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A Comparative Quantification of Existing Creep Models for Piezoactuators

Shabnam Tashakori[1,2,3](✉), Vahid Vaziri[1], and Sumeet S. Aphale[1]

[1] Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Aberdeen AB24 3UE, UK
[2] Department of Mechanical and Aerospace Engineering, Shiraz University of Technology, Shiraz, Iran
[3] Rahesh Innovation Center, Shiraz, Iran
shabnam.tashakori@alum.sharif.edu

Abstract. Piezoactuators are popularly employed in precise positioning applications at the micro- and nanometer scales. Their positioning performance, especially for low-frequency responses, is significantly impacted by creep - a phenomenon where the actuator deformation gradually changes in the presence of a persistently applied constant voltage. This change in deformation manifests itself in the gradual drifting of the end-effector position that the piezoactuator is driving. A significant research effort has therefore focussed on the accurate modelling of creep. This paper compares three popularly employed creep models against experimentally measured creep data obtained from a piezo-drive nanopositioner axis and quantifies their modelling accuracy. The quantification demonstrates that the fractional-order model (double logarithmic model) outperforms the other two integer-order models (Logarithmic and LTI models) along multiple, key performance indices.

Keywords: Creep dynamics · Nanopositioning · Piezoelectric actuator · Fractional-order modelling

1 Introduction

Nanopositioning encompasses a number of technologies that deliver nanometer-scale mechanical displacement with high precision. It underpins many conventional and emerging technical advances, e.g., scanning probe microscopy, disk-drive data storage, and nano-surgery. Piezoelectric actuation is most popularly employed to deliver nanopositioning due to several desirable characteristics, e.g., repeatability, lack of friction and stiction due to the absence of moving parts, easy control, and system integration. However, the positioning performance of piezoactuated nanopositioners is severely limited by the inherent linear resonance and nonlinear dynamics, i.e., hysteresis and creep.

Creep is a nonlinear phenomenon inherent to piezoactuators that results in drift over time in the output displacement despite fixed applied voltage; severely

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impacting slow-pace/start-from-previous-stop operations. To reduce this drift, the piezoelectric actuator should operate relatively fast and in a relatively short time interval [11]. However, even more accuracy is needed in many applications, e.g., while the measuring operation in scanning probe microscopes, the measuring sample should completely stay still without small movement, otherwise, the measured image will be distorted [6]. Another remedy can be using feedback methods. Nevertheless, these are not implementable in many situations as several displacement sensors should be mounted on the system, which is not always possible [11]. Therefore, precise modelling of the creep phenomenon is of great importance, whereas feedback methods can still be employed for further improvement in positioning. While Hysteresis is typically affecting all trajectories (operation speeds) and therefore is well studied [5,16], the creep is typically left unmodelled/unquantified in most nanopositioning literature [7,15]. With the advent of soft actuators, creep modelling has also seen renewed interest [8,14].

Extensive research efforts have focused on the mathematical modelling and open-/closed-loop compensation of the performance-limiting creep dynamics. Most modelling and control techniques proposed thus far lie within the span of integer-order calculus, categorized as: (i) logarithmic model [12], and (ii) linear time-invariant (LTI) model [17]. Recently, fractional-order (FO) calculus has emerged as a viable candidate capable of furnishing further improvements in the state-of-the-art in nanopositioning by allowing the formulation of improved models and controller designs [13]. Considering the PEA as a resistocaptance, a fractional-order model for creep phenomenon is proposed in [11]. In [10], another fractional-order model is presented by employing a physics-based fractional-order Maxwell resistive capacitor approach, and in [9], a simplified phenomenon-based fractional-order creep model is proposed.

In this paper, the integer-order and fractional-order modelling approaches describing the creep phenomenon are presented and numerically compared. Furthermore, experimental results are employed to further validate the fractional-order models, and demonstrate the superiority of this kind of modelling. Summarizing, the main contributions of this paper are (i) making a comparison between integer- and fractional-order models for creep, and (ii) validating a creep fractional-order model with experimental data.

The paper is organized as follows. In Sect. 2, two integer-order creep models as well as a fractional-order creep model are presented. Section 3 describes the experimental platform employed to record creep data that is subsequently used to quantify the accuracy of the three creep models being quantified in this work. Furthermore, the parameter choices of each model are explained and illustrative comparison is presented. Section 4 concludes this paper.

2 Creep Models

2.1 Integer-Order Logarithmic Model

In the logarithmic model, the output displacement $y(t)$ at time t is described with the following linear equation as a function of the logarithmic scaled time [11]:

$$y(t) = y_1(1 + \gamma log\frac{t}{t_1}), \tag{1}$$

with y_1 the displacement at time t_1 that is the time after which the creep occurs, and the constant γ specifies the creep rate.

2.2 Integer-Order Linear Time Invariant (LTI) Model

The LTI model considers a series of $n+1$ springs and n dampers to model the creep response. Consequently, the transfer function between the output response $Y(s)$ and the input voltage $U(s)$ is given by [11]:

$$G_c(s) = \frac{Y(s)}{U(s)} = \frac{1}{k_0} + \sum_{i=1}^{n} \frac{1}{c_i s + k_i}, \tag{2}$$

where $c_i,\ i = 1, ..., n$ are the damping coefficients and $k_i,\ i = 0, ..., n$ are the stiffness constants of the springs.

2.3 Fractional-Order Double Logarithmic Model

A fractional-order integrator, with an order between 0 and 1, causes drift, and hence, can be exploited to model the creep phenomenon. This is concluded based on two facts: (i) the PEA is a distributed-parameter component with memory effect, and (ii) it is proved that a fractional-order system is able to successfully model distributed-parameter systems which have memory effect [4]. Motivated by these facts, a piezoelectric actuator can be described as a resistocaptance (RC) as follows [2,4]:

$$\frac{Q(s)}{U(s)} = \frac{K}{s^\alpha}, \tag{3}$$

where $Q(s)$ and $U(s)$ are the input charge and driving voltage in frequency domain, respectively, and K and $0 < \alpha < 1$ are constants. The output displacement $y(t)$ is given by the following dynamics:

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F_p + F_{ext}, \tag{4}$$

with m, c, and k the mass, damping, and stiffness constants, respectively. Herein, $F_p = Tq(t)$, where T is the electro-mechanical transformer ratio and $q(t)$ is the input charge. When the external force F_{ext} is zero, the transfer function $G(s)$ is given by:

$$G(s) := \frac{Y(s)}{U(s)} = \frac{b}{s^\alpha(1 + a_1 s + a_2 s^2)}, \tag{5}$$

where $a_1 = \frac{c}{k}$, $a_2 = \frac{m}{k}$, and $b = \frac{TK}{k}$. For sufficiently large time, the above PEA model in (5) melts down to a fractional-order integrator, as follows:

$$lim_{s \to 0} G(s) = \frac{b}{s^{\alpha}}, \tag{6}$$

which actually shows that the mechanical response can be disregarded after a certain time t_c, so (5) can be written as:

$$G(s) = \frac{b}{s^{\alpha}}, \quad t \geq t_c. \tag{7}$$

Let $U(s) = \frac{1}{s}$, the an approximate displacement would be [3]:

$$y(t) = \mathscr{L}^{-1}\{G(s)U(s)\} = \frac{bt^{\alpha}}{\alpha \Gamma(\alpha)}, \quad t \geq t_c, \tag{8}$$

with $\Gamma(\alpha)$ the gamma function, which gives the following "double-logarithmic" creep model:

$$log\big(y(t)\big) = \alpha log(t) + log\big(\frac{b}{\alpha \Gamma(\alpha)}\big) \quad t \geq t_c. \tag{9}$$

3 Experimental Validation and Comparative Quantification

3.1 Experimental Setup

The experimental results in this paper are obtained by using a two-axis piezo-actuated serial kinematic nanopositioning stage, shown in Fig. 1. The actuation voltage, which is in the range of 0 V to 200 V, is supplied by two voltage amplifiers, PDL200 and Piezodrive, with an amplification factor of 20. The voltage off-set is 100 V, and its consecutive full-range displacement is $\pm 20 \mu$m along each axis. To gauge the real-time displacement in the matter of voltage signal, ranging from -10 V to 10 V, Microsense 4810, probe 2805, range of $\pm 50 \mu$m, is used, which is a high resolution capacitive sensor. The data acquisition system is performed via the National Instruments card (PCI-6621) on a PC that has a real-time module, also equipped with OPTIPLEX 780 with an Intel Core (TM)2 Duo Processor running at 3.167 GHz and 2 GB of DDR3 RAM memory. The cross-coupling between the two scanning axes is down to -40 dB. Therefore, two axes can be assumed decoupled and, consequently, treated as independent single-input-single-output systems. Moreover, the axis with its resonance at 716 Hz (i.e., the fast axis) is chosen as the test platform with 20 kHz sampling frequency [1].

3.2 Simulation Results

The logarithmic model (1) includes one parameter γ that needs to be identified. As shown in Fig. 2b, the steady-state error is zero for $\gamma = 0.56$. Moreover, the

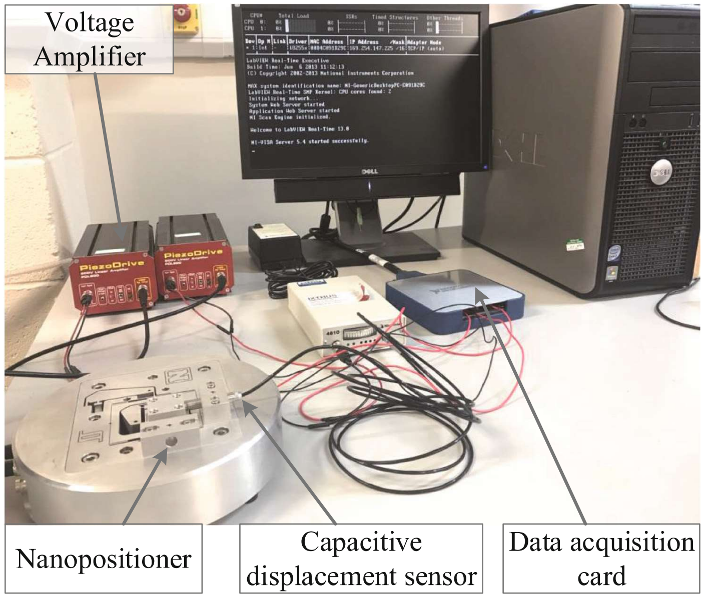


Fig. 1. Two-axis piezo-actuated serial kinematic nanopositioning stage, designed by the EasyLab, University of Nevada, Reno, USA [1].

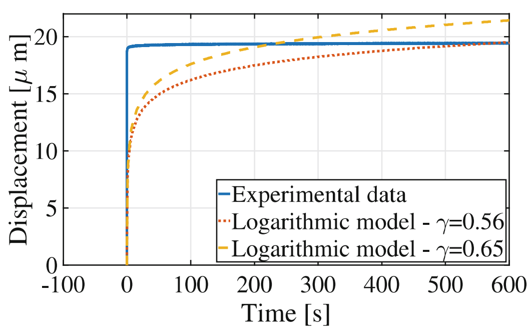
minimum of the Root Mean Square (RMS) error and zero mean error occurs at $\gamma = 0.65$, as illustrated. The displacement for each of these γ are demonstrated in Fig. 2a in comparison to the experimental data.

Figure 2c illustrates the following reduced-order LTI model (n is set to one in (2)):

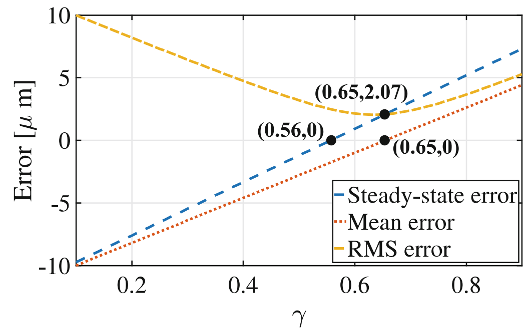
$$G_c(s) = \frac{1}{k_0} + \frac{1}{c_1 s + k_1}, \tag{10}$$

in which three constants k_0, k_1, and c_1 are to be identified. Since the displacement starts at 0µm, the constant k_0 should be selected large enough, e.g., 1000. The values for the stiffness constant k_1 can also be computed such that the final value at $t = 600$ s reaches 19.4191µm, which leads to $k_1 = 0.0515$. Therefore, only one parameter c_1 has to be selected. This constant specifies the sharpness of the unit response. Figure 2d shows the errors with respect to different values of c_1, from which $c_1 = 0.1$ is selected since it makes the mean error zero. Note that the steady-state error is zero for all values of c_1 since this error, as discussed above, is only influenced by k_1.

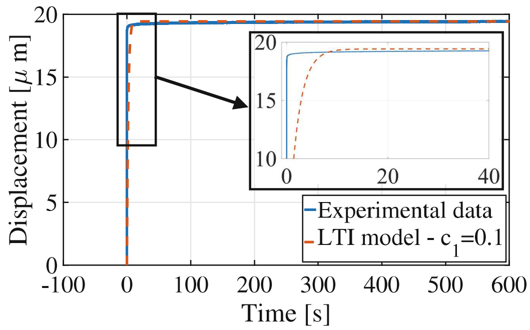
The fractional-order double logarithmic model (8) is illustrated in Fig. 2e. Two constants α and b are involved in this model, which should be identified. The constant α specifies the creep rate and the constant b can be selected afterwards such that the final model error at $t = 600$ s is zero. Consequently, as shown in Fig. 2f, the steady-state error is zero for all values of α. Therefore, this model also includes one independent constant α. The mean error is zero at $\alpha = 0.003$ where the RMS error is also minimum (with $b = 4.5684$). Using this α, the result of the double logarithmic model perfectly fits the experimental result, as can be seen in Fig. 2e.



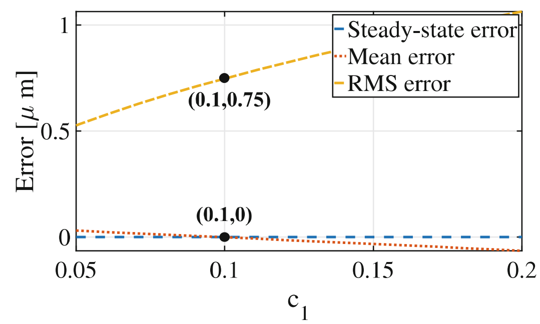
(a) The integer-order logarithmic model.



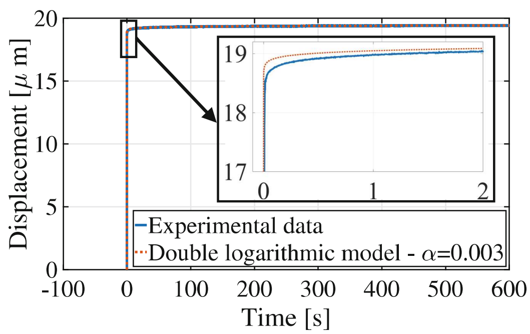
(b) Errors of the logarithmic model for different parameters γ.



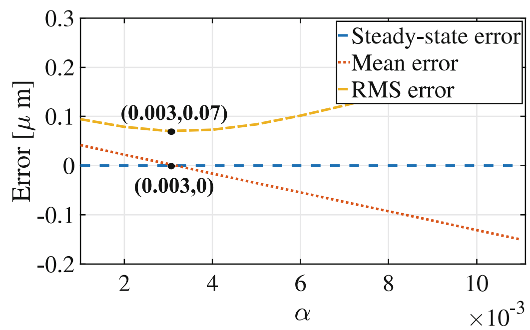
(c) The integer-order LTI model.



(d) Errors of the LTI model for different parameters c_1.



(e) The fractional-order double logarithmic model.



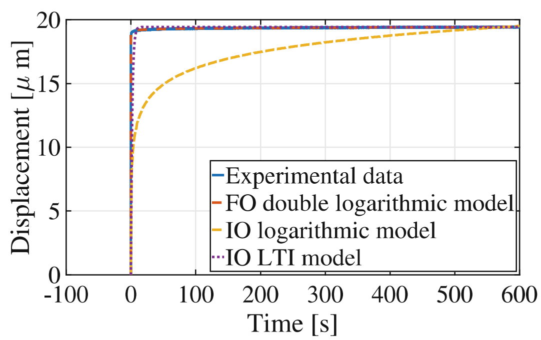
(f) Errors of the double logarithmic model for different parameters α.

Fig. 2. Choosing the model parameter that minimizes the modelling error and comparing the results with the experimental data.

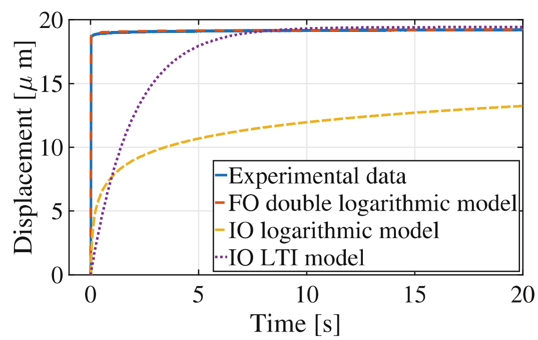
Remark 1. Although the selected values for c_1 and α makes the mean error of both the LTI and double logarithmic model zero, the corresponding RMS error is smaller in the latter (RMS error is 0.75 in LTI model and 0.07 for double logarithmic model, see Figs. 2d and 2f). Thus, the fractional-order double logarithmic model fits more precisely to the experimental data in comparison to the integer-order LTI model. To support this, an illustrative comparison is shown in Fig. 3.

Remark 2. Assuming a higher order LTI model ($n > 1$) may lead to a more accurate modelling but it also increases the number of parameters $(2n + 1)$ that need to be identified. Whereas, the fractional-order double logarithmic model results in an excellent accuracy, as shown in Fig. 3, with only one independent parameter.

Summarizing, in this paper, three different modelling schemes were studied and their modelling errors were compared based on measured system dynamics data for a specific input amplitude. These models can capture the creep dynamics for different input amplitudes.



(a) Time frame $t = (0 : 600)s$.



(b) Zoomed at $t = (0 : 20)s$.

Fig. 3. Comparison between two integer-order models and a fractional-order model to capture creep phenomenon.

4 Conclusions

Creep is a key limiting factor in the controlled performance of artificial muscles/soft actuators/dielectric actuators/electroactive polymers that have significant additional nonlinearities such as hysteresis. Establishing accurate creep models will allow for improved control schemes for these actuators which enable a spectrum of exciting research avenues such as bioinspired robotics, flexible prosthetics and life-like haptic interfaces.