Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
ABCDEF		1111111

Use this template to begin typing the first page (summary page) of your electronic report. This template uses a 12-point Times New Roman font. Submit your paper as an Adobe PDF electronic file (e.g. 1111111.pdf), typed in English, with a readable font of at least 12-point type.

Do NOT include the name of your school, advisor, or team members on this or any page.

Papers must be within the 25 page limit.
(Includes everything: Summary Sheet, Table of Contents, References, Appendices)

Be sure to change the **Team Control Number** and **Problem Choice** above.
You may delete these instructions as you begin to type your report here.

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Enter your summary here by replacing the (red) text. Do not center.

What should you put here? Basically, you want a brief restatement of the problem, followed by a largely *non-technical* description of how the problem was solved, and a concise account of final results and conclusions. You should describe your results in complete sentences that can stand on its own, without using heavy jargon, non-standard variables, or uncommon abbreviations. References should be avoided in the summary.

Here is an excerpt from [**?**]:

> The summary is an essential part of your MCM/ICM paper and should appear as the first page of your solution report. The judges place considerable weight on the summary, and winning papers are often distinguished from other papers based on the quality of the summary.
>
> - To write a good summary, imagine that a reader will choose whether to read the body of the paper based on your summary: Your concise presentation in the summary should inspire a reader to learn about the details of your work.
> - You should write the summary last, as it should clearly describe your approach to the problem and, most prominently, your most important conclusions. Ensure you plan time after solving your problem to write a comprehensive and articulate summary.
> - Summaries that are mere restatements of the contest problem, or are a cut-and-paste boilerplate from the Introduction are generally considered to be weak.

Start your solution on the next page.

Contents

General guidlines

Your solution should be articulate, concise, and organized in order to allow the reader to easily follow the solution process and conclusions. Key statements should present major ideas and results.

- A Table of Contents assists the reader in previewing the organization of your report.
- Present a clarification or restatement of the problem as appropriate.
- Present a clear exposition of all variables and hypotheses.
- State and justify reasonable assumptions that bear on the problem.
- Present an analysis of the problem, motivating or justifying the model being used.
- Summarize derivations, computations, or illustrative examples in the main body of the solution, and leave lengthy derivations and/or calculations and data in appropriate appendices.
- Include a design of the model. Discuss how the model could be tested, to include error analysis, sensitivity, and/or stability.
- Discuss any apparent strengths or weaknesses to your model or approach.
- Provide a conclusion and report results explicitly.
- Document resources and references.

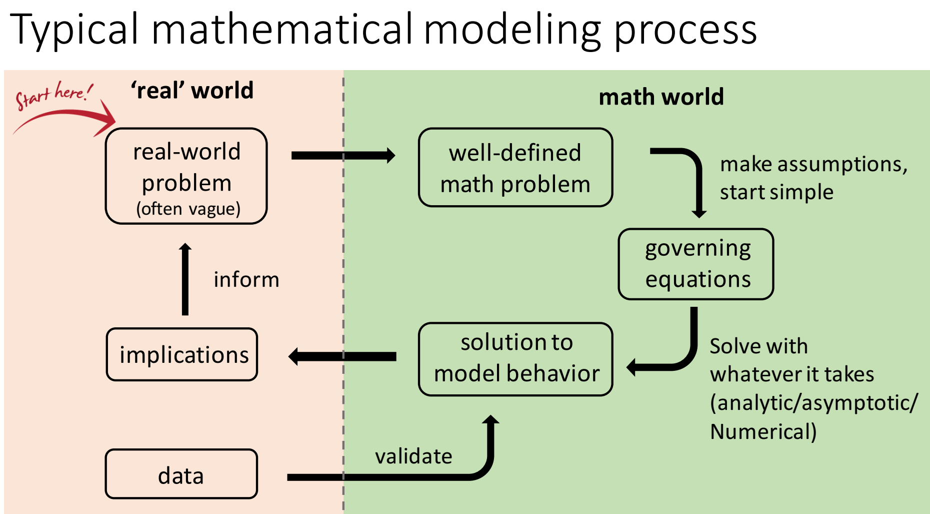

Figure 1: A flowchart outlining a typical mathematical modelling process. Use a flowchart to outline your mathematical model.

1 Introduction

Write an introduction to your report here. What to include here? Restate and refine the problem. Real-world problems can be broad and complex. It is important to refine the conceptual idea into *a concise problem statement* that indicates exactly what the output of the model will be. Your introduction should be more detailed and technical than your summary. You may include a brief history and context of the problem. If you cite works that have tackled this problem or similar ones, briefly describe their approach or results. Relate this to your approach/results. In the last paragraph you may also want to include a a brief outline of your report, along the lines of *"In Section 1 we give our definitions and notation. Section 2 describes our numerical experiments. . . .. We analyze/discuss the output of our model in Section 5. . . ."* Replace the numbers in that example with appropriate `\ref` commands pointing to the correct `\labels` in your document.

Remember to Break Things Up Into Logical Sections. You may consult [1, 2] for some additional tips on modelling approaches and communicating your solution.

A nice addition would a flowchart for your model. While this can be done directly in LaTeX (using the the TikZ package, see this tutorial) it is perhaps easier to use a third party app. The flowchart in Figure 1 was taken from this webpage.

2 Some tips on using LaTeX

2.1 Overall aspect

Aim for a clean and consistent layout of your report. Remember the *2-3-4 rule*: break up sentences longer than *two* lines, sentences that have more than *three* verbs, and paragraphs that have more than *four* long sentences. Start new paragraphs by leaving a blank line. Use `\noindent` at the beginning of a line that shouldn't be indented (not a new paragraph line, e.g., following a centered math environment).

Use ~ between the last two words in a paragraph , or use `\hbox{..}` to deal with running words (i.e. lonely words on the last line of a paragraph). Use ~ before `\ref{..}` to avoid instances (e.g., *Figure* #) where *the figure number* is displayed at the beginning of the next line. Remember to use `\label{..}` and `\ref{..}` for cross-referencing.

2.2 Math - tips for new users

In this section we collect some general tips for *new* LaTeX users.

2.2.1 Math Mode

Always type in-line mathematics in math mode using `$..$` or `\(..\)`. This includes names of functions, variables and parameters. For example, write "n-dimensional" and not "n-dimensional". For displayed equations (centered) use `\[..\]` or `\begin{equation} .. \end{equation}`.

2.2.2 Punctuation

Punctuation should appear outside math mode, for inline equations, otherwise the spacing will be incorrect. For displayed equations, punctuation should appear as part of the display. All equations *must* be punctuated, as they are part of a sentence.

2.2.3 Math functions

Mathematical functions should be typeset in roman font. This is done automatically for the many standard mathematical functions that LaTeX supports, such as sin, tan, exp, log, max, etc. If the function you need is not built into LaTeX, create your own by adding in the preamble of your .tex file `\def\diag{\mathop{\mathrm{diag}}}`.

2.2.4 Math expressions

- Ellipses (dots) are never explicitly typed as "...". Instead they are typed as `\dots` for baseline dots, as in $x_1, x_2, \dots, x_n$ or as `\cdots` for vertically centered dots, as in $x_1 + x_2 + \cdots + x_n$.
- Type ith instead of $i'th$ or i^{th}.
- Avoid using `\frac` to produce stacked fractions in the text. Write $n^3/3$ flops instead of $\frac{n^3}{3}$ flops.
- If you are using angle brackets to denote an inner product use `\langle` and `\rangle`: the correct way is $\langle x, y\rangle$ and not <x,y>, typed as $< x, y >$.

2.2.5 Text in Displayed Equations

When a displayed equation contains text such as "subject to $x \geq 0$?, instead of putting the text in `\mathrm` put the text in an `\mbox`, as in subject to $x \geq 0$. Note that `\mbox` switches out of math mode, and this has the advantage of ensuring the correct spacing between words. If you are using the amsmath package you can use the `\text` command instead of `\mbox`. See equation 1 for an example.

$$\min\{\, \|A - X\|_F : X \text{ is a correlation matrix} \,\}. \tag{1}$$

When a displayed equation contains (staked) fractions in parenthesis or brackets, use `\left( .. \right)` and `\left[ .. \right]`, respectively, to automatically adjust their height.

$$\text{Correct } \left(\frac{x-1}{x^2+1}\right)^{\alpha}; \text{ incorrect } (\frac{x-1}{x^2+1})^{\alpha}.$$

For displayed equations occupying more than one line it is best to use the environments provided by the `amsmath` package. Of these, `align` (and `align*` if equation numbers are not wanted) is the preferred one. Example:

$$\begin{aligned} \cos(A) &= I - \frac{A^2}{2!} + \frac{A^4}{4!} + \cdots, \\ \sin(A) &= A - \frac{A^3}{3!} + \frac{A^5}{5!} - \cdots, \end{aligned}$$

Others, such as `gather` and `aligned`, are occasionally needed. See also Appendix A.

Avoid using the standard LaTeX environment `eqnarray`, because it doesn't produce as good results as the `amsmath` environments, nor is it as versatile.

Use `\begin{cases}..\end{cases}` to display piecewise defined functions:

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & \text{otherwise.} \end{cases}$$

Do not use * for multiplication, e.g., write Area $= \pi r^2$ and not Area $= \pi * r^2$.

2.3 Tables

Captions go above tables, so place the `\caption{..}` command at the start of the table. Remember to include sufficient details in the captions. Aim for simple and clear tables (less rules is better, and avoid vertical rules).

Table 1: A table

	A			B		
	$mv(units)$	Rel. err	Time (s)	$mv(units)$	Rel. err	Time (s)
1st parameter	11034	1.3e-7	3.9	15846	2.7e-11	5.6
2nd parameter	21952	1.3e-7	6.2	31516	2.7e-11	8.8
3rd parameter	15883	5.2e-8	7.1	32023	1.1e-11	1.4e1
4th parameter	11180	8.0e-9	4.3	17348	1.5e-11	6.6

Try to avoid phrases like "as shown in the table above". LaTeX will place tables (and figures) in the best position on the current or next page. Use `\label{..}` and `\ref{..}` as done in the source document for Table 2. To try to “force" LaTeX to place tables and figures as close as possible to where they are mentioned, use the option [!htp], e.g., `\begin{table}[!htp]` or `\begin{figure}[!htp]`.

Table 2: A table with many numbers, aligned at the decimal point. The value of $\delta_{1,2}$ in experiment 7 was not available.

n	c_1	c_2	$\delta_{1,2}$	c_3	Δ	$T_1(°C)$	$T_2(°C)$
1	16.128	8.872	16.128	1.402	1.373	−146.6	−137.6
2	3.442	−2.509	3.442	0.299	0.343	133.2	152.4
3	1.826	−0.363	1.826	0.159	0.119	168.5	−161.1
4	0.993	−0.429	0.993	0.086	0.08	25.6	90
5	1.29	0.099	1.29	0.112	0.097	−175.6	−114.7
6	0.483	−0.183	0.483	0.042	0.063	22.3	122.5
7	0.766	−0.475	NA	0.067	0.039	141.6	−122
8	0.624	0.365	0.624	0.054	0.04	−35.7	90
9	0.641	−0.466	0.641	0.056	0.045	133.3	−106.3
10	0.45	0.421	0.45	0.039	0.034	−69.4	110.9
11	0.598	−0.597	0.598	0.052	0.025	92.3	−109.3

See Appendix C for a wide table rotated to fit in landscape mode. Notice that Table 2 did not fit before this line, so it is displayed on the next page.

2.4 Figures

Captions go *below* figures. So put the caption at the end of a figure environment. The `\label` statement should go after the `\caption` statement (or it can be put inside it), otherwise references to that label will refer to the subsection in which the label appears rather than the figure or table.
Example: We fit a deterministic trend model containing seasonal means together with a linear time trend to the logarithms of the electricity values. Figure 2 shows the data from the the last five years of the series together with two years of forecasts and the 95% forecast limits. The code for Figure 2 is included in Appendix B.

2.5 Bibliographies

Produce your bibliographies using BibTeX, creating your own bib file. Note that “Export citation" options on journal websites rarely produce perfect bib entries. More often than not the entry has an improperly cased title, an incomplete or incorrectly accented author name, improperly typeset maths in the title, or some other error, so always check and improve the entry. You may use https://zbib.org to generate a bib entry from a ORL, ISBN, DOI, PMID, arXiv ID, or title, or add the information manually (recommended for online data and resources).

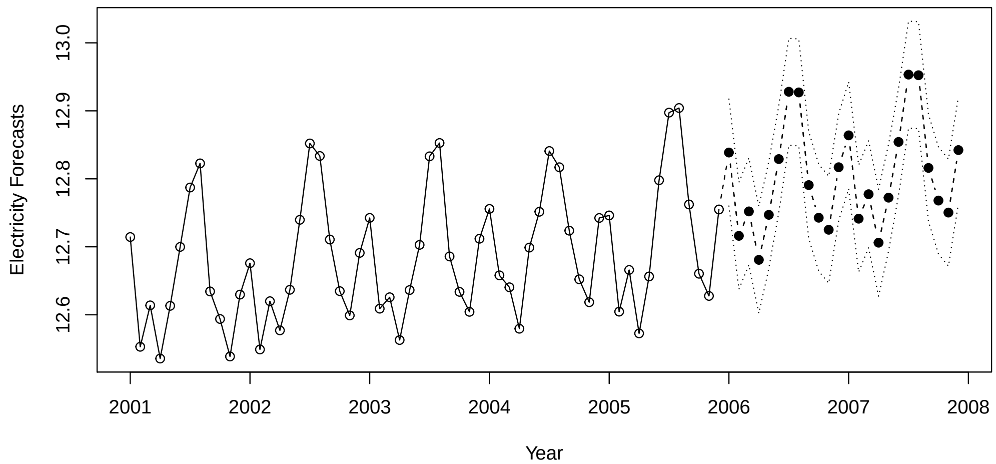

Figure 2: Log(electricity) forecast for 2006-2008 and the 95% forecast limits. Forecasts follow the upward trend and the width of the 95% confidence band is quite narrow.

3 Conclusion

Here goes your big ending. Read the *MCM/ICM Contest Rules, Registration and Instructions* at https://www.comap.com/undergraduate/contests/mcm/instructions.php.

References

[1] K. M. Bliss, K. R. Fowler, and B. J. Galluzzo, *Math Modeling: Getting Started and Getting Solutions*, SIAM, Philadelphia, 2014.

[2] K. M. Bliss, K. F. Kavanagh, B. J. Galluzzo, and R. Levy, *Math Modeling: Computing and Communicating*, SIAM, Philadelphia, 2018.

A Appendix: Lengthly calculations

Only if necessary. Included below is a sample lengthly derivation.

Suppose that $\{Y_t\}$ is a stationary process with autocovariance function γ_k and let $\overline{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t$. Then

$$Var(\overline{Y}) = Var\left(\frac{1}{n}\sum_{t=1}^{n} Y_t\right) = \frac{1}{n^2}Var\left(\sum_{t=1}^{n} Y_t\right) = \frac{1}{n^2}Cov\left(\sum_{t=1}^{n} Y_t, \sum_{s=1}^{n} Y_s\right) = \frac{1}{n^2}\sum_{t=1}^{n}\sum_{s=1}^{n}\gamma_{t-s}$$

$$= \frac{1}{n^2}\sum_{t=1}^{n}(\gamma_{t-1} + \gamma_{t-2} + \ldots + \gamma_{t-(n-1)} + \gamma_{t-n})$$

$$
\begin{aligned}
&= \frac{1}{n^2}(\gamma_0 + \gamma_{-1} + \gamma_{-2} + \ldots + \gamma_{2-n} + \gamma_{1-n} \\
&\qquad + \gamma_1 + \gamma_0 + \gamma_{-1} + \ldots + \gamma_{3-n} + \gamma_{2-n} \\
&\qquad\qquad \vdots \\
&\qquad + \gamma_{n-1} + \gamma_{n-2} + \gamma_{n-3} + \ldots + \gamma_1 + \gamma_0) \\
&= \frac{1}{n^2}\left[n\gamma_0 + 2(n-1)\gamma_1 + 2(n-2)\gamma_2 + \ldots + 2(n-(n-1))\gamma_{n-1}\right] \\
&= \frac{\gamma_0}{n} + \frac{2}{n}\sum_{k=1}^{n-1}\left(\frac{n-k}{n}\right)\gamma_k \\
&= \frac{1}{n}\sum_{k=-n+1}^{n-1}\left(1 - \frac{|k|}{n}\right)\gamma_k
\end{aligned}
$$

B Appendix: Snippets of code

Only if necessary and relevant to your *modelling approach*. Use `\begin{verbatim} .. \end{verbatim}` if you copy-paste from your programming console. Here is an example:

```
> data(electricity)
> month. = season(electricity)
> determ = as.matrix(model.matrix(~month.+time(electricity)-1))[,-1]
> model = arima(log(electricity), order=c(0,0,0), xreg=determ)
> model

Call:
arima(x = log(electricity), order = c(0, 0, 0), xreg = determ)

Coefficients:
      intercept  month.February  month.March  month.April  month.May  month.June
       -37.8299         -0.1246      -0.0908      -0.1642    -0.1000     -0.0202
s.e.     0.4212          0.0099       0.0099       0.0099     0.0099      0.0099
      month.July  month.August  month.September  month.October  month.November
          0.0768        0.0737          -0.0647        -0.1148         -0.1346
s.e.      0.0099        0.0099           0.0099         0.0099          0.0099
      month.December  time(electricity)
             -0.0448             0.0253
s.e.          0.0099             0.0002

sigma^2 estimated as 0.00161:  log likelihood = 711.56,  aic = -1397.11

> newmonth. = season(ts(rep(1,24), start=c(2006,1), freq=12))
> trend = time(electricity)[length(electricity)]+(1:24)*deltat(electricity)
> plot(model, n.ahead = 24, n1 = c(2001,1), pch = 19,
+      xlab = ‘Year’, ylab = ‘Electricity Forecasts’,
+      newxreg = as.matrix(model.matrix(~newmonth.+trend-1))[,-1])
```

C Appendix: Wide tables

Avoid such tabes if possible. Present the information in a plot or split the table in smaller tables.

Table 3: A wide table presented in landscape mode.

	$w = 8$			$w = 16$			$w = 32$		
	$t = 0$	$t = 1$	$t = 2$	$t = 0$	$t = 1$	$t = 2$	$t = 0$	$t = 1$	$t = 2$
$dir = 1$									
c	0.0790	0.1692	0.2945	0.3670	0.7187	3.1815	-1.0032	-1.7104	-21.7969
c	-0.8651	50.0476	5.9384	-9.0714	297.0923	46.2143	4.3590	34.5809	76.9167
c	124.2756	-50.9612	-14.2721	128.2265	-630.5455	-381.0930	-121.0518	-137.1210	-220.2500
$dir = 0$									
c	0.0357	1.2473	0.2119	0.3593	-0.2755	2.1764	-1.2998	-3.8202	-1.2784
c	-17.9048	-37.1111	8.8591	-30.7381	-9.5952	-3.0000	-11.1631	-5.7108	-15.6728
c	105.5518	232.1160	-94.7351	100.2497	141.2778	-259.7326	52.5745	10.1098	-140.2130