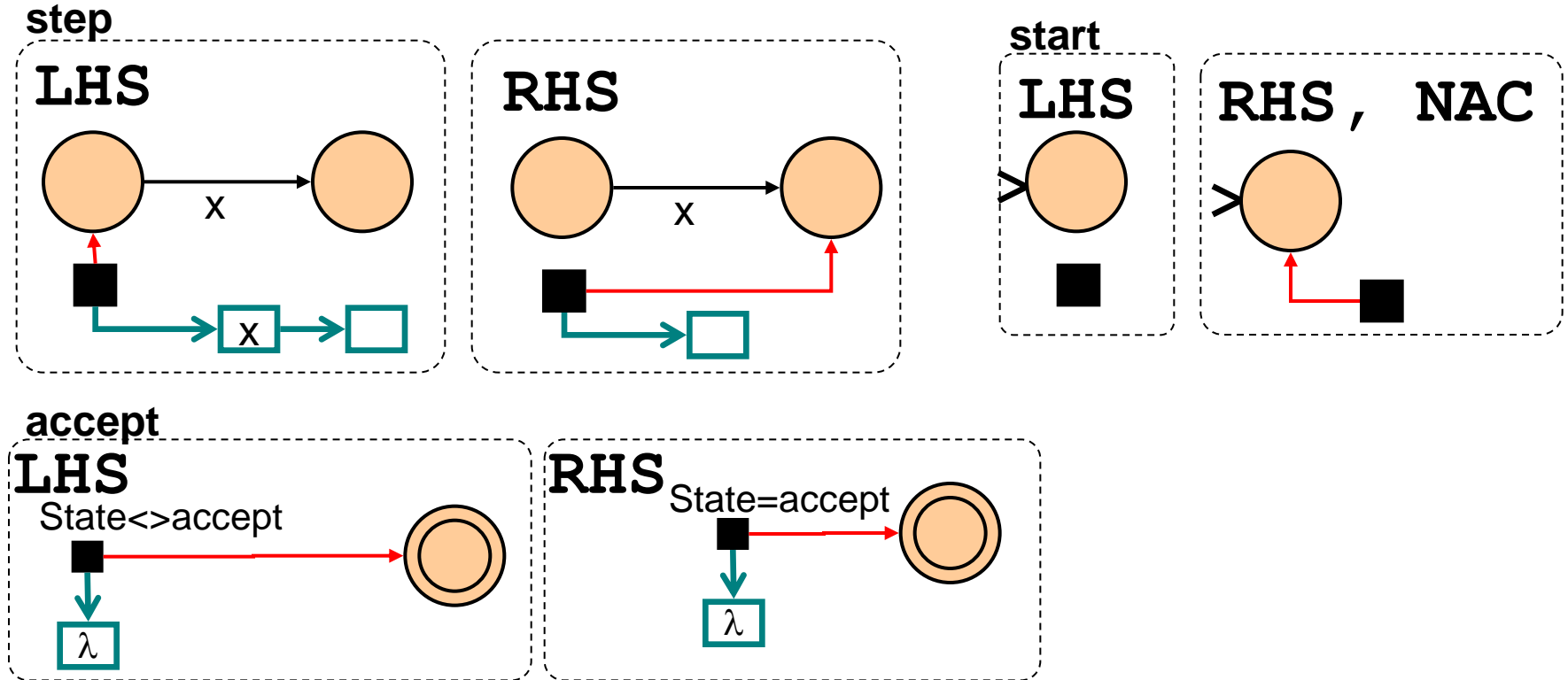


H'



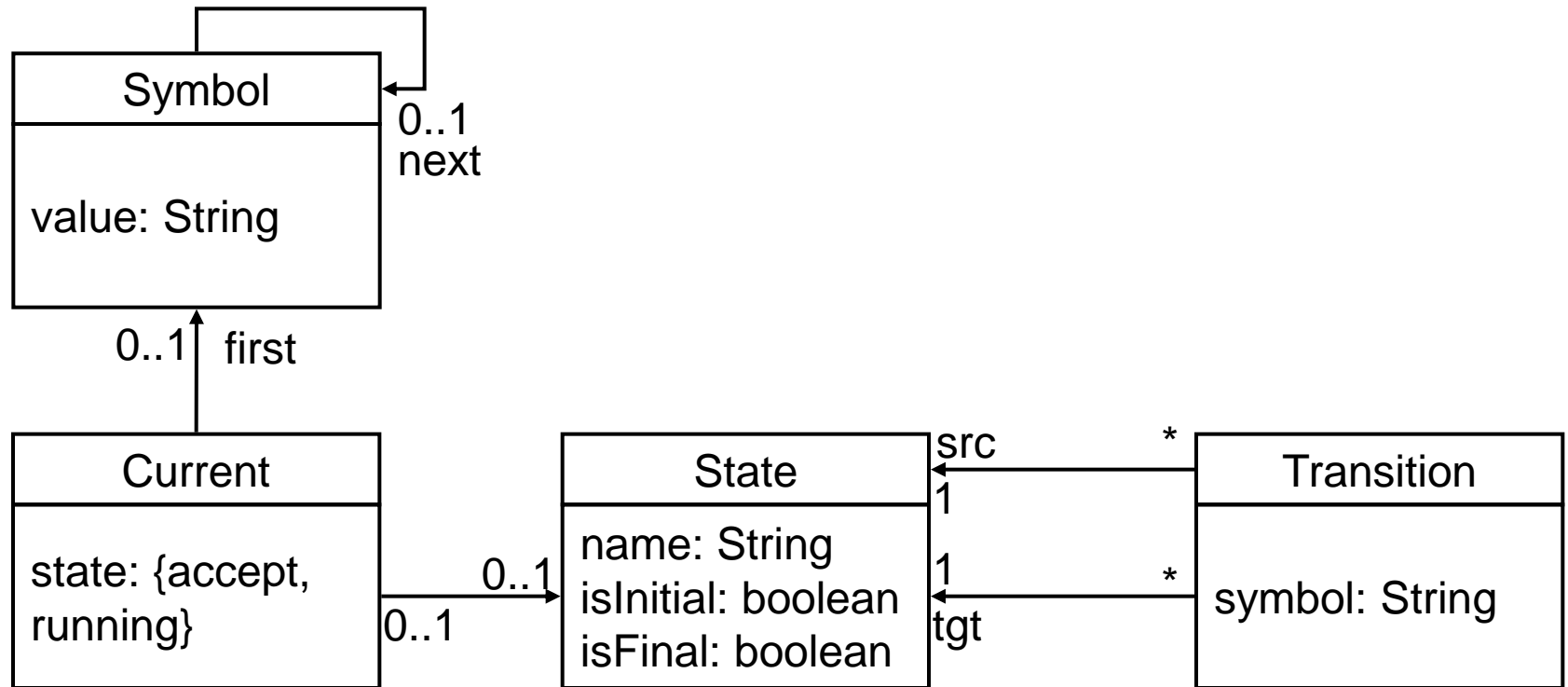
Non-Injective Match

Example: Simulating a State Automaton



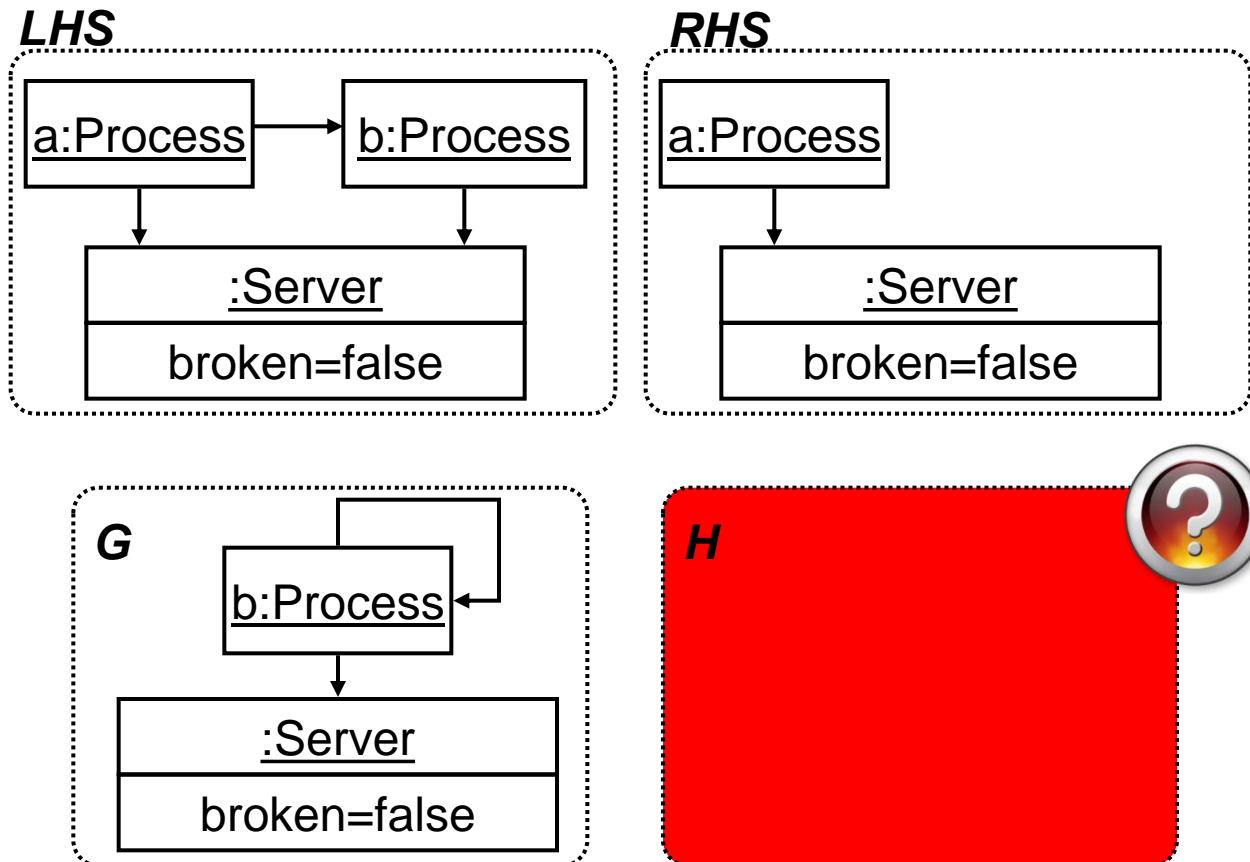
Non-Injective Match

Example: Simulating a State Automaton



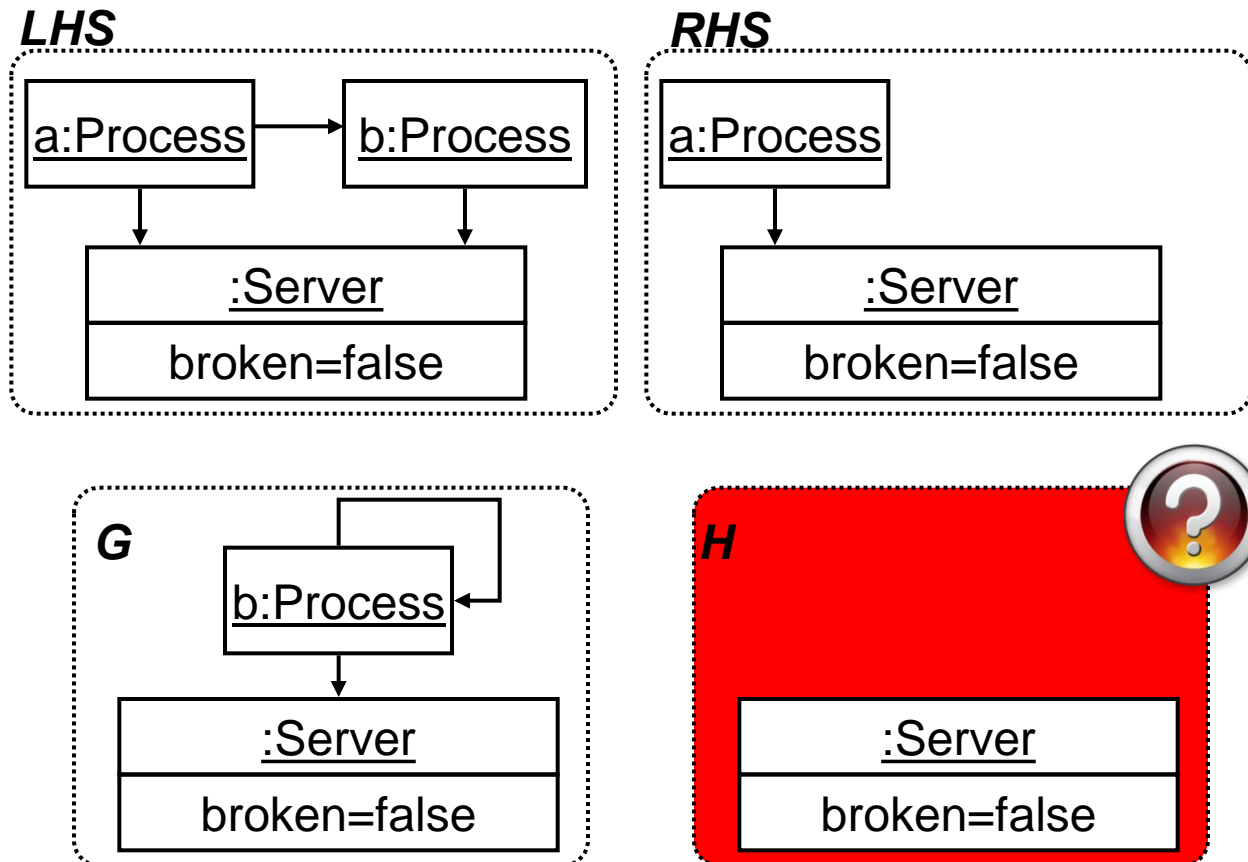
Non-Injctive Matches

- **Identification condition:** What happens if two elements are identified into one, and the rule states that one should be deleted and the other preserved?



Non-Injctive Matches

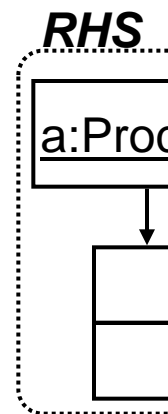
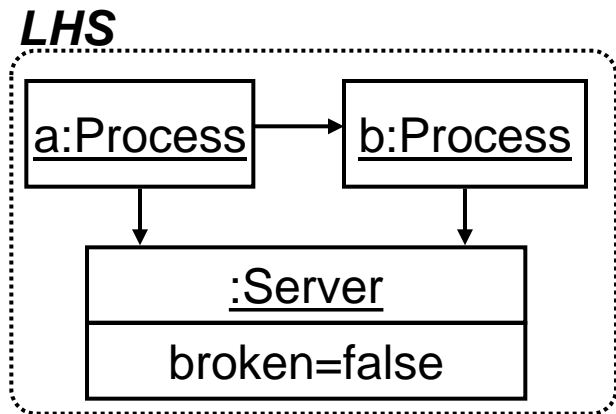
- Option 1 [**SPO**]: All elements are deleted.



Non-Injective Matches

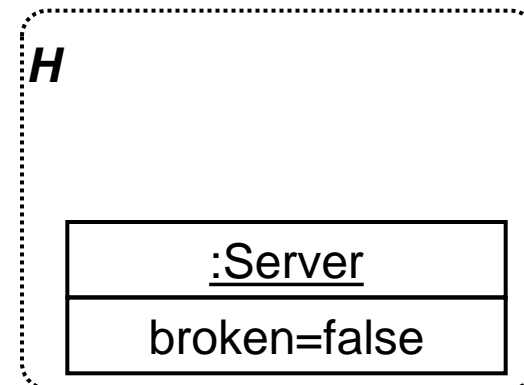
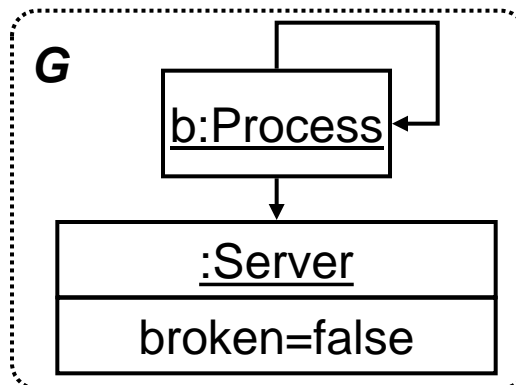
- **Option 1 [SPO]:** All elements are deleted

In SPO dangling edges are deleted.



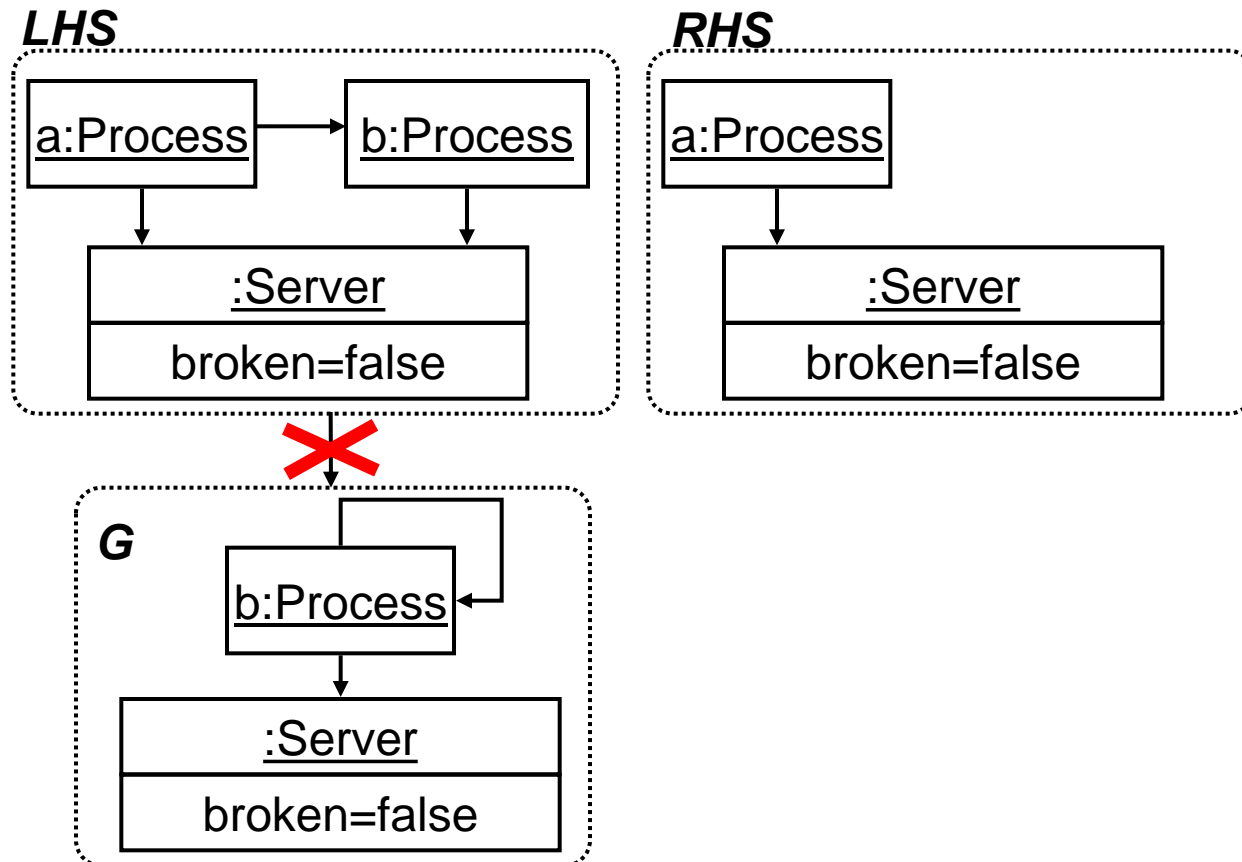
✗ the rule may have secondary effects, not included in its definition.

✓ flexibility.




Non-Injctive Matches

- **Option 2 [DPO]:** the rule cannot be applied at that match.



Index

- Introduction.
- Overview of Graph Transformation.
- **Exercises** 
- Control Languages.
- Meta-modelling and Graph Transformation.
- Applications.
- Tools.
- Conclusions.

Exercise

- Model the dynamics of a simple messaging protocol:
 - The network is made of nodes and routers.
 - Nodes are connected to routers, and routers can be connected to other routers.
 - Nodes can create messages, directed to a given target node.
 - Nodes propagate messages to routers, and these to one of the routers they are connected with, randomly. But if the destination node is connected to a given router, the message is forwarded to the target node.
 - When a node receives a message directed to it, it sends back an ack message to the sender.

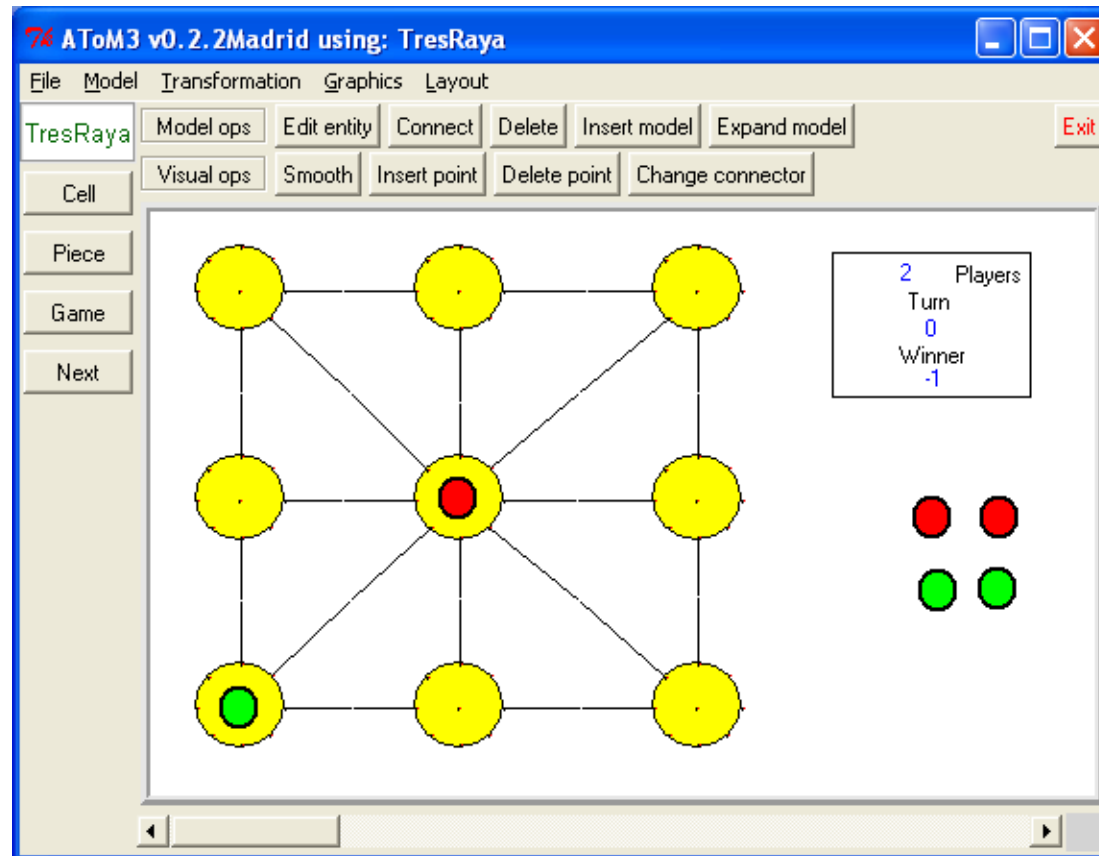
Exercise



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 - Nodes propagate messages to routers, and these to one of the routers they are connected with, randomly. But if the destination node is connected to a given router, the message is forwarded to the target node.
 - When a node receives a message directed to it, it sends back an ack message to the sender.
 - Routers have a maximum message capacity, so that they cannot receive more messages if they reach such capacity.

Exercise

- Model the dynamics of TIC-TAC-TOE.

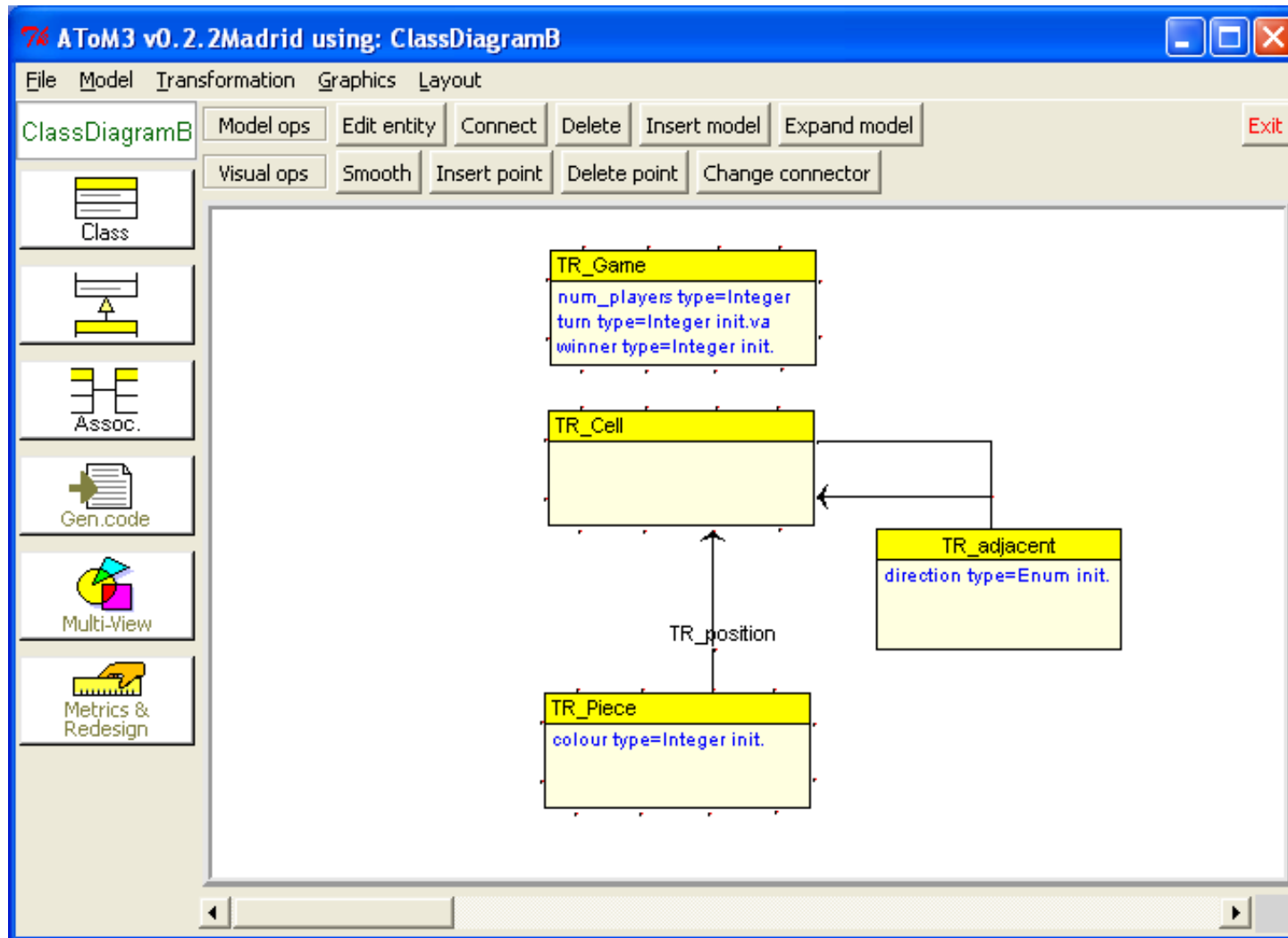


Exercise



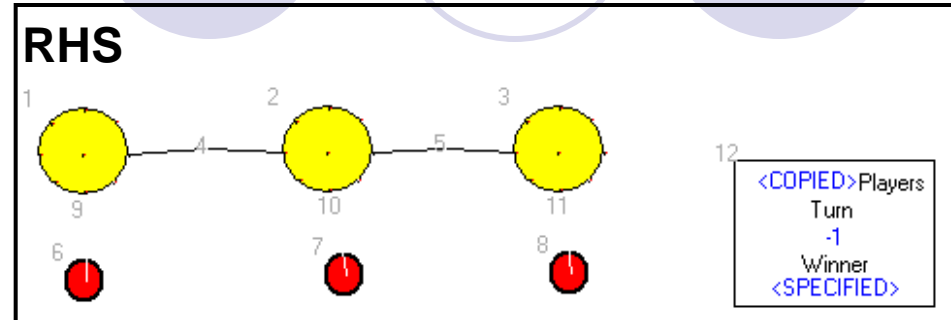
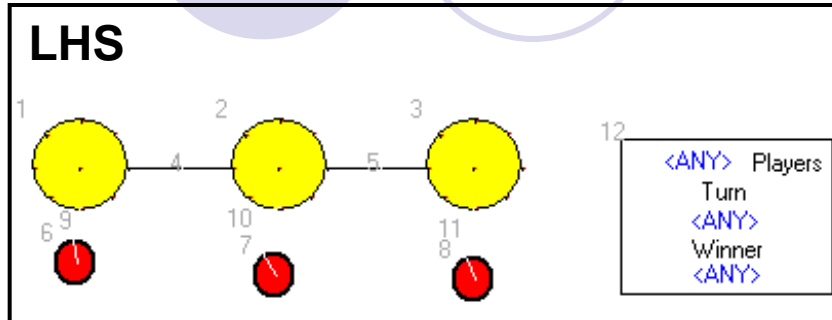
- Step 1: Meta-Model
- Step 2: GT rules.

Exercise



Rules

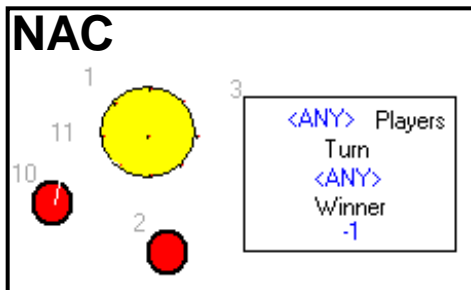
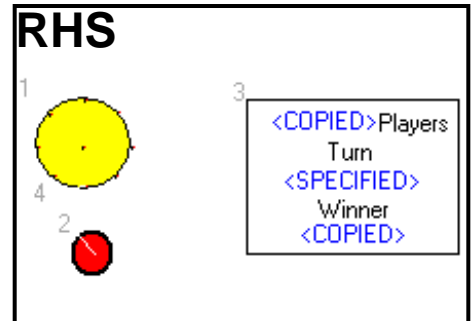
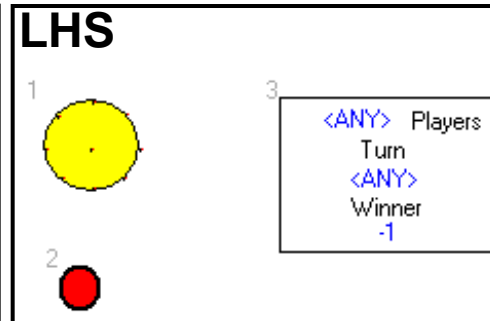
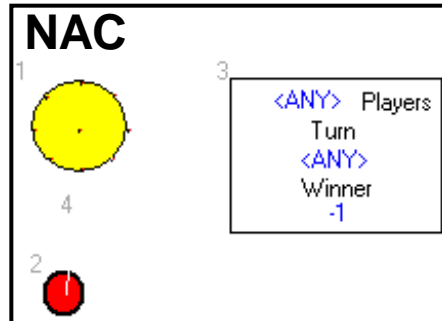
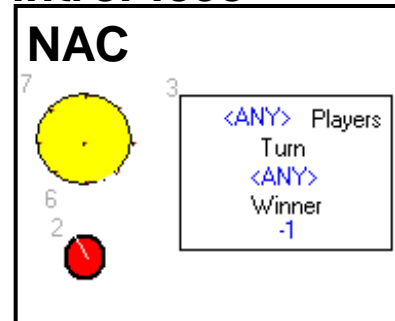
winGame



ATTRIBUTE CONDITIONS

(n(4).direction==n(5).direction) and (n(12).winner== -1) and (n(6).colour==n(7).colour) and (n(7).colour==n(8).colour)

introPiece




ATTRIBUTE CONDITIONS

n(3).turn==n(2).colour

ATTRIBUTE COMPUTATIONS

n(3).turn = (n(3).turn+1)%n(3).num_players

Index

- Introduction.
- Overview of Graph Transformation.
- Exercise: Playing tic-tac-toe.
- **Control Language.** 
- Meta-modelling and Graph Transformation.
- Applications.
- Tools.
- Conclusions.

Control Languages



- Putting all rules into a set is not practical for complex problems.
- A means to control the order of rule execution:
 - Rule priorities.
 - Rule layers.
 - Full-fledged control languages:
 - Transformation Units.
 - Story diagrams (similar to activity diagrams).

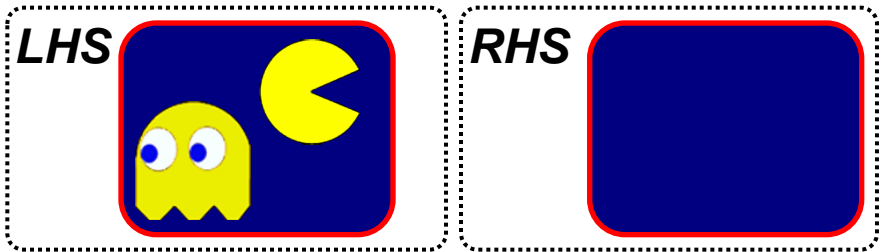
Priorities

- If several rules are applicable, take the one with higher priority.

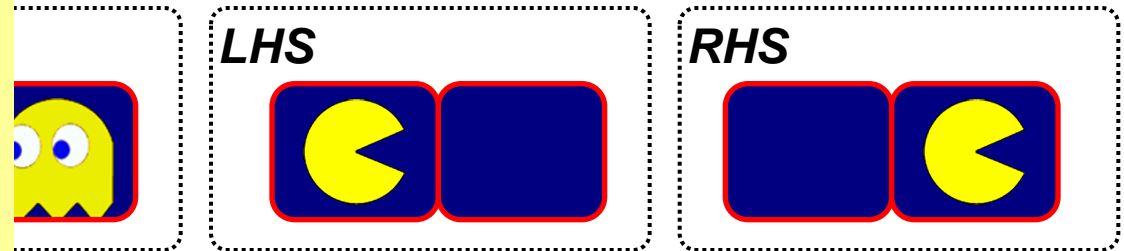
Simplifies the grammar, we do not need NACs to make rules mutually exclusive.

Used in tools like AToM³

Rule: GAME Over, PRIO: 0 (Higher)



Rule: Ghost moves, PRIO (1)



Rule: Ghost moves, PRIO (1)



Layers

- Phasing mechanism, rules are assigned to layers.
 - Start with layer 0. Apply all its rules as long as possible until none is applicable anymore.
 - Follow with next layer, until all layers are visited.

Useful to express pre-processings and processes to be done in sequential order.

Used in tools like AGG

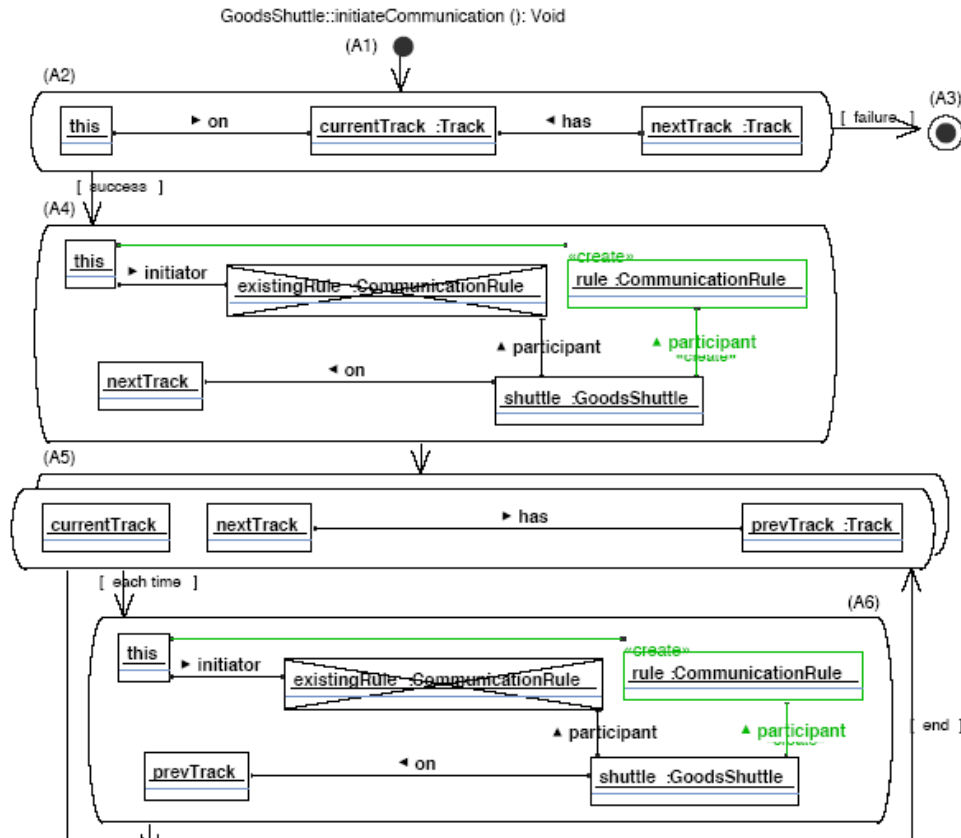
Transformation Units.

- Basic control structures (sequencing, as long as possible, etc).
 - Similar to regular expressions, where symbols are rule names.
 - It is also possible to control the conditions for graphs to be terminal.
 - Strong theoretical basis.
- Used in tools like Henshin
 - Independent unit: choose an applicable subunit at random
 - Loop unit: *
 - Priority unit: executes the subunit with highest priority
 - ...

Hans-Jörg Kreowski, Sabine Kuske, Grzegorz Rozenberg. 2008. “Graph Transformation Units – An Overview”. In *Concurrency, Graphs and Models*, pp.: 57–75, 2008, vol 5065 of LNCS (Springer)

Story diagrams.

- Similar to activity diagrams, each state has associated a rule.



Complex execution flow.
Conditional branchings.

Used in tools like Fujaba
(<http://www.fujaba.de/>)

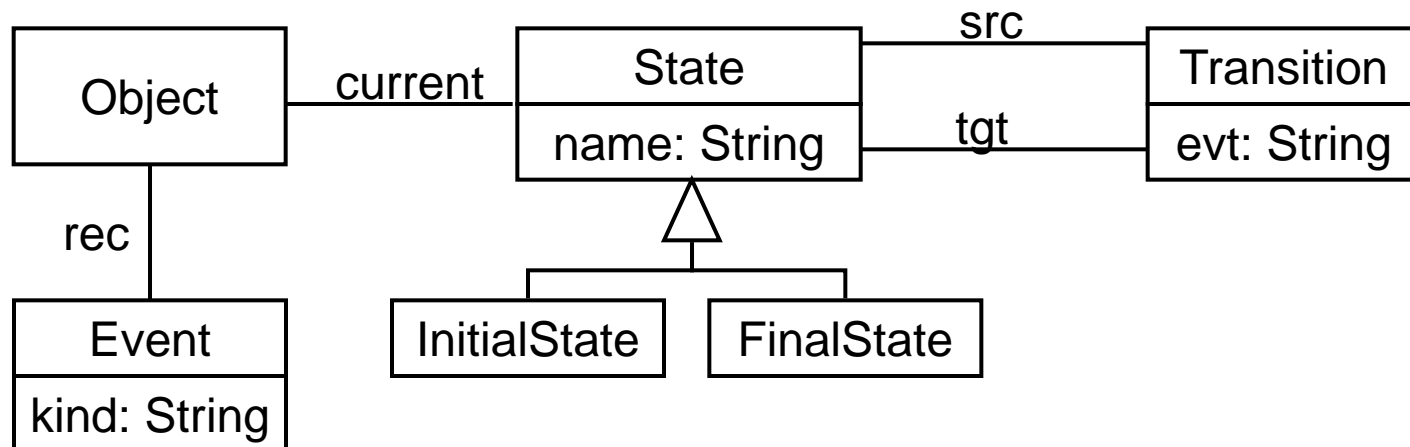
Index

- Introduction.
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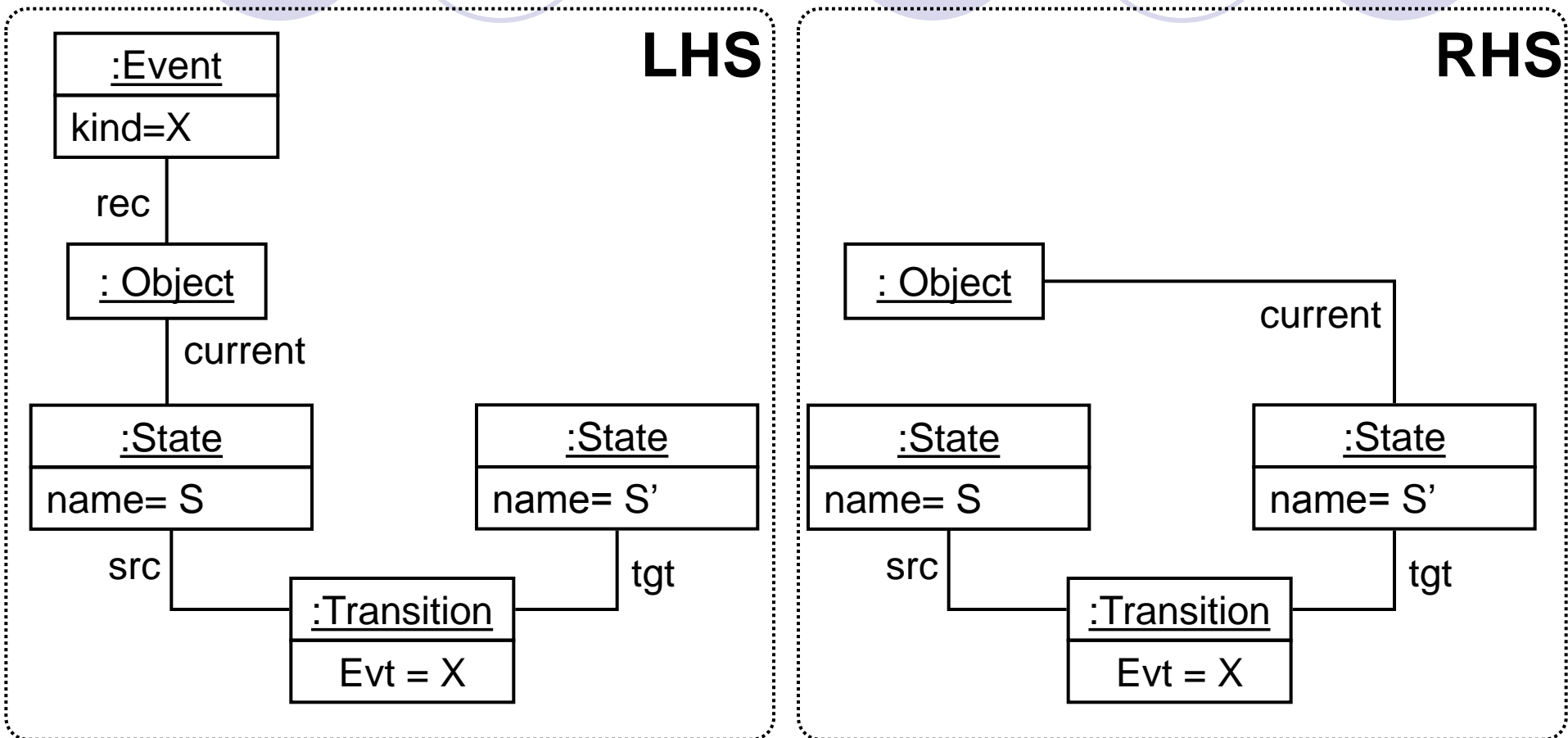


Meta-Modelling and GT

- Use the inheritance hierarchy of the meta-model in rules.
- Objects in rules can be matched by objects of any subclass in host graphs.
- More compact rules.

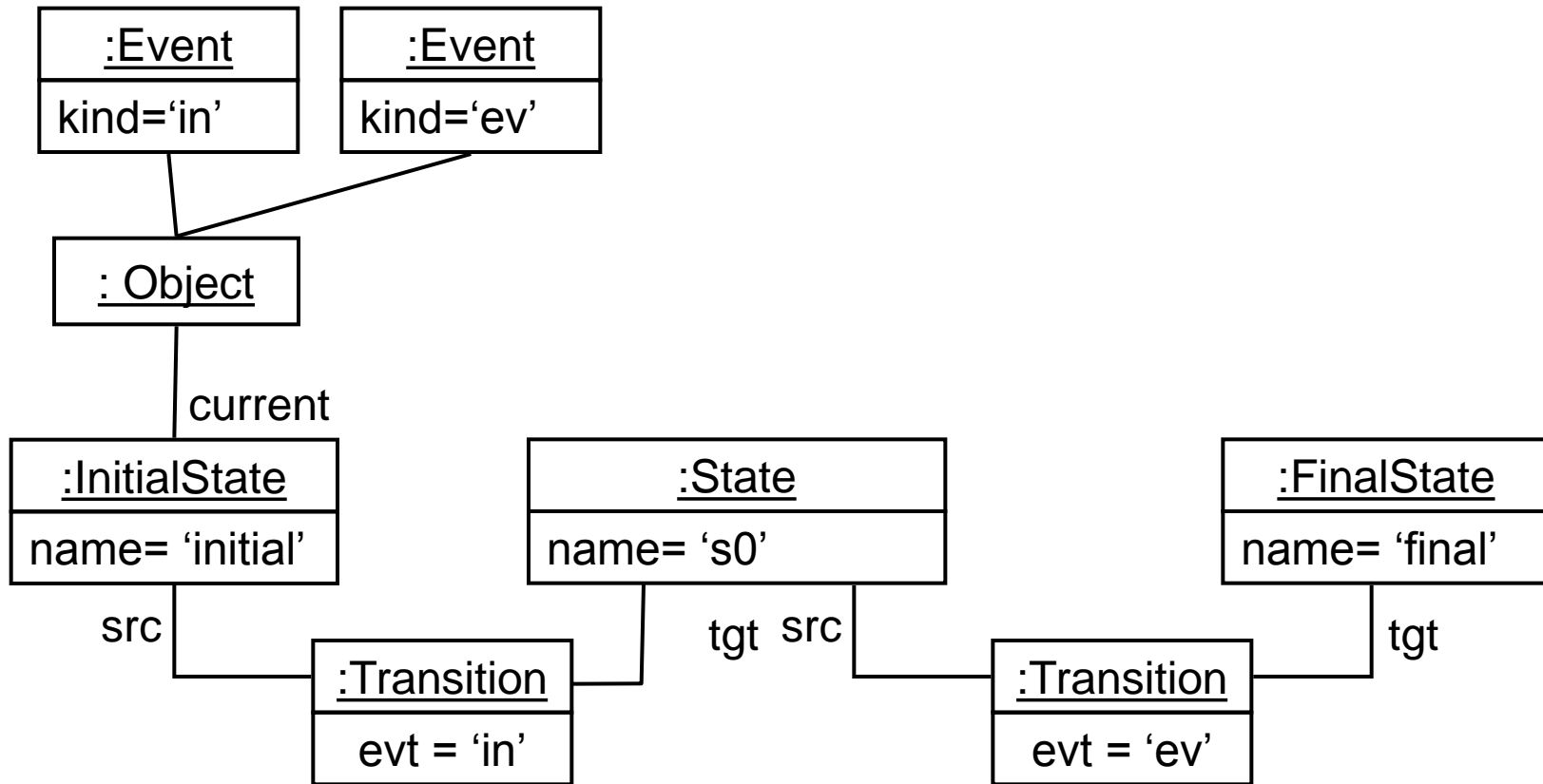


Meta-Modelling and GT

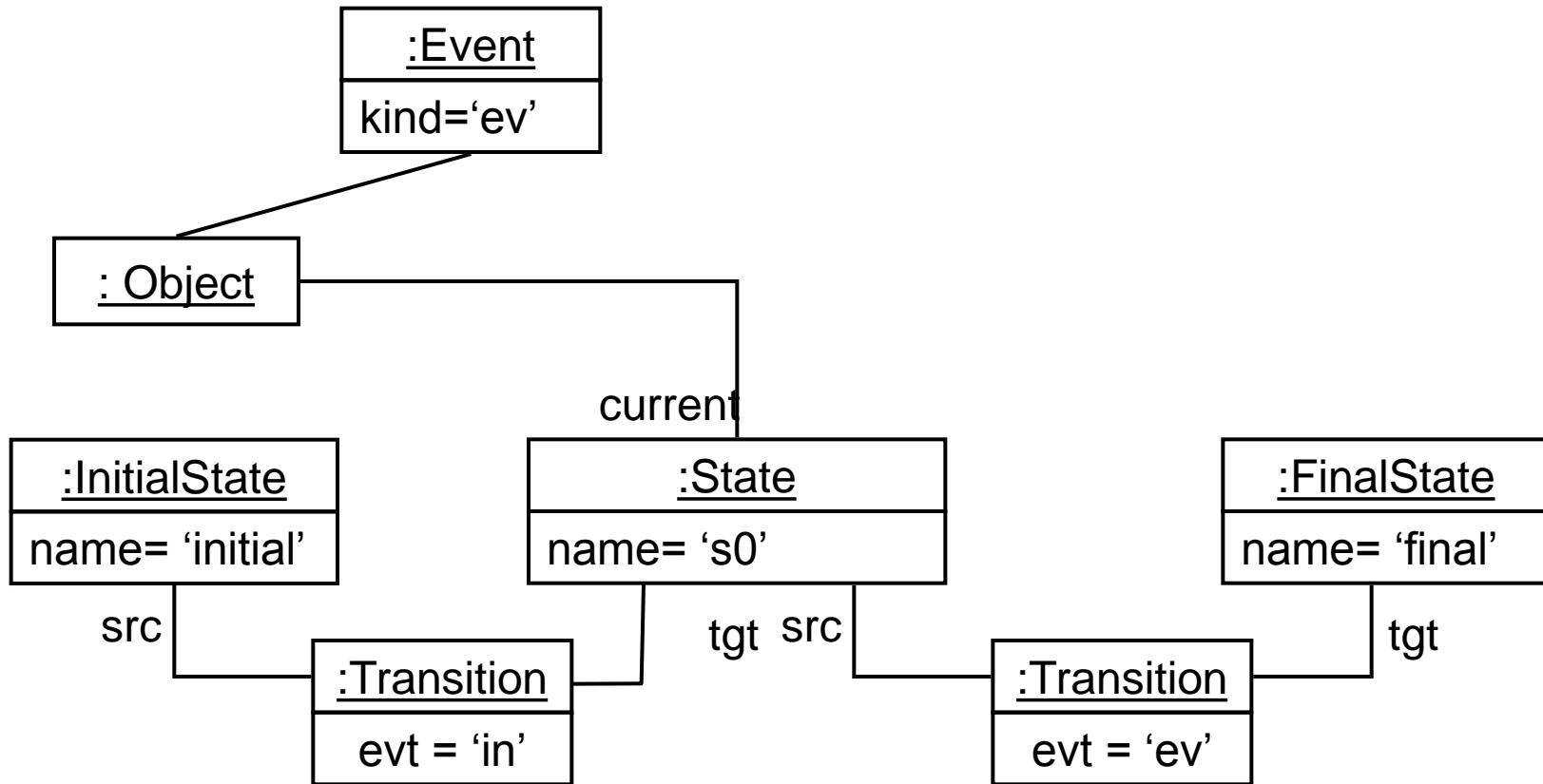


- The rule is equivalent to a set of 9 rules, by substituting the type of both State objects by all subtypes of State.

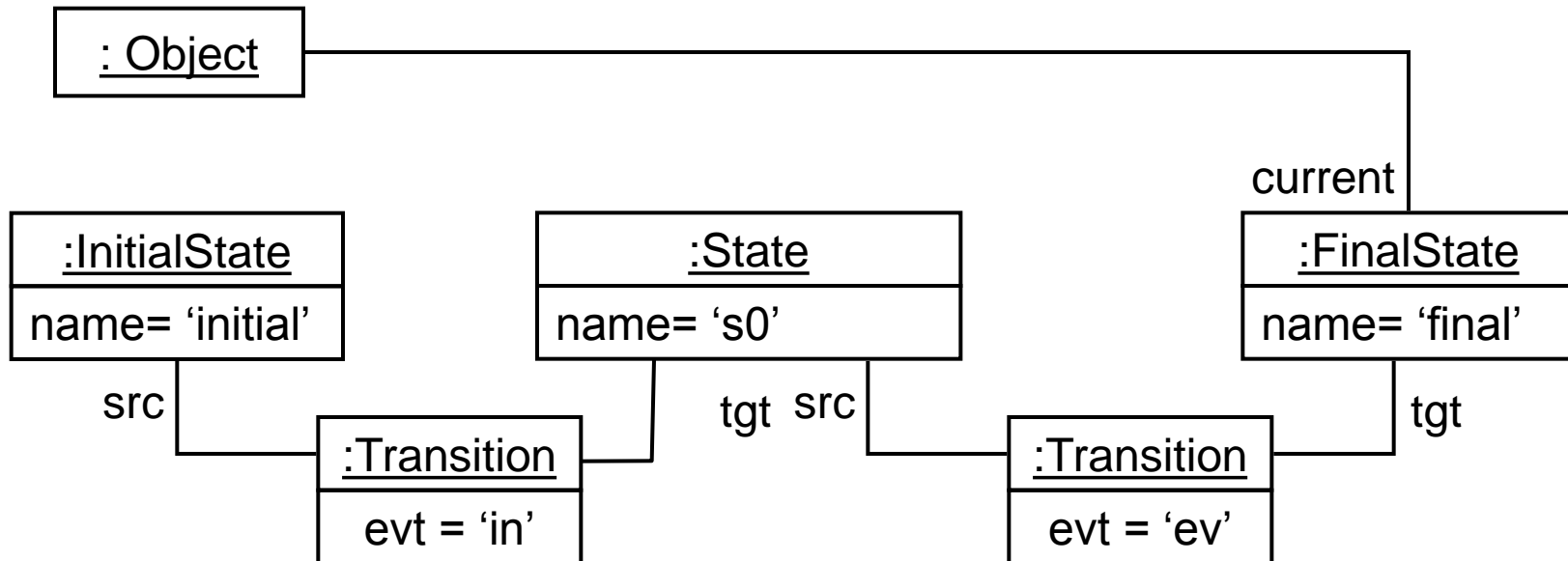
Meta-Modelling and GT




Meta-Modelling and GT



Meta-Modelling and GT



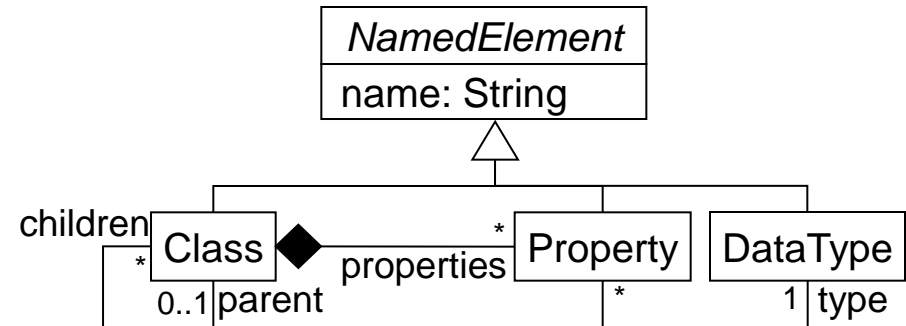
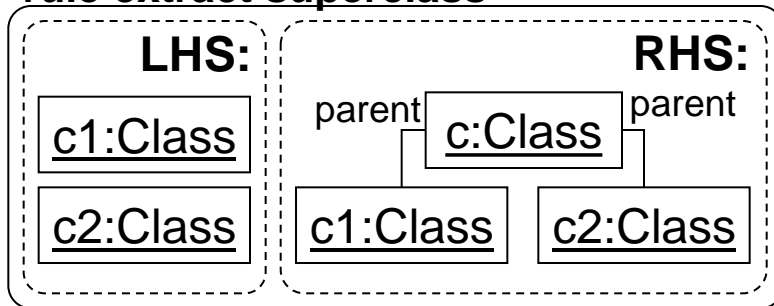
Index

- Introduction.
- Overview of Graph Transformation.
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- Control Language.
- **Applications.** 
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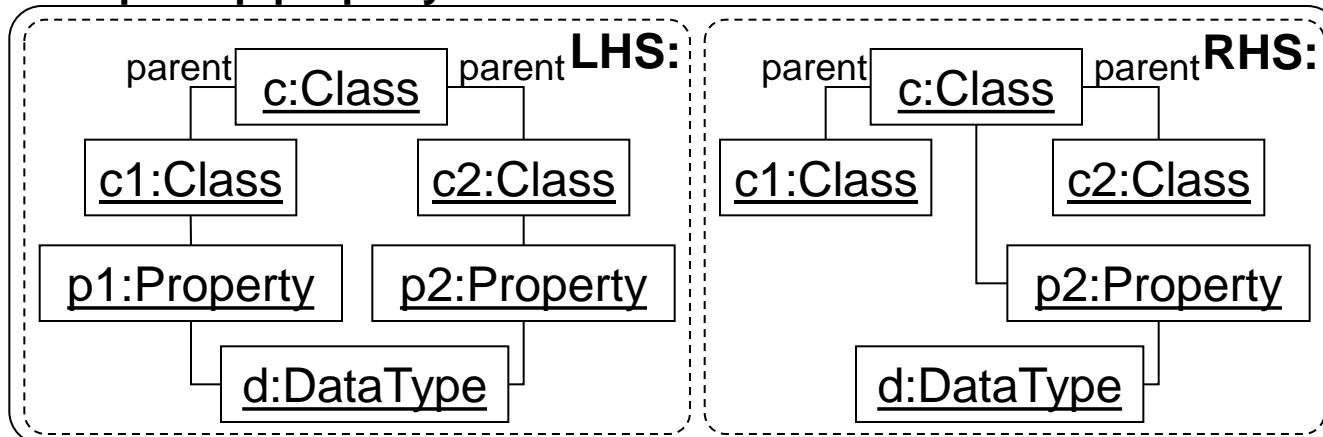
Applications

Refactoring of OO models.

rule extract superclass



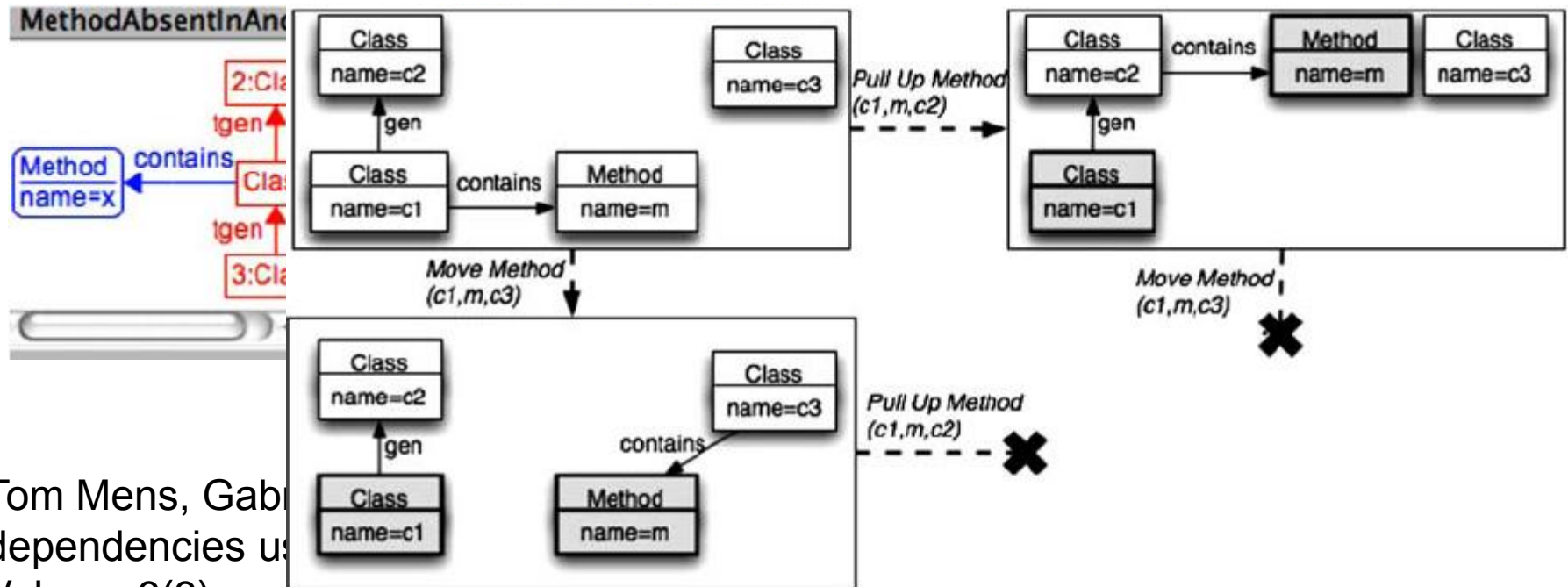
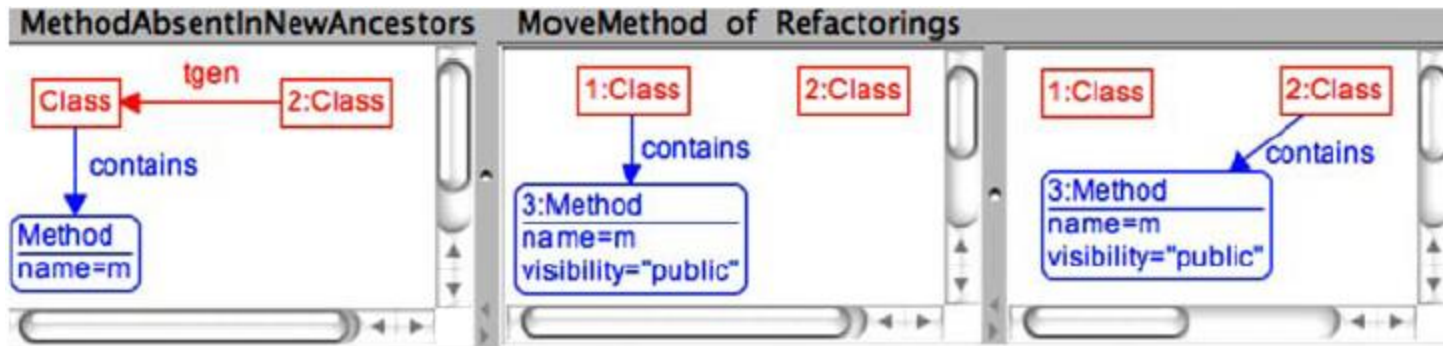
rule pull-up property



Application condition: p1.name=p2.name

Applications

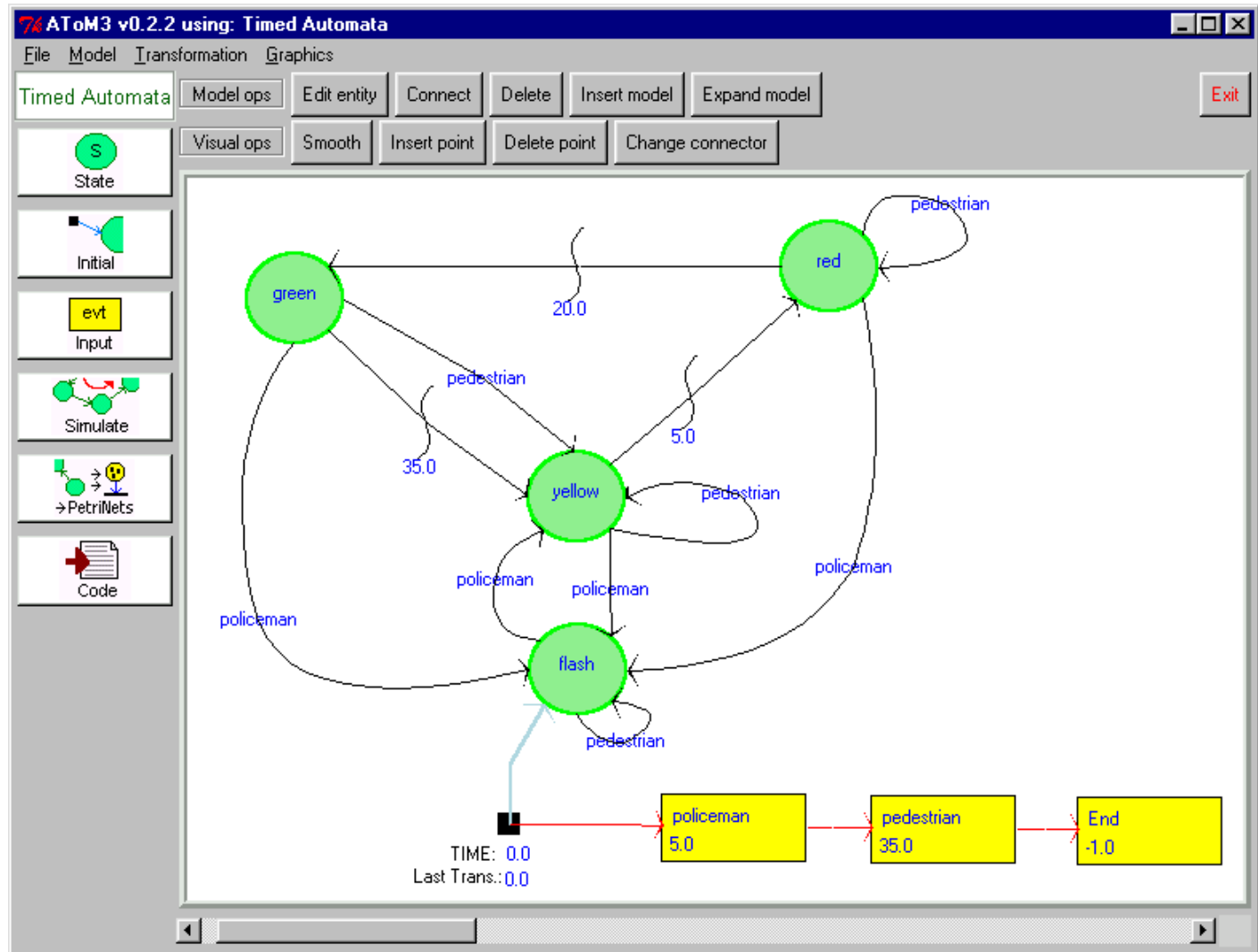
Analysis of Refactoring.



Tom Mens, Gabriel
dependencies us
Volume 6(3), pp. 203—208, Springer.

Simulation

AToM³

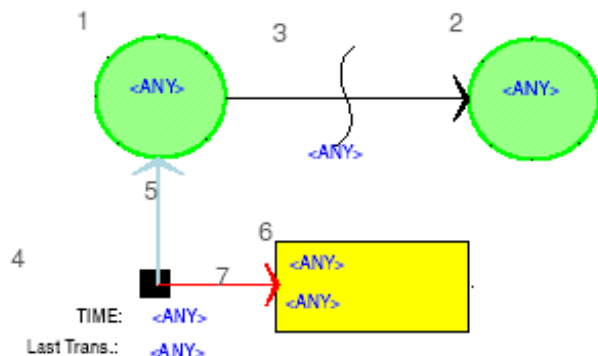


Simulation

AToM³

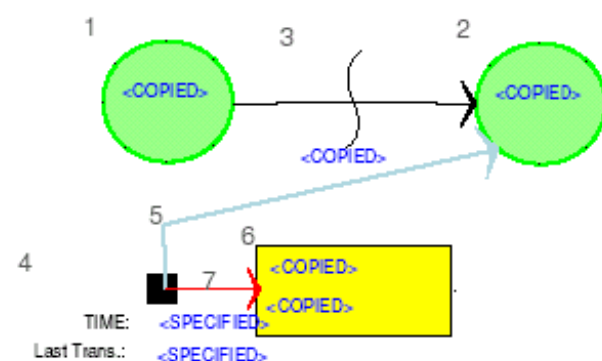
TA_MoveTimeTrans

Priority 1



CONDITION: `node(6).ArrivalTime > node(4).Time + node(3).Time`

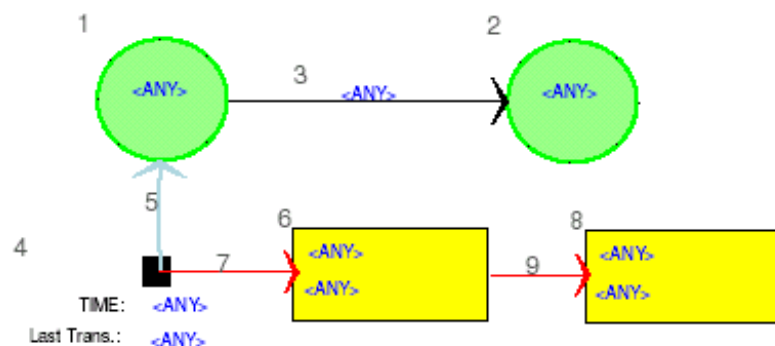
⋮=



TIME += `node(3).Time`
Last Trans = TIME

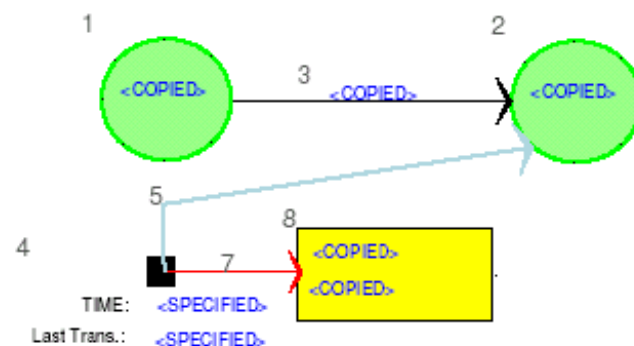
TA_MoveTrans

Priority 3



CONDITION: `node(6).Value == node(3).Value`

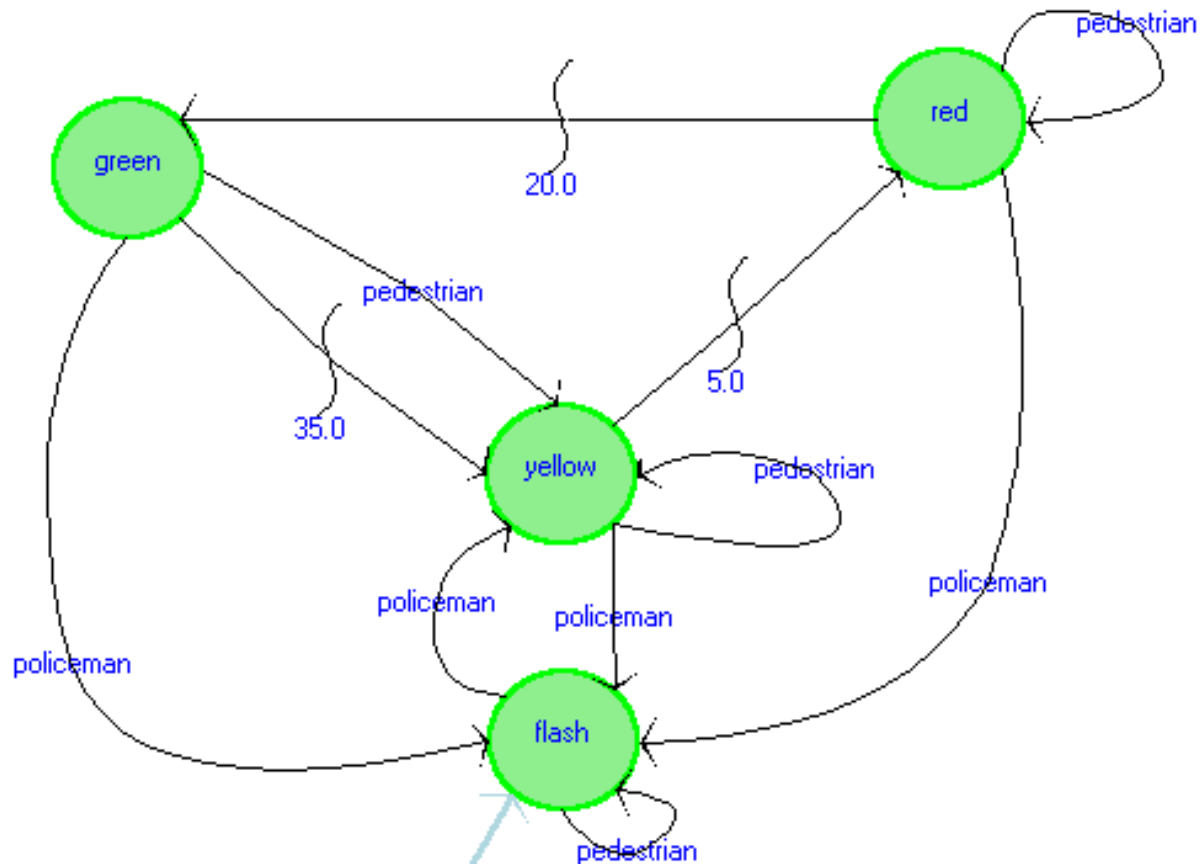
⋮=



TIME = `node(6).ArrivalTime`
Last Trans = TIME

Simulation

AToM³



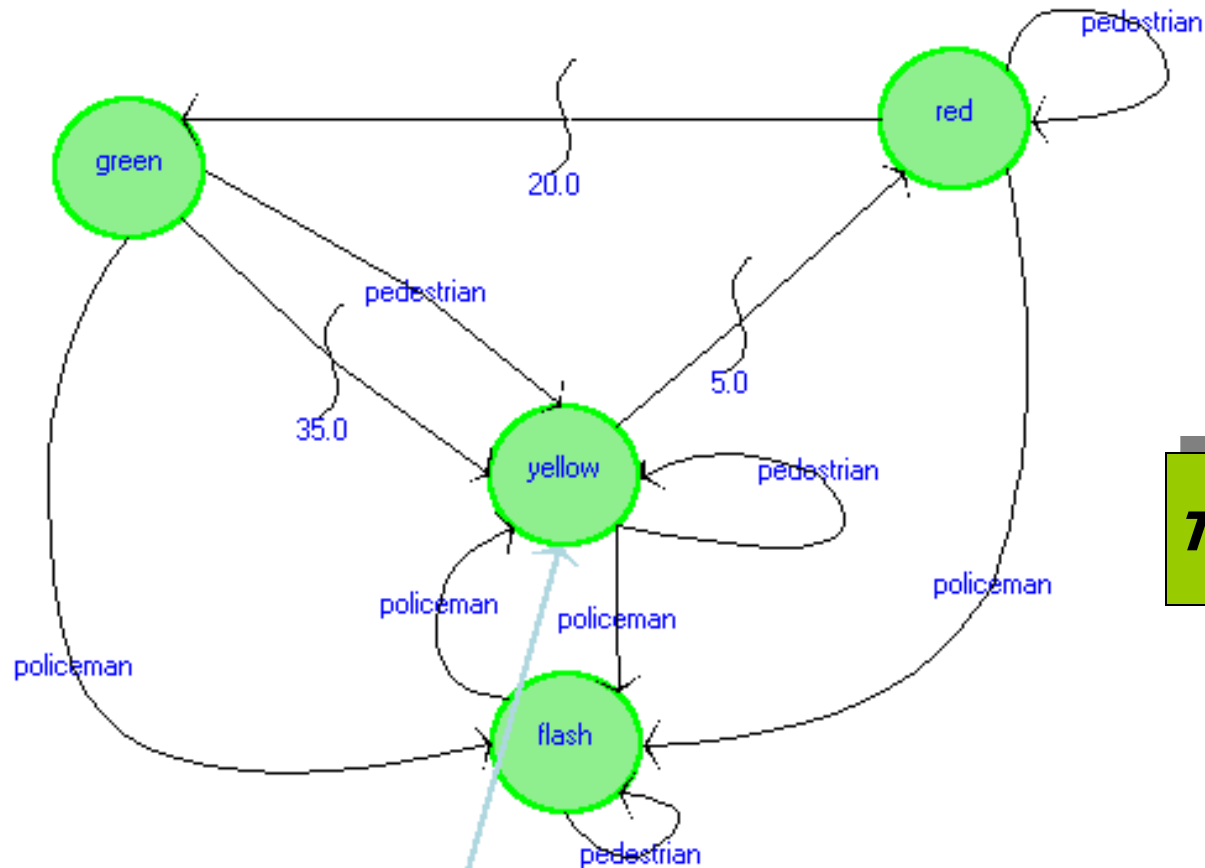
TA_MoveTrans

TIME: 0.0
Last Trans.: 0.0



Simulation

AToM³



TA_MoveTimeTrans

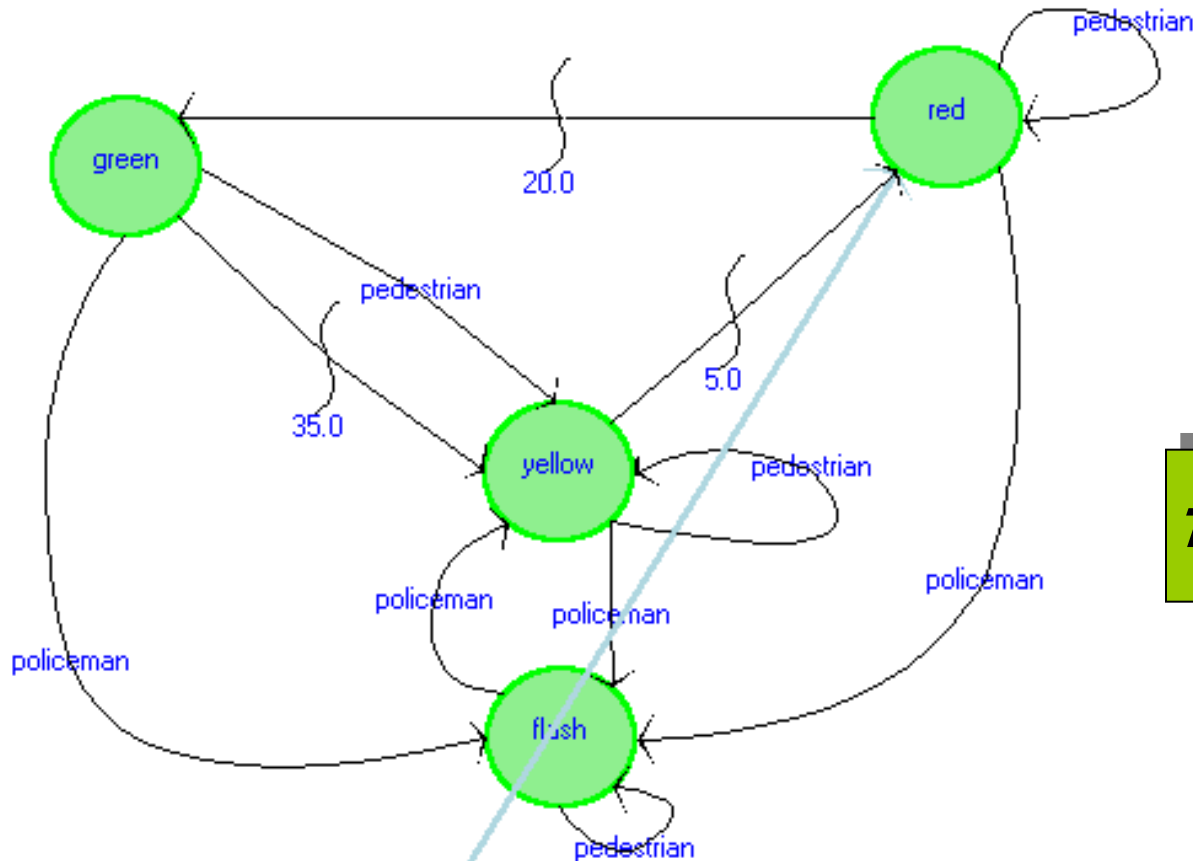
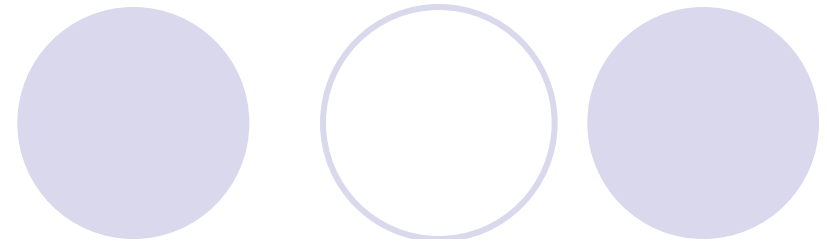
TIME: 5.0
Last Trans.: 5.0

pedestrian
35.0

End
-1.0

Simulation

AToM³



TA_MoveTimeTrans

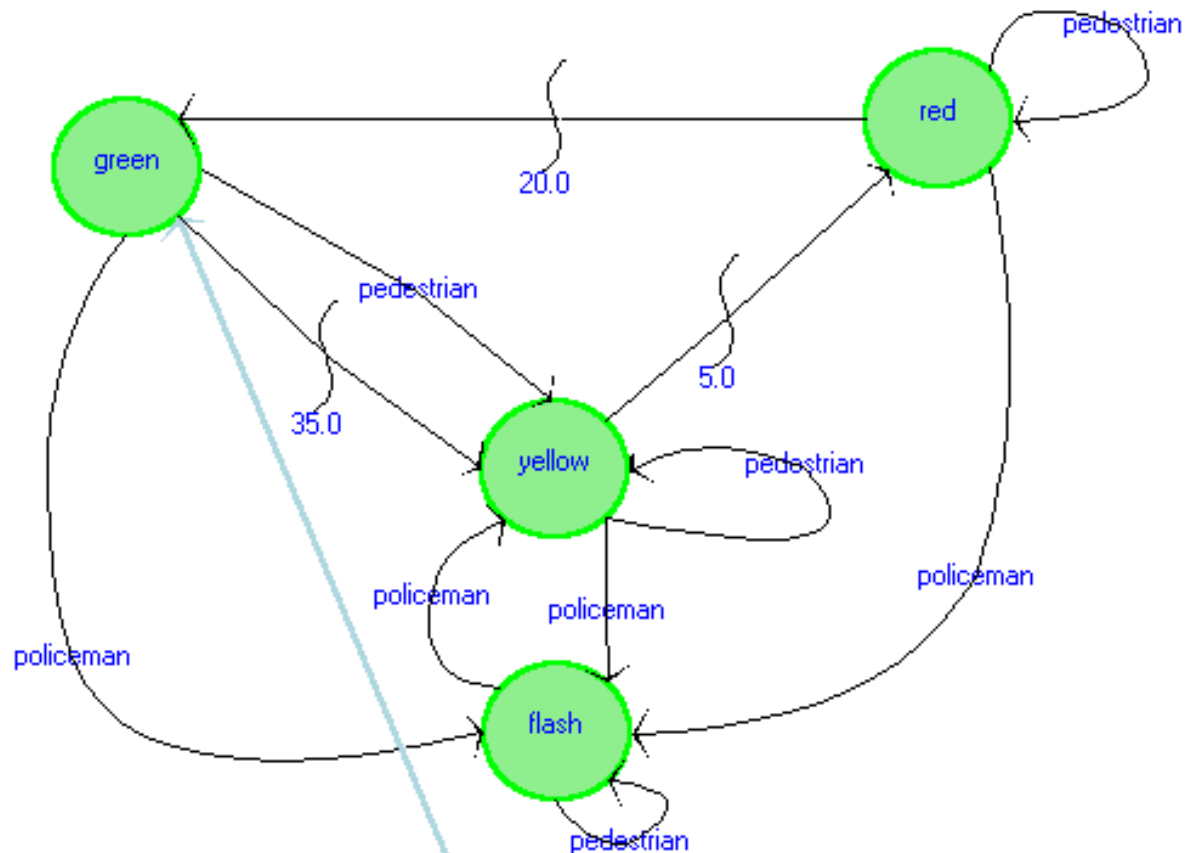
TIME: 10.0
Last Trans.: 10.0

pedestrian
35.0

End
-1.0

Simulation

AToM³



TIME: 30.0
Last Trans.: 30.0

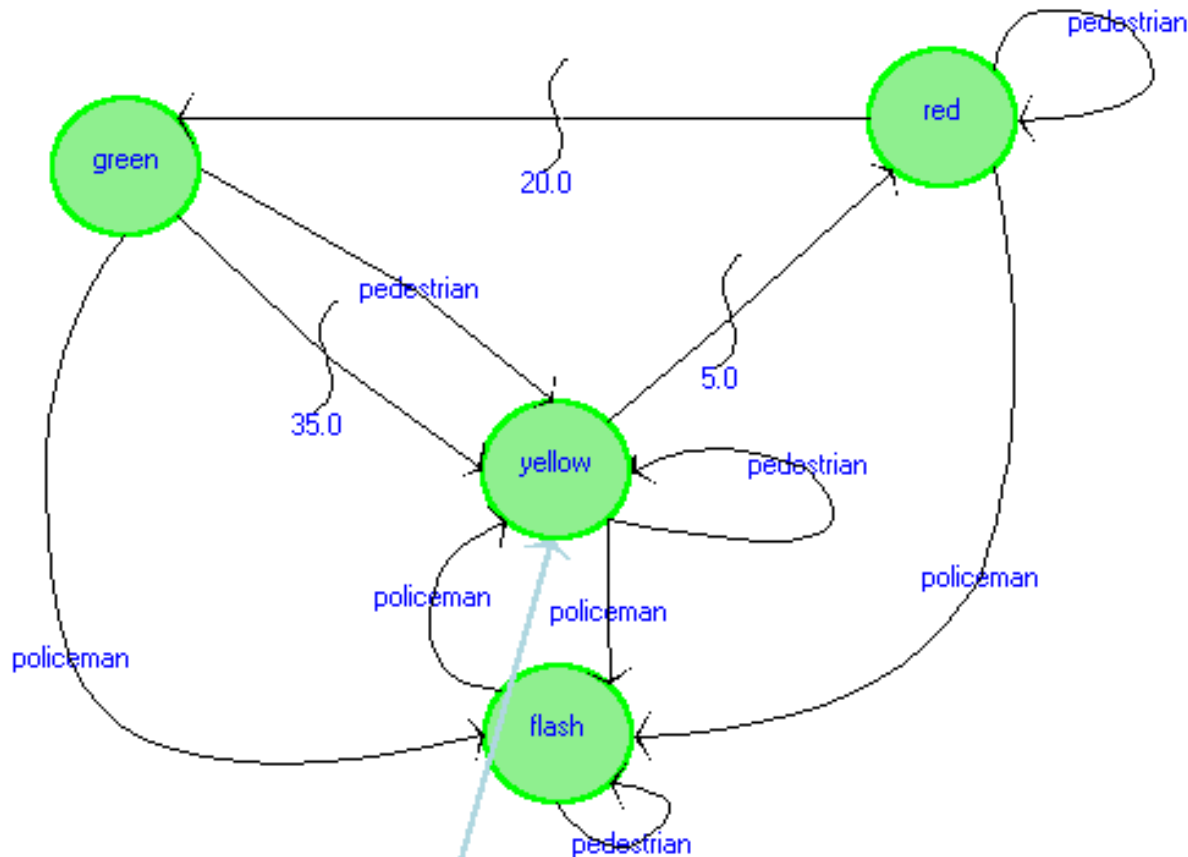
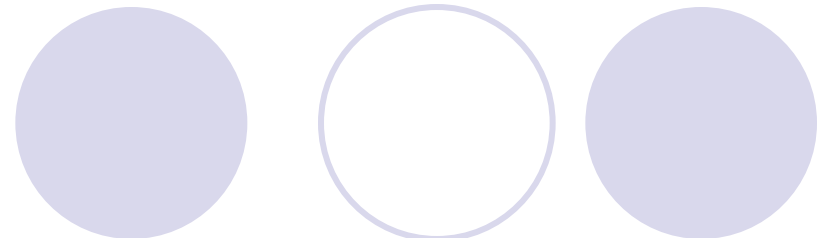
pedestrian
35.0

End
-1.0

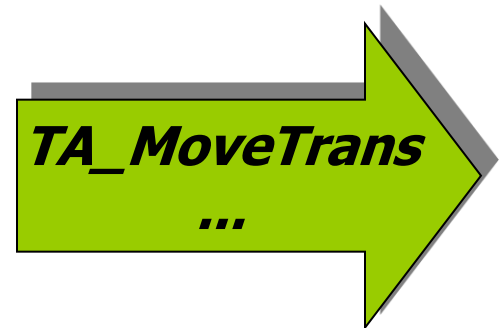
TA_MoveTrans

Simulation

AToM³



TIME: 35.0
Last Trans.: 35.0

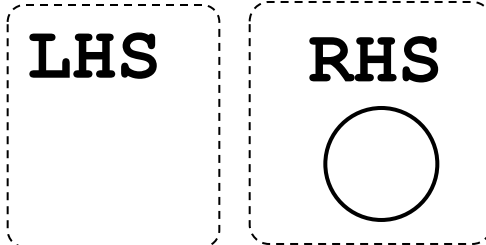


End
-1.0

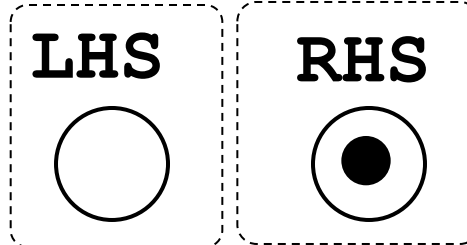
Generation of Languages

- Grammars can also be used as an alternative to meta-models to describe modelling languages.
 - Similar to programming language definition through Chomsky grammars.
- Example: Defining the language of Petri nets.

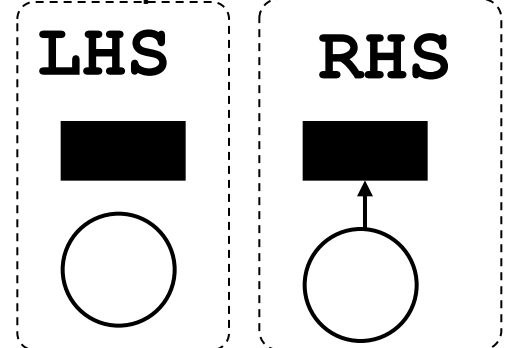
createPlace



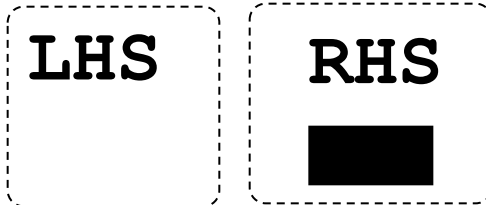
createToken



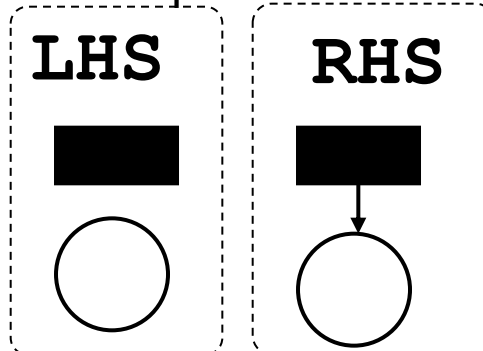
addInput



createTransition

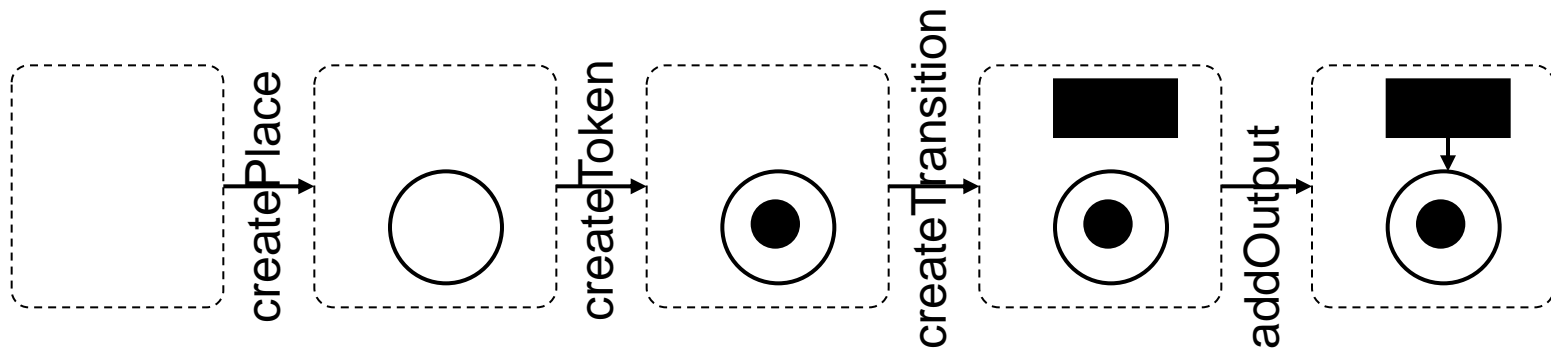


addOutput

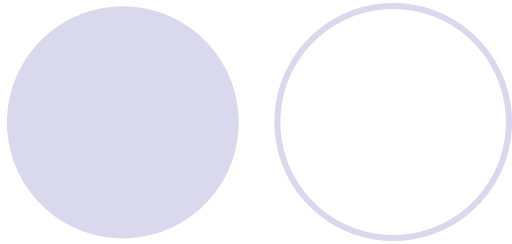


Generation of Languages

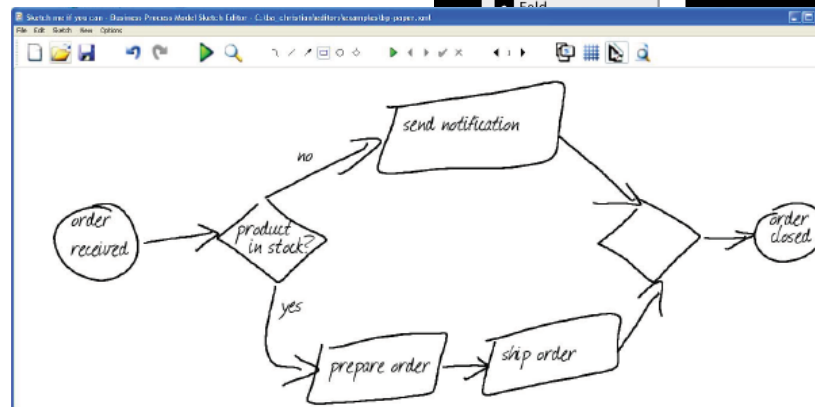
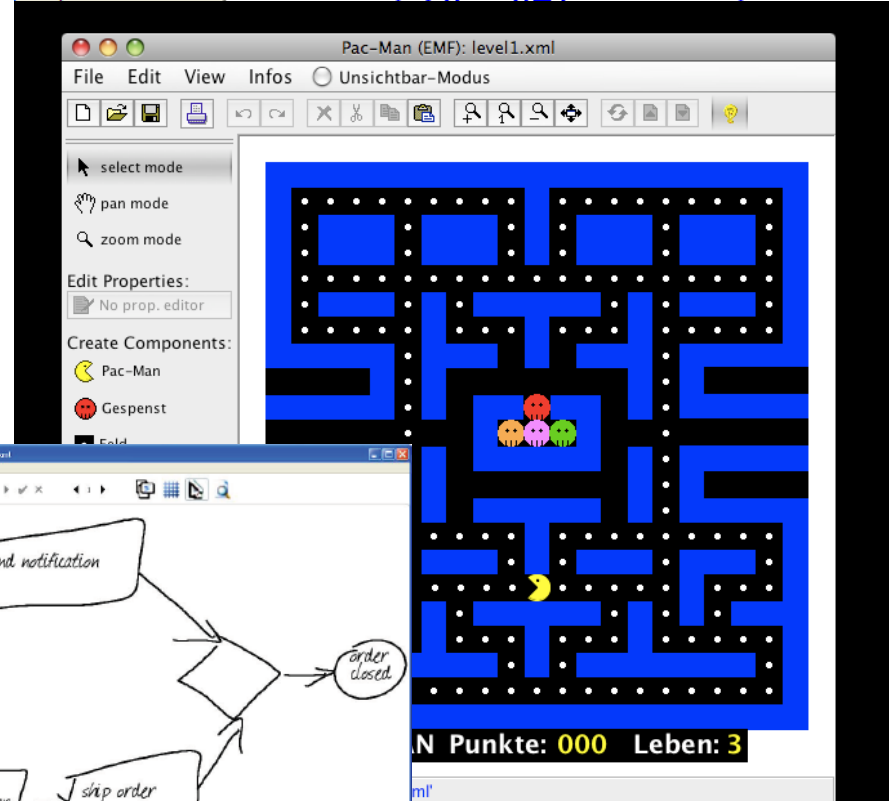
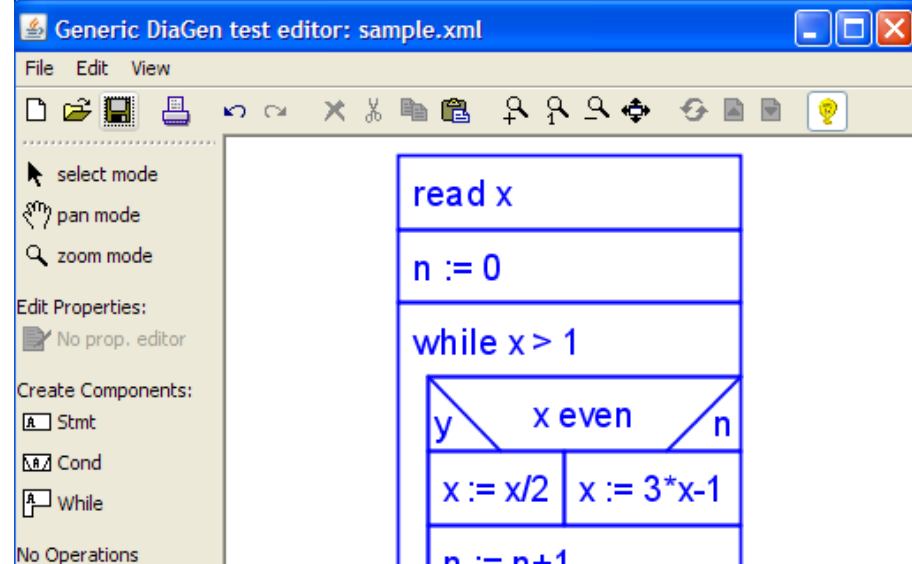
- Derivation example:



- The semantics of the grammar SEM(PN) are all possible reachable models: the language of Petri nets.
- For recognition, one uses parsing, applying rules backwards trying to reduce the model to the empty graph.
- Also useful to generate random, synthetic models (e.g., for testing model transformations, code generators, etc)



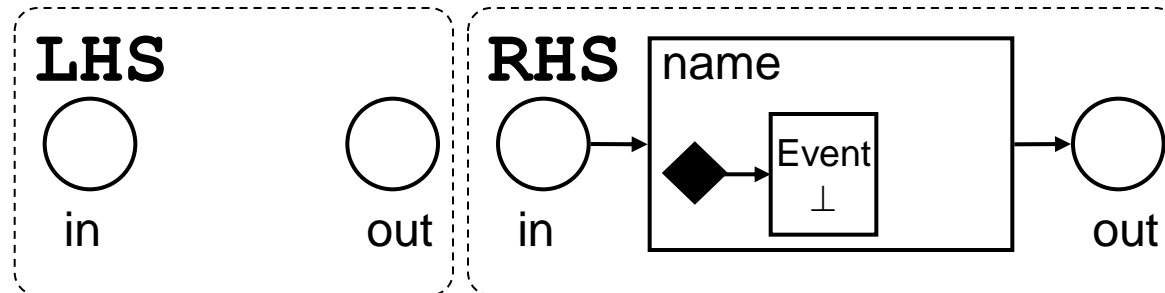
- DiaGen/DiaMeta.
(<https://www.unibw.de/inf2/diagen/diagen-diameta>).
First version in 1993.
- Based on hypergraph grammars.
- Sketching.
- Mark Minas (Munich).




Complex Editing Commands

- Express by means of rules complex editing commands.
- The match in which the rule has to be applied can be partially given by the user by selecting elements in the model.
- Assign the execution of such rule to a button in the GUI.

createComponent (name: String)

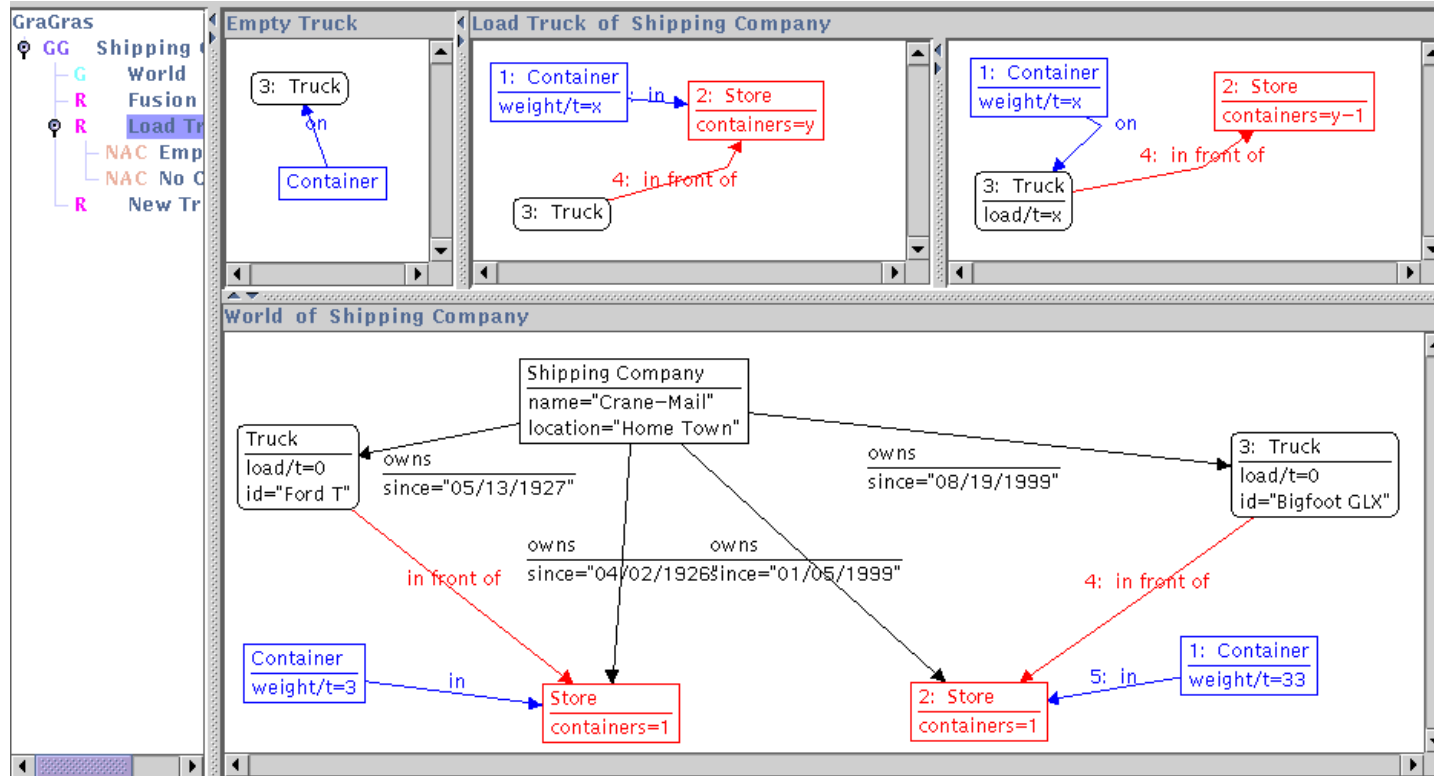


Index

- Introduction.
- Overview of Graph Transformation.
- Exercise: Playing tic-tac-toe.
- Control Language.
- Applications.
- **Tools.** 
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AGG

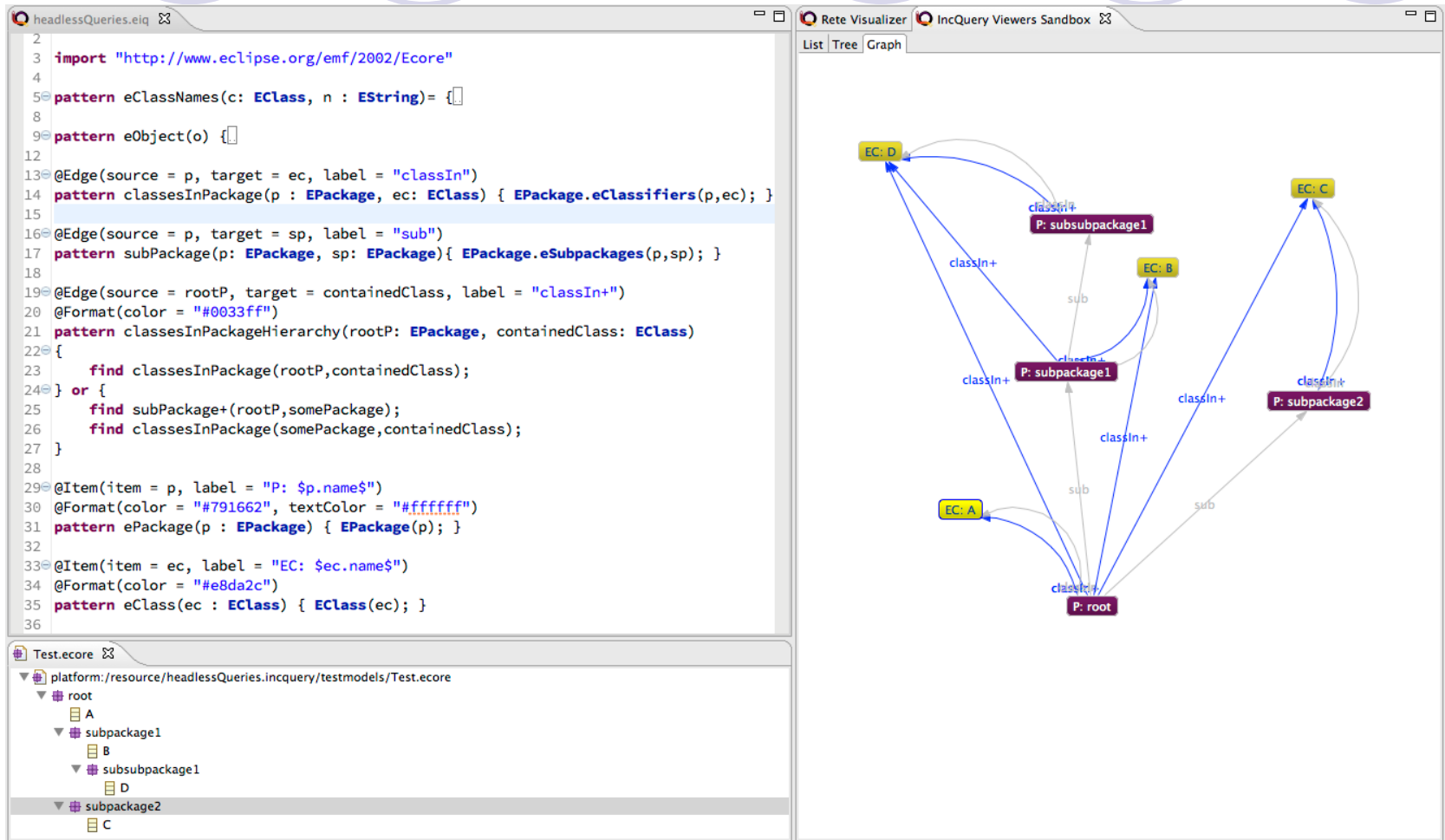
<http://tfs.cs.tu-berlin.de/agg/>



- Definition and execution of grammars.
- Type Graph.
- Analysis (Critical Pairs – confluence)

VIATRA

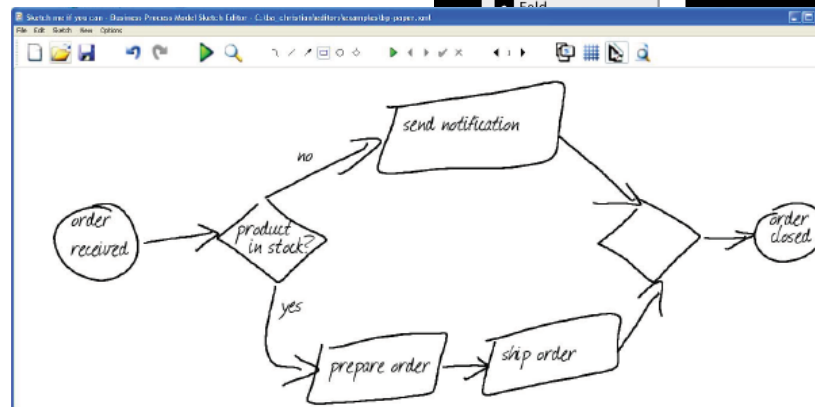
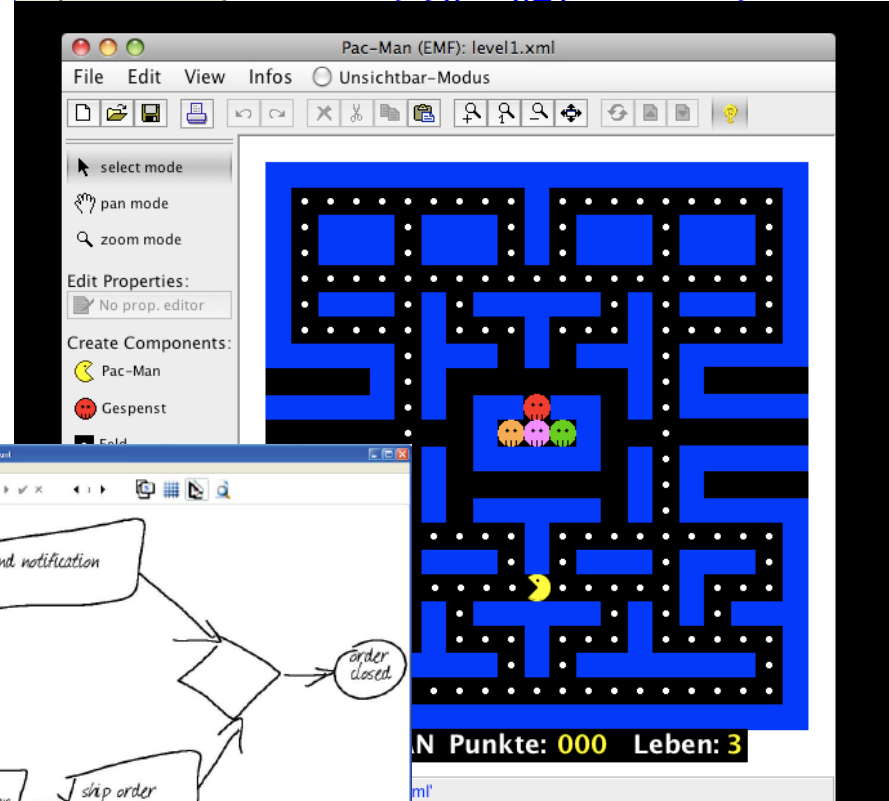
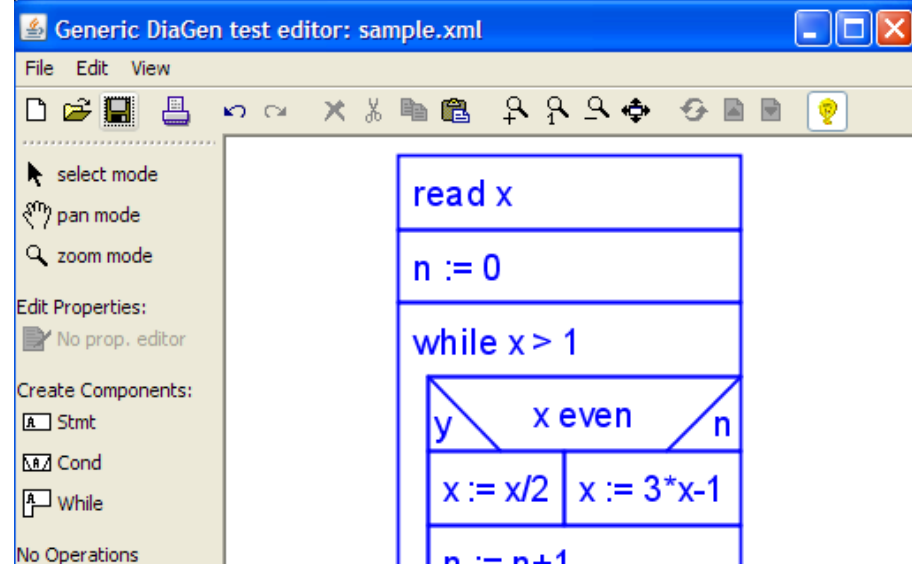
<https://www.eclipse.org/viatra/>



- Definition and execution of queries and (reactive) transformations.
- Models and meta-models
- Scalability

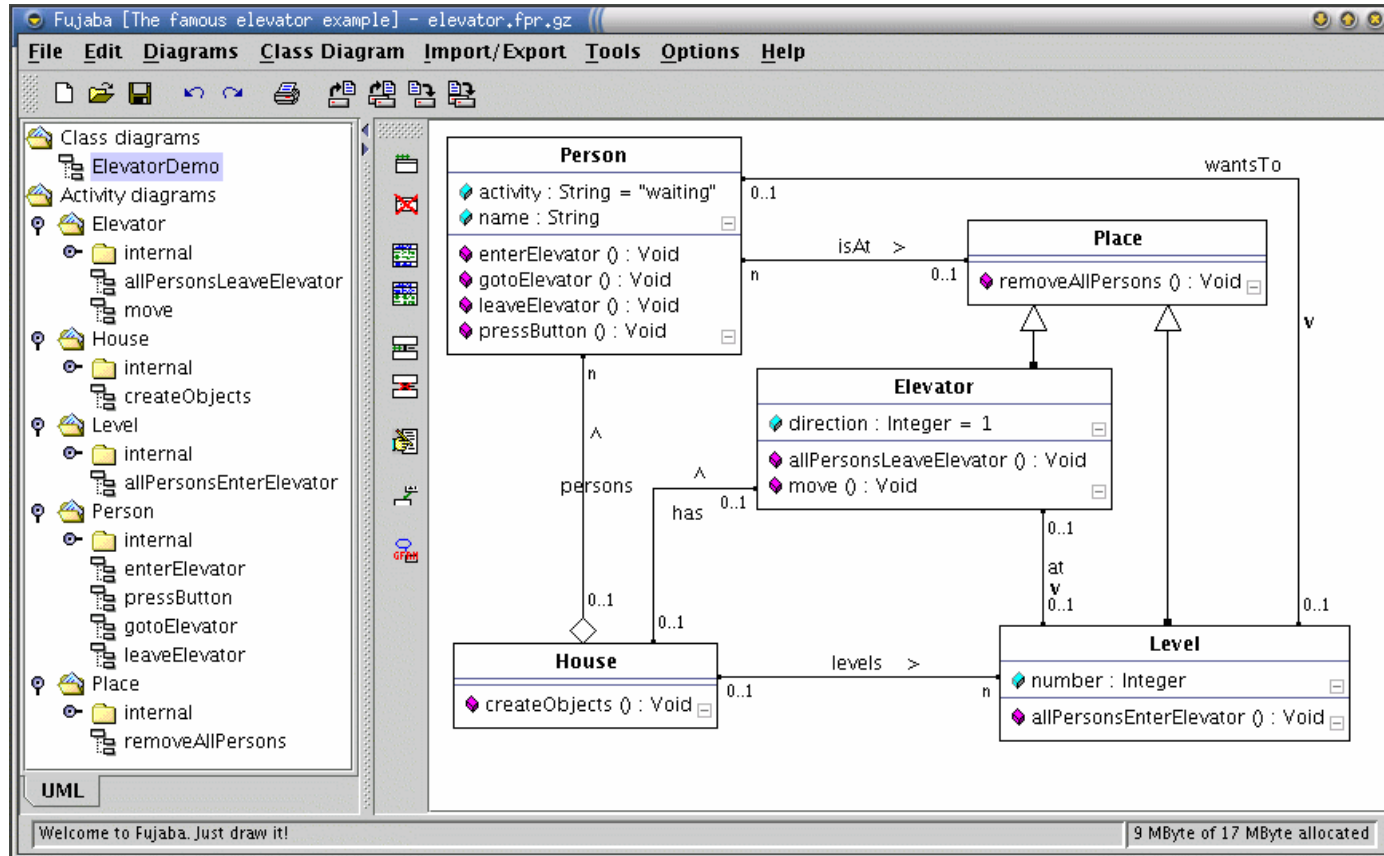
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FUJABA

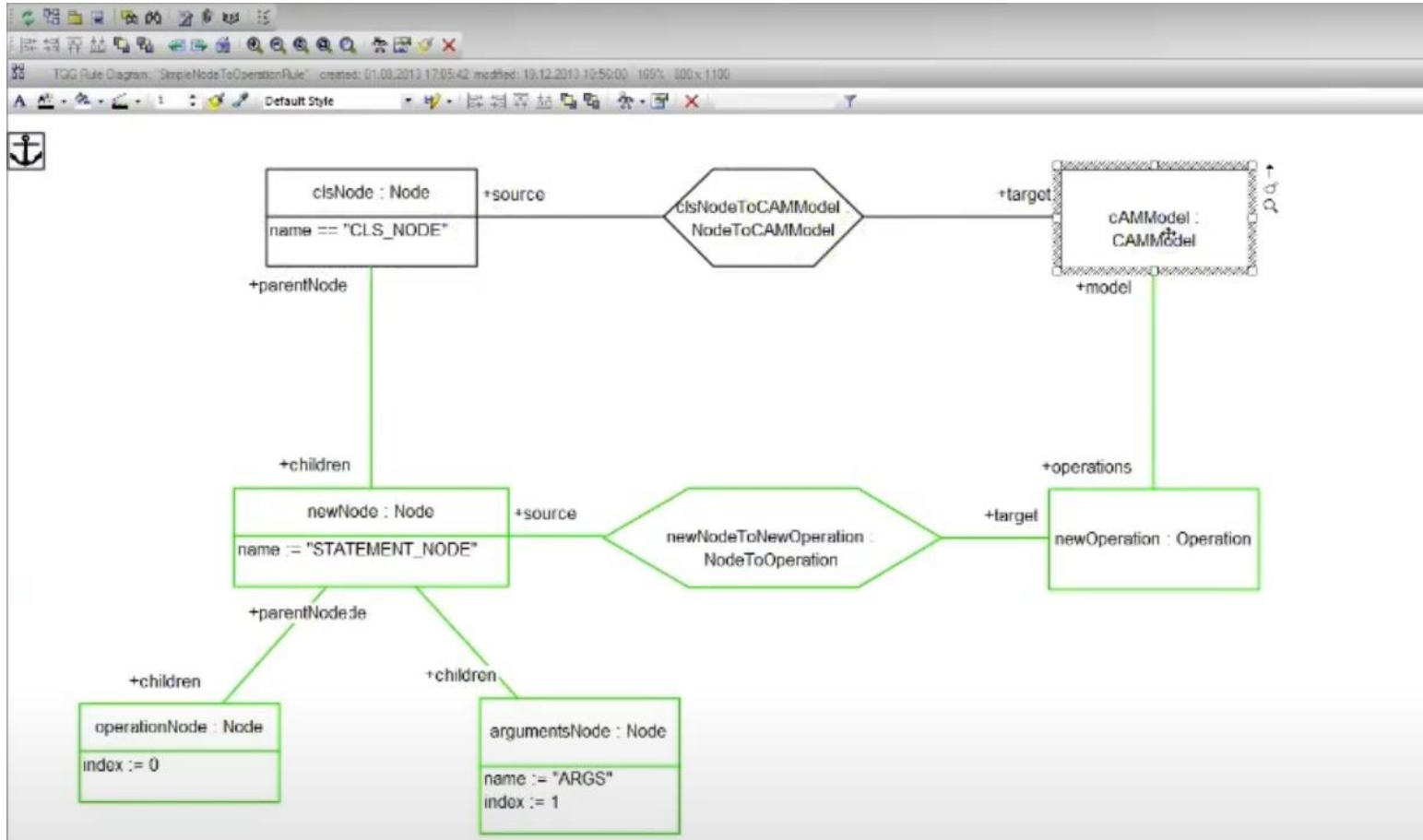
<https://github.com/fujaba>



- Story diagrams (Activity diagrams+rules)
- From UML to Java and back.

eMOFLON

<https://emoflon.org/>

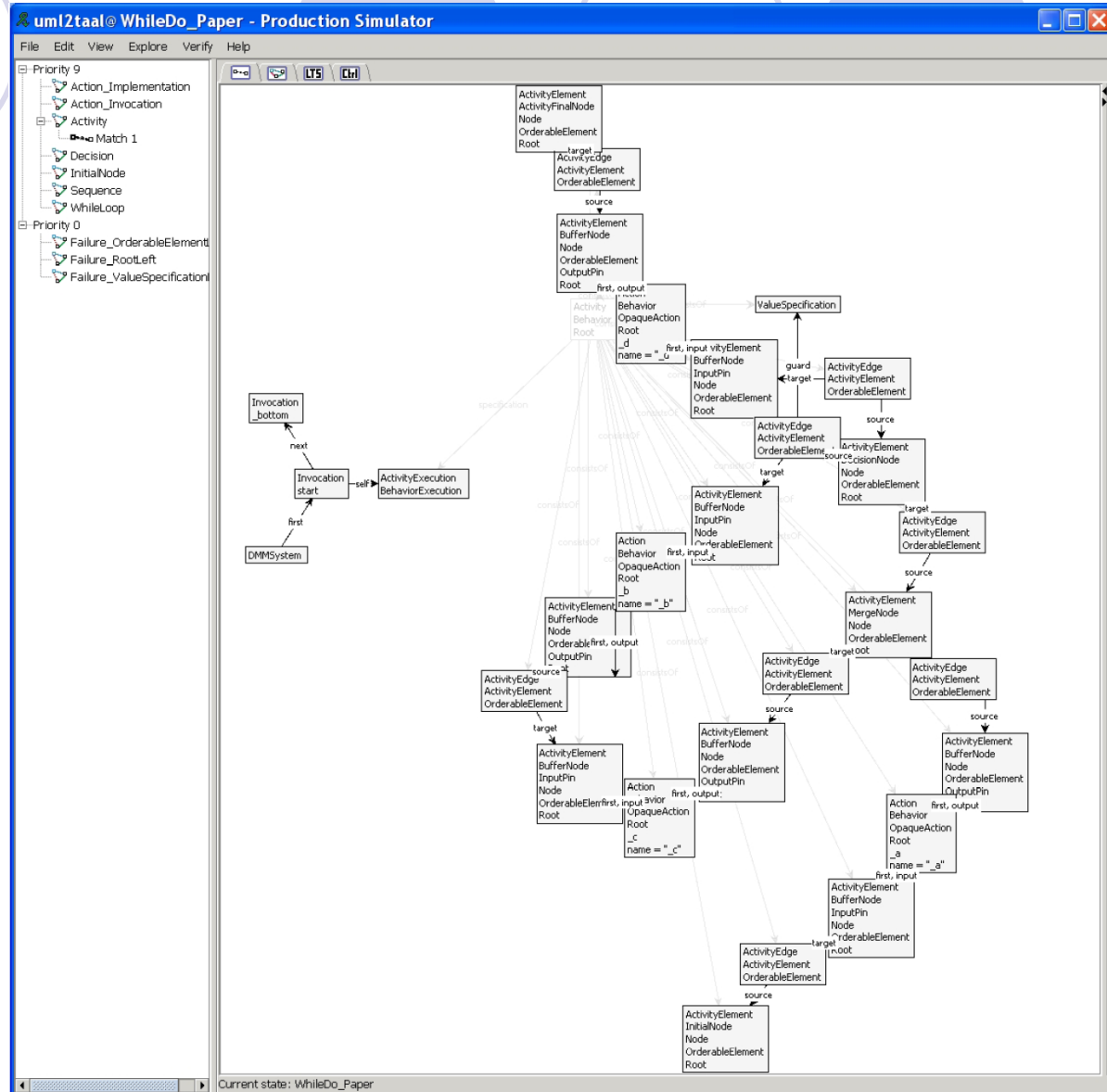


- Rules for Model to Model transformation (Triple graph grammars)
- Within Enterprise Architect

GROOVE

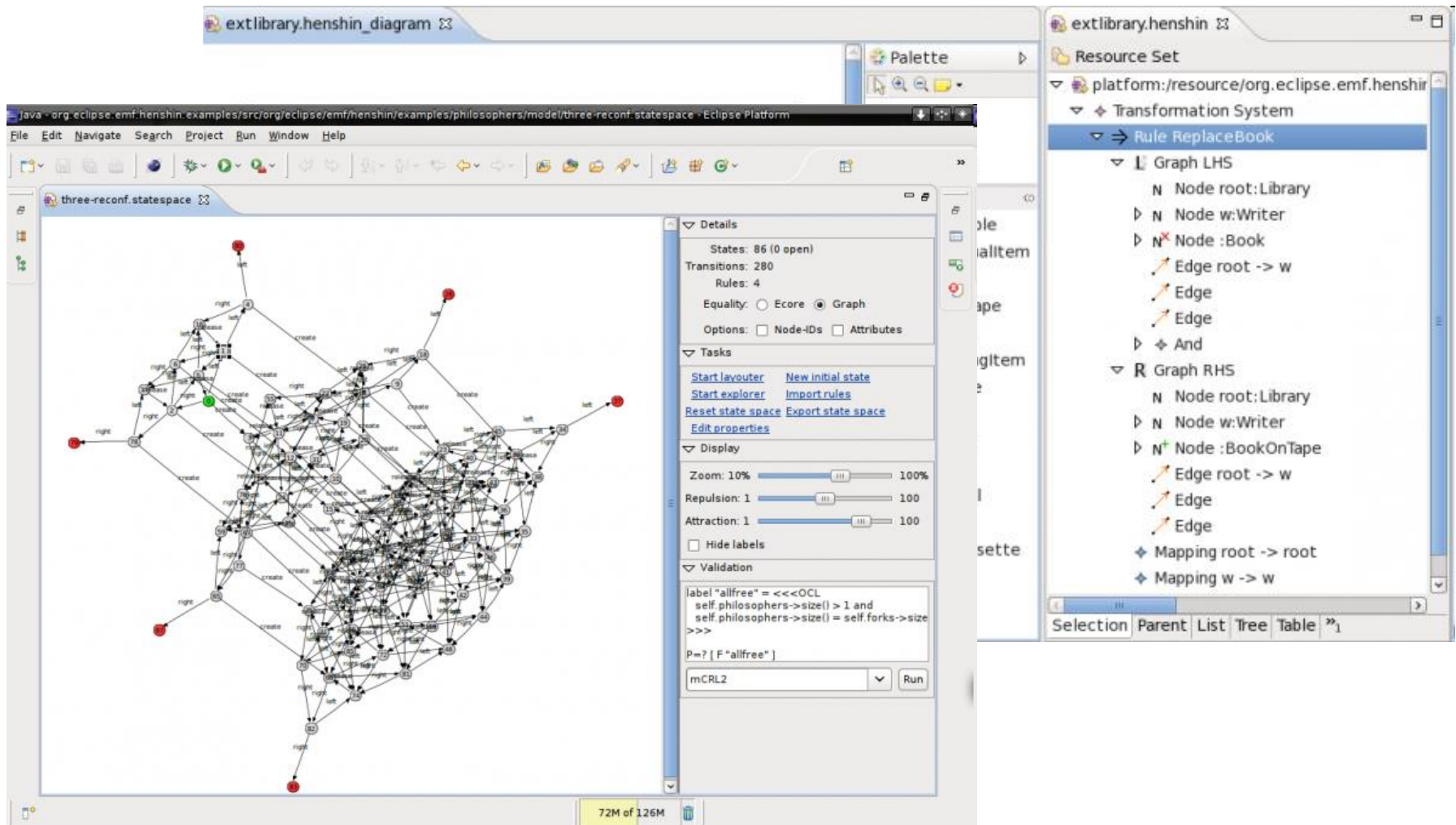
<http://groove.cs.utwente.nl/>

- Quantified Rules.
- Model Checking.



Henshin

<http://www.eclipse.org/modeling/emft/henshin/>



- State Space Analysis for Verification.

Index

- Introduction.
- Overview of Graph Transformation.
- Exercise: Playing tic-tac-toe.
- Control Language.
- Applications.
- Tools.
- **Conclusions.**



Conclusions

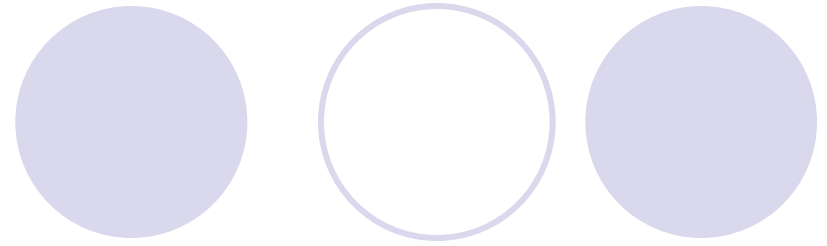


- An overview of Graph Transformation.
 - Rule, application conditions.
 - Grammar, derivation.
 - Dangling edges, non-injectivity.
 - Control languages for rule execution.
- Based on pre- and post-conditions.
- Use the language of the DSVL.

Remaining Issues

- Some words on formalizations.
 - Analysis.
- Specifying invariants:
 - Graph constraints and application conditions.
- Use of grammars to specify model-to-model transformations.

Bibliography.



- Handbook of Graph Grammars and Computing by Graph Transformation. 3 Vols. 1997. World Scientific.
- Ehrig, H., Ehrig, K., Prange, U., Taentzer, G. 2006. “Fundamentals of Algebraic Graph Transformation”.Springer.