

THE UNIVERSITY OF CHICAGO

DYNAMIC MODULATION OF ACTOMYOSIN CONTRACTILITY AND  
CORTICAL FLOW BY ACTIN FILAMENT RECYCLING

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For Claire, you keep me going.

## ABSTRACT

Actomyosin-based cortical flow is a fundamental engine for cellular morphogenesis. Cortical flows are generated by cross-linked networks of actin filaments and myosin motors, in which active stress produced by motor activity is opposed by passive resistance to network deformation. Continuous flow requires local remodeling through crosslink unbinding and and/or filament disassembly. But how local remodeling tunes stress production and dissipation, and how this in turn shapes long range flow, remains poorly understood. In this thesis, I attempt to describe the dynamics that underlie cortical flow in a specific cellular system, and extended the observations to a general modeling framework to understand how flows arise in many systems.

To begin, I first characterize the dynamics of cortical flow in the *C. elegans* embryo, and show that existing active fluid theories are adequate to describe the flow behavior of cell polarization and maintenance. I next investigate the turnover of cortical actin by using a recently developed method of monitoring individual actin monomer lifetimes *in vivo*. Additionally, I analyze existing data where filament turnover has been disrupted to show that cortical flows are abrogated. Instead, the networks only transiently contract, indicating that filament turnover is indeed crucial for cortical flows.

I next introduce a computational model for cross-linked networks with active motors based on minimal requirements for production and dissipation of contractile stress, namely asymmetric filament compliance, spatial heterogeneity of motor activity, reversible cross-links and filament turnover.

Finally, I characterize how the production and dissipation of network stress depend, individually, on cross-link dynamics and filament turnover, and how these dependencies combine to determine overall rates of cortical flow. My analysis predicts that filament turnover is required to maintain active stress against external resistance and steady state flow in response to external stress. Steady state stress increases with

filament lifetime up to a characteristic time  $\tau_a$ , then decreases with lifetime above  $\tau_a$ . Effective viscosity increases with filament lifetime up to a characteristic time  $\tau_c$ , and then becomes independent of filament lifetime and sharply dependent on crosslink dynamics. These individual dependencies of active stress and effective viscosity define multiple regimes of steady state flow. In particular my model predicts the existence of a regime, when filament lifetimes are shorter than both  $\tau_c$  and  $\tau_a$ , in which dependencies of effective viscosity and steady state stress cancel one another, such that flow speed is insensitive to filament turnover, and shows simple dependence on motor activity and crosslink dynamics. These results provide a framework for understanding how animal cells tune cortical flow through local control of network remodeling. Importantly, we learn that there must be some amount of local control of actomyosin activity in order for cortical flows to persist. As an additional project, I developed a model of upstream regulators of actin and myosin, showing how local recruitment through actin assembly can generate patterns of actomyosin contractility.

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# **CHAPTER 1**

## **INTRODUCTION**

## 1.1 Biological Context of Cortical Flow

Cortical flow is a fundamental and ubiquitous form of cellular deformation that underlies cell polarization, cell division, cell crawling and multicellular tissue morphogenesis[9, 40, 6, 101, 75, 68]. These flows originate within a thin layer of cross-linked actin filaments and myosin motors, called the actomyosin cortex, that lies just beneath the plasma membrane [82]. Local forces produced by bipolar myosin filaments are integrated within cross-linked networks to build macroscopic contractile stress[70, 5, 43]. At the same time, cross-linked networks resist deformation and this resistance must be dissipated by network remodeling to allow macroscopic deformation and flow. How force production and dissipation depend on motor activity and network remodeling remains poorly understood.

Recent work has also begun to reveal mechanisms for active stress generation in disordered actomyosin networks. Theoretical studies suggest that spatial heterogeneity in motor activity along individual filaments, and asymmetrical filament compliance (stiffer in extension than in compression), are sufficient for macroscopic contraction [51, 50], although other routes to contractility may also exist [50]. Local interactions among actin filaments and myosin motors are sufficient to drive macroscopic contraction of disordered networks *in vitro* [71], and the kinematics of contraction observed in these studies support a mechanism based on asymmetrical filament compliance and filament buckling. However, in these studies, the filaments were preassembled and network contraction was transient, because of irreversible network collapse[1], or buildup of elastic resistance[69], or because network rearrangements (polarity sorting) dissipate the potential to generate contractile force [24, 76, 74, 88]. This suggests that network turnover may play essential role(s) in allowing sustained production of contractile force. Recent theoretical and modeling studies have begun to explore how this might work [39, 61, 105], and to explore dynamic behaviors that can emerge when contractile material undergoes turnover [81, 21]. However, it remains a challenge to understand how force production and dissipation depend individually on the local interplay of network architecture, motor activity and filament turnover, and how these dependencies combine to mediate tunable control of long range cortical flow.

Studies in living cells reveal fluid-like stress relaxation on timescales of 10-100s [64, 40, 9, 41, 25, 3], which is thought to arise through a combination of cross link unbinding and actin filament turnover [19, 18, 82]. Theoretical [11, 67] and computational [47, 54, 52] studies reveal that cross-link unbinding can endow actin networks with complex time-dependent viscoelasticity. However, while cross-link unbinding is sufficient for viscous relaxation (creep) on very long timescales *in vitro*, it is unlikely to account for the rapid cortical deformation and flow observed in living cells [95, 53, 54, 103, 57]. Experimental studies in living cells reveal rapid turnover of cortical actin filaments on timescales comparable to stress relaxation (10-100s) [77, 29, 28, 13, 49]. Perturbing turnover can lead to changes in cortical mechanics and in the rates and patterns of cortical flow[93, 28]. However, the specific contributions of actin turnover to stress relaxation and how these depend on network architecture remain unclear.

## 1.2 Active fluid models of pattern formation and cortical flow

One successful approach to modeling cortical flow has relied on coarse-grained phenomenological descriptions of actomyosin networks as active fluids, whose motions are driven by gradients of active contractile stress and opposed by an effectively viscous resistance[64]. In these models, spatial variation in active stress is typically assumed to reflect spatial variation in motor activity and force transmission[8], while viscous resistance is assumed to reflect the internal dissipation of elastic resistance due to local remodeling of filaments and/or cross-links[82, 19]. Models combining an active fluid description with simple kinetics for network assembly and disassembly, can successfully reproduce the spatiotemporal dynamics of cortical flow observed during polarization [64], cell division [91, 81], cell motility [45, 62] and tissue morphogenesis [4]. However, it remains a challenge to connect this coarse-grained description of cortical flow to the microscopic origins of force generation and dissipation within cross-linked actomyosin networks.

Much of the recent theoretical interest in cortical flow centers on the biological

context of pattern formation. In particular, in the developing *C. elegans* embryo, an initial small asymmetry is magnified by cortical flow to produce a global polarized cell[68]. This global polarization from mechanical flow constitutes a mechanism of macroscopic pattern formation beyond the conventional Turing reaction-diffusion mechanism[42]. Therefore, it becomes all the more important to elucidate the specific micromechanical mechanisms that make it possible for a system to exhibit active fluid-like properties.

To understand how cells exert physiological control over cortical deformation and flow, or to build and tune networks with desired properties *in vitro*, it is essential to connect this coarse-grained description to the microscopic origins of force generation and dissipation within cross-linked actomyosin networks. Both active stress and effective viscosity depend sensitively on microscopic parameters including densities of filaments, motors and cross-links, force-dependent motor/filament interactions, cross-link dynamics and network turnover rates. Thus a key challenge is to understand how tuning these microscopic parameters controls the dynamic interplay between active force generation and passive relaxation to control macroscopic dynamics of cortical flow.

### 1.3 Rheology and Theory of Semi-flexible Cross-linked Networks

Cross-linked networks of semi-flexible polymers are a class of materials with poorly understood but highly interesting properties. Early studies of semi-flexible polymer networks reconstituted *in vitro* revealed novel, nonlinear rheology, spurring interest from materials scientists[12]. Cross-linked networks of cytoskeletal polymers have been a subject of great interest to biologists because of their importance as structural components of cells[27, 83].

On shorter timescales, the response of cross-linked polymer networks to applied stress can be well-described theoretically in terms of purely elastic mechanical resistance. On longer timescales, the network's elastic resistance begins to give way to a viscous relaxation of stored stress, but the mechanisms that govern this viscous

relaxation remain poorly understood. It is important to understand the mechanism behind this long timescale relaxation of cross-linked polymer networks both for understanding their novel material properties as well as understanding how this effect may govern physiologically important cellular processes[86].

For *in vitro* reconstitutions, this viscous relaxation is thought to result from transient unbinding and rebinding of intermolecular cross-links[95, 11]. However, there is still no clear understanding of how local relaxations of network connectivity would give rise to a global viscous relaxation. In our work, we wish to expand upon a well-established mechanical picture of cross-linked semi-flexible polymer networks to incorporate slippage of cross-links over longer timescales.

## 1.4 Short Timescale Mechanics of Cross-linked Actin Filament Networks

Early *in vitro* studies of cross-linked actin filament networks revealed strikingly different elastic behaviors compared to the already well-understood flexible polymer gels [89]. The complexity of these behaviors drove a surge in both experimental and theoretical studies of semi-flexible networks. For a comprehensive review of this field we recommend [12], but we will shortly repeat some important milestones here.

### 1.4.1 Theories of Semi-flexible Filament Networks

Diversity and discrepancy in observations led a drive toward systematic *in vitro* experimental explorations of the rheology of cross-linked semi-flexible polymer networks at short timescales. In studies with rigid irreversibly cross-linked networks, it was found that differences in network structure could lead to remarkably different elastic moduli, suggesting distinct phases of mechanical response [33]. These discoveries in turn begat theoretical work on the basic implications of the semi-flexible nature of filaments on network mechanics.

Prior work on the basic physics of individual semi-flexible polymers [63, 22], and comprehensive theories of semi-flexible filament solutions, [66] laid a groundwork

for theoretical considerations of cross-linked networks. Beginning with the so-called mikado model descriptions[37, 100], it was determined that there should exist a minimum rigidity percolation threshold, and that the connectivity of the network determined whether the mechanical response was dominated by non-affine bending or affine stretching of filaments. Continuing to more explicit theories[87], the mechanics of rigidly cross-linked networks were shown to be well-described in terms of purely elastic stretching of filaments between cross-linked points.

#### *1.4.2 Incorporating Effects of Cross-link Compliance*

Despite the success of the theory for rigid cross-links, early studies showed that surprising qualitative differences in mechanical response could be traced to differences in the chosen cross-linker[97, 55]. In addition, many studies using more compliant cross-linkers showed that cross-linker compliance could give rise to different nonlinear rheological properties on short timescales[31, 32, 44, 56]. Making matters even more complicated, ongoing research has begun to uncover added complexity from more highly complex issues such as filament bundling[67, 17] and the effects of active cross-linking by molecular motors[48].

While theorists have built a number of largely successful models that help characterize different aspects of the cross-link dominated response[38, 102, 10], the diversity of behaviors of these networks makes a precise yet general theory more difficult.

### **1.5 Long Timescale Stress Relaxation from Transient Cross-link Unbinding**

At long timescales, the purely elastic behavior of cross-linked networks gives way to fluid-like stress relaxation. Additionally, fluid-like flows have been observed in a number of cellular processes[64, 40, 9, 41, 25, 3]. In *in vitro* studies, long timescale creep behaviors are thought to arise predominantly from the transient nature of filament binding for most biologically relevant cross-linkers[53, 54, 103, 57]. While the importance of cross-link dynamics in determining the mechanical response of semi-flexible

polymer networks has been known for at least 20 years[95], there is still a gap in our understanding of how microscopic cross-link unbinding relates to viscous flows.

### *1.5.1 Models of Stress Relaxation with Transient Cross-links*

The dependence of network rheology on cross-link unbinding is an active subject of theoretical research[67]. Several theoretical methods have addressed cross-link binding and unbinding directly [11, 67] in analytical approaches that allowed well-constrained fits for specific cross-linkers. These theories have therefore focused conceptually at the level of the cross-linked filament and were extended analytically to macroscopic networks. In another approach, modelers have taken cross-links as extended springlike structures [47] that are able to bind and unbind in simulated filament networks. Finally, other more ambitious simulations have even sought to interrogate the effects of cross-link unbinding in combination with the more complex mechanics of filament bundles[54, 52].

Ultimately, the complexity of the many theoretical approaches that have been applied to this problem have made it difficult to distinguish what, if any, core physical mechanisms may be sufficient to explain the observed forms of stress relaxation. We believe that serious qualitative understanding can be generated by focusing on some of the common elements exhibited in the aforementioned literature.

## **1.6 Goals of Thesis**

### *1.6.1 Elucidating the Importance of Filament Turnover*

The fundamental importance of actin filament turnover in setting the mechanical properties of a cell has been known to biologists for decades [16]. Nevertheless, theoretical investigators have only begun to explore the role of turnover quite recently [39, 61, 105]. A major factor contributing to the slow advancement of our understanding of the role of turnover in cell mechanics is the difficulty in experimentation. However, recent advances in experimental techniques of reconstituted systems should make turnover a viable avenue of study in the near future.

Another major factor preventing the advancement of our understanding of turnover's impact on cell mechanics and dynamics is the complexity already inherent to our computational models of polymer systems at many length and time scales[60]. In this work we wish to accelerate the interest in filament turnover as a fundamental contributor to cellular mechanics. To do so, we will deemphasize the particularities of the polymer and cross-linker models and create a simplified generic model. With this simplified model we will then be in a position to analyze the fundamental role of filament turnover.

### *1.6.2 Novelty of Cross-link Slip Approach*

We introduce a coarse-grained representation of filament cross-linking in which cross-linked filaments are able to slide past each other as molecular bonds form and rupture, akin to coarse-grained models of molecular friction[94, 85, 26]. This drag-like coupling has been shown to be an adequate approximation in the case of ionic cross-linking of actin[96, 14], and can be found in the theoretical basis of force-velocity curves for myosin bound filaments[2]. We propose that it will form a suitable bulk approximation in the presence of super molecular cross-links as well.

Importantly, this simplification allows us to extend our single polymer models to dynamical systems of larger network models for direct comparison between theory and modeling results. This level of coarse graining will therefore make it easier to understand classes of behavior for varying compositions of cross-linked filament networks. In addition, it allows us to compute a new class of numerical simulations efficiently, which gives us concrete predictions for behaviors in widely different networks with measurable dependencies on molecular details.

### *1.6.3 Bridging the Theoretical Gap Between Active Fluids and Polymer Models*

The goal of this work is to build a computational bridge between the microscopic description of cross-linked actomyosin networks and the coarse grained macroscopic description of an active fluid. We seek to capture the essential microscope features

(dynamic cross-links, active motors and semi flexible actin filaments with asymmetric compliance and continuous filament recycling), but in a way that is sufficiently simple to allow systematic exploration of how parameters that govern network deformation and flow in an active fluid theory depend on microscopic parameters. To this end, we introduce several coarse-grained approximations into our representation of filament networks. First, we represent semi-flexible actin filaments as simple springs with asymmetric compliance (stronger in extension than compression). Second, we replace dynamic binding/unbinding of elastic cross-links with a coarse-grained representation in terms of molecular friction [94, 85, 26], such that filaments can slide past each other against a constant fictional resistance. Third, we used a similar scheme to introduce active motors at filament crossover points with a simple linear force/velocity relationship, and we introduce dispersion of motor activity by making only a subset of filament overlaps active [2]. Finally, we model filament turnover by allowing entire filaments to appear and disappear with a fixed probabilities per unit time. Importantly, these simplifications allow us to extend our single polymer models to dynamical systems of larger network models for direct comparison between theory and modeling results. This level of coarse graining will therefore make it easier to understand classes of behavior for varying compositions of cross-linked filament networks. In addition, it allows us to compute a new class of numerical simulations efficiently, which gives us concrete predictions for behaviors in widely different networks with measurable dependencies on molecular details.

We use this model first to characterize the passive response of a cross-linked network to externally applied stress, then the buildup and maintenance of active stress against an external resistance, and finally the steady state flows produced by an asymmetric distribution of active motors in which active stress and passive resistance are dynamically balanced across the network. Our results reveal how network remodeling can tune cortical flow through simultaneous effects on active force generation and passive resistance to network deformation.

# CHAPTER 2

## SINGLE MOLECULE MEASUREMENTS OF ACTIN

### FILAMENT TURNOVER

#### 2.1 Work that I did

##### 2.1.1 Fitting algorithm for SMPreSS

The SMPreSS technique works by fitting timecourses of counts of single molecules. The equation to which it is fit gives estimates of the model parameters. Francois Robin and myself worked to devise the form of the model equation to be fit. Francois created one simplification for computation, and I found another one that was easier to use. I implemented the fitting function and implemented techniques to extract the errors from the curve fitting results in MATLAB.

##### 2.1.2 Control experiments to determine the accuracy of SMPreSS

Using the SMPreSS technique, we believed it was possible to independently measure the disassociation constant and the photobleaching rate of the system. To test this, Francois and I set out to systematically vary the laser intensity and determine if our fitting scheme found changes in the photobleaching rate, but not the disassociation constant. I collected and processed the majority of the data for these control experiments.

##### 2.1.3 Taking the proper mean for Par-3 measurements

We wished to use measurements of Par-3 disassociation constants as a benchmark with which to compare our technique to the literature. Originally, the group was taking a pure mean of single molecule lifetime over the total count of the number of

events, which led to an estimate of the lifetime that was far shorter than what had been measured previously by FRAP. The problem was simply that the way we were taking the mean was technically incorrect. In a given period of time, many more particles with a short lifetime will appear compared to relatively few with a long lifetime. Meanwhile if you were to observe at a single instance, there would be closer to an exponential distribution of short and long time events. Taking the average over a wide swath of time therefore, skews the measurement to including more and more short duration events while only counting the long duration events once (even though they appear in many time bins).

The correct way to take the mean is to weight the appearance duration of each event by the duration of the event. Another way to put this is that you only want to add the mean of the long time over and over for every observed timepoint that you have. After I incorporated this correction, our measurement lined up with previous measurements for the disassociation constant for Par-3, and we were able to publish this result as a corroboration of our techniques validity.

## 2.2 Contents of Paper

# CHAPTER 3

## FILAMENT RECYCLING AND SUSTAINED CONTRACTILE FLOWS IN AN ACTOMYOSIN NETWORK

### 3.1 Results

The goal of this study is to understand how cortical flow is shaped by the simultaneous dependencies of active stress and effective viscosity on filament turnover, crosslink drag and on “network parameters” that control filament density, elasticity and motor activity. We approach this in three steps: First, we analyze the passive deformation of a cross-linked network in response to an externally applied stress; we identify regimes in which the network response is effectively viscous and characterize the dependence of effective viscosity on network parameters and filament turnover. Second, we analyze the buildup and dissipation of active stress in cross-linked networks with active motors, as they contract against an external resistance; we identify conditions under which the network can produce sustained stress at steady state, and characterize how steady state stress depends on network parameters and filament turnover. Finally, we confirm that the dependencies of active stress and effective viscosity on network parameters and filament turnover are sufficient to predict the dynamics of networks undergoing steady state flow in response to spatial gradients of motor activity.

#### *3.1.1 Filament turnover allows and tunes effectively viscous steady state flow.*

**Networks with passive cross-links and no filament turnover undergo three stages of deformation in response to an extensional force.** To characterize the passive response of a cross-linked filament network without filament recycling,

we simulated a simple uniaxial strain experiment in which we pinned the network at one end, imposed an external stress at the opposite end, and then quantified network strain and internal stress as a function of time (Fig. ??E). The typical response occurred in three qualitatively distinct phases (Fig. 3.1A,C). At short times the response was viscoelastic, with a rapid buildup of internal stress and a rapid  $\sim$ exponential approach to a fixed strain (Fast viscoelastic response to extensional stress. Plots of normalized strain vs time during the elastic phase of deformation in passive networks under extensional stress. Measured strain is normalized by the equilibrium strain predicted for a network of elastic filaments without crosslink slip  $\gamma_{eq} = \sigma/G_0 = \sigma/(2\mu/l_c)$ . A), which represents the elastic limit in the absence of cross-link slip predicted by [37]. At intermediate times, the local stress and strain rate were approximately constant across the network (Fig. 3.1B), and the response was effectively viscous; internal stress remained constant while the network continued to deform slowly and continuously with nearly constant strain rate (shown as dashed line in Fig. 3.1C) as filaments slip past one another against the effective cross-link drag. In this regime, we can quantify effective viscosity,  $\eta_c$ , as the ratio of applied stress to the measured strain rate. Finally, as the network strain approached a critical value ( $\sim 30\%$  for the simulation in Fig. 3.1), strain thinning lead to decreased network connectivity, local tearing, and rapid acceleration of the network deformation (see inset in Fig. 3.1C).

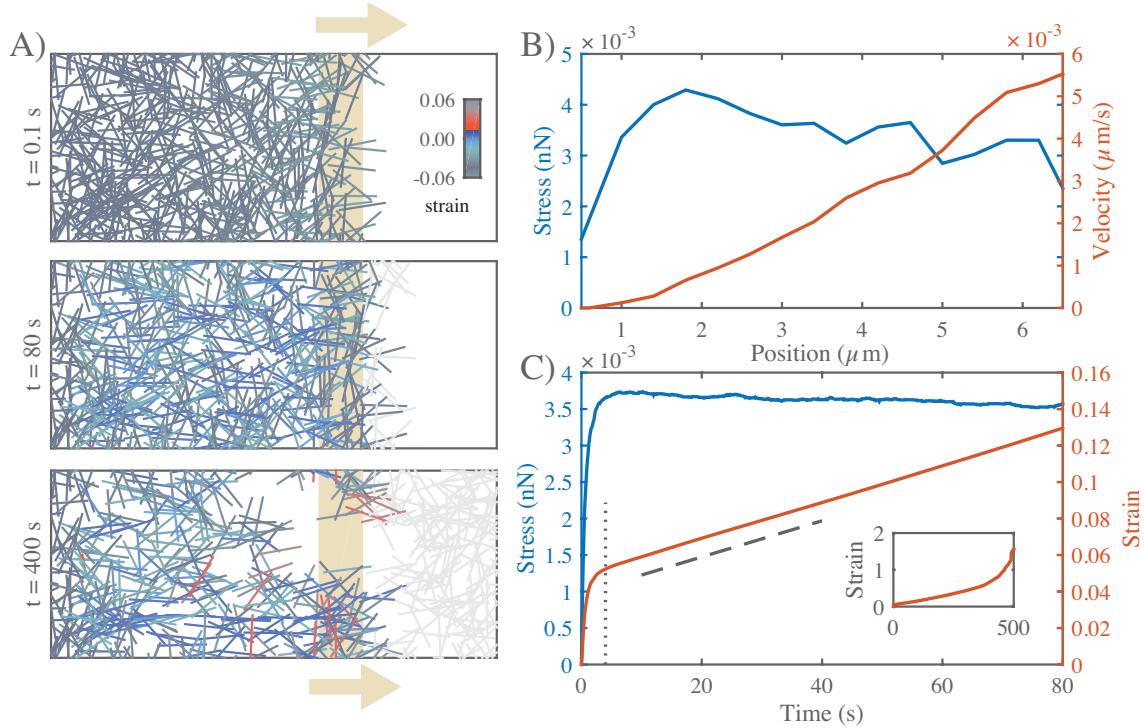


Figure 3.1: Networks with passive cross-links and no filament turnover undergo three stages of deformation in response to an extensional stress. **A)** Three successive time points from a simulation of a  $4 \times 6.6 \mu\text{m}$  network deforming under an applied stress of  $0.005 \text{ nN}/\mu\text{m}$ . Stress (tan arrows) is applied to filaments in the region indicated by the tan bar. In this and all subsequent figures, filaments are color-coded with respect to state of strain (blue = tension, red = compression). Network parameters:  $L = 1 \mu\text{m}$ ,  $l_c = 0.3 \mu\text{m}$ ,  $\xi = 100 \text{ nN} \cdot \text{s}/\mu\text{m}$ . **B)** Mean filament stress and velocity profiles for the network in (a) at  $t=88\text{s}$ . Note that the stress is nearly constant and the velocity is nearly linear as predicted for a viscous fluid under extension. **C)** Plots of the mean stress and strain vs time for the simulation in (a), illustrating the three stages of deformation: (i) A fast initial deformation accompanies rapid buildup of internal network stress; (ii) after a characteristic time  $\tau_c$  (indicated by vertical dotted line) the network deforms at a constant rate, i.e. with a constant effective viscosity,  $\eta_c$ , given by the slope of the dashed line; (iii) at long times, the network undergoes strain thinning and tearing (see inset)

**Network architecture sets the rate and timescales of deformation.** To characterize how effective viscosity and the timescale for transition to effectively viscous behavior depend on network architecture and cross-link dynamics, we simulated a uniaxial stress test, holding the applied stress constant, while varying filament length  $L$ , density  $l_c$ , elastic modulus  $\mu_e$  and cross link drag  $\xi$  (see S1 Table). We measured the elastic modulus,  $G_0$ , the effective viscosity,  $\eta_c$ , and the timescale  $\tau_c$  for transition from viscoelastic to effectively viscous behavior, and compared these to theoretical predictions. We observed a transition from viscoelastic to effectively viscous deformation for the entire range of parameter values that we sampled. Our estimate of  $G_0$  from simulation agreed well with the closed form solution  $G_0 \sim \mu/l_c$  predicted by a previous theoretical model [37] for networks of semi-flexible filaments with irreversible cross-links (Fig. 3.2B).

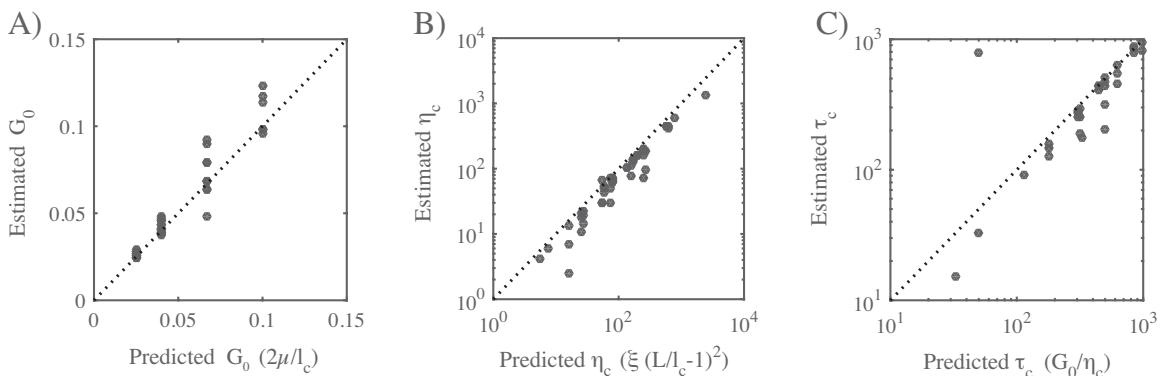


Figure 3.2: Network architecture sets the rate and timescales of deformation. **(a-c)** Comparison of predicted and simulated values for: **A)** the bulk elastic modulus  $G_0$ , **B)** the effective viscosity  $\eta_c$  and **C)** the timescale for transition from viscoelastic to viscous behavior  $\tau_c$ , given by the ratio of the bulk elastic modulus  $G_0$  to effective viscosity,  $\eta_c$ . Dotted lines indicate the relationships predicted by theory.

A simple theoretical analysis of filament networks with frictional cross link slip, operating in the intermediate viscous regime (see S1 Appendix A.2), predicted that the effective viscosity  $\eta_c$  should be proportional to the cross-link drag coefficient and to the square of the number of cross-links per filament:

$$\eta_c = 4\pi\xi \left( \frac{L}{l_c} - 1 \right)^2 \quad (3.1)$$

As shown in Fig. 3.2B, our simulations agree well with this prediction for a large range of sampled network parameters. Finally, for many linear viscoelastic materials, the ratio of effective viscosity to the elastic modulus  $\eta_c/G_0$  sets the timescale for transition from elastic to viscous behavior[65]. Combining our approximations for  $G_0$  and  $\eta_c$ , we predict a transition time,  $\tau_c \approx L^2\xi/l_c\mu$ . Measuring the time at which the strain rate became nearly constant (i.e.  $\gamma \sim t^n$  with  $n > 0.8$ ) yields an estimate of  $\tau_c$  that agrees well with this prediction over the entire range of sampled parameters (Fig. 3.2C). Thus the passive response of filament networks with frictional cross link drag is well-described on short (viscoelastic) to intermediate (viscous) timescales by an elastic modulus  $G_0$ , an effective viscosity  $\eta_c$ , and a transition timescale  $\tau_c$ , with well-defined dependencies on network parameters. However, without filament turnover, strain thinning and network tearing limits the extent of viscous deformation to small strains.

**Filament turnover allows sustained large-scale viscous flow and defines two distinct flow regimes.** To characterize how filament turnover shapes the passive network response to an applied force, we introduced a simple form of turnover in which entire filaments disappear at a rate  $k_{diss}\rho$ , where  $\rho$  is the filament density, and new unstrained filaments appear with a fixed rate per unit area,  $k_{app}$ . In a non-deforming network, filament density will equilibrate to a steady state value,  $\rho_0 = k_{ass}/k_{diss}$ , with time constant  $\tau_r = 1/k_{diss}$ . However, in networks deforming under extensional stress, the density will be set by a competition between strain thinning and density equilibration via turnover.

We simulated a uniaxial stress test for different values of  $\tau_r$ , while holding all other parameters fixed (Fig. 3.3A-C). For large  $\tau_r$ , as described above, the network undergoes strain thinning and ultimately tears. Decreasing  $\tau_r$  increases the rate at which the network equilibrates towards a steady state density  $\rho_0$ . However, it also increases the rate of deformation and thus the rate of strain thinning (Fig. 3.3B). We found that the former effect dominates, such that below a critical value  $\tau_r = \tau_{crit}$ , the network can achieve a steady state characterized by a fixed density and a constant strain rate (Filament turnover rescues strain thinning. **a)** Plots of strain vs time

for different turnover times (see inset in (b)). Note the increase in strain rates with decreasing turnover time. **b)** Plots of filament density vs strain for different turnover times  $\tau_r$ . For intermediate  $\tau_r$ , simulations predict progressive strain thinning, but at a lower rate than in the complete absence of recycling. For higher  $\tau_r$ , densities approach steady state values at longer times. ). Simple calculations (S1 Appendix A.3) show that the critical value of  $\tau_r$  is approximately:

$$\tau_{crit} = \frac{\xi \left( \sqrt{\frac{L}{l_c}} - 1 \right)^3}{\sigma}. \quad (3.2)$$

where  $\sigma$  is the applied stress,  $L/l_c$  the linear cross link density, and  $\xi$  is the effective crosslink drag.

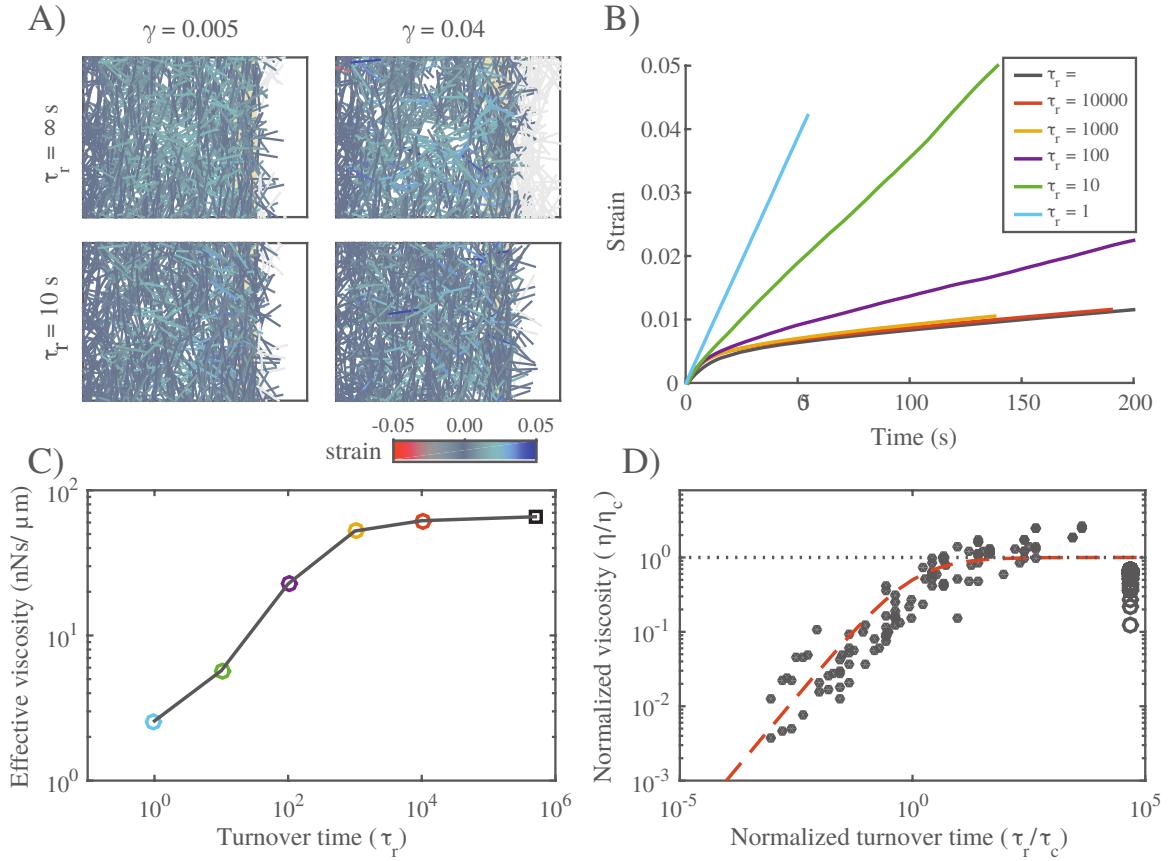


Figure 3.3: Filament recycling defines two regimes of effectively viscous flow. **A)** Comparison of  $20 \times 12 \mu\text{m}$  networks under  $0.001 \text{ nN}/\mu\text{m}$  extensional stress without (top) and with (bottom) filament turnover. Both images are taken when the networks had reached a net strain of 0.04. For clarity, filaments that leave the domain of applied stress are greyed out. **B)** Plots of strain vs time for identical networks with different rates of filament turnover. Network parameters:  $L = 5 \mu\text{m}$ ,  $l_c = 0.5 \mu\text{m}$ ,  $\xi = 10 \text{ nN} \cdot \text{s}/\mu\text{m}$ . **C)** Plot of effective viscosity vs turnover time derived from the simulations shown in panel b. Square dot is the  $\tau_r = \infty$  condition. **D)** Plot of normalized effective viscosity ( $\eta/\eta_c$ ) vs normalized turnover time ( $\tau_r/\tau_c$ ) for a large range of network parameters and turnover times. For  $\tau_r \ll \tau_c$ , the viscosity of the network becomes dependent on recycling time. Red dashed line indicates the approximation given in equation 3.3 for  $m = 3/4$ .

For  $\tau_r < \tau_{crit}$ , we observed two distinct steady state flow regimes (Fig. 3.3B,C). For intermediate values of  $\tau_r$ , effective viscosity remains constant with decreasing  $\tau_r$ . However, below a certain value of  $\tau_r$  ( $\approx 10^3$  for the parameters used in Fig. 3.3C), effective viscosity decreased monotonically with further decreases in  $\tau_r$ . To

understand what sets the timescale for transition between these two regimes, we measured effective viscosity at steady state for a wide range of network parameters ( $L, \mu, l_c$ ), crosslink drags ( $\xi$ ) and filament turnover times (Fig. 3.3D). Strikingly, when we plotted the normalized effective viscosity  $\eta_r/\eta_c$  vs a normalized recycling rate  $\tau_r/\tau_c$  for all parameter values, the data collapsed onto a single curve, with a transition at  $\tau_r \approx \tau_c$  between an intermediate turnover regime in which effective viscosity is independent of  $\tau_r$  and a high turnover regime in which effective viscosity falls monotonically with decreasing  $\tau_r/\tau_c$  (Fig. 3.3D).

This biphasic dependence of effective viscosity on filament turnover can be understood intuitively as follows: As new filaments are born, they become progressively stressed as they stretch and reorient under local influence of surrounding filaments, eventually reaching an elastic limit where their contribution to resisting network deformation is determined by effective crosslink drag. The time to reach this limit is about the same as the time,  $\tau_c$ , for an entire network of initially unstrained filaments to reach an elastic limit during the initial viscoelastic response to uniaxial stress, as shown in Fig. 2b. For  $\tau_r < \tau_c$ , individual filaments do not have time, on average, to reach the elastic limit before turning over; thus the deformation rate is determined by the elastic resistance of partially strained filaments, which increases with lifetime up to  $\tau_r = \tau_c$ . For  $\tau_r > \tau_c$ , the deformation rate is largely determined by cross-link resistance to sliding of maximally strained filaments, and the effective viscosity is insensitive to further increase in  $\tau_r$ .

These results complement and extend a previous computational study of irreversibly cross-linked networks of treadmilling filaments deforming under extensional stress[46]. Kim et al identified two regimes of effectively viscous deformation: a “stress-dependent” regime in which filaments turnover before they become strained to an elastic limit and deformation rate is proportional to both applied stress and turnover rate; and a “stress-independent” regime in which filaments reach an elastic limit before turning over and deformation rate depends only on the turnover rate. The fast and intermediate turnover regimes that we observe here correspond to the stress-dependent and independent regimes described by Kim et al, but with a key difference. Without filament turnover, Kim et al’s model predicts that a network

cannot deform beyond its elastic limit. In contrast, our model predicts viscous flow at low turnover, governed by an effective viscosity that is set by cross-link density and effective drag. Thus our model provides a self-consistent framework for understanding how crosslink unbinding and filament turnover contribute separately to viscous flow and connects these contributions directly to previous theoretical descriptions of cross-linked networks of semi-flexible filaments.

In summary, our simulations predict that filament turnover allows networks to undergo viscous deformation indefinitely, without tearing, over a wide range of different effective viscosities and deformation rates. For  $\tau_r < \tau_{crit}$ , this behavior can be summarized by an equation of the form:

$$\eta = \frac{\eta_c}{1 + (\tau_c/\tau_r)^m} \quad (3.3)$$

For  $\tau_r \gg \tau_c$ ,  $\eta \approx \eta_c$ : effective viscosity depends on crosslink density and effective crosslink drag, independent of changes in recycling rate. For  $\tau_r \ll \tau_c$ , effective viscosity is governed by the level of elastic stress on network filaments, and becomes strongly dependent on filament lifetime:  $\eta \sim \eta_c(\tau_r/\tau_c)^m$ . The origins of the  $m = 3/4$  scaling remains unclear (see Discussion).

### *3.1.2 Filament turnover allows persistent stress buildup in active networks*

**In the absence of filament turnover, active networks with free boundaries contract and then stall against passive resistance to network compression.** Previous work [51, 71, 48] identifies asymmetric filament compliance and spatial heterogeneity in motor activity as minimal requirements for macroscopic contraction of disordered networks. To confirm that our simple implementation of these two requirements (see Models section) is sufficient for macroscopic contraction, we simulated active networks that are unconstrained by external attachments, varying filament length, density, crosslink drag and motor activity. We observed qualitatively similar results for all choices of parameter values: Turning on motor activity in an initially unstrained network induced rapid initial contraction, followed by a slower buildup

of compressive stress (and strain) on individual filaments, and an  $\sim$ exponential approach to stall (Fig. 3.4). The time to stall,  $\tau_s$ , scaled as  $L\xi/v$  (Mechanical properties of active networks. **a)** Time for freely contracting networks to reach maximum strain,  $\tau_s$ , scales with  $L\xi/v$ . **b)** Free contraction requires asymmetric filament compliance, and total network strain increases with the applied myosin force  $v$ . Note that the maximum contraction approaches an asymptotic limit as the stiffness asymmetry approaches a ratio of  $\sim 100$ . **c)** Maximum stress achieved during isometric contraction,  $\sigma_m$ , scales approximately with  $\sqrt{\mu_e v}/l_c$ . **d)** Time to reach max stress during isometric contraction scales approximately with  $L\xi/\sqrt{\mu_e v}$ . Scalings for  $\tau_s$ ,  $\sigma_m$  and  $\tau_m$  were determined empirically by trial and error, guided by dimensional analysis. A). On even longer timescales, polarity sorting of individual filaments, as previously described [76, 69, 74, 88] lead to network expansion (see S2 Video).

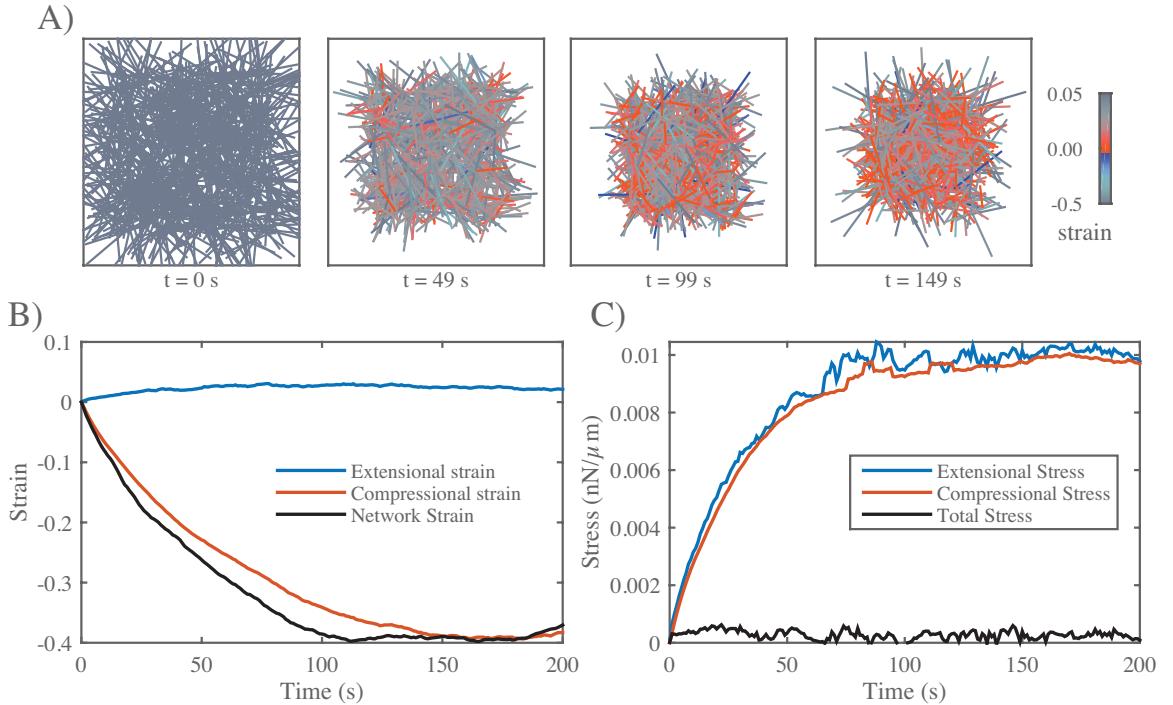


Figure 3.4: In the absence of filament turnover, active networks with free boundaries contract and then stall against passive resistance to network compression. **A)** Simulation of an active network with free boundaries. Colors represent strain on individual filaments as in previous figures. Note the buildup of compressive strain as contraction approaches stall between 100 s and 150 s. Network parameters:  $L = 5 \mu\text{m}$ ,  $l_c = 0.3 \mu\text{m}$ ,  $\xi = 1 \text{nN} \cdot \text{s}/\mu\text{m}$ ,  $v = 0.1 \text{nN}$ . **B)** Plots showing time evolution of total network strain (black) and the average extensional (blue) or compressive (red) strain on individual filaments. **C)** Plots showing time evolution of total (black) extensional (blue) or compressive (red) stress. Note that extensional and compressive stress remain balanced as compressive resistance builds during network contraction.

During the rapid initial contraction, the increase in network strain closely matched the increase in mean compressive strain on individual filaments Fig. 3.4B, as predicted theoretically [51, 50] and observed experimentally[71]. Contraction required asymmetric filament compliance and spatial heterogeneity of motor activity ( $\mu_e/\mu_c > 1$ ,  $\phi < 1$ , Mechanical properties of active networks. **a)** Time for freely contracting networks to reach maximum strain,  $\tau_s$ , scales with  $L\xi/v$ . **b)** Free contraction requires asymmetric filament compliance, and total network strain increases with the applied myosin force  $v$ . Note that the maximum contraction approaches an asymptotic limit

as the stiffness asymmetry approaches a ratio of  $\sim 100$ . **c)** Maximum stress achieved during isometric contraction,  $\sigma_m$ , scales approximately with  $\sqrt{\mu_e v}/l_c$ . **d)** Time to reach max stress during isometric contraction scales approximately with  $L\xi/\sqrt{\mu_e v}$ . Scalings for  $\tau_s$ ,  $\sigma_m$  and  $\tau_m$  were determined empirically by trial and error, guided by dimensional analysis. B). Thus our model captures a minimal mechanism for bulk contractility in disordered networks through asymmetric filament compliance and dispersion of motor activity. However, in the absence of turnover, contraction is limited by internal buildup of compressive resistance and the dissipative effects of polarity sorting.

**Active networks cannot sustain stress against a fixed boundary in the absence of filament turnover.** During cortical flow, regions with high motor activity contract against a passive resistance from neighboring regions with lower motor activity. To understand how the active stresses that drive cortical flow are shaped by motor activity and network remodeling, we analyzed the buildup and maintenance of contractile stress in active networks contracting against a rigid boundary. We simulated active networks contracting from an initially unstressed state against a fixed boundary (Fig. 3.5A,B), and monitored the time evolution of mean extensional (blue), compressional (red) and total (black) stress on network filaments (Fig. 3.5C,D). We focused initially on the scenario in which there is no, or very slow, filament turnover, sampling a range of parameter values controlling filament length and density, motor activity, and crosslink drag.

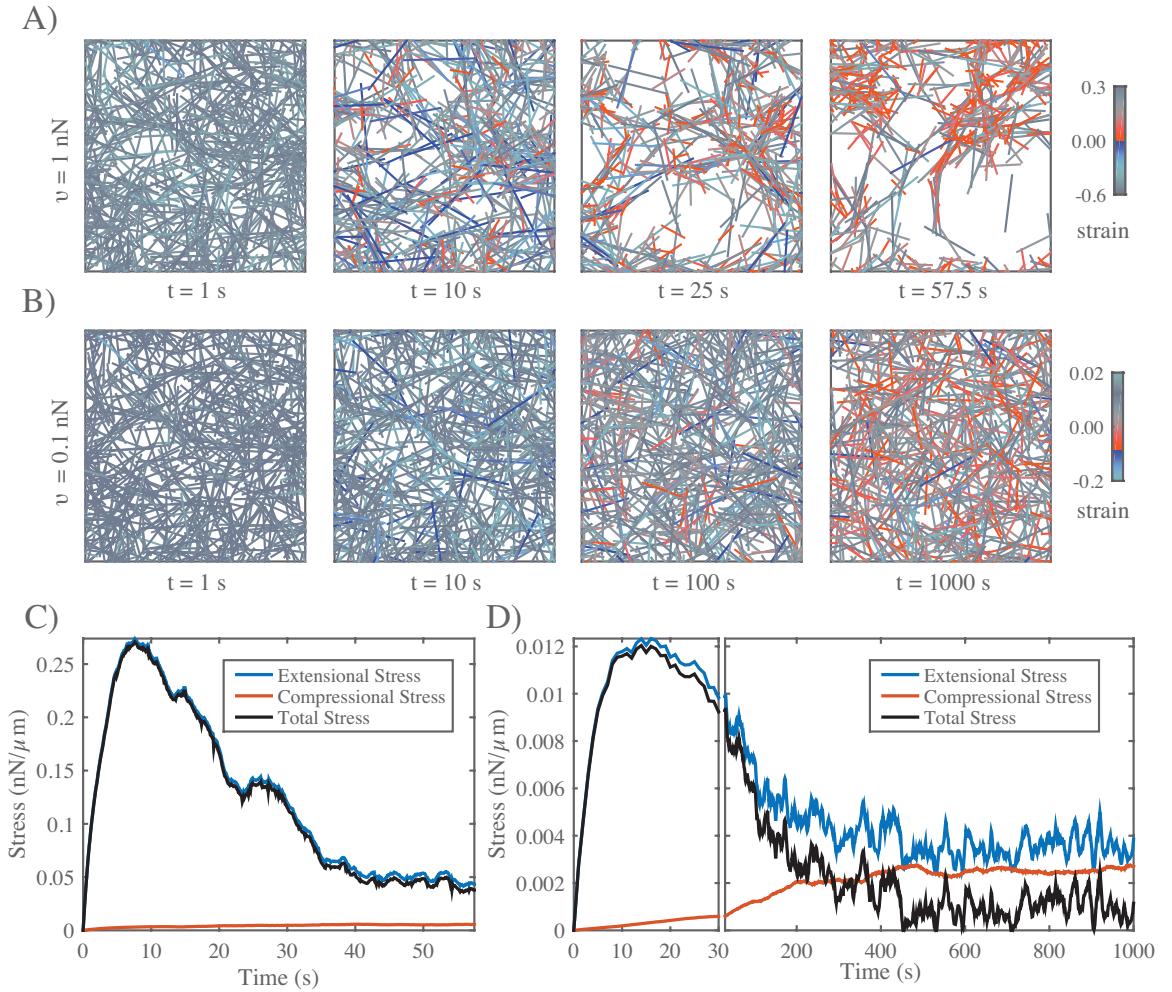


Figure 3.5: In the absence of filament turnover, active networks cannot sustain continuous stress against a fixed boundary. **A)** Simulation of an active network with fixed boundaries. Rearrangement of network filaments by motor activity leads to rapid loss of network connectivity. Network parameters:  $L = 5 \mu\text{m}$ ,  $l_c = 0.3 \mu\text{m}$ ,  $\xi = 1 \text{ nN} \cdot \text{s}/\mu\text{m}$ ,  $v = 1 \text{ nN}$ . **B)** Simulation of the same network, with the same parameter values, except with ten-fold lower motor activity  $v = 0.1 \text{ nN}$ . In this case, connectivity is preserved, but there is a progressive buildup of compressive strain on individual filaments. **C)** Plots of total network stress and the average extensional (blue) and compressive (red) stress on individual filaments for the simulation shown in (a). Rapid buildup of extensional stress allows the network transiently to exert force on its boundary, but this force is rapidly dissipated as network connectivity breaks down. **D)** Plots of total network stress and the average extensional (blue) and compressive (red) stress on individual filaments for the simulation shown in (b). Rapid buildup of extensional stress allows the network transiently to exert force on its boundary, but this force is dissipated at longer times as decreasing extensional stress and increasing compressive stress approach balance. Note the different time scales used for plots and subplots in **C)** and **D)** to emphasize the similar timescales for force buildup, but very different timescales for force dissipation.

We observed a similar behavior in all cases: total stress built rapidly to a peak value  $\sigma_m$ , and then decayed towards zero (Fig. 3.5C,D). The rapid initial increase in total stress was determined largely by the rapid buildup of extensional stress (Fig. 3.5C,D) on a subset of network filaments (Fig. 3.5A,B  $t = 10s$ ). The subsequent decay involved two different forms of local remodeling: under some conditions, e.g. for higher motor activity (e.g. Fig. 3.5A,C), the decay was associated with rapid local tearing and fragmentation, leading to global loss of network connectivity as described previously both in simulations[61] and *in vitro* experiments [1]. However, for many parameters, (e.g. for higher motor activity as in Fig. 3.5B,D), the decay in stress occurred with little or no loss of global connectivity. Instead, local filament rearrangements changed the balance of extensile vs compressive forces along individual filaments, leading to a slow decrease in the average extensional stress, and a correspondingly slow increase in the compressional stress, on individual filaments (see Fig. 3.5D).

Combining dimensional analysis with trial and error, we were able to find empirical scaling relationships describing the dependence of maximum stress  $\sigma_m$  and the time to reach maximum stress  $\tau_m$  on network parameters and effective crosslink drag ( $\sigma_m \sim \sqrt{\mu_e v}/l_c$ ,  $\tau_m \sim L\xi/\sqrt{\mu_e v}$ , Mechanical properties of active networks. **a)** Time for freely contracting networks to reach maximum strain,  $\tau_s$ , scales with  $L\xi/v$ . **b)** Free contraction requires asymmetric filament compliance, and total network strain increases with the applied myosin force  $v$ . Note that the maximum contraction approaches an asymptotic limit as the stiffness asymmetry approaches a ratio of  $\sim 100$ . **c)** Maximum stress achieved during isometric contraction,  $\sigma_m$ , scales approximately with  $\sqrt{\mu_e v}/l_c$ . **d)** Time to reach max stress during isometric contraction scales approximately with  $L\xi/\sqrt{\mu_e v}$ . Scalings for  $\tau_s$ ,  $\sigma_m$  and  $\tau_m$  were determined empirically by trial and error, guided by dimensional analysis. C,D). Although these relationships should be taken with a grain of salt, they are reasonably consistent with our simple intuition that the peak stress should increase with motor force ( $v$ ), extensional modulus ( $\mu_e$ ) and filament density ( $1/l_c$ ), and the time to reach peak stress should increase with crosslink drag ( $\xi$ ) and decrease with motor force ( $v$ ) and extensional modulus ( $\mu_e$ ).

**Filament turnover allows active networks to exert sustained stress on a fixed boundary.** Regardless of the exact scaling dependencies of  $\sigma_m$  and  $\tau_m$  on network parameters, these results reveal a fundamental limit on the ability of active networks to sustain force against an external resistance in the absence of filament turnover. To understand how this limit can be overcome by filament turnover, we simulated networks contracting against a fixed boundary from an initially unstressed state, for increasing rates of filament turnover (decreasing  $\tau_r$ ), while holding all other parameter values fixed (Fig. 3.6A-C). While the peak stress decreased monotonically with decreasing  $\tau_r$ , the steady state stress showed a biphasic response, increasing initially with decreasing  $\tau_r$ , and then falling off as  $\tau_r \rightarrow 0$ . We observed a biphasic response regardless of how stress decays in the absence of turnover, i.e. whether decay involves loss of network connectivity, or local remodeling without loss of connectivity, or both (Filament turnover prevents tearing of active networks. **a)** An active network undergoing large scale deformations due to active filament rearrangements. **b)** The same network as in (a) but with a shorter filament turnover time. **c)** Plots of internal stress vs time for the network in (a). **d)** Plots of internal stress vs time for the network in (b). and not shown). Significantly, when we plot normalized steady state stress ( $\sigma/\sigma_m$ ) vs normalized recycling time ( $\tau_r/\tau_m$ ) for a wide range of network parameters, the data collapse onto a single biphasic response curve, with a peak near  $\tau_r/\tau_m = 1$  (Fig. 3.6D). In particular, for  $\tau_r < \tau_m$ , the scaled data collapsed tightly onto a single curve representing a linear increase in steady state stress with increasing  $\tau_r$ . For  $\tau_r > \tau_m$ , the scaling was less consistent, although the trend towards a monotonic decrease with increasing  $\tau_r$  was clear. These results reveal that filament turnover can rescue the dissipation of active stress during isometric contraction due to network remodeling, and they show that, for a given choice of network parameters, there is an optimal choice of filament lifetime that maximizes steady state stress.

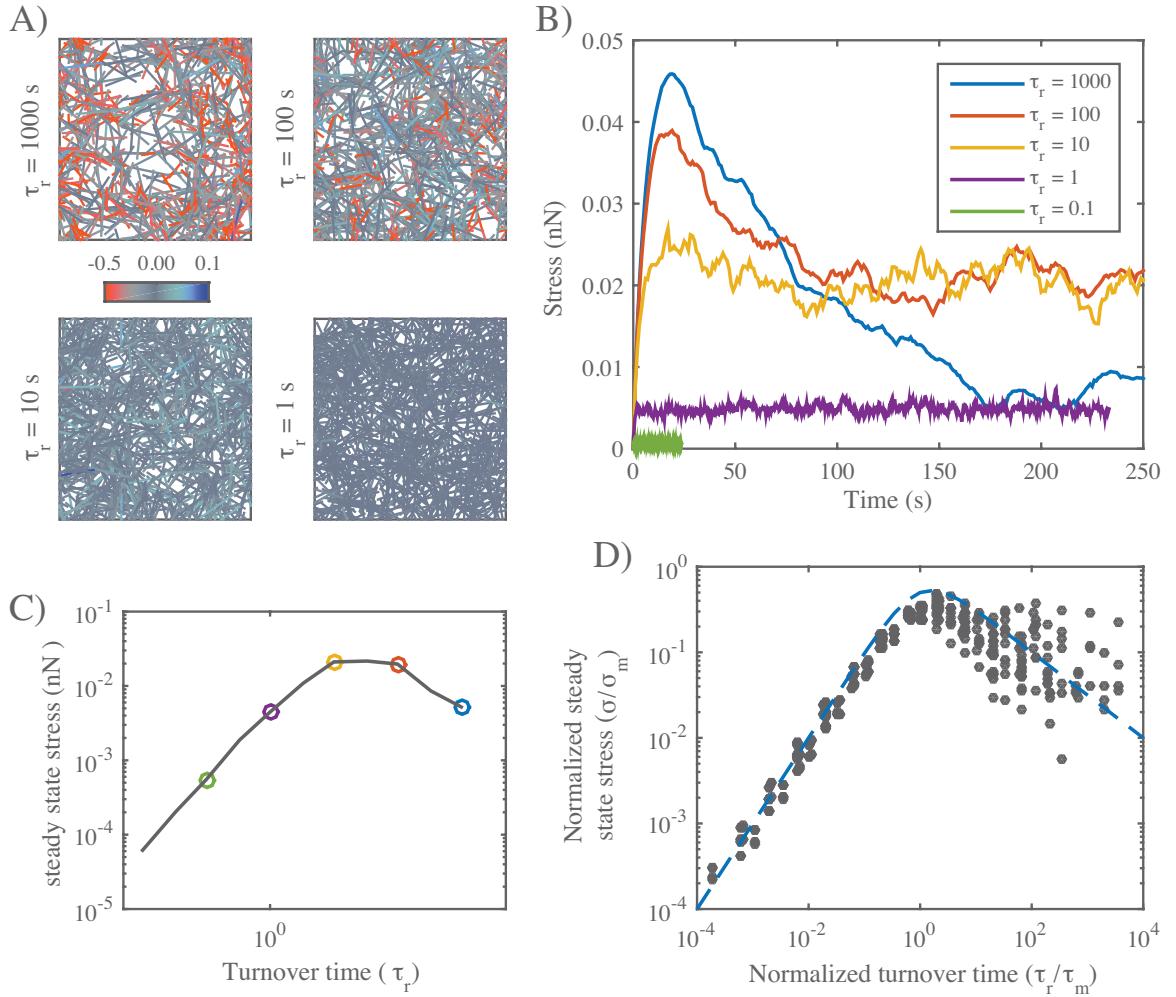


Figure 3.6: Filament turnover allows active networks to exert sustained stress on a fixed boundary. **A)** Snapshots from simulations of active networks with fixed boundaries and different rates of filament turnover. All other parameter values are the same as in Fig. 3.5A. Note the significant buildup of compressive strain and significant remodeling for longer, but not shorter, turnover times. **B)** Plots of net stress exerted by the network on its boundaries for different recycling times; for long-lived filaments, stress is built rapidly, but then dissipates. Decreasing filament lifetimes reduces stress dissipation by replacing compressed with uncompressed filaments, allowing higher levels of steady state stress; for very short lifetimes, stress is reduced, because individual filaments do not have time to build stress before turning over. **C)** Plots of  $\approx$ steady state stress estimated from the simulations in **B**) vs turnover time. **D)** Plot of normalized steady state stress vs normalized recycling time for a wide range of network parameters and turnover times. Steady state stress is normalized by the predicted maximum stress  $\sigma_m$  achieved in the absence of filament turnover. Turnover time is normalized by the predicted time to achieve maximum stress  $\tau_m$ , in the absence of filament turnover. Predictions for  $\sigma_m$  and  $\tau_m$  were obtained from the phenomenological scaling relations shown in (Fig. 3.12C,D). Dashed blue line indicates the approximation given in equation 3.4 for  $n = 1$ .

We can understand the biphasic dependence of steady state stress on filament lifetime using the same reasoning applied to the case of passive flow: During steady state contraction, the average filament should build and dissipate active stress on approximately the same schedule as an entire network contracting from an initially unstressed state (Fig. 3.6B). Therefore for  $\tau_r < \tau_m$ , increasing lifetime should increase the mean stress contributed by each filament. For  $\tau_r > \tau_m$ , further increases in lifetime should begin to reduce the mean stress contribution. Directly comparing the time-dependent buildup and dissipation of stress in the absence of turnover, with the dependence of steady state stress on  $\tau_r$ , supports this interpretation (Bimodal dependence on turnover time matches bimodal buildup and dissipation of stress in the absence of turnover. **a)** Bimodal buildup of stress in a network with very slow turnover ( $\tau_r = 1000s$ ). **b)** Steady state stress for networks with same parameters as in (a), but for a range of filament turnover times. )

As for the passive response (i.e. Equation 3.3), we can describe this biphasic dependence phenomenologically with an equation of the form:

$$\sigma_{ss} = \frac{\sigma_m}{(\tau_r/\tau_m)^n + \tau_m/\tau_r} \quad (3.4)$$

where the origins of the exponent  $n$  remain unclear.

### 3.1.3 Filament turnover tunes the balance between active stress buildup and viscous stress relaxation to generate flows

Thus far, we have considered independently how network remodeling controls the passive response to an external stress, or the steady state stress produced by active contraction against an external resistance. We now consider how these two forms of dependence will combine to shape steady state flow produced by spatial gradients of motor activity. We consider a simple scenario in which a network is pinned on either side and motor activity is continuously patterned such that the right half network has uniformly high levels of motor activity (controlled by  $v$ , with  $\psi = 0.5$ ), while the left half network has none. Under these conditions, the right half network will contract continuously against a passive resistance from the left half network. Because

of asymmetric filament compliance, the internal resistance of the right half network to active compression should be negligible compared to the external resistance of the left half network. Thus the steady state flow will be described by:

$$\dot{\gamma} = \frac{\sigma_{ss}}{\eta} \quad (3.5)$$

where  $\sigma_{ss}$  is the active stress generated by the right half-network (less the internal resistance to filament compression),  $\eta$  is the effective viscosity of the left half network and strain rate  $\dot{\gamma}$  is measured in the left half-network. Note that strain rate can be related to the steady state flow velocity  $v$  at the boundary between right and left halves through  $v = \dot{\gamma}Dx$ . Therefore, we can understand the dependence of flow speed on filament turnover and other parameters using the approximate relationships summarized by equations 3.3 and 3.4 for  $\eta$  and  $\sigma_{ss}$ . As shown in Fig. 3.7, there are two qualitatively distinct possibilities for the dependence of strain rate on  $\tau_r$ , depending on the relative magnitudes of  $\tau_m$  and  $\tau_c$ . In both cases, for fast enough turnover ( $\tau_r < \min(\tau_m, \tau_c)$ ), we expect weak dependence of strain rate on  $\tau_r$  ( $\dot{\gamma} \sim \tau_r^{1/4}$ ). For all parameter values that we sampled in this study (which were chosen to lie in a physiological range),  $\tau_m > \tau_c$ . Therefore we predict the dependence of steady state strain rate on  $\tau_r$  shown in Fig. 3.7A.

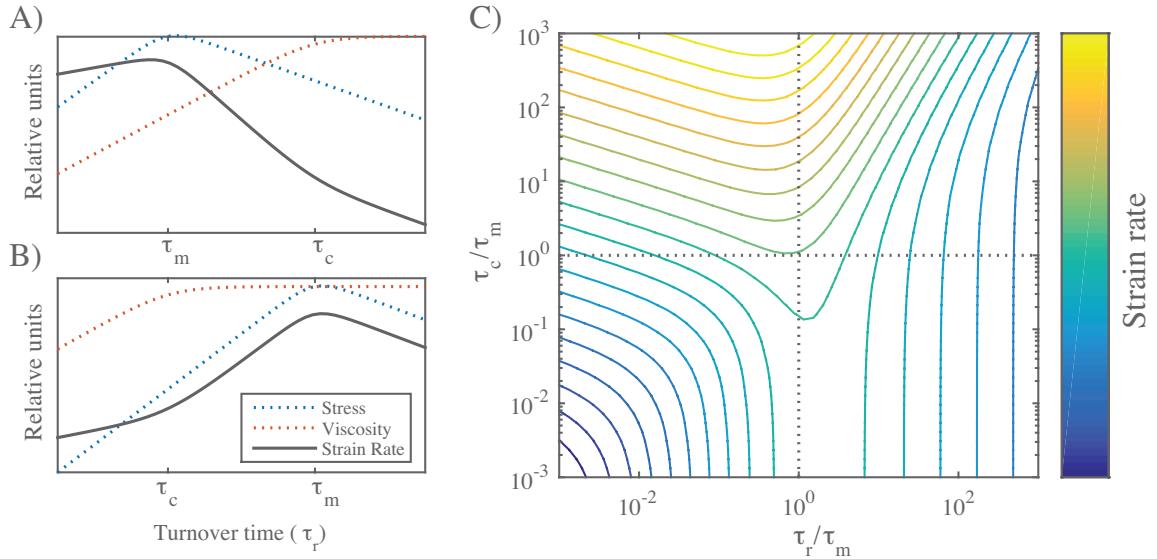


Figure 3.7: Filament recycling tunes the magnitudes of both effective viscosity and steady state stress. **A)** Dependence of steady state stress, effective viscosity, and resulting strain rate on recycling time  $\tau_r$  under the condition  $\tau_m < \tau_c$ . **B)** Same as a) but for  $\tau_c < \tau_m$ . **C)** State space of flow rate dependence relative to the two relaxation timescales,  $\tau_r$  and  $\tau_c$  normalized by the stress buildup timescale,  $\tau_m$ .

To confirm this prediction, we simulated the simple scenario described above for a range of values of  $\tau_r$ , with all other parameter values initially fixed. As expected, we observed a sharp dependence of steady state flow speeds on filament recycling rate (Fig. 3.8B,C). For very long recycling times, ( $\tau_r = 1000s$ , dark blue line), there was a rapid initial deformation (contraction of the active domain and dilation of the passive domain), followed by a slow approach to a steady state flow characterized by slow contraction of the right half-domain and a matching dilation of the left half-domain (see Dynamics of steady state flow. Plots of stress and strain vs position for networks in which motor activity is limited to the right-half domain and filament turnover time is either **a)**  $\tau_r = 10000$  or **b)**  $\tau_r = 10s$ . Blue indicates velocity while orange represents total stress, measured as described in the main text. ). However, with decreasing values of  $\tau_r$ , steady state flow speeds increased steadily, before reaching an approximate plateau on which flow speeds varied by less than 15 % over more than two decades of variation in  $\tau_r$  (Fig. 3.8C).

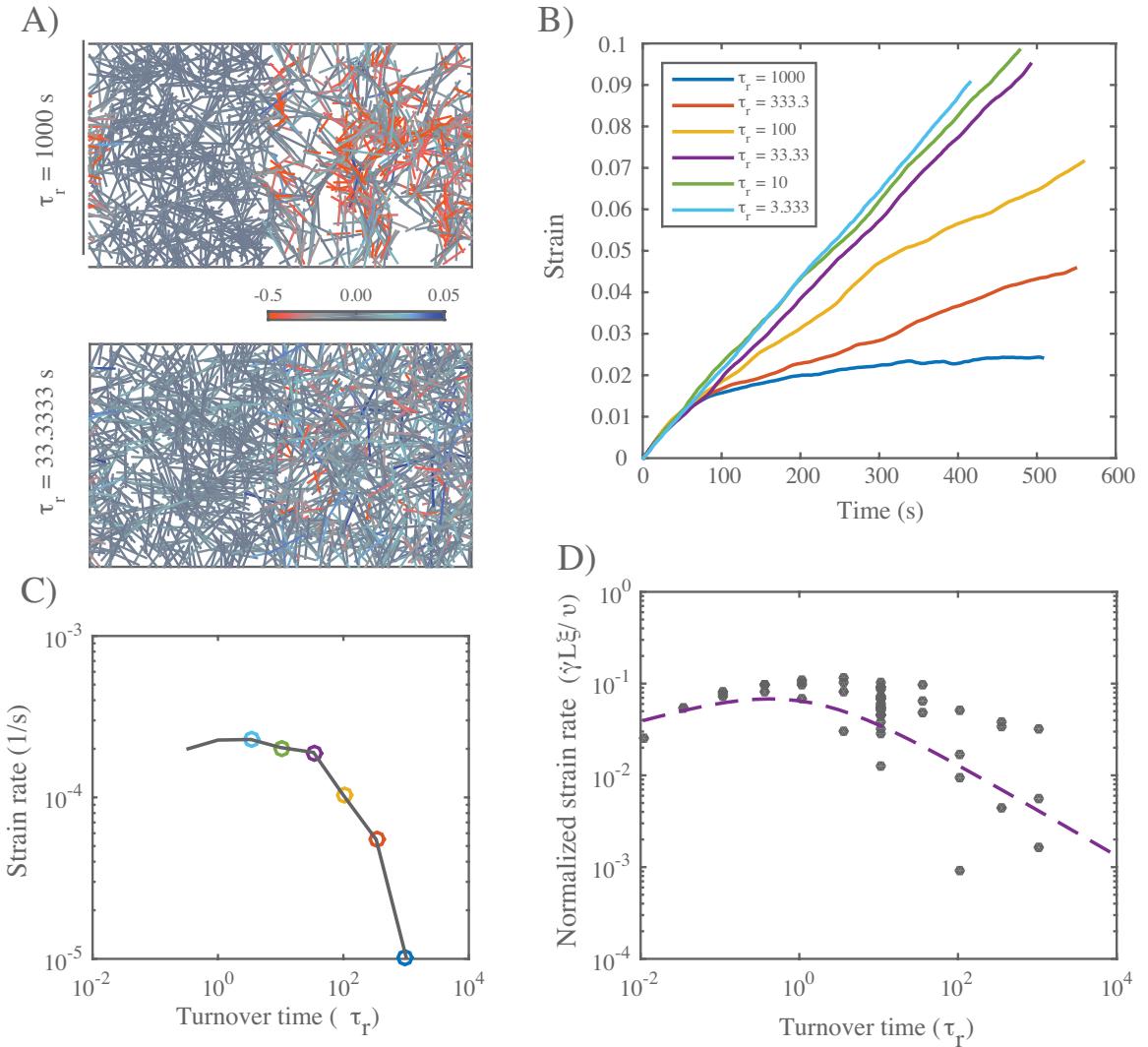


Figure 3.8: Filament recycling allows sustained flows in networks with non-isotropic activity. **A)** Example simulations of non-isotropic networks with long ( $\tau_r = 1000$ ) and short ( $\tau_r = 33$ ) recycling timescales. In these networks the left half of the network is passive while the right half is active. Network parameters are same as in Figs 3.5 and 3.6. Importantly, in all simulations  $\tau_m < \tau_c$ . **B)** Graph of strain for identical networks with varying recycling timescales. With long recycling times, the network stalls; reducing the recycling timescale allows the network to persist in its deformation. However, for the shortest recycling timescales, the steady state strain begins to slowly return to 0 net motion. Measurements are based on the passive side of the network. **C)** Steady state strain rates for the networks in (b). **D)** Graph of network's long-term strain rate as a function of recycling timescale. Dashed line is form of dependence predicted by the theoretical arguments shown in Fig. 3.7.

We repeated these simulations for a wider range of parameter values, and saw similar dependence of  $\dot{\gamma}$  on  $\tau_r$  in all cases. Using equation 3.3 with  $\tau_r < \tau_c$  and equation 3.4 with  $\tau_r < \tau_m$ , and the theoretical or empirical scaling relationships found above for  $\eta_c$ ,  $\tau_c$ ,  $\sigma_m$  and  $\tau_m$ , we predict a simple scaling relationship for  $\dot{\gamma}$  (for small  $\tau_r$ ):

$$\dot{\gamma} = \frac{v}{\xi L} (\tau_r)^{1/4} \quad (3.6)$$

Indeed, when we plot the steady state measurements of  $\dot{\gamma}$ , normalized by  $v/\xi L$ , for all parameter values, the data collapse onto a single curve for small  $\tau_r$ . Thus, our simulations identify a flow regime, characterized by sufficiently fast filament turnover, in which the steady state flow speed is buffered against variation in turnover, and has a relatively simple dependence on other network parameters.

### 3.2 Discussion

Cortical flows arise through a dynamic interplay of force production and dissipation within cross-linked actomyosin networks. Here we combined computer simulations with simple theoretical analysis to explore how this interplay depends on motor activity, crosslink dynamics, network architecture and filament turnover. Our results reveal two essential requirements for filament turnover during cortical flow: (a) to allow the continuous relaxation of elastic resistance without catastrophic loss of network connectivity and (b) to prevent the dissipation of active stress through local network rearrangements. We find that biphasic dependencies of active stress and passive relaxation on filament lifetime define multiple modes of steady state flow with distinct dependencies on network parameters and filament turnover.

We identify two distinct modes of passive response to uniaxial stress: a low turnover mode in which filaments strain to an elastic limit before turning over, and effective viscosity depends on crosslink density and effective crosslink friction, and a high turnover mode in which filaments turn over before reaching an elastic limit and effective viscosity is proportional to elastic resistance and  $\approx$  proportional to filament

lifetime. We note that the weakly sub-linear dependence of effective viscosity on filament lifetime that we observe in the high turnover regime may simply reflect a failure to capture very local modes of filament deformation, since a previous study [46] in which filaments were represented as connected chains of smaller segments predicted linear dependence of effective viscosity on filament lifetime. While previous studies have emphasized individual roles for cross-link unbinding or filament turnover in stress relaxation [19, 18, 82], here we have captured their distinct contributions within a single self-consistent modeling framework.

Our simulations confirm the theoretical prediction [51, 71, 48] that spatial heterogeneity of motor activity and asymmetric filament compliance are sufficient to support macroscopic contraction of unconstrained networks. However, under isometric conditions, and without filament turnover, our simulations predict that active stress cannot be sustained. On short timescales, motor forces drive local buildup of extensional stress, but on longer timescales, active local filament rearrangements drive local changes in connectivity that lead, invariably, to a decay in active stress. Under some conditions, contractile forces drive networks towards a critically connected state, leading to tearing and fragmentation, as previously described [1, 61]. However, we find that stress decay can also occur without any global loss of connectivity, through a gradual decrease in extensile force and a gradual increase in compressive force along individual filaments. Our results suggest that when filaments can slide relative to one another, the motor forces that produce active stress will inevitably lead to local changes in connectivity that decrease active stress. Thus for contractile networks to maintain isometric tension on long timescales, they must either form stable crosslinks to prevent filament rearrangements, or they must continuously recycle network filaments (or active motors) to renew the local potential for production of active stress.

Indeed, our simulations predict that filament turnover is sufficient for maintenance of active stress. As in the passive case, they predict biphasic dependence of steady state stress on filament turnover: For short-lived filaments ( $\tau_r < \tau_m$ ), steady state

stress increases linearly with filament lifetime because filaments have more time to build towards peak extensional stress before turning over. For longer lived filaments ( $\tau_r > \tau_m$ ), steady state stress decreases monotonically with filament lifetime because local rearrangements decrease the mean contributions of longer lived filaments. These findings imply that for cortical networks that sustain contractile stress under approximately isometric conditions, tuning filament turnover can control the level of active stress, and there will be an optimal turnover rate that maximizes the stress, all other things equal. This may be important, for example in early development, where contractile forces produced by cortical actomyosin networks maintain, or drive slow changes in, cell shape and tissue geometry [82, 35].

For cortical networks that undergo steady state flows driven by spatial gradients of motor activity, our simulations predict that the biphasic dependencies of steady state stress and effective viscosity on filament lifetime define multiple regimes of steady state flow, characterized by different dependencies on filament turnover (and other network parameters). In particular, the  $\sim$ linear dependencies of steady state stress and effective viscosity on filament lifetime for short-lived filaments define a fast turnover regime in which steady state flow speeds are buffered against variations in filament lifetime, and are predicted to depend in a simple way on motor activity and crosslink resistance. Measurements of F-actin turnover times in cells that undergo cortical flow [90, 72, 98, 36, 30, 77] suggests that they may indeed operate in this fast turnover regime, and recent studies in *C. elegans* embryos suggests that cortical flow speeds are surprisingly insensitive to depletion of factors (ADF/Cofilin) that govern filament turnover [64], consistent with our model’s predictions. Stronger tests of our model’s predictions will require more systematic analyses of how flow speeds vary with filament and crosslink densities, motor activities, and filament lifetimes.

### 3.2.1 Summary of Supplemental Materials

**S1 Appendix. Code Reference and Supplementary Methods** **A.1)** Reference to simulation and analysis code. **A.2)** Derivation of effective viscosity. **A.3)**

Derivation of critical turnover timescale for steady state flow

**S1 Table. Parameter values.** List of parameter values used for each set of simulations.

**S1 Fig. Fast viscoelastic response to extensional stress.** Plots of normalized strain vs time during the elastic phase of deformation in passive networks under extensional stress. Measured strain is normalized by the equilibrium strain predicted for a network of elastic filaments without crosslink slip  $\gamma_{eq} = \sigma/G_0 = \sigma/(2\mu/l_c)$ .

**S2 Fig. Filament turnover rescues strain thinning.** **A)** Plots of strain vs time for different turnover times (see inset in (b)). Note the increase in strain rates with decreasing turnover time. **B)** Plots of filament density vs strain for different turnover times  $\tau_r$ . For intermediate  $\tau_r$ , simulations predict progressive strain thinning, but at a lower rate than in the complete absence of recycling. For higher  $\tau_r$ , densities approach steady state values at longer times.

**S3 Fig. Mechanical properties of active networks.** **A)** Time for freely contracting networks to reach maximum strain,  $\tau_s$ , scales with  $L\xi/v$ . **B)** Free contraction requires asymmetric filament compliance, and total network strain increases with the applied myosin force  $v$ . Note that the maximum contraction approaches an asymptotic limit as the stiffness asymmetry approaches a ratio of  $\sim 100$ . **C)** Maximum stress achieved during isometric contraction,  $\sigma_m$ , scales approximately with  $\sqrt{\mu_e v}/l_c$ . **D)** Time to reach max stress during isometric contraction scales approximately with  $L\xi/\sqrt{\mu_e v}$ . Scalings for  $\tau_s$ ,  $\sigma_m$  and  $\tau_m$  were determined empirically by trial and error, guided by dimensional analysis.

**S4 Fig. Filament turnover prevents tearing of active networks.** Plots of normalized strain vs time during the elastic phase of deformation in passive networks under extensional stress. Measured strain is normalized by the equilibrium strain predicted for a network of elastic filaments without crosslink slip  $\gamma_{eq} = \sigma/G_0 = \sigma/(2\mu/l_c)$ .

**S5 Fig. Bimodal dependence on turnover time matches bimodal buildup and dissipation of stress in the absence of turnover.** **A)** Bimodal buildup of stress in a network with very slow turnover ( $\tau_r = 1000s$ ). **B)** Steady state stress for networks with same parameters as in (a), but for a range of filament turnover times.

**S6 Fig. Dynamics of steady state flow.** Plots of stress and strain vs position for networks in which motor activity is limited to the right-half domain and filament turnover time is either **A)**  $\tau_r = 10000$  or **B)**  $\tau_r = 10s$ . Blue indicates velocity while orange represents total stress, measured as described in the main text.

**S1 Video. Extensional strain in passive networks.** Movie of simulation setup shown in Fig. 3.1. Colors are the same as in figure.

**S2 Video. Active networks contracting with free boundaries.** Movie of simulation setup shown in Fig. 3.4. Colors are the same as in figure.

### 3.3 Supplemental Materials

#### 3.3.1 *Simulation and Analysis Code Available Online*

All of the simulation and analysis code for generating the figures in this paper is available online. To find the source code please visit our Github repository at

<https://github.com/wmcfadden/activnet>

#### 3.3.2 *Steady-state Approximation of Effective Viscosity*

We begin with a calculation of a strain rate estimate of the effective viscosity for a network described by our model in the limit of highly rigid filaments. We carry this out by assuming we have applied a constant stress along a transect of the network. With moderate stresses, we assume the network reaches a steady state affine creep. In this situation, we would find that the stress in the network exactly balances the

sum of the drag-like forces from cross-link slip. So for any transect of length D, we have a force balance equation.

$$\sigma = \frac{1}{D} \sum_{\text{filaments}} \sum_{\text{crosslinks}} \xi \cdot (\mathbf{v}_i(\mathbf{x}) - \mathbf{v}_j(\mathbf{x})) \quad (3.7)$$

where  $\mathbf{v}_i(\mathbf{x}) - \mathbf{v}_j(\mathbf{x})$  is the difference between the velocity of a filament,  $i$ , and the velocity of the filament,  $j$ , to which it is attached at the cross-link location,  $\mathbf{x}$ . We can convert the sum over cross-links to an integral over the length using the average density of cross-links,  $1/l_c$  and invoking the assumption of (linear order) affine strain rate,  $\mathbf{v}_i(\mathbf{x}) - \mathbf{v}_j(\mathbf{x}) = \dot{\gamma}x$ . This results in

$$\begin{aligned} \sigma &= \frac{1}{D} \sum_{\text{filaments}} \int_0^L \xi \cdot (\mathbf{v}_i(\mathbf{s}) - \mathbf{v}_j(\mathbf{s})) \frac{ds \cos \theta}{l_c} \\ &= \sum_{\text{filaments}} \frac{\xi \dot{\gamma} L}{l_c} \cos \theta \cdot (x_l + \frac{L}{2} \cos \theta) \end{aligned} \quad (3.8)$$

Here we have introduced the variables  $x_l$ , and  $\theta$  to describe the leftmost endpoint and the angular orientation of a given filament respectively. Next, to perform the sum over all filaments we convert this to an integral over all orientations and endpoints that intersect our line of stress. We assume for simplicity that filament stretch and filament alignment are negligible in this low strain approximation. Therefore, the max distance for the leftmost endpoint is the length of a filament, L, and the maximum angle as a function of endpoint is  $\arccos(x_l/L)$ . The linear density of endpoints is the constant  $D/l_c L$  so our integrals can be rewritten as this density over  $x_l$  and  $\theta$  between our maximum and minimum allowed bounds.

$$\sigma = \frac{1}{D} \int_0^L dx_l \int_{-\arccos(x_l/L)}^{\arccos(x_l/L)} \pi d\theta \frac{\xi \dot{\gamma} L}{l_c} \cdot \frac{D}{L l_c} \cdot (x_l \cos \theta + \frac{L}{2} \cos^2 \theta) \quad (3.9)$$

Carrying out the integrals and correcting for dangling filament ends leaves us with a relation between stress and strain rate.

$$\sigma = 4\pi \left( \frac{L}{l_c} - 1 \right)^2 \xi \dot{\gamma} \quad (3.10)$$

We recognize the constant of proportionality between stress and strain rate as a viscosity (in 2 dimensions). Therefore, our approximation for the effective viscosity,  $\eta_c$ , at steady state creep in this low strain limit is

$$\sigma = 4\pi \left( \frac{L}{l_c} - 1 \right)^2 \xi \quad (3.11)$$

### 3.3.3 Critical filament lifetime for steady state filament extension

We seek to determine a critical filament lifetime,  $\tau_{crit}$ , below which the density of filaments approaches a stable steady state under constant extensional strain. To this end, let  $\rho$  be the filament density (i.e. number of filaments per unit area). We consider a simple coarse grained model for how  $\rho$  changes as a function of filament assembly  $k_{ass}$ , filament disassembly  $k_{diss}$ ,  $\rho$  and strain thinning  $\dot{\gamma}\rho$ . Using  $\rho_0 = \frac{k_{ass}}{k_{diss}}$ ,  $\tau_r = \frac{1}{k_{diss}}$ , and  $\dot{\gamma} = \frac{\sigma}{\eta_c}$ .

$$\frac{d\rho}{dt} = \frac{1}{\tau_r} \left( \rho_0 - \rho - \frac{\sigma\tau_r}{\eta_c(\rho)} \rho \right) \quad (3.12)$$

where  $\eta_c = \eta_c(\rho)$  on the right hand side reflects the dependence of effective viscosity on network density. The strength of this dependence determines whether there exists a stable steady state, representing continuous flow. Using  $\eta_c(\rho) \sim \xi \left( \frac{L}{l_c(\rho)} - 1 \right)^2$  from above (ignoring the numerical prefactor) and  $\rho \sim \frac{2}{Ll_c(\rho)}$ , we obtain:

$$\frac{d\rho}{dt} = \frac{1}{\tau_r} \left( \rho_0 - \rho - \frac{\sigma\tau_r}{\xi(\rho L^2/2 - 1)^2} \rho \right) \quad (3.13)$$

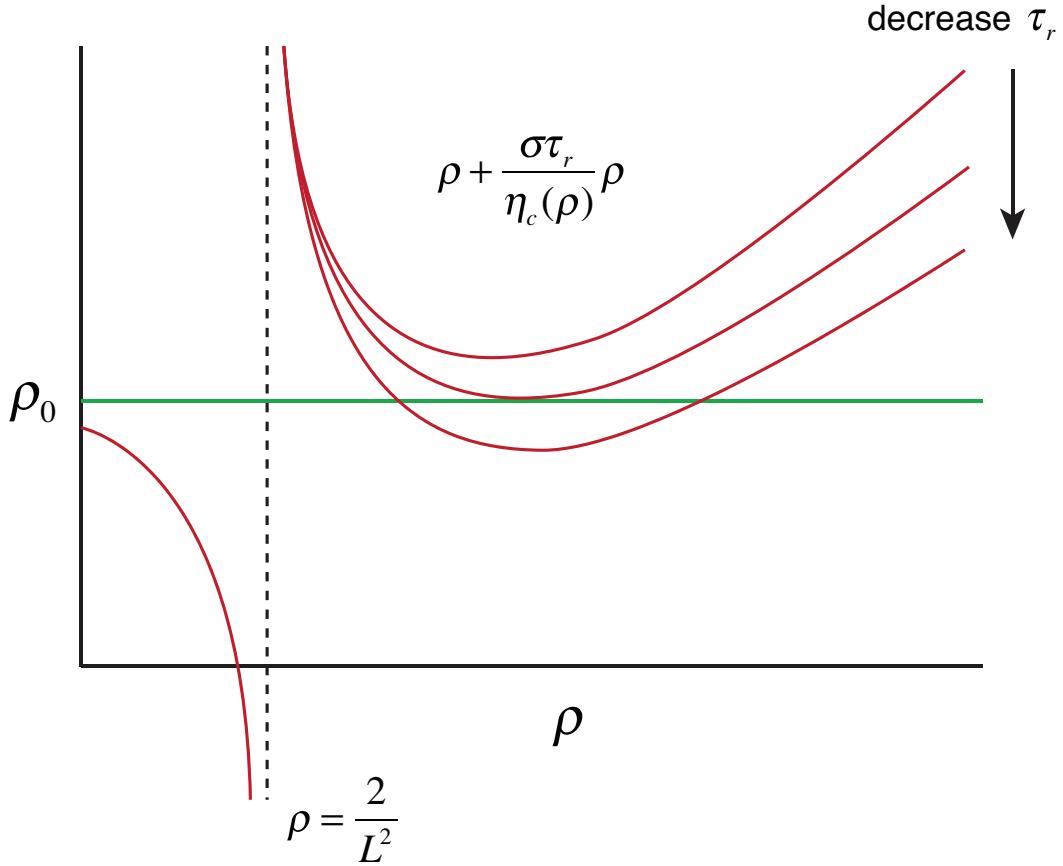


Figure 3.9: Flux balance analysis of network density. Qualitative plots of  $\rho + \frac{\sigma\tau_r}{\eta_c(\rho)}\rho$  (red curves) vs  $\rho_0$  (green line) for different values of  $\tau_r$ . For sufficiently large  $\tau_r$ , there are no crossings. For  $\tau_r < \tau_{crit}$ , there are two crossings: The rightmost crossing represents a stable steady state.

Figure 3.9 sketches the positive ( $\rho_0$ ) and negative ( $\rho + \frac{\sigma\tau_r}{\eta_c(\rho)}\rho$ ) contributions to the right hand side of Equation 6 for different values of  $\tau_r$ . For sufficiently large  $\tau_r$ , there is no stable state, i.e. strain thinning will occur. However, as  $\tau_r$  decreases below a critical value  $\tau_{crit}$ , a stable steady state appears. Note that when  $\tau_r = \tau_{crit}$ ,  $\rho + \frac{\sigma\tau_r}{\eta_c(\rho)}\rho$  passes through a minimum value  $\rho_0$  at  $\rho = \rho^*$ . Accordingly, to determine  $\tau_{crit}$ , we solve:

$$0 = \frac{d}{d\rho} \left( \rho + \frac{\sigma\tau_r}{\eta_c(\rho)} \rho \right) = 1 - \frac{\sigma\tau_r}{\xi(\rho L^2/2 - 1)^3} \quad (3.14)$$

From this, with some algebra, we infer that

$$\rho^* = \frac{2}{L^2} \left( 1 + \left( \frac{\sigma\tau_r}{\xi} \right)^{1/3} \right) \quad (3.15)$$

and

$$\frac{\sigma\tau_r}{\eta_c(\rho^*)} = \left( \frac{\sigma\tau_r}{\xi} \right)^{1/3} \quad (3.16)$$

We seek a value for  $\tau_r = \tau_{crit}$  at which

$$\rho^* + \frac{\sigma\tau_{crit}}{\eta_c(\rho^*)} \rho^* = \rho_0 \quad (3.17)$$

Substituting from above, and using  $\rho_0 = \frac{2}{Ll_c}$ , we have:

$$\frac{2}{L^2} \left( 1 + \left( \frac{\sigma\tau_{crit}}{\xi} \right)^{1/3} \right) \left( 1 + \left( \frac{\sigma\tau_{crit}}{\xi} \right)^{1/3} \right) = \frac{2}{Ll_c} \quad (3.18)$$

Finally, rearranging terms, we obtain

$$\tau_{crit} = \frac{\xi}{\sigma} \left( \sqrt{\frac{L}{l_c}} - 1 \right)^3 \quad (3.19)$$

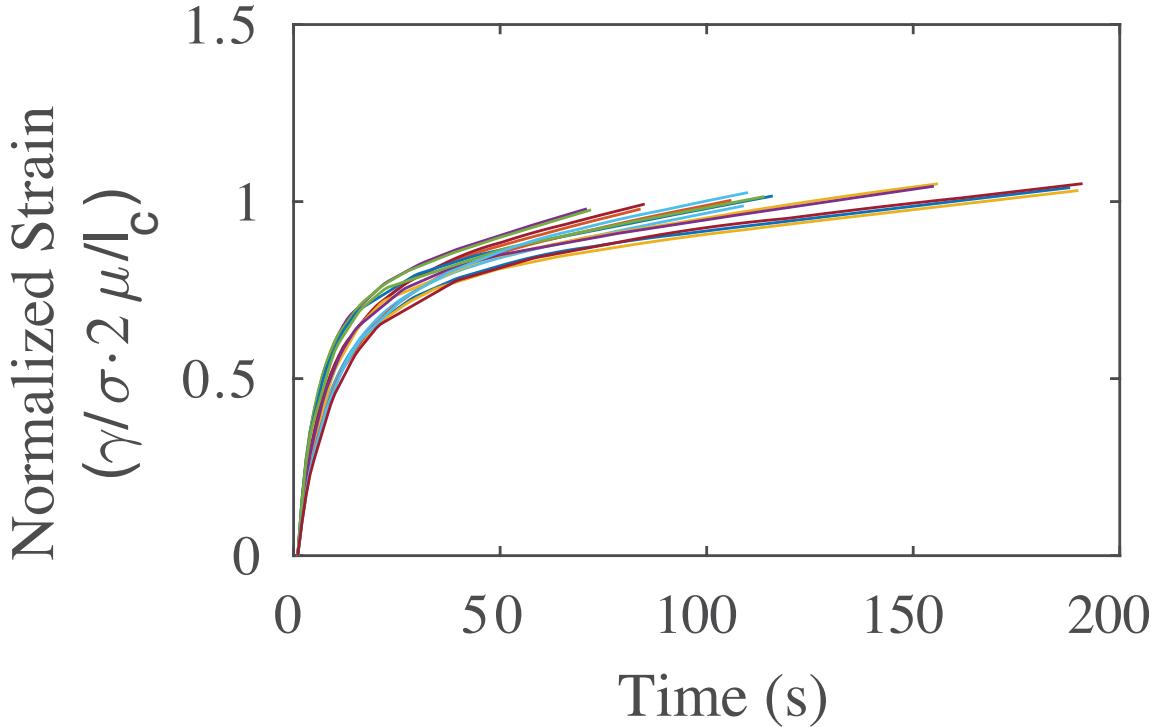


Figure 3.10: Fast viscoelastic response to extensional stress. Plots of normalized strain vs time during the elastic phase of deformation in passive networks under extensional stress. Measured strain is normalized by the equilibrium strain predicted for a network of elastic filaments without crosslink slip  $\gamma_{eq} = \sigma/G_0 = \sigma/(2\mu/l_c)$ .

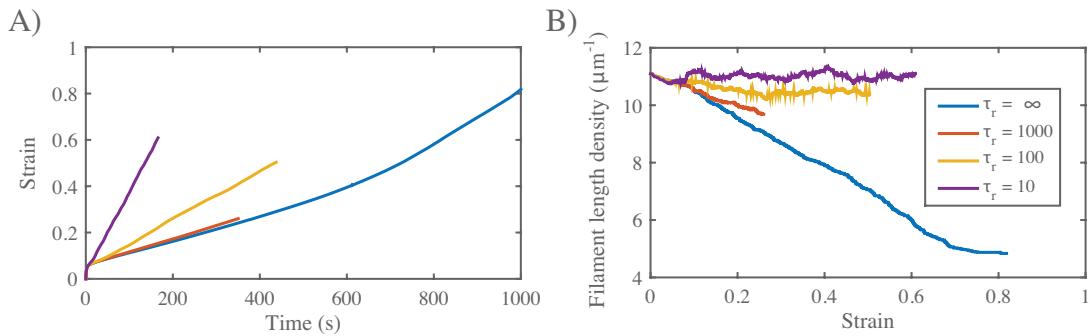


Figure 3.11: Filament turnover rescues strain thinning. **a)** Plots of strain vs time for different turnover times (see inset in (b)). Note the increase in strain rates with decreasing turnover time. **b)** Plots of filament density vs strain for different turnover times  $\tau_r$ . For intermediate  $\tau_r$ , simulations predict progressive strain thinning, but at a lower rate than in the complete absence of recycling. For higher  $\tau_r$ , densities approach steady state values at longer times.

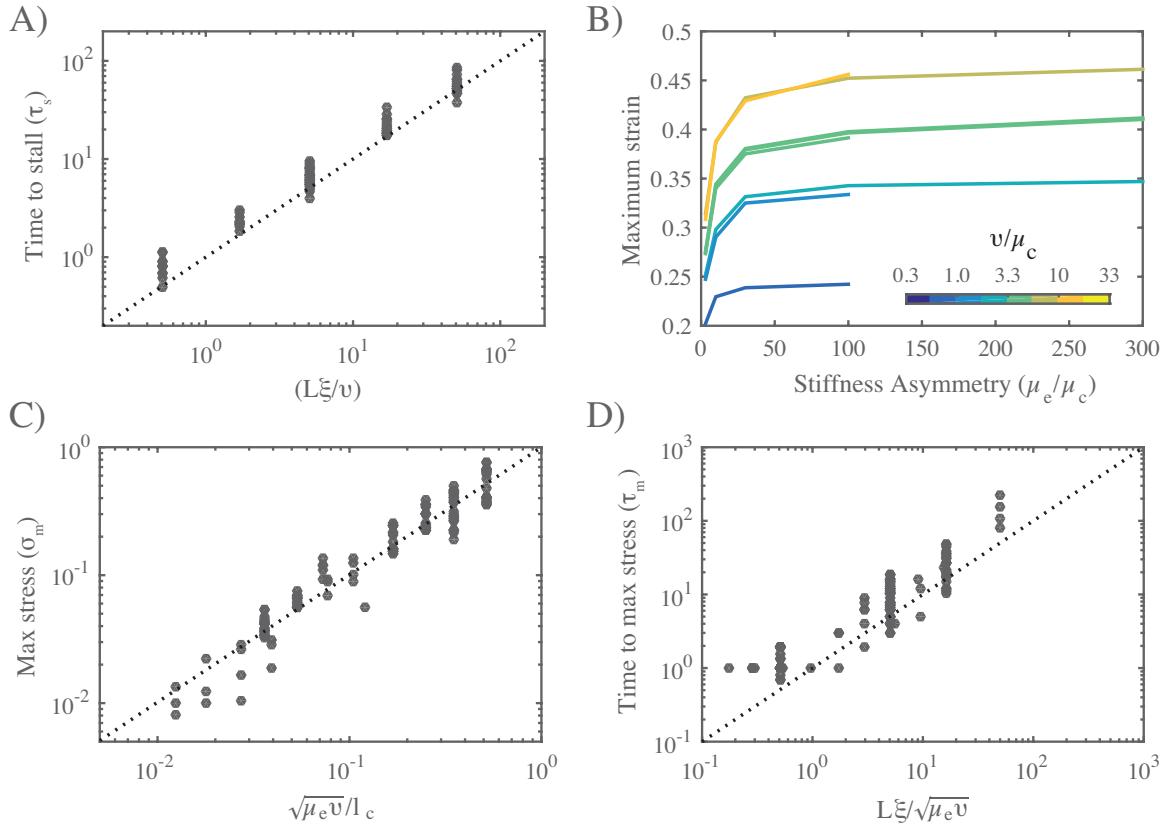


Figure 3.12: Mechanical properties of active networks. **a)** Time for freely contracting networks to reach maximum strain,  $\tau_s$ , scales with  $L\xi/v$ . **b)** Free contraction requires asymmetric filament compliance, and total network strain increases with the applied myosin force  $v$ . Note that the maximum contraction approaches an asymptotic limit as the stiffness asymmetry approaches a ratio of  $\sim 100$ . **c)** Maximum stress achieved during isometric contraction,  $\sigma_m$ , scales approximately with  $\sqrt{\mu_e v}/l_c$ . **d)** Time to reach max stress during isometric contraction scales approximately with  $L\xi/\sqrt{\mu_e v}$ . Scalings for  $\tau_s$ ,  $\sigma_m$  and  $\tau_m$  were determined empirically by trial and error, guided by dimensional analysis.

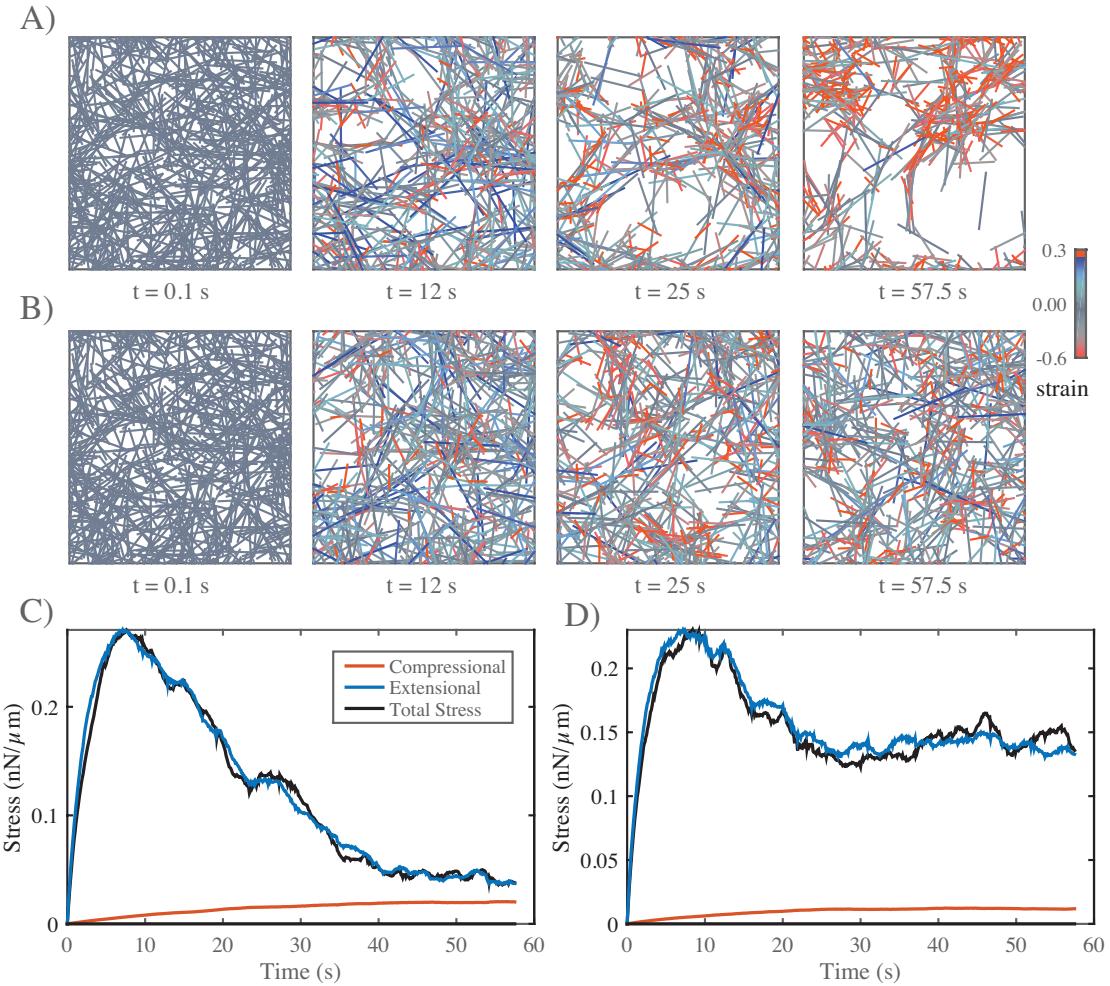


Figure 3.13: Filament turnover prevents tearing of active networks. **a)** An active network undergoing large scale deformations due to active filament rearrangements. **b)** The same network as in (a) but with a shorter filament turnover time. **c)** Plots of internal stress vs time for the network in (a). **d)** Plots of internal stress vs time for the network in (b).

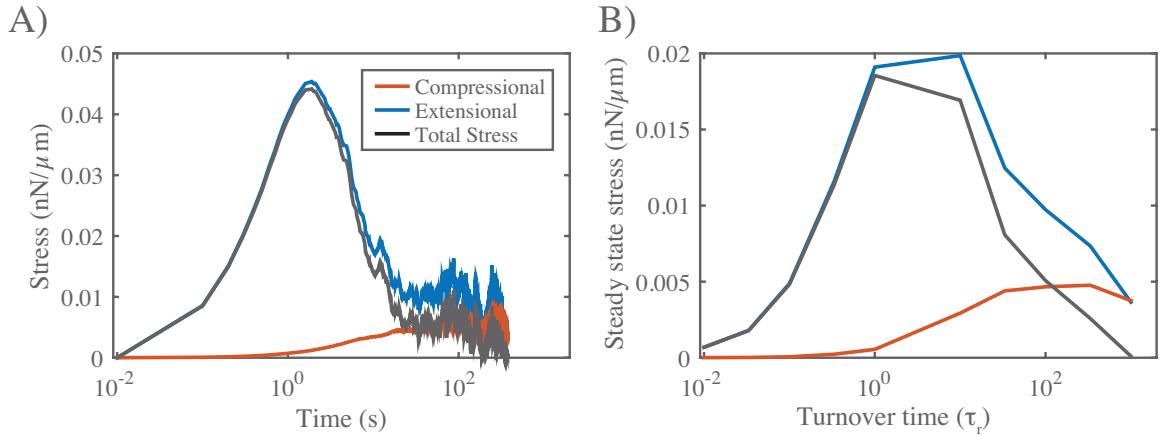


Figure 3.14: Bimodal dependence on turnover time matches bimodal buildup and dissipation of stress in the absence of turnover. **a)** Bimodal buildup of stress in a network with very slow turnover ( $\tau_r = 1000\text{s}$ ). **b)** Steady state stress for networks with same parameters as in (a), but for a range of filament turnover times.

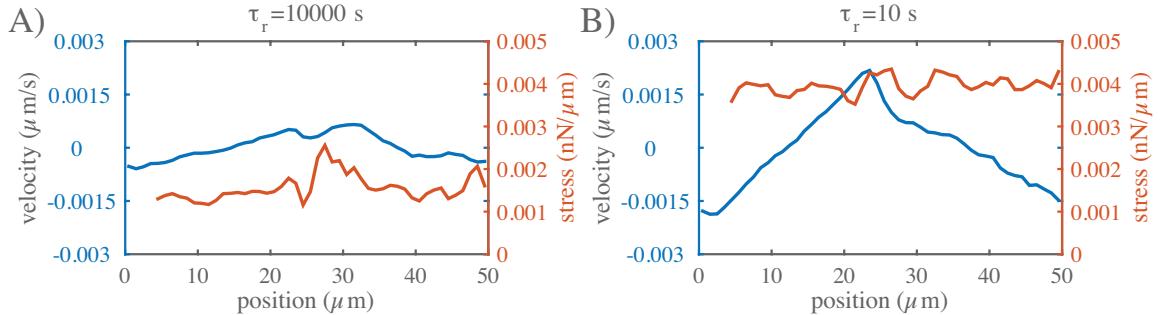


Figure 3.15: Dynamics of steady state flow. Plots of stress and strain vs position for networks in which motor activity is limited to the right-half domain and filament turnover time is either **a)**  $\tau_r = 10000\text{s}$  or **b)**  $\tau_r = 10\text{s}$ . Blue indicates velocity while orange represents total stress, measured as described in the main text.

Table 3.1: Simulation Parameter Values

Parameter	<b>Fig 3</b>	<b>Fig 4</b>	<b>Fig S2a,b</b>	<b>Fig S2c,d</b>	<b>Fig 7</b>	<b>Fig 9</b>
$L$	1, 3, 5, 7, 10	3	3, 5	3, 5	5	3, 5, 8
$l_c$	0.2, 0.3, 0.5, 0.8	0.3, 0.5	0.3	0.15, 0.2, 0.3, 0.4	0.2, 0.3	0.15, 0.2, 0.3, 0.4
$\mu_e/\mu_c$	100	100	3 – 300	100	100	100
$\mu_c$	0.01	0.01	0.01 – 0.3	0.001 – 0.03	0.01	0.01
$\xi$	0.1, 1	0.05, 0.1, 1	0.01, 0.1, 1	0.1, 1	0.1, 1, 3, 3	0.1, 1
$v$			0.1, 0.3, 1	0.1, 1	0.1, 1, 3	0.1
$\phi$			0.25	0.5	0.25, 0.75	0.25
$\tau_r$	0.0002 – 0.01	0.1 – 10 <sup>4</sup>			0.01 – 10 <sup>3</sup>	0.01 – 10 <sup>3</sup>
$\sigma$		0.00003 – 0.005				

# CHAPTER 4

## A MODEL OF UPSTREAM ACTOMYOSIN REGULATORS IN PULSED CONTRACTIONS

### 4.1 Motivation and experimental context

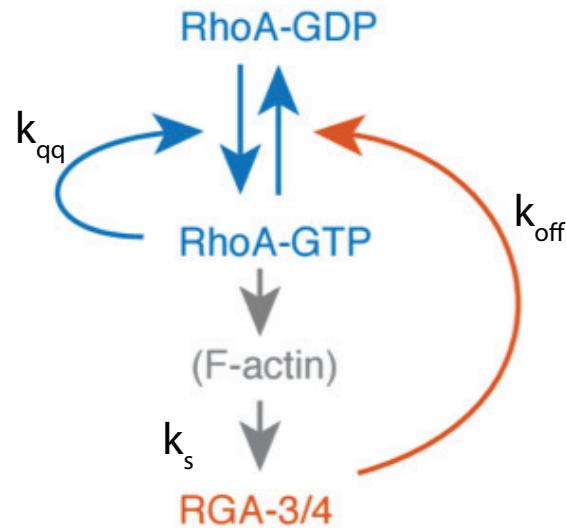


Figure 4.1: Reaction pathway with labels for coefficients associated with feedback strengths.

In many systems, cortical flows are driven not by continuous contraction of active material, but by repeated rounds of pulsatile contraction. While it was once believed that these pulsatile behaviors could be an emergent property of actomyosin contractility itself, it is now more probably suspected that these behaviors are caused by upstream regulators of local actomyosin activity.

It is still unclear why so many systems exhibit this type of behavior, but it is nonetheless important to understand the origins of these behaviors and what differences may arise due to their presence or absence in a contractile system. Therefore,

we have begun exploring how to model both the upstream regulators that govern contractile systems as well as the downstream effects of coupling these to regulators to contractile flows themselves.

The majority of our data comes from the work of Francois Robin and Jon Michaux [79]. They have shown that upstream of both actin and myosin is a separate pulsatile biochemical circuit consisting of a combination of positive and negative feedback between a pair of proteins called Rho and RGA as depicted in Figure 4.1.

To explain the dynamics I attempted to build ordinary differential equation models of local variation in active Rho and RGA concentrations. We found that many models are effectively equivalent at producing the qualitative results. However, this model in all of its forms is inconsistent with producing robust pulsatile behavior once the models were constrained by parameter fitting to the data.

## 4.2 A model for pulsatile actomyosin accumulation in *C. elegans*

I defined  $\rho$  as the concentration of Rho and  $r$  as the concentration of RGA, and generated association, dissociation and feedback parameters for the model.

$$\frac{d\rho}{dt} = k_{on}^\rho \left( 1 + k_{on}^{\rho\rho} \frac{\rho^n}{\rho_0^n + \rho^n} \right) - (k_{off}^\rho + k_{off}^{\rho r})\rho \quad (4.1)$$

$$\frac{dr}{dt} = k_{on}^{r\rho} \rho - k_{off}^r r \quad (4.2)$$

Next we can nondimensionalize the equation with  $q = \rho/\rho_0$ ,  $s = k_{off}^{\rho r}/k_{off}^r$ , and  $\tau = k_{off}^r t$ , and rename parameters for simplicity.

$$\frac{dq}{d\tau} = k_q \left( 1 + k_{qq} \frac{q^n}{1 + q^n} \right) - (k_{off} + s)q \quad (4.3)$$

$$\frac{ds}{d\tau} = k_s q - s \quad (4.4)$$

#### 4.2.1 Analyzing parameter space of the model

Just from analyzing the null-clines of this model we can gather a lot of information about the qualitative behavior of this model.

$$s_q = \frac{k_q}{q} \left( 1 + k_{qq} \frac{q^n}{1 + q^n} \right) - k_{off} \quad (4.5)$$

$$s_s = k_s q \quad (4.6)$$

The  $s_q$  null-cline appears as an inverse relation between  $s$  and  $q$ , with a hump around  $q = 1$  whose height is dictated by the strength of the  $q$  positive feedback,  $k_{qq}$ . The  $s_s$  null-cline is simply a straight line whose slope is governed by  $k_s$ . The dynamical system is only able to attain pulsatile behavior when the  $s_s$  null-cline runs parallel and to the left of the hump in the  $s_q$  null-cline. This gives two effective conditions on pulsatility: 1) If  $k_{qq}$  isn't much larger than 1, there will not be any hump, and so there cannot be any pulsatility. 2) If  $k_s$  and  $k_{off}$

To test this prediction, I implemented an automated search of the model's parameter space. The test worked by adding perturbations of fixed strength to the equilibrium state, and observing whether the response of the system exceeded the perturbation. As shown in Figure 4.2 for a number of simulation parameters, the response function could vary from having just a stable fixed point ( $k_{off} = 10$  and  $k_s = 25$ ) to having stable oscillations ( $k_{off} = 10$  and  $k_s = 63$ ) to having pulses ( $k_{off} = 20$  and  $k_s = 63$ ). Figure 4.2 also shows the null-clines for each case, which can be used to interpret whether pulsation is possible.

Using this automated system, I was able to generate 1600 simulations and classify them automatically based on the ratio of the magnitude of the perturbation to maximum response (to determine if positive feedback drove excitation) and whether the maximum was attained multiple times (to differentiate pulses and stable oscillations). This resulted in a phase diagram of the behavior as shown in Figure 4.3, with the three colors indicating the behavior of dynamical systems in that regime. The size and shape of the pulsing region of phase space was a direct result of the strength of

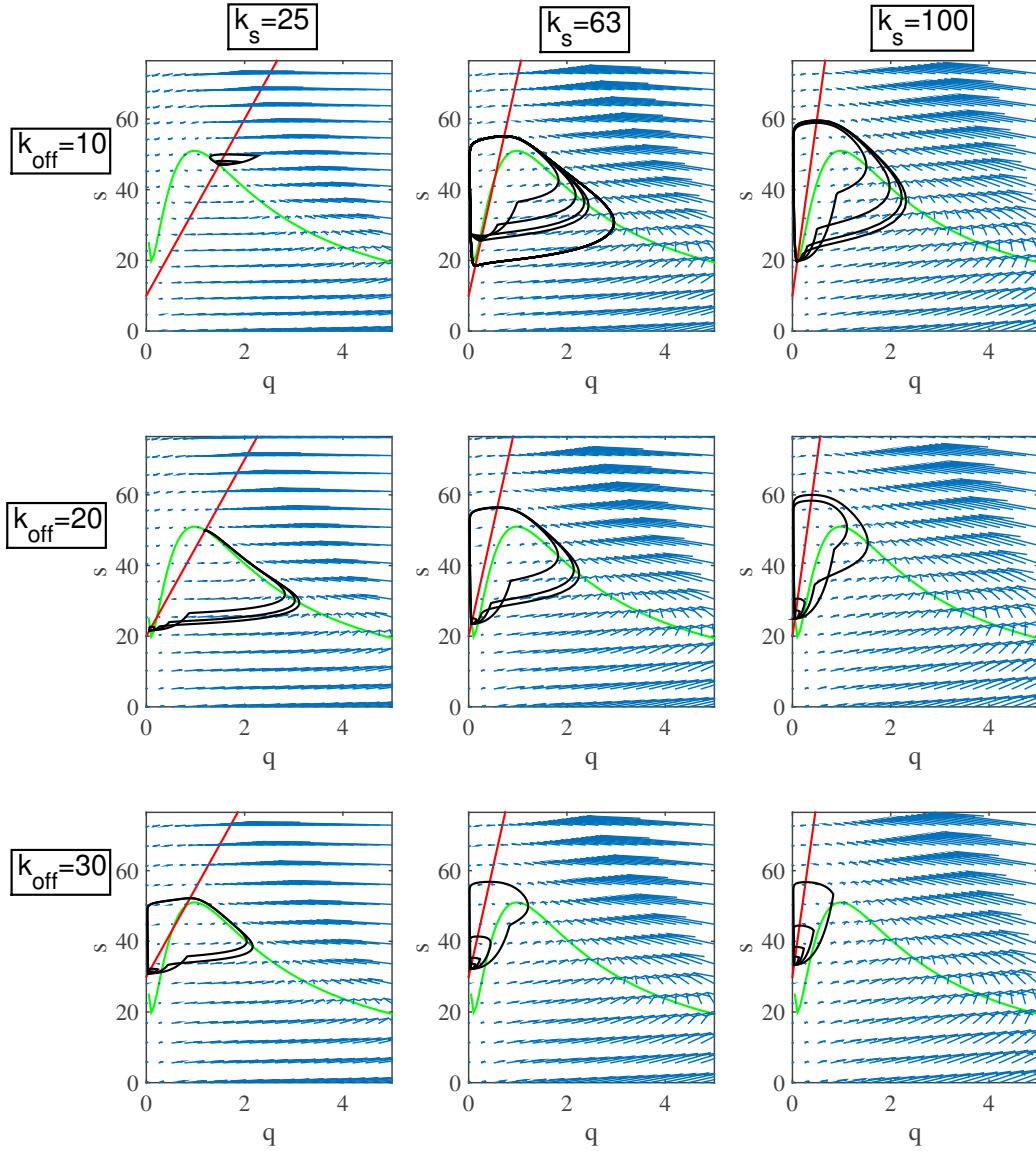


Figure 4.2: Perturbation simulation results plotted on phase planes for a variety of parameters.

positive feedback ( $k_{qq}$ ) with stronger feedback allowing more of phase space to permit pulsing and oscillations.

Finally, using this understanding of the phases behavior of the system, I implemented a stochastic dynamical equation to test the response to noise. As indicated

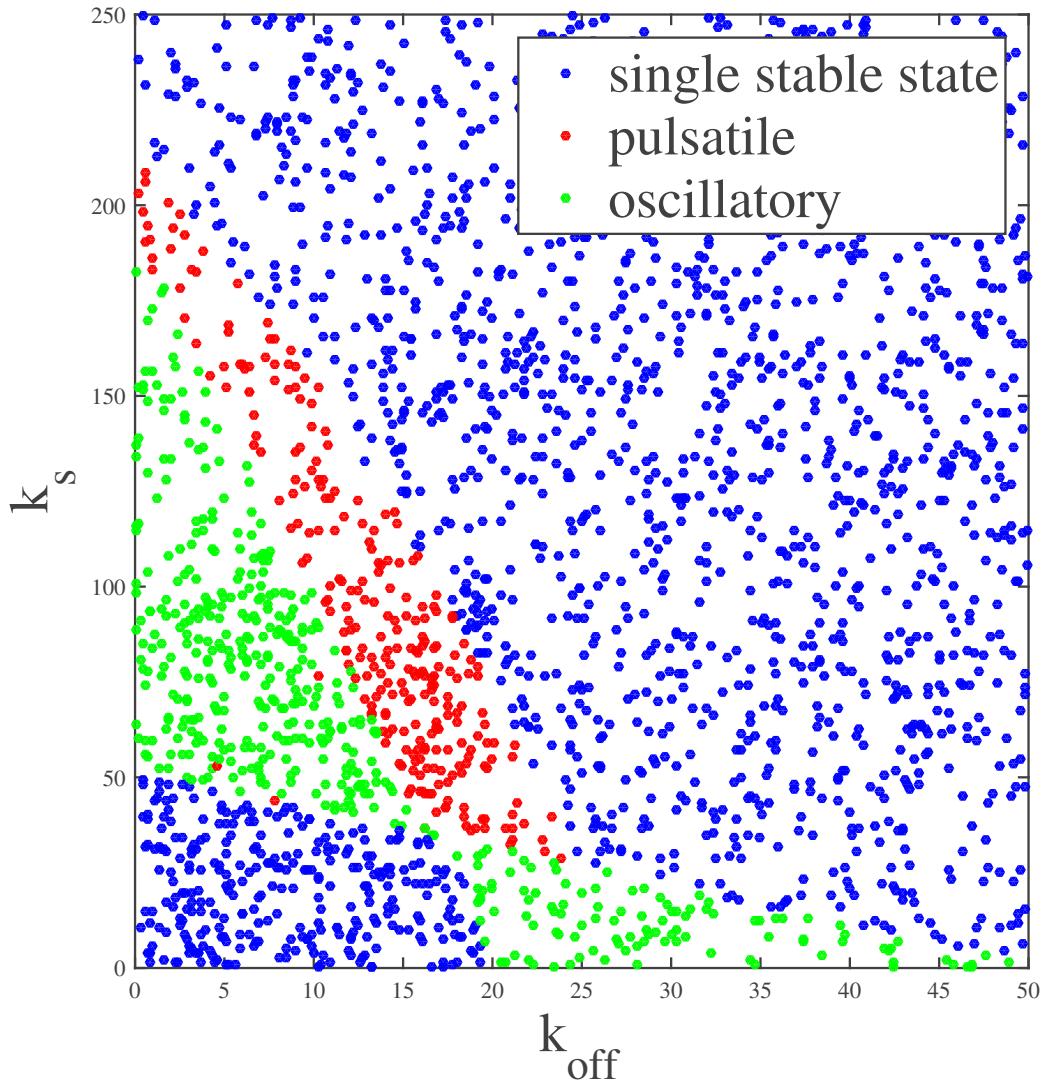


Figure 4.3: Phase diagram of for  $k_q = 1$  and  $k_q q = 100$ .

in Figure 4.4, a basal level of noise was able to trigger robust pulses in both Rho and RGA for certain parameters. Taken together this analysis indicated that our biochemical reaction circuit for Rho and RGA feedback could in principle account for the excitable dynamics exhibited in the system.

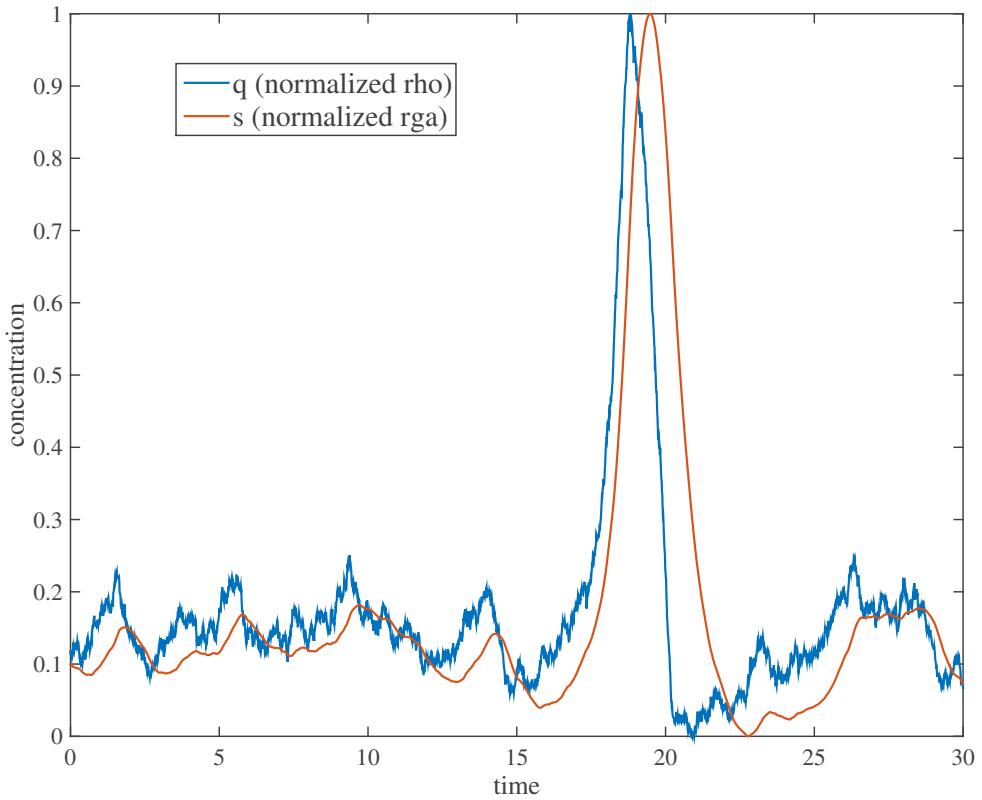


Figure 4.4

### 4.3 Simplified model and data fitting

#### 4.3.1 Fitting techniques

Although this model accounted for the qualitative behavior, we wished to determine whether it could be corroborated by fitting to the data presented in the paper. To experiment with fitting the normalized data, I created a reduced model, where the equilibrium concentration had been renormalized to 0 for both Rho and RGA, and the feedbacks between Rho and RGA are allowed to be non-linear.

$$\frac{dq}{d\tau} = \alpha \frac{q^n}{1 + q^n} - sq^k \quad (4.7)$$

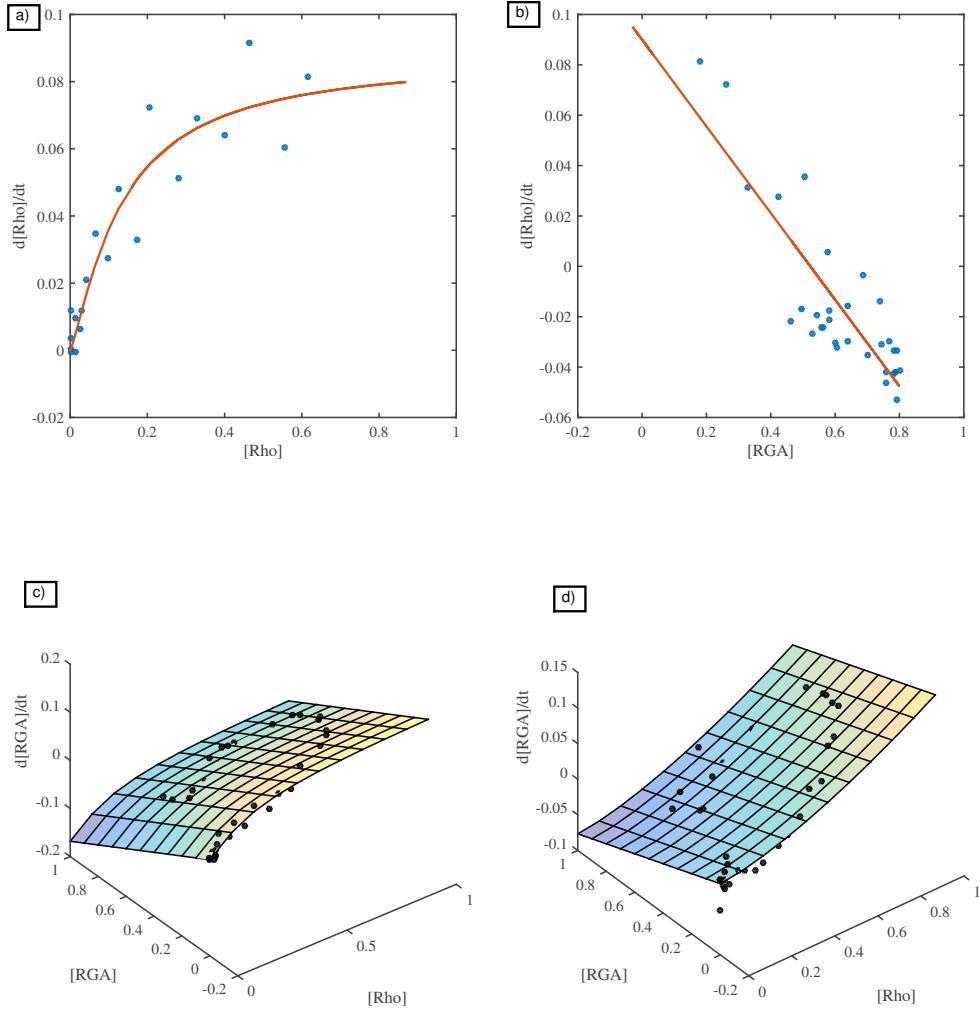


Figure 4.5: Multiple methods of fitting.

$$\frac{ds}{d\tau} = \beta q^{(m-k)} - s \quad (4.8)$$

These equations resulted in a family of models with different levels of feedback, depending on the values of  $n$ ,  $m$ , and  $k$ . I fit models with a variety of exponents and found that any model with  $k \approx 0$ ,  $n > 1$ , and  $m \approx 2$  gave the best overall agreement with the greatest robustness. I used two different methods to fit the Rho and

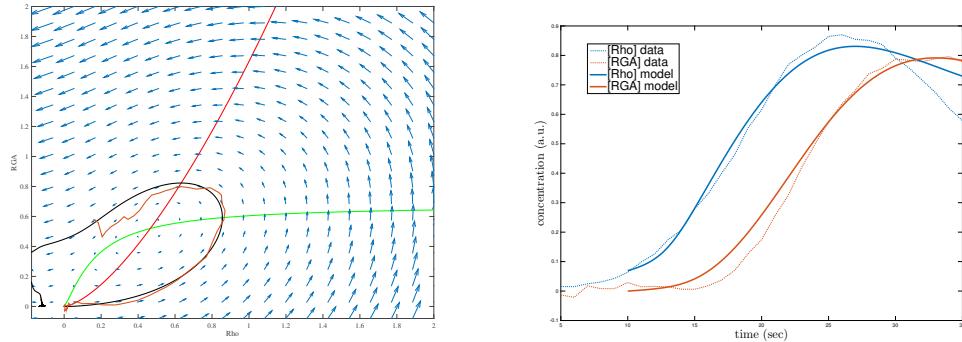


Figure 4.6: Simulation results and fitted data for model with  $m = 2$  and  $k = 0$ . Displaying simulation results and the data used to fit the simulation on a phase plane (left) and as pairs of time plots (right).

RGA data to determine the most appropriate model parameters. The first required subsecting the data to fit only datapoints where some terms in the equation were presumably close to constant (Figure 4.5a,b). For example, as long as RGA concentration remained relatively small, the equation for Rho could be taken to consist of only the Rho self-feedback function. The second method allowed fitting all of the data the equations at once (Figure 4.5c,d). The second method is clearly more accurate, but it comes at the cost of being slightly more difficult to explain to the lay-biologist.

The resulting fitted models could then be used to run simulations. As shown in Figure 4.6, the models and fitted data were largely indistinguishable relative to the error in the data. Although the model was quite successful at recapitulating the original data, I found that the resulting models were not necessarily. For example by changing the value of the parameter  $\alpha$  by 25% and rerunning the fits, I could tune the system from a stable system to pulsatility and into an oscillatory regime (Figure 4.7). Therefore, it appears that this model is not incredibly robust to minor variations in parameters.

#### 4.4 Conclusion

Ultimately, a model similar to 4.1 and 4.2 was utilized in the final paper in combination with at the 2D fitting routine outlined in Figure 4.5c,d. In short, all models

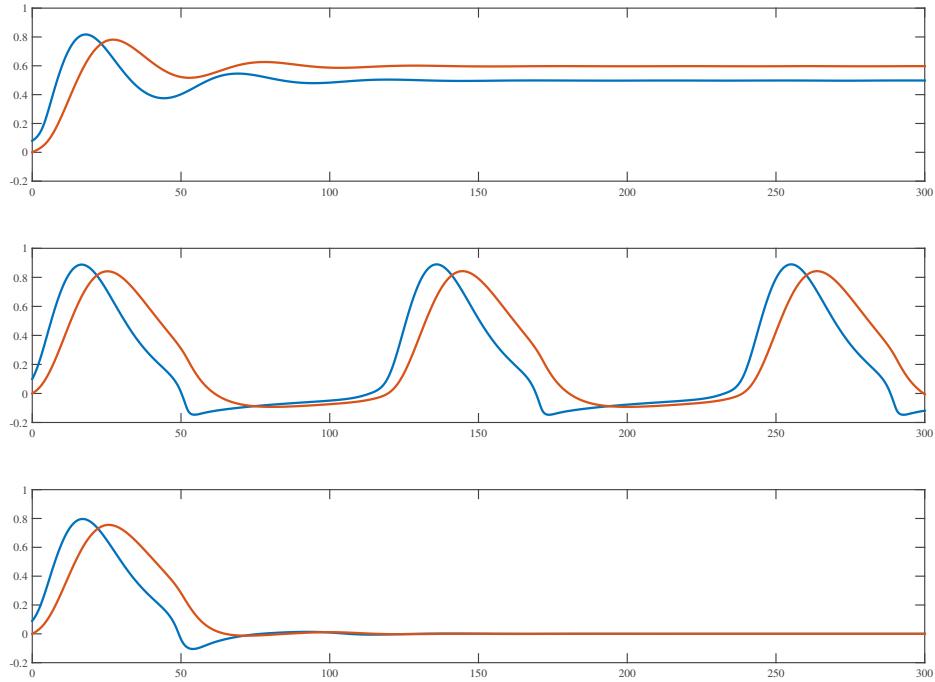


Figure 4.7: Small variation in simulation parameters can have large qualitative effects.

performed fairly well at describing the qualitative features of the data, but none were very effective at generating a quantitative match with great robustness. We believe this is indicative of the need to perhaps extend to incorporating spatial effects in driving the dynamics of active material pulses. Based on the theoretical results of [8], we attempted to incorporate our upstream regulatory model into a 1D active fluid. This ongoing work will hopefully tie together the spatial and temporal interdependencies driving this system into its interesting non-equilibrium state.

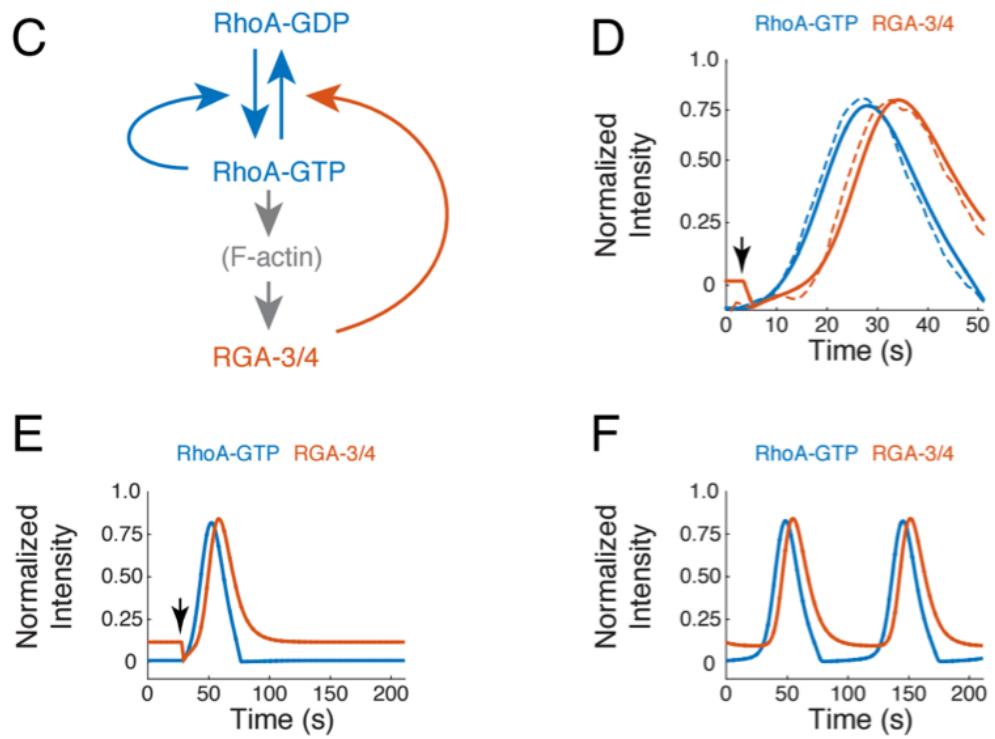


Figure 4.8: Final figure as it appears in Reference [79].

# CHAPTER 5

## PHASES OF DEFORMATION IN FILAMENT NETWORKS WITH CROSS-LINK SLIP

### 5.1 Results

#### 5.1.1 Steady-state Approximation of Effective Viscosity

We begin with a calculation of a strain rate estimate of the effective viscosity for a network described by our model in the limit of highly rigid filaments. We carry this out by assuming we have applied a constant stress along a transect of the network. With moderate stresses, we assume the network reaches a steady state affine creep. In this situation, we would find that the stress in the network exactly balances the sum of the drag-like forces from cross-link slip. So for any transect of length  $D$ , we have a force balance equation.

$$\sigma = \frac{1}{D} \sum_{\text{filaments}} \sum_{\text{crosslinks}} \xi \cdot (\mathbf{v}_i(\mathbf{x}) - \mathbf{v}_j(\mathbf{x})) \quad (5.1)$$

where  $\mathbf{v}_i(\mathbf{x}) - \mathbf{v}_j(\mathbf{x})$  is the difference between the velocity of a filament,  $i$ , and the velocity of the filament,  $j$ , to which it is attached at the cross-link location,  $\mathbf{x}$ . We can convert the sum over cross-links to an integral over the length using the average density of cross-links,  $1/l_c$  and invoking the assumption of (linear order) affine strain rate,  $\mathbf{v}_i(\mathbf{x}) - \mathbf{v}_j(\mathbf{x}) = \dot{\gamma}x$ . This results in

$$\begin{aligned} \sigma &= \frac{1}{D} \sum_{\text{filaments}} \int_0^L \xi \cdot (\mathbf{v}_i(\mathbf{s}) - \mathbf{v}_j(\mathbf{s})) \frac{ds \cos \theta}{l_c} \\ &= \sum_{\text{filaments}} \frac{\xi \dot{\gamma} L}{l_c} \cos \theta \cdot (x_l + \frac{L}{2} \cos \theta) \end{aligned} \quad (5.2)$$

Here we have introduced the variables  $x_l$ , and  $\theta$  to describe the leftmost endpoint and the angular orientation of a given filament respectively. Next, to perform the sum over all filaments we convert this to an integral over all orientations and endpoints that intersect our line of stress. We assume for simplicity that filament stretch and filament alignment are negligible in this low strain approximation. Therefore, the max distance for the leftmost endpoint is the length of a filament,  $L$ , and the maximum angle as a function of endpoint is  $\arccos(x_l/L)$ . The linear density of endpoints is the constant  $D/l_c L$  so our integrals can be rewritten as this density over  $x_l$  and  $\theta$  between our maximum and minimum allowed bounds.

$$\sigma = \frac{1}{D} \int_0^L dx_l \int_{-\arccos(\frac{x_l}{L})}^{\arccos(\frac{x_l}{L})} \frac{d\theta}{\pi} \frac{\xi \dot{\gamma} L}{l_c} \cdot \frac{D}{L l_c} \cdot (x_l \cos \theta + \frac{L}{2} \cos^2 \theta) \quad (5.3)$$

Carrying out the integrals and correcting for dangling filament ends leaves us with a relation between stress and strain rate.

$$\sigma = \frac{(L - 2l_c)^2 \xi}{4\pi l_c^2} \dot{\gamma} \quad (5.4)$$

We recognize the constant of proportionality between stress and strain rate as a viscosity. Therefore, our approximation for the effective viscosity,  $\eta_{eff}$ , at steady state creep in this low strain limit is

$$\eta_{eff} = \frac{(L - 2l_c)^2 \xi}{4\pi l_c^2}. \quad (5.5)$$

As illustrated in Figure 5.1, under moderate strains ( $\gamma < 0.2$ ), our simulations show that in the high density limit, our theoretical approximation from Eqn 5.5 is highly accurate at explaining the network behavior. Aside from a geometrical factor, our approximation is valid for both shear and extensional stresses applied to the network.

As the density of the network approaches the breakdown limit, the effective viscosity diverges from our expected value. At the low connectivities, our expected viscosity goes to 0, but the medium viscosity begins to take over as we cross the percolation threshold at  $L/l_c \sim 6$ .

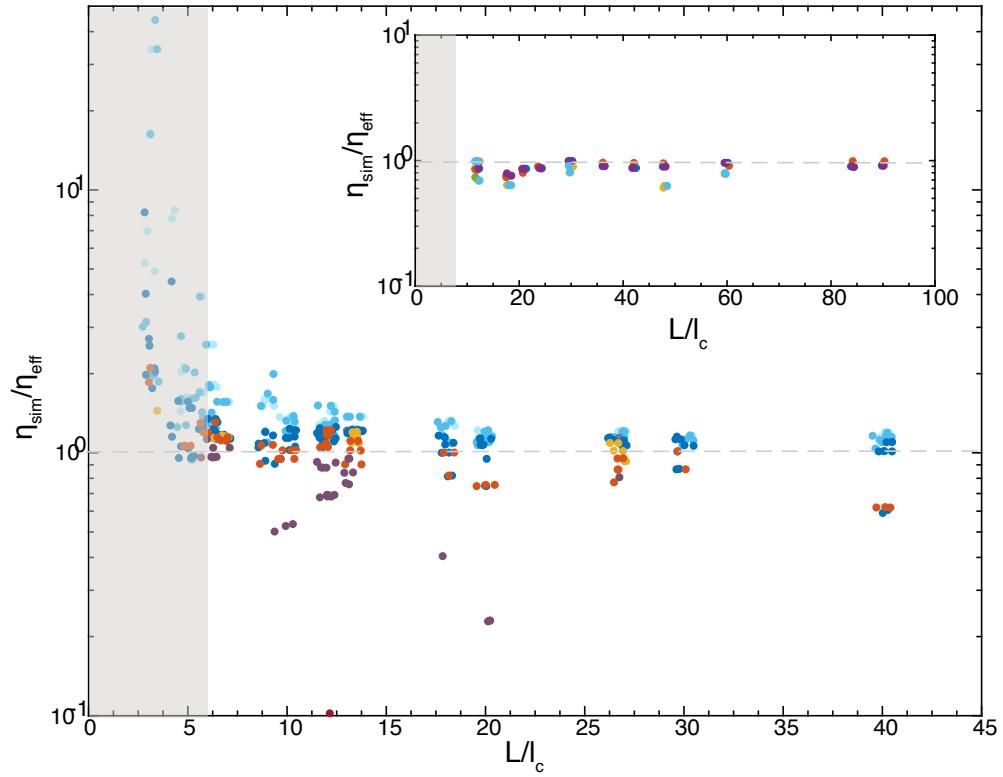


Figure 5.1: Ratio of effective viscosity measured by shear simulation to predicted effective viscosity as a function of connectivity,  $L/l_c$ . Inset: Same measurement for extensional simulations

In addition to changing the architecture and effective drag coefficient, we also validated the generality of our approximation by varying simulation size, medium viscosity, filament stiffness, and applied stress. We were able to find a slight trend that depended on filament stiffness as indicated in the difference between blue and red data points in Figure 5.1. The deviation from our approximation and variability in results manifested itself more strongly when filaments were highly compliant. To investigate this effect further, we next performed a more detailed analysis of the creep response while varying filament compliances.

### 5.1.2 Effects of Filament Compliance

The effect of filament compliance on cross-linked networks under strain is a subject of active research at the moment [Sayantan]. Therefore, we wished to use our computational approach to extend our understanding of filament networks in the regime of non-negligible filament compliance.

In irreversibly cross-linked polymer networks, filament compliance is known to give rise to elastic deformation of the network as described in[37, 100].

During the initial affine deformation immediately after the application of an external stress, we see a rapid stretch of filaments,  $\langle \delta L/L \rangle_0$ , in response to the affine purely mechanical strain,  $\gamma_{xy}$ , which closely follows  $\langle \delta L/L \rangle_0 = \gamma_{xy} \sin(\theta) \cos(\theta)$ . As shown in Figure 5.2, during the first phase in our simulations, the total network strain (solid) is described almost entirely by the strain of the filaments (dotted).

However, in the presence of cross-link slip, the filaments are not permanently constrained to remain at  $\langle \delta L/L \rangle_0$ . Interestingly, although the mean filament strain stays approximately constant, the distribution of individual filament strains broadens around the affine approximation as shown in the inset of Figure 5.2.

During the period where crosslink slip allows changes in the filament length distribution, we also find a long-lived intermediate relaxation phase that deviates from both the initial purely elastic relaxation and the later purely viscous behavior of section 5.1.1. In Panel B of Figure 5.2, we show that the standard deviation of the filament stretch distribution continues to increase throughout the period that the strain rate is non constant.

Approximating this broadening as a normally distributed variation in filament stretched length throughout the network ( $\mathcal{N}$ ) with a time varying standard deviation,  $\sigma(t)$ , we have  $\delta L/L = \langle \delta L/L \rangle_0 + \sigma(t) \cdot \mathcal{N}$ . This has an effect on the total mechanical energy stored in the network  $\mathcal{H} \sim \langle \delta L/L \rangle^2 = \langle \delta L/L \rangle_0^2 + \sigma(t)^2$ . Therefore, the network will deform further while some strain energy is being stored in the further stretching filaments.

Eventually the contribution from slow filament stretching will become negligible compared to that from pure cross-link slip on rigid rods. This occurs on a timescale

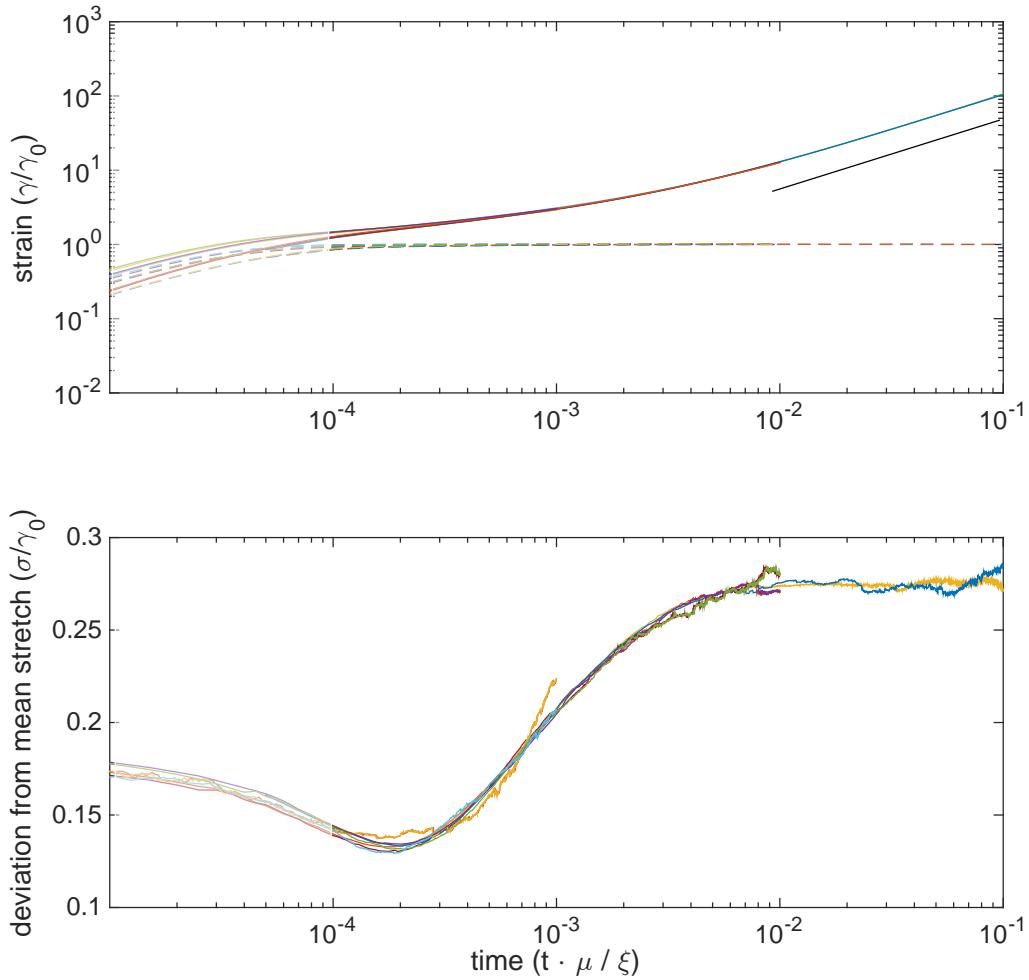


Figure 5.2: Network and filament strain for different filament drag coefficient parameters. (top) Plot of total strain normalized by the final mean filament strain,  $\delta L/L$ . Dashed lines show the amount of strain from affine mechanical stretching. (bottom) Standard deviation of filament extension for the networks in A. Note that the creep compliance in A becomes constant (slope 1) only after the spread in filament extension in B stops increasing. Colors indicate unique experimental conditions.

similar to that of cross-link slip and causes the effective viscosity to decay back toward the rigid limit. This gives rise to a less-than-linear creep response during times after the initial elastic relaxation but before full filament relaxation from cross-link slip. As shown in Figure 5.3, the transition begins to take place as network strain reaches

10 to 100 times the strain from pure mechanical stretching,  $\gamma_0 = \delta L/L$ , and this property is independent of the magnitude of the rate of strain.

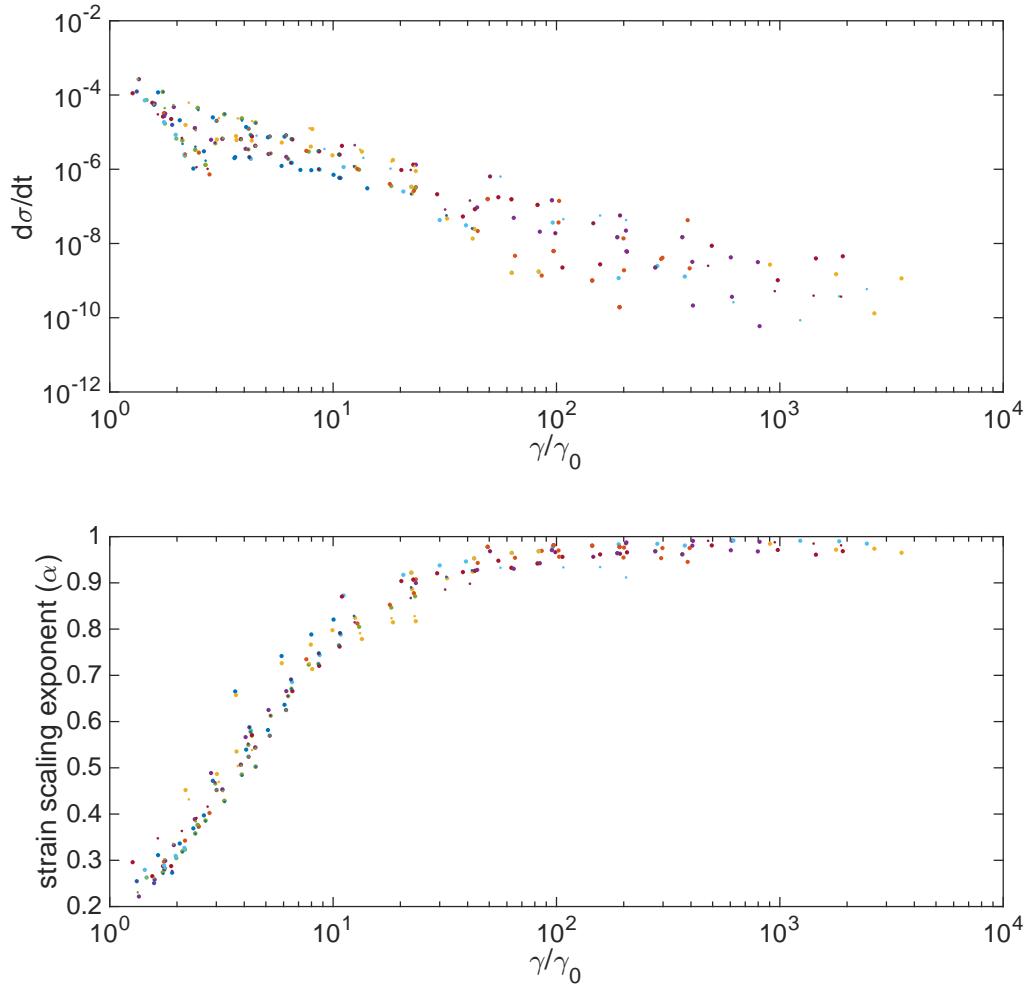


Figure 5.3: Sublinear network strain ends as change in filament strain decays. (top) Change in standard deviation of filament strain,  $\sigma$ , as a function of strain relative to pure mechanical strain. (bottom) Dependence of strain rate exponent as a function of strain relative to pure mechanical strain,  $\gamma_0$ . Colors indicate unique experimental conditions.

### 5.1.3 Alignment at High Strain and Network Tearing

Once the network is able to accumulate a large strain, the assumption of nearly uniform distributions of filament orientations begins to break down.

At this point the filament orientations become unevenly distributed  $\langle \delta L/L \rangle \neq \gamma_{xy} \sin(\theta) \cos(\theta)$ , with a larger number of filaments aligning in the direction of extension rather than compression. Filament alignment, conceptually, causes the formation of subdomains that no longer span the space of the network. To the authors' knowledge an exact derivation of the dependence of network connectedness on filament alignment has not been carried out, but Monte Carlo simulations have been used to show that alignment does indeed lead to lower connectedness[23].

We find that over time, the orientational distribution of the filaments begins to peak around 45 degrees as the large strain induces alignment. In Figure 5.4, we see that as the angular standard deviation falls, this reorientation eventually leads to fewer bonds bridging the network perpendicular to the line of strain. As this connectivity begins to noticeably decrease, the observed effective viscosity decreases as well, giving rise to greater than linear creep. From the inset of Figure 5.4 we can also see that the onset of phase D occurred before the network had completely reached phase C, leading to a rapid transition between sub-linear and super-linear creep. Finally, it should be noted that the end of this simulation resulted in the network tearing apart.

### 5.1.4 Phase Diagram of Dominant Behavior

In Figure 5.5, we illustrate the four stereotyped phases of the general mechanical behavior that we observed in our networks. A deforming network typically undergoes a rapid filament stretching, a slower relaxation of elastic constraints, a phase of purely viscous cross-link slippage, and an eventual alignment and breakdown of network connectivity.

Finally, to explore the transitions between the various phases, we measured the creep response for a computationally tractable network ( $L/l_c = 25$ ), as we varied the filament extensional modulus,  $\mu$ , and the cross-link friction coefficient,  $\xi$ . In Figure

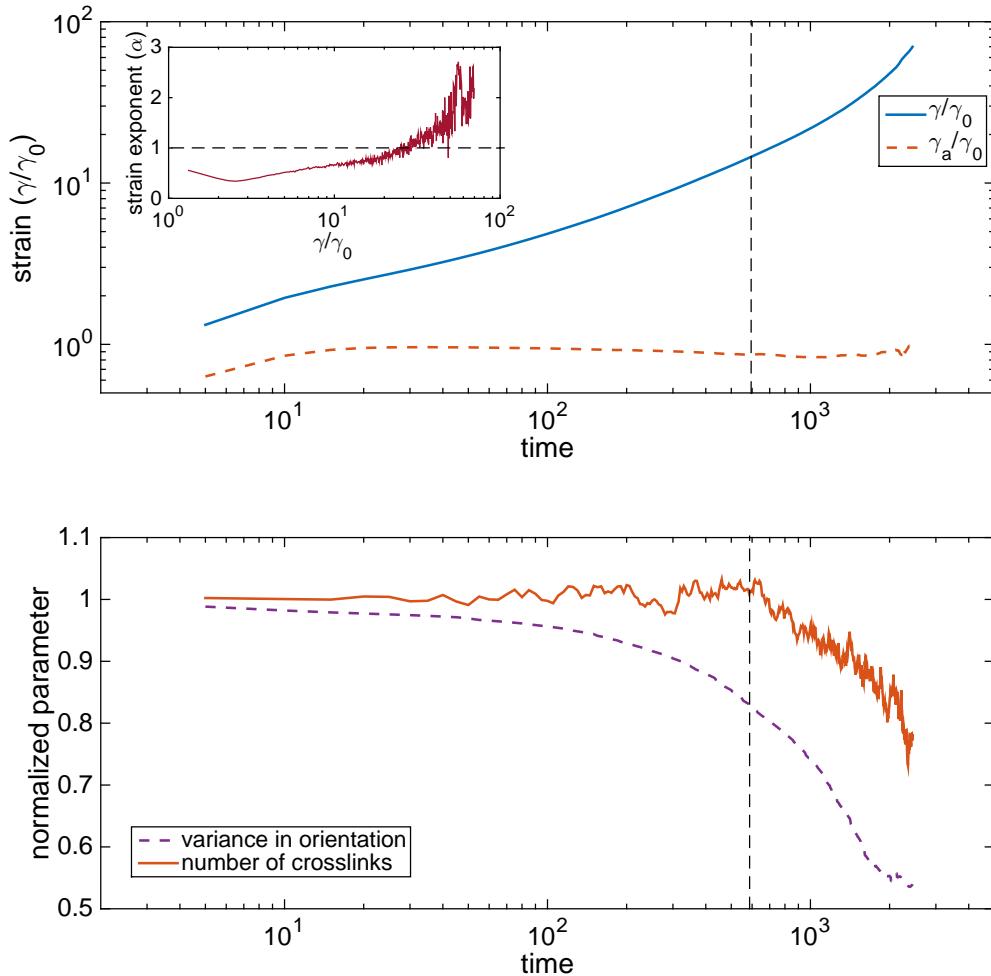


Figure 5.4: Creep response of a network transitioning to phase D. (top) Strain curves for a network undergoing large scale deformation. Inset shows strain exponent as a function of strain (exponent passes 1). (bottom) Traces for the variance in filament orientation and number of cross links. Vertical dashed line shows the point where the strain exponent becomes greater than one.

5.6, we classified parameter sets based on their strain exponent. We can see the trends for the transitions between phases A, B, and C. The line for the transition to D is still speculative at this time.

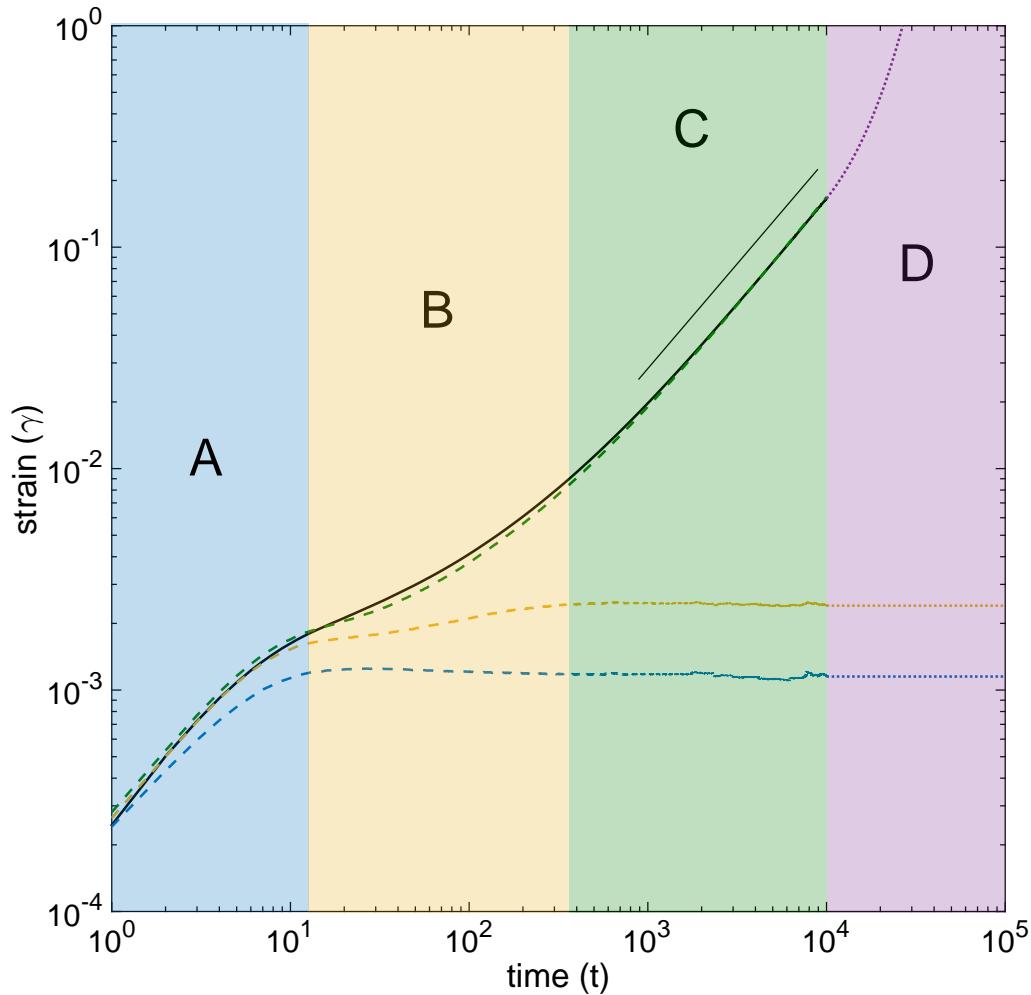


Figure 5.5: Schematic of the general creep response of compliant filament networks illustrating the 4 phases of deformation: A) rapid mechanical response, B) combination of slow filament stretching and cross-link slip, C) cross-link slip dominated (line indicates slope of one), D) network tearing from filament alignment. Note that the portion of the curve in section D is only a hypothetical continuation of the actual data.

### 5.1.5 Frequency dependent modulus

This section will include a figure on the frequency dependent modulus once I get those.

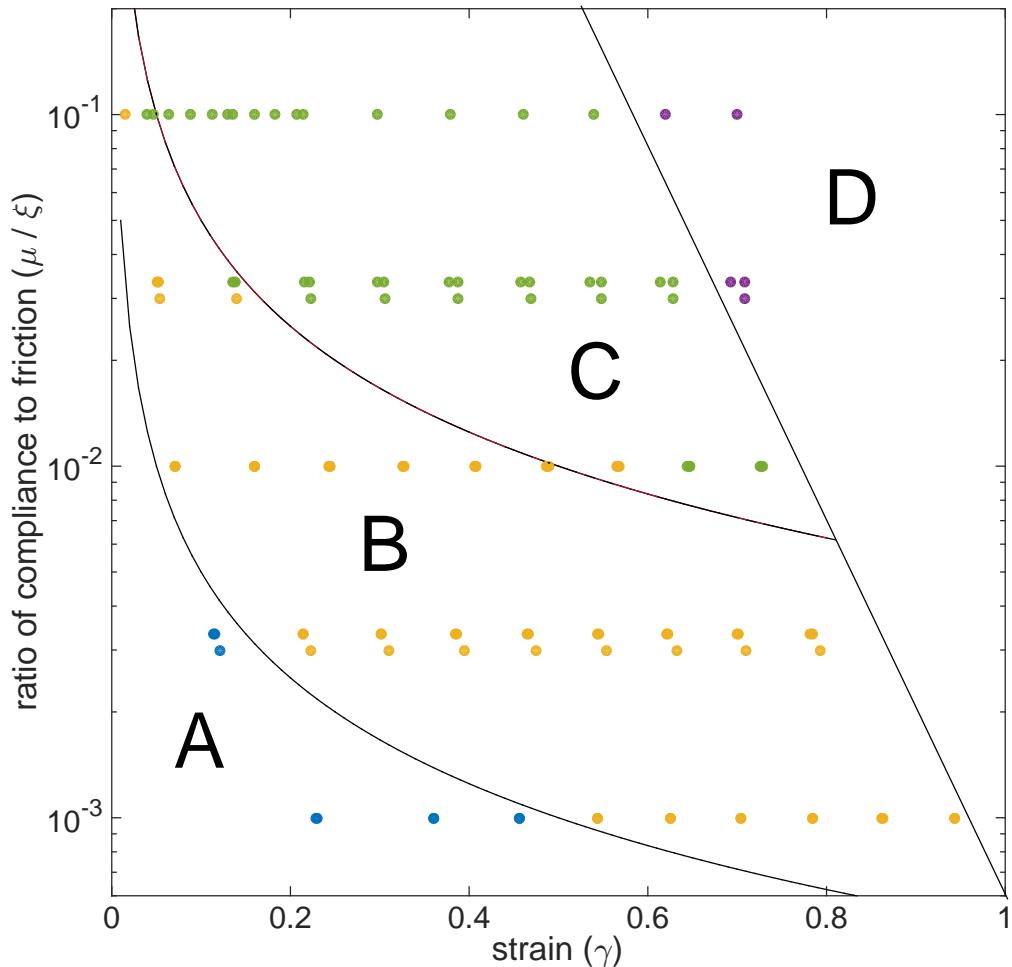


Figure 5.6: phase diagram of creep response for different filament extension,  $\mu$  and cross-link friction,  $\xi$ . Yellow, green, and purple dots correspond to creep measurements  $\gamma \sim t^\alpha$  with  $\alpha < 0.92$ ,  $0.92 < \alpha < 0.98$ , or  $\alpha > 0.98$  respectively. Blue dots represent creep measurements where  $\gamma_{total} < 2\gamma_{mechanical}$

### 5.1.6 Strain Memory

Finally, we found an interesting behavior when we introduced non-linear extensional stiffness into our filaments. When the network is allowed to relax to its unstrained state, there is generally a time comparable to the period of strain storage over which the energy in the network is relaxed away.

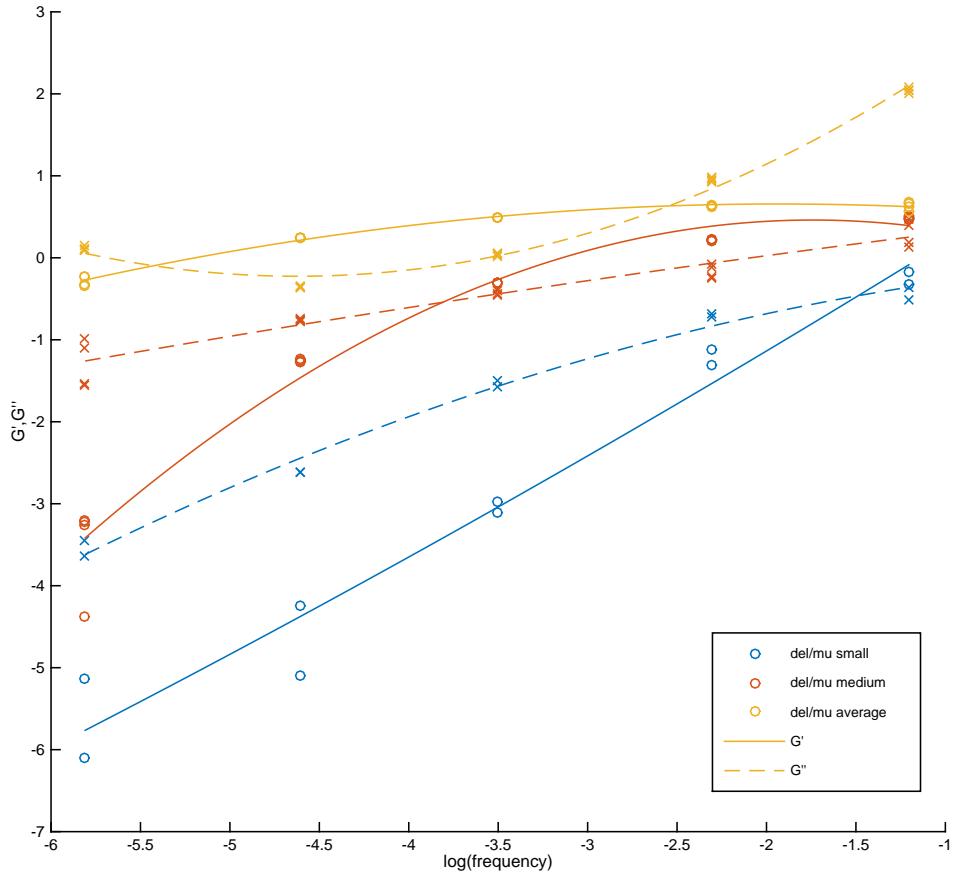


Figure 5.7: Frequency dependent moduli for networks.

We observed deviations from this behavior by applying stepwise stress pulses to simulated networks, and observing whether the network behaves identically upon reversal of the applied stress direction. If the network has no strain memory then each reversal will result in an identically shaped creep curve. However, when we include nonlinear filament extension in our model, we find that the mechanical strain can be stored for longer periods of time than it took to entrain the network.

This behavior mimics recent experiments in filamin cross-linked networks. Filamin provides a high level of compliance to a network ( $\gamma_0 > 0.5$ ) without substantial cross-link unbinding. This allows large scale rearrangements to take place without driving

very much cross-link slip, similar to the conditions in section 5.1.2. However, if we force individual filaments to undergo a strongly nonlinear stiffening at strains above 5%, we find an interesting long term strain storage.

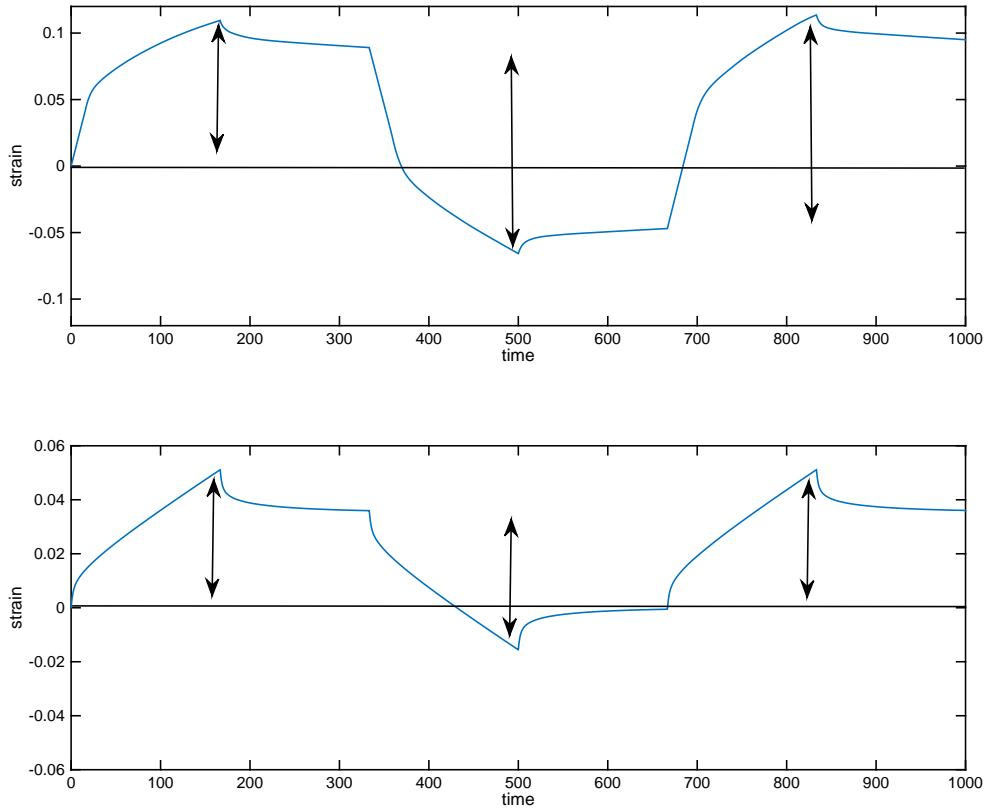


Figure 5.8: Creep curves in the presence of reversing applied stress for (a) nonlinear extension or (b) linear extension. Note that for linear filaments the induced strain returns to approximately 0 after a complete cycle, while in the nonlinear case the cycle is not completely reversible.

Figure 5.8 shows that the strain storage occurs, but I will need time for further study to build the full analytical picture of where when and why this happens in my model.

## 5.2 Summary and Conclusions

We have proposed a simplified effective friction model for understanding 2D cross-linked networks. Our model extends previous Mikado and lattice models to include effects of cross-link relaxation. We expect that our model can confer insights into mechanisms of network stress relaxation in quasi-2D networks such as those found in *in vitro* actin monolayer experiments[71] as well as in eukaryotic actomyosin cortices[64].

Our model is the first to address the plausible dependence of network effective viscosity on network structural properties. This led to a derivation of an estimate for the long timescale creep rate of networks under constant stress. Although this derivation neglects possible frequency dependence at short timescales, this finding offers a potential framework for addressing the dependence of network deformation rate on filament concentration and length.

Additionally, our simulations suggest that, in the presence of constant shear stress, cross-link friction will also produce a long-lived phase of sublinear creep as filaments relax from their affine stretched position. While this phase may transiently resemble more explicit 3D models such as [11], it is clear that our model differs by predicting that network will achieve a constant effective viscosity more rapidly. In particular, we predict that this relaxation will occur at a rate similar to that of rate of cross-link slip derived strain and will therefore be negligible after the network has slipped by roughly ten times the magnitude of the purely affine mechanical deformation.

In building our model we have neglected any other sources of potential mechanical relaxation in order to simplify our analysis. In the future, we hope to extend our model to include biochemically driven forms of relaxation such as filament turnover or regulated cross-link unbinding.

This model forms a basis for addressing 2D filament network deformation, and it proposes a simplified formulation of important qualitative properties. In this way we are able to address potentially general phases of network deformation and delineate what network properties may give rise to them. This may provide an important starting point for addressing the general importance of network structure in more complex networks containing active elements.

### 5.3 Network Tearing under Extensional Stress

#### 5.3.1 Extensional Thinning and Network Tearing

For moderate extensional stresses, the rigid filament approximation of the effective viscosity simply picks up a different geometrical factor out front.

However, at higher stress and in the presence of different things happen.

$$\frac{\partial l_c}{dt} = l_c \dot{\gamma} = \frac{l_c \sigma}{\eta} \sim l_c^3 \frac{\sigma}{L^2 \xi} \quad (5.6)$$

We can see that the rate of network thinning accelerates as we would expect. When the network reaches some minimum connectivity we assume that it stops behaving as a continuum material and the network tears irreversibly.

$$\tau_{break} = \frac{\eta_{eff}}{2\sigma} \cdot \left( 1 - \frac{l_c^2}{l_{break}^2} \right) \quad (5.7)$$

This provides us with an estimate of the timescale of catastrophic breakdown for a network with a given initial architecture and molecular drag.

#### 5.3.2 Tearing Events During Extensional Strain

This behavior is caused primarily by the low density network undergoing tearing events that interrupt global connectedness.

### 5.4 Deriving Molecular Drag Coefficients

Thus far, the idea of a molecular drag coefficient was taken as a phenomenological, measured parameter for a given experimental setup. While this is a sufficient pragmatic justification, it's useful to try to motivate the quantitative value of this drag coefficient by connecting it to the underlying cross-link properties of binding affinity, concentration, and extensibility.

To do this we'll imagine the simplified case of two cross linkers sliding past each other in one dimension. In this case, assume that we have an equilibrium number

of bound cross-linkers,  $n_B$ , each of which is displaced from its equilibrium length by some distance  $x$ . Each cross linker unbinds with rate  $k_{off}$  and rebinds at it's relaxed position ( $x = 0$ ) with rate  $k_{on}$ . At the same time, all the cross linkers are being pulled from their relaxed position at a rate,  $v$ , which is simply the rate at which the filaments are sliding past each other.

We can write the differential equation for the change in the density of cross-links,  $\rho$ , at displacement  $x$  as they are pulled upon, bind, and unbind.

$$\frac{\partial \rho}{\partial t} = -k_{off}\rho(x) - v\frac{\partial \rho}{\partial x} + k_{on}\delta(x) \quad (5.8)$$

Recognizing that  $\int \rho(x) dx = n_B$  implies  $k_{on} = k_{off}n_B$ , we can find the steady state solution

$$\rho(x) = \frac{n_b k_{off}}{v} \cdot \exp\left(-\frac{k_{off}}{v}x\right) \quad (5.9)$$

If each cross-link has a spring constant  $\mu_c$ , then we can equate the force on all cross-links to the applied force that is sliding the filaments past each other. Realistically, the spring constant and binding affinity would be functions of the cross-link stretch, but here we are taking them as approximately constant.

$$\int_0^\infty \rho(x)\mu_c x dx = v\frac{\mu_c n_B}{k_{off}} = F_{app} \quad (5.10)$$

a

Therefore, the term next to  $v$ , (i.e.  $\frac{\mu_c n_B}{k_{off}}$ ) would be equal to our molecular drag coefficient,  $\xi$ . Assuming approximately 1 cross link per filament overlap, and using parameter estimates culled from Ferrer et al., we build the following table of estimates for  $\xi$ .

<b>cross-linker type</b>	$\alpha$ -actinin	filamin-A
<b>dissociation constant (<math>s^{-1}</math>)</b>	0.4	0.6
<b>spring constant (<math>nN/\mu m</math>)</b>	455	820
<b>drag coefficient, <math>\xi</math> (<math>\frac{nN\cdot s}{\mu m}</math>)</b>	182	492

This molecular description assumed both a constant off-rate and linear force ex-

tension of cross-links. In the event that binding kinetics are regulated by the state of extension, we would expect (based on  $R_f$ ) to find a region that exhibits a stick-slip behavior instead of the smooth. Depending on the nature of any coupling between cross-links local stick-slip could either give rise to a global stick-slip behavior or a heterogenous mixture of stuck and sliding cross-links. It would be interesting to explore this topic further in the future, but in the present analysis, we choose to ignore complications from these nonlinear effects.

# CHAPTER 6

## IMPACT OF FILAMENT RECYCLING ON CORTICAL FLOW IN ANIMAL CELLS

### 6.1 Active Fluid Model Fitting of Cortical Flow

#### Measurement in *C. elegans* embryos

To ascertain the manner in which active stress and effective viscosity depend upon actin and myosin densities, I studied cortical flow in the *C. elegans* single cell embryo during polarity maintenance phase. The single cell *C. elegans* embryo poses a viable model system wherein to determine how active stress and effective viscosity depend on constitutive properties of the cytoskeleton. During the *C. elegans* first division cycle, cell fate determinants and small regulatory GTPases are segregated between distinct anterior and posterior domains, and these asymmetries are actively sustained through regulatory feedback for 5 minutes during the polarity maintenance phase. [11] As displayed in Fig. 6.1 this system is characterized by a spatially varying distribution of myosin motors in the presence of persistent unidirectional cortical flow from the posterior pole towards a band of cytoskeletal aggregation near the midline.

In addition to the steady state velocity profile, the *C. elegans* embryo provide a number of advantages that made these measurements easier. The events of polarization are highly stereotyped and reproducible, allowing for comparison between experiments. Transgenic strains expressing fluorescently-tagged variants of myosin-II and F-actin binding proteins (e.g. utrophin and moesin), allowing for simultaneous visualization of F-actin and myosin by fluorescence microscopy. In addition, the superficial cortical layer allows for high quality microscopy necessary for the experiments, and the cell geometry allows simplification to a 2D system. Finally, an important factor necessary for constraining the theory is a fine level of control over the densities

of both actin and myosin, which can be supplied in the *C. elegans* embryo by RNAi depletion of nearly any gene expressed in the germline. [12]

### *6.1.1 Obtaining spatially resolved simultaneous measurement of actin and myosin densities.*

Rationale: Actin and myosin are assumed to be the drivers of cortical flow and their density profiles and gradients are therefore important variables necessary to parametrize the functions of active stress and effective viscosity upon which the theory is based.

To obtain the density profile, I used interleaved HILO microscopy timelapses to obtain simultaneous images of myosin and actin in the *C. elegans* embryo during the single celled polarity maintenance phase. Next, I coarse grained the image using image processing blurs or a fixed square bin so that microscopic inhomogeneity is removed from the measurement. Limiting the analysis to one dimension improved the measurement quality and made the mathematics simpler so an average density taken along cross section lines running perpendicular to the embryo AP axis was computed. The example in Fig. 6.1 displays a typical fluorescence image of the myosin localization in the embryo along with the AP projection axis. The myosin fluorescence profile changes gradually over the course of five minutes with a period of particular stability lasting roughly two minutes.

### *6.1.2 Obtaining spatially resolved measurement of cortical flow velocities.*

To obtain a flow profile, I analyzed the myosin interframe displacements with particle image velocimetry (PIV). I averaged a set of contiguous frames from the videos prior to blurring or binning in order to obtain a more clear image and provide a larger flow displacement between PIV analyzed frames. The PIV algorithm is assumed to maintain a nearly constant absolute margin of error so maximizing interframe flow distance while still maintaining consistent results minimizes relative error. I

selected a region of interest that remained consistent throughout all experiments and encompassed the entirety of the dilating and contracting regions.

The PIV algorithm was an FFT multipass with window deformation and a post-processing clean up that rejects and interpolates any velocity with a speed greater than 3 times the standard deviation.[13] Only the on axis motion is relevant at this time so the output velocity vectors will be projected onto a line parallel to the AP axis. This component of the flow velocity was averaged along the cross sections running perpendicular to the AP axis as before. The diagram below illustrates a typical PIV flow analysis on the left. On the right, the projection of the flow velocity onto the AP axis is shown to vary slowly in magnitude over the course of 5 minutes and to maintain the same overall shape.

### 6.1.3 Using the data to constrain a basic active fluid model

I compared these measurements to the theoretical predictions by setting up a few non-linear functional fits of the velocity, effective viscosity and active stress as a function of myosin density using a nonlinear least-squares fitting solver with the equation

$$v(x) = C_1 e^{x/l} + C_2 e^{-x/l} + \int_a^b \frac{\delta\mu}{\delta x} (e^{(x'-x)/l} + e^{(x-x')/l}) \quad (6.1)$$

where  $l$  is a length scale of force falloff and the constants  $C_1$ ,  $C_2$  are constrained by the boundary conditions at the points  $a$  and  $b$  in the embryo,  $v(a) = v_a$  and  $v(b) = v_b$ . See [Bois et al] for more details on this equations derivation.

Data fitting was carried out in MATLAB by passing a starting guess for the fitting coefficients to lsqnonlin along with the function defined above.

The results of this analysis showed that, indeed, an active fluid function akin to that used in [Mayer and Grill] is capable of explaining the observed flow profiles in *C. elegans* embryos.

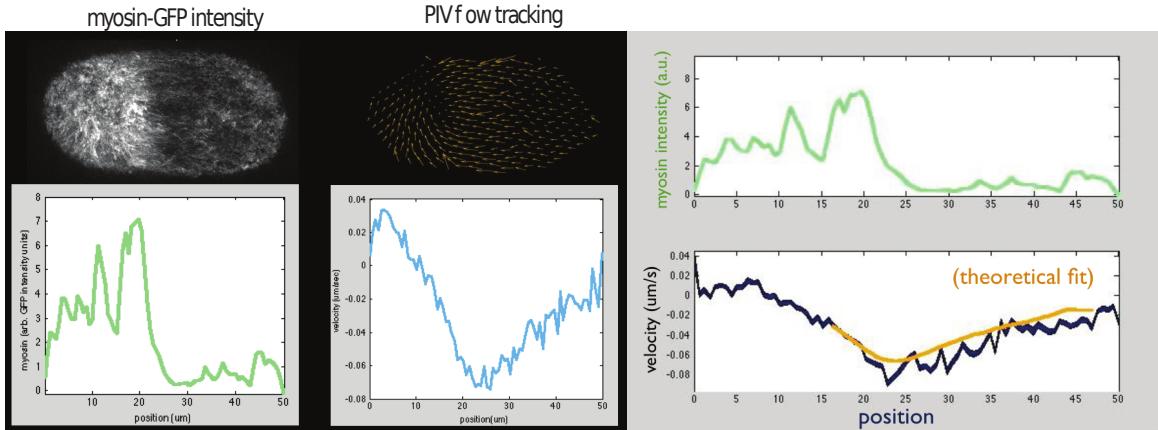


Figure 6.1: Measurement of myosin intensity and flow profile. These measurements fit a theoretical model of active fluid flow.

#### 6.1.4 Disruption of Flow by Myosin Depletion

The above experiment only sampled a limited region of myosin-actin state space if only analyzed in wild type. Using only data from this limited region does not uniquely determine the form of the functions that satisfy the equations of motion. I needed to test outside this range by varying the concentration of actin and myosin independently in order to adequately constrain the fitted equations of active stress and effective viscosity.

RNAi depletion is a common practice used to decrease the cellular concentration of endogenously expressed proteins in *C. elegans*. By feeding ssRNA containing *E. coli* to mother nematodes, the mRNA in the nematode germline is progressively silenced and the remaining protein decays away. I used this technique to lower the expression of myosin regulatory light chain kinase (MRCK), which is an upstream regulator of myosin. Overall myosin density decreases with a depletion of MRCK. We assume that the degree of depletion can be assessed by integrating the myosin density profile over the length of the cell.

Gene depleted embryos were imaged and processed in the exact same manner as described above for wild type.

As shown in Fig. 6.2, the actin densities were largely unaffected. Nevertheless, the myosin density (not shown) and flow profile (Fig. 6.2, right) were reduced. Progressive

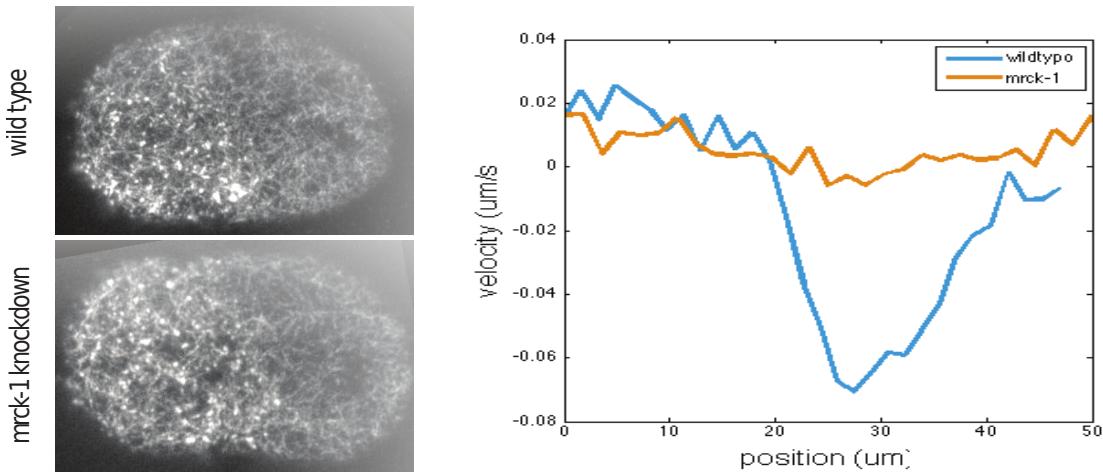


Figure 6.2: Measurement of actin distributions and flow profiles for *mrck-1* knock-down embryos. MRCK-1 is an upstream regulator of myosin.

depletion of MRCK also showed a graded response to myosin depletion, indicating that myosin activity imparts internal force in a dose dependent manner.

## 6.2 Stabilization of Filament Turnover with Jasplakinolide Treatment

Jasplakinolide is a small molecule inhibitor of actin depolymerization. When treated with jasplakinolide, actin filaments can be stabilized to have very long filament lifetimes. To study the effect of filament stabilization on cortical flows, Jon Michaux treated embryos with jasplakinolide and imaged the actin cytoskeleton with utrophin::GFP. The results were striking, and clearly showed that loss of turnover resulted in disrupted flows. Moreover, in some cases, the treatment didn't just result in a stalling of flows, but also led to a tearing of the cortex followed by further transient contraction. Nevertheless, all embryos with jasplakinolide treatment eventually stalled and failed to continue with normal polarization and division.

I made an attempt to quantify the degree of contraction possible in jasplakinolide treated embryos using hand tracking of fiducial markers in Jon's videos. As can be seen in Fig. 6.3, whether the cortex stalled or tore, the resulting strain asymptotically approached some limiting strain. This is in sharp contrast to the persistent strains

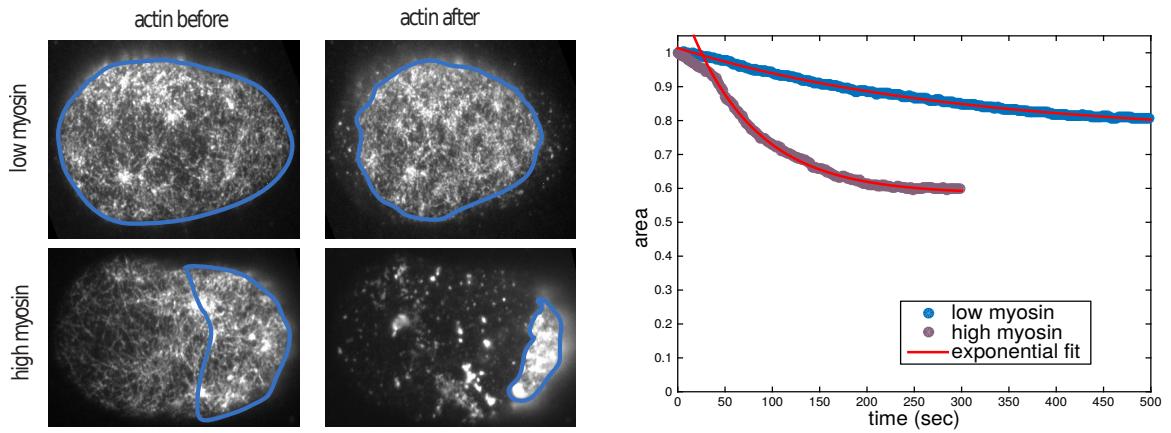


Figure 6.3

$(\gamma > 1)$  possible in the steady state flows found when turnover is present.

# CHAPTER 7

## OPEN ISSUES & FUTURE DIRECTIONS

### 7.1 Comparison with Recent Modeling Publications

There have been several recent publications incorporating various mechanisms of turnover in cross-linkers, filaments, and motors, while probing timescales of stress generation and dissipation. In the following two sections, I will compare my work with the results found in the two most pertinent recent papers.

#### *7.1.1 Role of Turnover in Active Stress Generation in a Filament Network by Tetsuya Hiraiwa and Guillaume Salbreux*

In their paper from May of 2016, Hiraiwa and Salbreux showed that their model networks of filaments and active and passive cross-linkers were capable of generating transient stresses, and that adding filament turnover was able to cause those stresses to be maintained indefinitely. They also showed that there needed to be a critical number of cross-linkers present in order for the network to generate stress. The specifics of how and why these conclusions were dependent on the specific model they used so I will next explain the specifics of their model and how it differed from my own.

Their model consisted of rigid filaments and rigid motors in the presence of passive cross-linkers. The passive cross-linkers were considered to be point-like and to bind the rigid filaments together directly at their point of contact. From this it follows that the cross-linking constraint only allows deformation through filaments rotating around their points of contact. In addition, it should be clear in this mechanical picture that 3 filaments attached into a triangle will not be able to undergo any deformation. According to the paper, the main driver of force generation is when a motor filament walks along two actin filaments that are cross-linked together at

one point. The motor filament exerts a force on the two filaments which can serve to either contract or extend the filaments relative to each other. Averaging over the total number of configurations before the motor detaches, they find that one motor with two filaments has a net contractile effect. In contrast, in my simulations, this effect would not take place at all. The geometrical argument requires myosin motors to be fairly large relative to the actin filaments in order for this effect to be significant as was shown in [50]. In my simulations, the contractile asymmetry followed directly from the asymmetry between the spring constant of extension vs. compression.

Next, their model predicted that there would be a minimum number of cross-linkers required before there could be a net stress generated within the network. Their critical number of cross-linkers turned out to be equal to the number of filaments. This seems reasonable, as with fewer cross-linkers, there would on average be less than one cross-linker per filament, meaning that most filaments would on average only be attached to one other filaments and would therefore be unable to transmit force between two filaments. In my model, cross-linking was assumed to take place at every filament overlap point, but if I relaxed this requirement of my model, I presume that reducing the number of cross-linking points would result in very nearly the same effect as was found in this study.

These networks generated and maintained their stress, but the addition of cross-linker turnover prevented stresses from persisting. Once cross-links were allowed to turn over, filaments and motors were able to freely rearrange. The authors showed that this free rearrangement led to loss of connectivity and clumping of filaments and motors which prevented the sustainment of stress. This is highly similar to the result from my work where a form of viscous cross-link slipping also resulted in a global loss of stress buildup over time. In my work, however, this was baked into the form of the cross-linking from the beginning so all that could be varied was the timescale over which stress dissipation took place. Entirely preventing stress dissipation by making the cross-link binding permanent was not possible in my framework.

The most pertinent conclusion from this paper was that the addition of filament turnover into the network restored the ability to maintain a non-zero stress indefinitely even when cross-links were turning over. Their explanation for this is similar to the

reasoning from my paper: active motors rearrange filaments, which causes a loss of connectivity, but this can be prevented by putting filaments back into the rarefied regions of the network. They show examples of this to suggest that this is indeed the case, and continue to show that their finding for the critical number of cross-linkers qualitatively holds for the case of turnover as well.

The authors were able to present a phase diagram that summarized their main conclusions. In the diagram, they essentially just showed that there was an optimum in turnover time and that the optimum varied with the number of cross-linkers. The more cross-linkers the network contained, the faster the turnover had to occur in order to reach the optimum. Because the number of cross-linkers was not varied in my simulations there was no similar conclusion in my paper.

The authors in this paper did not look at the passive dissipation of stress in the absence of active stress generation. However, it seems safe to assume that at least two conclusions from their work would hold in their simulation framework if they were to probe the passive properties using an external force as I did. First, the network would not be able to maintain global connectivity if the number of cross-linkers was less than the number of filaments. The reasoning is identical to the active case where having fewer than one cross-link per filament will almost certainly lead to a global loss of connectivity over a large enough spatial scale. Second, in the presence of cross-link turnover the network would not retain global connectivity.

### *7.1.2 Interplay of active processes modulates tension and drives phase transition in self-renewing, motor-driven cytoskeletal networks by Michael Mak, Muhammad H. Zaman, Roger D. Kamm & Taeyoon Kim*

The model of Mak et al. is probably one of the most intricate models used to simulate actomyosin mechanics in the field. As such, it is very useful for suggesting the origins of emergent properties in these networks. On the other hand, its complexity can make it difficult to pin down precisely which effects lead to which outcomes. Nevertheless,

in this paper, the authors were able to draw conclusions about what mechanisms led to their observations and bulk measurements. Importantly, this was the first work to show that networks without turnover can only generate transient net stress and that turnover allows the network to persistently maintain stress.

Their modeling framework consists of segmented actin filaments where each filament has an extensional and a bending spring constant. These filaments are connected by cross-linkers that are also small springs, which can bind and unbind randomly with a characteristic timescale. Finally, motors are implemented as cross-linkers with the ability to periodically hop from one location to the next along the filament. This modeling framework is very useful due to its high molecular similarity with a number of known properties about actin and myosin networks. In particular, the model tries to base its conclusions firmly on a realistic picture of actin and myosin mechanics by choosing simulation parameters that match closely with real measured values.

Their work presents many modeling scenarios where the end result is a network with a short-term buildup of stress followed by a global loss of connectivity and a falloff in the global stress generation. They vary a number of physiologically relevant parameters and monitor the sustainability of stress. Similar to the above work by Hiraiwa and Salbreux, they focus on varying the number of cross-linkers and the filament turnover rate, but they also explicitly vary the percent of crosslinkers that are active. By doing this they were able to map a phase diagram, which showed that the ability to sustain force was possible in one region of parameter space and that the sustained stress was only large in another region of parameter space. Taken together, the overlap of those two domains shows where the network is able to sustain large stresses. This occurs in a confined domain similar to that shown in the work of Hiraiwa and Salbreux. Their explanation for the inability of some networks to sustain stress is that networks deform and generate clusters of material which prevent global stresses to be sustained. They measure the clustering and show that it is important.

They end their work with a generalization of their findings to a so-called active spring model of network contraction. This model represents a simplified view of their simulation results, and recapitulates the rising and falling timecourse of network stress buildup. Finally, they address a handful of experiments that loosely corroborate their

findings. It will be interesting to see more in-depth experimental validations of these models in the future.

This work did not include an analysis of the passive properties of the network, but the authors had already inspected the network in the passive case in their prior work [46]. In fact, their prior conclusions of the importance of filament turnover for tuning the viscosity of simulated networks was the main influence on my current work.

### *7.1.3 Shared conclusion of all three works*

I think it is important to point out that all three of these works have shown that there is in some sense an optimal turnover time for producing a maximal steady state stress. Because the methods of producing the simulations took account of different assumptions, the exact form of the maximal time is different, however, it is remarkable that this property was found to be general to all three modeling forms. It is fairly clear from a mechanical perspective why this would be the case in hindsight, but it appears that this outcome wasn't predicted before these attempts were made at generating models.

However, it should be pointed out that in contrast to the other two papers, my model allows for a subtler mechanism underlying the transience of stress generation in networks without filament turnover. In my work, I attempted to emphasize that the global loss of stress will occur even if the network does not undergo visible thinning and tearing. In particular, there can be a persistent global stress coming from contractile and extensile segments in the network, but these effects will cancel each other to generate no net stress. This observation goes a long way to show that the absence of a permanent stress buildup is not possible even when tearing doesn't occur and that indeed it is reasonable to believe that there should be no way to maintain a permanent stress

## 7.2 Incorporating multi-segment filaments and bending degrees of freedom

For simplicity, notable aspects of semi-polymer mechanics have been ignored throughout the entirety of this thesis. In developing this work, I chose to limit my analysis to single springlike filaments in order to focus attention on the most prominent properties of semi-flexible polymers. In effect, I took the minimal number of model elements (and accompanying free parameters) that would suffice to produce the 2D network flows of interest.

While this choice greatly simplified the analyses performed and allowed me to focus my results, it does ignore aspects of filament mechanics that may play an observable role in macroscopic cell mechanics. In particular, there are two clear oversimplifications that are introduced by using single springs: uniform strain along filaments and absence of bending.

Uniform strain along filaments results from the model's current insistence that forces add to produce a net strain in the filament. Because all forces are transmitted merely to the ends of each spring, there can be no internal regions of variable strain anywhere else along the filament. This necessarily overlooks the local deformations that could be driven by internal motor forces. The net result will be that deformations on small length scales. As such, certain measurements that were made in the above analysis are most probably over-averaged and not indicative of what would be found in a real system. It is still unclear what impact this will have on the macroscopic dynamics of the system, but nevertheless, this is one of the largest missing pieces that could invalidate some of the model predictions.

The absence of bending degrees of freedom is probably of less concern than the imposition of uniform filament strain. To begin, the fact that bending causes filament stiffness asymmetries has already been incorporated in the asymmetric extensional stiffness imposed on filaments in this model. Thus, adding bending will only serve to double count this asymmetry and will probably not provide much benefit. A second aspect of filament network mechanics is more problematic. It has been shown previously that the mechanical picture of 2D networks can transition from extension

dominated to bending dominated when network densities are sufficiently sparse. The net result of this is that at low enough densities, the main mechanical resistance will be dependent on filaments resisting bending. My model will neglect this transition to bending dominated mechanics, and therefore, the model network should become completely pliable despite the fact that the real network will still maintain some resistance to deformation. However, analyzing networks at such sparse densities will present numerous challenges to the continuum models presented in this work so worrying about the specifics of bending dominated mechanics may be of lesser concern.

Nevertheless, it is important to note that the current implementation can easily allow the introduction of multisegment bending elements. If one uses segment sizes that are shorter than the total filament length, joints will automatically be introduced that separate the filament into multiple regions that are free to deform on their own. However, with  $\kappa = 0$ , these joints will be free to rotate, which will cause the model to create effectively separated springs that are merely forced to share one attached end. However,  $\kappa > 0$ , the model will introduce a bending spring that tries to keep individual filaments straight. The magnitude of the bending modulus can then be varied to change the bending stiffness of the filament.

### 7.3 Probing more complex mechanisms of turnover

Another notable simplification in this work is the method of incorporating filament turnover by causing entire filaments to be reset at random. This is actually not the microscopic means by which filaments are depolymerized and repolymerized in the cell. In fact, both filament depolymerization and repolymerization are governed by more complex and intricate processes that have been studied to painstaking detail. The net result of these events does cause rapid and complete strain resetting on long enough timescales, but there can be subtleties that change the exact form of the strain and orientation resetting of filaments.

The actual mechanism for filament depolymerization relies on a balance of slow filament treadmilling, accompanied by faster timescale filament severing events. The combined action of these two mechanisms should cause a quite rapid removal of the

filament from the physically connected network and thus cause an effectively immediate stress dissipation much like the one demonstrated in our model. Nevertheless, there may be all kinds of regulatory factors which act to make the stress dissipation less idealized than in the simplified model. One such complicating factor would be stress dependent severing rates, which would cause non-uniform dissipation of stress or preservation of stress depending on whether filaments were preferentially severed based on being high stress or low stress state, respectively. Any mechanism could in principle be added to this model, however, it would add to the difficulty of interpreting results and therefore should be incorporated only if some aspect of the model is found to be deficient in its explanatory ability of a specific result.

In addition, to regulation causing non-uniformity in depolymerization, there are molecular details that impact our assumptions about repolymerization as well. In cells, much of the structural actin cytoskeleton is aided in polymerization by formins, which recruit actins to rapidly accelerate the polymerization process. While many aspects of the polymerization are still under active study there is at least some preliminary evidence that the specific nature of formin polymerization provides biases in the orientation of newly polymerized filaments. Specifically, formins appear to follow existing actin filaments preferentially, thereby serving to lay down new actin along a template of existing actin. In addition, crosslinking proteins can preferentially align newly polymerized actin as well. This may be an important part of regulating stress assymetries and as such may need to be incorporated into some aspects of active network models in the future.

The end result of these complex processes allows stress resetting to occur independently from orientational resetting. In this sense, the simplifications in the current implementation tend to conflate the processes of stress resetting and orientation resetting. In effect, there may be different timescales over which different aspects of network memory relax. As such this may be an important avenue of future work.

### *7.3.1 Which is more important, filament relocation or stress resetting?*

It seems very clear that the global disruption of connectivity will necessarily lead to an inability to maintain global net stresses. However, in this work, I've argued that one need not develop global loss of connectivity in order for the net. I found that it was possible for networks to remain macroscopically connected, but for local rearrangements and the internal balance of extension and compression to cause the global stress to dissipate. Therefore, there are actually two distinct activities occurring when filaments are recycled in my model: 1) they are relocated to locations where there may be fewer filaments, and 2) they are reset to have 0 net strain. From this basis, Ron Rock on my committee asked an interesting question: Which is more important, filament relocation or stress resetting.

In my opinion, it is stress resetting which is ultimately more important than filament relocation. Overall, if the thinning from global rearrangement was apparent, then one would have to relocate filaments to maintain connectivity. However, without resetting strain, the filaments would still be able to reach internal balance between extension and compression. In other scenarios, where there is no global thinning, the net stress is still lost over time due to the local rearrangements and extensional and compressive balancing.

It would be easy to test this with the framework I have put in place. One could decouple the two mechanisms by relocating filaments without changing their strain state, and to reset their strain without moving them to new areas. If I were to relocate filaments to regions where connectivity was being lost, but I was to retain them in their stressed state, I predict that the net stress would be lost. This might be useful for future work although it appears pretty clear how it would work out.

### *7.3.2 Overlooking the subtleties of myosin turnover*

My model notably lacks any direct mechanism by which motor turnover can be implemented—i.e. any pair of segments that interact actively at the beginning of the simulation will continue to do so any time they meet each other throughout the

simulation. In a more realistic scenario it should be possible for an intersection of two filaments to undergo intermittent transitions from active to passive interaction. This would mean that the over time filaments would be able to be driven to be stressed and then even without local rearrangement, start to dissipate that stress after the myosin turns off.

Importantly, this oversight leaves the possibility that filament turnover is not actually required to allow stresses to persist. Instead, it would be possible for the filament stresses to be reset by the constant deactivation and reactivation of motor activity. In principle, this could give rise to a form of stress resetting similar to the mechanism found with filament recycling. As such, filament recycling could end up being redundant. Nevertheless, it's possible that myosin turnover would not be capable of completely resetting filament stress because the filament stress relaxation would not be instantaneous as it is with filament recycling.

It would be easy to address this concern by modifying the simulation framework. When originally contemplating this modeling framework I considered incorporating a time varying active force, but decided this would be too contrived to use. Perhaps a better implementation would simply allow random switching of motor activity on and off with yet another characteristic timescale, say  $\tau_\tau$ . In this case, it would seem reasonable that the stress state of the network would be dependent on the minimum of the filament recycling timescale mentioned above ( $\tau_r$ ) and the myosin turnover timescale ( $\tau_\tau$ ).

## 7.4 A novel cell squishing technique to measure timescales of relaxation *in vivo*

At the moment, there are very few possible experiments to probe the stress relaxation possible from filament turnover. By squishing the cell into a hot dog shape and then letting it freely relax to a sphere, one can approximate the viscosity of the cell's surface. This is because the timescale of relaxation to spherical for a purely viscous droplet embedded in a medium with much lower viscosity is  $\tau \sim T/\eta$  where  $T$  is the surface tension and  $\eta$  is the droplet viscosity.

Some of my preliminary experiments showed that this technique was highly reproducible. In addition, by carrying out the experiments on days with different barometric pressures and in rooms with different temperatures, I was able to get a consistent shift in the relaxation timescale between sets of samples. Finally, in an experiment with Latrunculin A, a factor that largely depolymerizes the entire actin cytoskeleton, I could determine that the timescale of relaxation for cells in the absence of cortical structure was effectively instantaneous. Thus, changes in cortical viscosity should be easily assessed.

I made several attempts to perform the experiment myself, but, alas, my experimental chops were not up to the task. When treating with jasplakinolide to stabilize the cortex, the embryos underwent a global irreversible contraction. This is to be expected from our earlier observations where myosin acting on a network without turnover causes network contraction and tearing of the cortex. Therefore, in order to perform the experiments properly, one needs to first knock down myosin in the cortex. This presents some experimental difficulty, but can easily be overcome by anyone with sufficient training in *C. elegans* biology.

## APPENDIX A

### SOURCE CODE AND DOCUMENTATION

#### A.1 Practical Implementation of Model Framework

Although the prior description is useful for communicating the mathematical principles underlying my modeling framework, I also want to provide a practical explanation of the logic of the code structure. This will help to clarify the implementation details that may be ambiguous in the above formulation and make it easier for others to approach and modify my code.

The simulation code is entirely built on top of MATLAB's ode solving functions. Therefore, at the highest level, the entire simulation framework simply boils down to providing functions to define the ode's left and right hand side, along with the initial conditions for the system. However, because MATLAB's solvers do not allow discontinuities, discontinuous filament recycling events cannot be implemented directly in the ODE. Effectively, it is necessary to stop the simulation solver at fixed times, reset some subset of filaments, and restart the solver with the new state every time a turnover event happens.

In the following sections, I'll walk through the code's logic in more detail, emphasizing the packages and files that modularly summarize the core functions. First I will begin with an explanation of the command line interface used to set model parameters and deploying simulations.

To find the *activnet* simulation code, visit:

<https://github.com/wmcfadden/activnet/tree/master/simulation>

##### *A.1.1 Launching simulations from the command line*

The *activnet* package includes a function called *activnet\_gen.m*, which can be used to launch simulations on any MATLAB system. The following is the documentation of

the parameters passed to this function.

```

function p = activnet_gen(zet,L,mu,kap,lc,xi,ups,phi,psi,
                           r,sig,Dx,Dy,Df,Dw,ls,lf,tinc,tfin,nonlin)
% generates an active network simulation and prints node positions
% at time steps. Parameters are defined as follows:
%
%   zet - medium viscosity
%   L - length of the filament
%   mu - compressional modulus of the filament
%   kap - bending modulus of a filament if ls<L
%   lc - average distance between filament overlaps
%   xi - frictional resistance between two overlapping segments
%   ups - motor force at filament overlaps
%   phi - fraction of overlaps that receive a motor force
%   psi - spatial variation in motor force
%           (if external force applied, psi sets the periodicity of
%           force application, and psi<0 sets square wave.)
%   r - recycling rate
%   sig - applied stress in the x direction applied at Df*Dx
%           (sig<0 sets stress to be applied in y direction)
%   Dx - x-dimension of domain
%   Dy - y-dimension of domain
%   Df - position at which external force is applied
%           (if Df<0, also position in x dimension where network stops)
%   Dw - width of window in x-dimension where forces are applied
%   ls - length of filament segments
%   lf - length of force falloff at end of filament
%           (just ensures continuous forces)
%   tinc - time increment to return solutions
%           (will be decreased automatically if r is too large)

```

```
% tfin - end time of simulation
% nonlin - factor by which to make filament stiffer for extension
%           (if nonlin<0, add spacing so filaments dont reach edge)
```

These parameters will be explained in more detail below, but here I'll provide a brief practical explanation for their use in setting up simulations. The parameters `zet`, `L`, `mu`, `kap`, `lc`, `xi`, `ups`, `phi`, `r`, `sig`, `Dx`, and `Dy` are defined in the mathematical methods section above as  $\zeta$ ,  $L$ ,  $\mu_c$ ,  $\kappa$ ,  $l_c$ ,  $\xi$ ,  $v$ ,  $\phi$ ,  $1/\tau_r$ ,  $\sigma$ ,  $D_x$ , and  $D_y$ , respectively. The parameter `nonlin` is used to internally calculate the extensional modulus  $\mu_e = \text{nonlin} \times \mu_c$  (i.e. if `mu` is 1 and `nonlin` is 100, then  $\mu_c = 1$  and  $\mu_e = 100$ ). The parameter `psi` is used to generate a spatial gradient in internal activity or temporal periodicity if an external force is applied. The spatial gradient is linear with a maximum at the center of the domain. The parameter `Df` defines the fraction along the x-domain where any external stress will be applied (i.e. stress is applied at  $Df^*Dx$ ).

All of these terms are positive by definition so setting certain terms negative is used to trigger certain behavior in the simulation environment:

- Setting `mu` < 0 enables the extensional spring constant to be different than the compressive.
- Setting `sig` < 0 causes the external stress to be applied in the y direction rather than the x direction, resulting in shear stress simulations.
- Setting `nonlin` < 0 results in space being added to the edge of the domain so that no filaments intersect with the boundaries of the simulation, allowing the network to undergo unconstrained contraction.
- Setting `psi` < 0 results in oscillating positive and negative stresses being applied to the network (as opposed to a sinusoidal pattern regularly).
- Setting `Df` < 0 will cause the rightward edge of the initially generated network to end at the location where the force is applied.

The parameter `ls` sets the segment size of the filament. Theoretically this segment size is very small (the size of a single actin monomer perhaps), but for computational

reasons this value is set much larger. The parameters `Dw` and `lf` set the regions over which forces will fall off for the domain and individual filaments, respectively. These are just there to ensure there are no discontinuous changes in the ODE equation as filaments move around, and should be set to some smallish number, like 0.05, but don't affect the results too much overall. Finally, `tfin` and `tinc` set the end time of the simulation and the timesteps at which to print results, respectively. The program will automatically shrink `tinc` if the recycling rate, `r` is large enough that manual position updates must occur on timescales smaller than `tinc`.

To run a simulation with an external stress the following code could be used.

```
activnet_gen(0.001, 1, -0.01, 0, 0.8, 10, 0, 0, 0,
0, -0.002, 20, 4, -0.33, 0.05, 1, 0.025, 1, 10000, 100)
```

To run a simulation with internal filament activity the following code could be used.

```
activnet_gen(0.001, 5, -0.01, 0, 0.3, 10, 0.1, 0.25, 0,
0.001, 0, 25, 25, 0.5, 0.05, 5, 0.025, 1, 10000, -100)
```

Both of these have nonlinear filament extension because the third argument (`mu`) is less than 0. The results will be printed to the MATLAB console.

The currently deployed package can also be used to launch simulations on any linux system with MATLAB 2014b or the MATLAB 2014b Compiler Runtime installed (you will need to have a \$MATLAB environment variable enabled. Please contact your sysadmin.). In addition, the package can be recompiled on any system running MATLAB to create a new deployment that will run on the compiled MATLAB version and operating system (see the README.txt file for more information about deployment).

```
./run_activnet_gen.sh $MATLAB 0.001 1 -0.01 0 0.8 10 0 0 0 0
-0.002 20 4 -0.33 0.05 1 0.025 1 10000 100 > output.file
```

### *A.1.2 Package structure of activnet*

To understand the high-level overview of this code, you can analyze the basic package structure and the small number of important top level packages.

- activnet\_gen.m: topmost wrapping function to call for running simulations; confirms input, generates initial conditions, and calls activnet.m
- activnet.m: Chooses between which ODE implementation to run; stops ODE solver execution to perform filament recycling.
- odes folder: Contains all the ODE computation internals.
- helpers folder: Some additional functions written to perform line intersection tests.
- analysis folder: All the code used to generate the figures in the accompanying publication.

Note: the rest of this documentation is most useful if one is looking over the internals of the code. Trying to understand this without working with the code is probably not very useful.

### *A.1.3 ODE Wrapper functions*

As mentioned above, the initialization and launch of the simulation takes place using the function activnet\_gen.m. This function is quite simple. It just ensures proper input parameters, builds a randomized network of filaments, prints the network node positions, and then calls the function activnet.m to compute the simulation results. From there, activnet.m takes care of pausing solvers to update filament position for recycling as well as choosing between solvers depending on the type of problem being solved.

## Generating Initial Conditions

Before an ODE solve can be called one needs to set up the initial conditions of the network. Every solver implementation in MATLAB operates on 1D vectors where every element represents a variable to be integrated. Therefore, the data structure for this model is going to need to be set up to be encoded as a 1D vector of numbers. Because the model framework is based around the 2D position of points, I will continually be shifting between the 1D vector,  $z$  and the 2D vector  $p$  using the following transformations.

```
p = reshape(z0,[],2); % z -> p
z0 = reshape(p,1,[]); % z -> p
```

To select the number of filaments,  $N$ , to generate using the input parameters, we use the approximating formula  $N \approx \text{floor}(2*Dx*Dy/lc/L)$ . This formula is derived from the tiling of a  $Dx \times Dy$  domain with lines of length  $L$  and spacings between lines  $lc$ . This diverges from the exact number of filaments needed when  $L/lc$  is small, but we ignore this discrepancy because we mostly operate in the regime where  $L/lc > 10$ .

We therefore begin by creating an initial set of filament segment endpoints  $p$ , where we want to have  $N$  total filaments each with  $ncnt$  segment endpoints. We do this by selecting a random starting point in our domain ( $Dx$  by  $Dy$ ) with random orientation ( $\text{thet}$ ).

```
p = zeros(N*ncnt,2); %initialize all endpoints to 0
for i=1:N
    p((i-1)*ncnt+1,:) = [Dp*Dx*rand Dy*rand]; %random starting location
    thet = rand*2*pi; %random direction
    for j = 2:ncnt % iterate through remaining segments to add endpoints
        p((i-1)*ncnt+j,:) = p((i-1)*ncnt+j-1,:)+L/(ncnt-1.0)*[cos(thet) sin(thet)];
    end
end
```

In most cases, the network should be assembled such that the domain is filled entirely in the  $y$  dimension ( $y = 0$  and  $y = Dy$ ) and filled to the left edge ( $x = 0$ )

and up to the position  $x = Dp * Dx$  in the x-dimension. However, in the case that the parameter `nonlin` is less than 0, we set the initial conditions such that there is an empty space of size  $0.2Dx$  and  $0.2Dy$  at the x and y boundaries.

To keep our density the same we should adjust our calculation of  $N$  above, however, in the current implementation this adjustment is not made. This is a known bug that was corrected for after-the-fact in the publication by multiplication of  $l_c$  by a constant factor depending on the simulation setup.

## Choosing between active and driven simulations

During development I noticed that there were some cases where I could skip certain sections of the computation to speed up integration of the ODEs. In particular, when there is no internal forces generated by filament activity, the right hand side of the differential equation is easier to compute (see below). Therefore, in any situation where internal activity is set to 0, we use the more efficient code. The `activnet.m` code makes this decision by determining if there is pulling of filaments rather than internal force generation.

```
pull = (isempty(nu)&&sig^~=0)
```

In this case it runs a different pair of functions to compute the ODE right hand side and mass matrix (see below).

## Implementing Filament Recycling

The core function of `activnet.m` is to serve as a wrapper for the ODE solvers mentioned above and described in greater detail below. However, since the ODE solvers won't allow discontinuous changes to the positions of nodes, `activnet.m` must also carryout the task of stitching together smaller simulations and manually updating positions in between. The code in `activnet.m` implements a repeated loop with calls to the underlying ode solvers. If  $r > 0$ , a solver computes the integral between two adjacent time points in the vector of all time points to evaluate,  $tt$ . Then the positions are updated and the solver is used to evaluate integral for the next time interval. Logic

implemented in activnet\_gen.m ensures that the timesteps are selected small enough so that there will not be too many filaments disappearing at once.

The following code chunk implements the position update for a randomly selected subset of nodes whenever the recycling rate,  $r$ , is greater than 0. The logic implemented in activnet\_gen.m ensures that the timesteps are not so large that more than 5% of filaments will be undergoing a discontinuous jump at any one time (i.e  $r * istep * tinc < 0.05$ ).

```
% % select random indices
i = randi(N,floor(r*istep*tinc*N)+(rand<mod(r*istep*tinc*N,1)),1);
% % reset the position of the first segment endpoint
p((i-1)*ncnt+1,:) = [Dp*Dx*rand(size(i)) Dy*rand(size(i))];
% % pick random orientation for each filament
thet = rand(size(i))*2*pi;
for j = 2:ncnt % % iterate over rest of endpoints
    p((i-1)*ncnt+j,:) =
        p((i-1)*ncnt+j-1,:)+L/(ncnt-1.0)*[cos(theta) sin(theta)];
end
```

#### A.1.4 Overview of Numerical Integration Implementations

As mentioned above, the solution is numerically integrated using MATLAB's built-in ODE solvers. The ODE equation is the low-Reynolds limit of Newton's equation of motion for all the filaments. So to implement, I simply supply an ode solver with a function to compute the right hand side of a system of differential equations ( $\mathbf{f}(\mathbf{x})$ ) along with a function to compute the mass matrix ( $\mathbf{A}$ ) that connects the derivatives on the left hand side of the ODE system.

$$\mathbf{A} \cdot \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad (\text{A.1})$$

With those two matrices the system of differential equations can then be integrated to the desired level of precision by one of MATLAB's implicit ode solvers (e.g. ode15s

or `ode23`). In that equation,  $\mathbf{x}$  simply represents the positions of filament endpoints (nodes). Therefore  $\dot{\mathbf{x}}$  is just a way to represent the velocity of every endpoint.

The right hand side represents the forces (external or internal) driving the motion of the filaments. The mass matrix  $\mathbf{A}$  represents the coupling of filaments to each other (off diagonal elements) and to the solvent in which they are embedded (diagonal elements). Once those are provided, MATLAB's solvers take care of the integration (see MATLAB ode solver documentation for more info).

Right hand sides: *activnet\_act\_ode.m* vs. *activnet\_pull\_ode.m*

The right hand side of the equation constitutes all the non-viscous forces in the simulation. The most important and fundamental forces are those of the intrafilament spring mechanics. In these simulations, filaments are represented by chains of filament segments that act as springs with particular properties. As described above, each segment has a compressive (`mu`) and extensional (`mu * nonlin`) spring constant that governs the motion between endpoints along the length of each filament. There is also a bending spring constant (`kap`) which effectively tries to straighten adjoining segments if they are not already colinear. The internal mechanical forces of all filaments in the simulation is represented in the following code snippet.

```
%% compute intrafilament forces
10 = L/(ncnt-1); % length of segment
dp = zeros(size(p));
for n=1:ncnt:length(p)
    va_orth=[0 0];
    va = [0 0];
    la = 0;
    for i=0:ncnt-2
        % % custom subtraction minding boundaries
        vb = mydiff(p(n+i,:),p(n+i+1,:),Dx,Dy);
        lb = sqrt(vb*vb');
        gam = (lb-10)/10; % extension of the segment
```

```

f = mu*vb/lb*gam;           % longitudinal spring force
if(mu<0)                   % allow nonlinearity
    f = -f*(1+(muN-1)*double(gam>0));
end
dp(n+i,:) = dp(n+i,:) + f;
dp(n+i+1,:) = dp(n+i+1,:) - f;

% % below is for bending stiffness

vb_orth = [-vb(2) vb(1)];
if(i>0)
    if(va_orth*vb'>0)
        va_orth = -va_orth;
    end
    if(vb_orth*va'<0)
        vb_orth = -vb_orth;
    end
    tor = kap/10^2*acos(max(min(va*vb'/la/lb,1),0));
    dp(n+i-1,:)=dp(n+i-1,:)+tor*va_orth/la;
    dp(n+i,:)=dp(n+i,:)-tor*va_orth/la;
    dp(n+i+1,:)=dp(n+i+1,:)+tor*vb_orth/lb;
    dp(n+i,:)=dp(n+i,:)-tor*vb_orth/lb;
end
va = vb;
va_orth = vb_orth;
la = lb;
end
end

```

The code loops over every `ncnt` nodes, which constitute the nodes of a single filament. Next the code loops through each node of the individual filament. First, the

code evaluates the extensional force on each node given by  $f = \mu * v_b / l_b * \gamma$  with the modification that if  $\mu < 0$  and  $\gamma < 0$ , the force constant is modified to incorporate the nonlinear extensional stiffness. Next, the code evaluates the bending forces for adjacent segments that are not aligned using

```
tor = kap/10^2*acos(max(min(va*vb'/la/lb,1),0)).
```

The `max(min(...))` is used simply to ensure that there are no aberrantly large values due to errors in evaluation of `acos` with arguments very slightly greater than 1 or less than 0. The two vectors `va_orth` and `vb_orth` are calculated to direct the force orthogonal to the orientation of the filament segment. Finally, it should be noted that a bending force is generated from both the segment behind and the segment in front of the current node of interest and that the force from these need to be assigned at both the current node and the equal and opposite force assigned at the other end of each filament.

After the basic mechanical properties of the filament are accounted for, we need to add extra forces (otherwise the simulations won't be any more interesting than just a bunch of inert springs). What we do depends on whether the simulation is meant to represent active motors or a passive network with an external driver. In the next two sections we cover the specifics of the force equations for both of these cases.

## Active ODE

For the ODE with internal activity, we need to compute a very different force on each node. In particular, we have to check for intersections and add forces to filaments that intersect as if they are being pulled on by an active motor at their overlap point. To do this we will use one of the helper functions described below to return all of our overlapping lines. After that we will step through all pairs of intersecting lines and add the appropriate force in the appropriate direction. The following code snippet shows the process with documentation.

```
g = lineSegmentGrid(indL,XY,Dx,Dy,10); % find instersecting lines
```

```
% find distance of intersections from endpoints
```

```

f = min(1,max(0,(g-lf/2)/(1-lf)));
for ind=1:size(g,1)
    i = g(ind,3);
    j = g(ind,4);

    vm = mydiff(p(j,:),p(j+1,:),Dx,Dy);
    lm = sqrt(vm*vm');

    edg = 1;    % this will be used to reduce the force
    % as it gets closer to the edge of a filament

    if(g(ind,1)<lf)
        edg = edg*g(ind,1)/lf;
    elseif((1-g(ind,1))<lf)
        edg = edg*(1-g(ind,1))/lf;
    end

    if(g(ind,2)<lf)
        edg = edg*g(ind,2)/lf;
    elseif((1-g(ind,2))<lf)
        edg = edg*(1-g(ind,2))/lf;
    end

    mul = 1;    % this will be used to modulate the force if
    % psi says there should be a spatial gradient

    if(psi>0)
        mul = double(g(ind,5)>=psi*abs(-Dx*Df));
    end

tnu = nu(ceil(i/ncnt),ceil(j/ncnt))*mul;  % variation in

```

```
% filament force (phi)

dp(i:i+1,:) = dp(i:i+1,:)+edg*tnu/lm*[vm*(1-f(ind,1));vm*f(ind,1)];
dp(j:j+1,:) = dp(j:j+1,:)-edg*tnu/lm*[vm*(1-f(ind,2));vm*f(ind,2)];

end
```

The last line shows the net force that is added to both pairs of nodes that are intersecting ( $i:i+1$  and  $j:j+1$ ) with one positive and the other negative to conserve the net force to be 0. The term  $f$  is calculated as the distance of the overlap point between the two nearest node locations, and is used to set how much force is applied to the first vs second node. For each iteration of the loop, the force is calculated for the  $j:j+1$  filament, and each interaction is stepped through twice (once with each filament in the  $j:j+1$  role). The  $vm$  calculation determines the direction in which the forces will be applied. The terms  $edg$  and  $tnu$  simply modify the applied force by scalar quantities based on closeness to the end of the filament and spatial variation in motor force intensity.

## Pulling ODE

For the case of the system with an external stress, we will need to add forces at the desired location of stress, and we will need to constrain the filaments located at the edge of the domain. The following two code snippets show these two behaviors.

```
%% add external force at centerline
if(psi>0)
val = sig*sin(psi*t);
elseif(psi<0)
val = sig*round(mod(0.5+-psi*t,1)).*(round(mod(0.55+-psi*t/2,1))-0.5)*2;
else
val = sig;
end
```

```

subp = p(:,1)>(Df-Dw)*Dx&p(:,1)<(Df+Dw)*Dx;
ff = 1-abs(p(subp,1)-Df*Dx)/Dw/Dx;
if(sig<0)
    dp(subp,1)=dp(subp,1) - Dy*val.*ff/sum(ff);
else
    dp(subp,2)=dp(subp,2) - Dy*val.*ff/sum(ff);
end

```

The top section of this code calculates stress that should be applied and sets that as `val`. If `psi` is nonzero then we are looking to have a temporally varying applied stress. If `psi > 0`, then we want a sinusoidally varying stress with frequency `psi`. If `psi < 0`, then we want a square wave with frequency `psi`. The max stress in each case is still `sig`.

The bottom section selects the nodes that will have force added to them in `subp`. The force to be applied to each node is not equally distributed, but instead, `ff` sets the fraction of force to be proportional to the distance to the center of the region of applied stress (set by the width `Dw`). This distribution function is normalized so that the total amount of force is equal to the amount of force needed to set the stress to `val` (i.e.  $F_{total} = val \times Dy$ ). Finally the force is applied in the x direction or the y direction based on whether `sig` is greater or less than 0.

```

% % constrain edges
subp = p(:,1)<Dw*Dx;
dp(subp,:)=dp(subp,:).*repmat(4*p(subp,1)/Dw/Dx-3,1,2);

subp = p(:,1)>Dx*(1-Dw);
dp(subp,:)=dp(subp,:).*repmat(4*abs(p(subp,1)-Dx)/Dw/Dx-3,1,2);

subp = p(:,1)<3*Dx/4*Dw|p(:,1)>Dx-3*Dx/4*Dw;
dp(subp,:)=0;

```

This last segment merely constrains the force at the far left and right edges of the domain to be 0 (within  $0.75D_w \times D_x$  from each edge). There is a linear transition to  $dp=0$  in the remaining 25% of the region.

It should be noted that in the pulling simulations, there is no need to compute the intersection of filaments. This means that there are far fewer computations than in the active case. I wrote two separate functions for each of the different cases simply to avoid having to perform redundant checks on the cases repeatedly (i.e. the choice is made once in the outer wrapping functions rather than having to repeatedly make the choice for which code to run on each iteration of the solver).

#### *A.1.5 Left hand side: The mysterious mass matrix*

To understand the logic of coupling filaments together by their velocities it might be helpful to look at a simple example. Imagine that you have a single particle in some one-dimensional fluid with viscosity,  $\zeta$ , and with a force,  $F$ , acting on it. Assuming low Reynolds limit so there is no acceleration of the particle, the equation of motion would look like the following.

$$\zeta \dot{x} = F \quad (\text{A.2})$$

Now assume you have two particles in that fluid, but (for now) we assume the particles can't interact in any way. Therefore, we can express the equation of motion for both particles pretty simply with the following equation.

$$\zeta \dot{\mathbf{x}} = \mathbf{F} \quad (\text{A.3})$$

Obviously, all we did was replace the scalars with vectors. To make things a little neater we could replace the scalar  $\zeta$  with a so called mass matrix that would simply look like the following matrix.

$$\mathbf{A} = \begin{bmatrix} \zeta & 0 \\ 0 & \zeta \end{bmatrix} \quad (\text{A.4})$$

Now, if we had some drag-like coupling between the velocities of our two particles with a drag coefficient,  $\xi$ , we could simply add a term to the off diagonal.

$$\mathbf{A} = \begin{bmatrix} \zeta & -\xi \\ -\xi & \zeta \end{bmatrix} \quad (\text{A.5})$$

And carrying out our math we can see that this just gives a frictional coupling between the two particles.

$$\begin{aligned} \zeta v_1 - \xi v_2 &= F_1 \\ \zeta v_2 - \xi v_1 &= F_2 \end{aligned} \quad (\text{A.6})$$

This is the entire logic behind the construction of the `activnet_mass.m` function. After finding which filament segments are overlapping, I simply add off-diagonal terms to the mass matrix that couple the nodes of those filament segments together.

Similar to the previous section, there are computational efficiencies to be gained in computing the mass matrix,  $\mathbf{A}$ , if the system is being driven externally (as opposed to being driven by internal activity).

### *activnet\_mass.m vs. activenet\_mass\_sp.m*

When we are constraining filaments to remain motionless we run into one little problem with our mass matrix formulation as presented thus far. Essentially, the right hand side of the equation is being set to 0, but the left hand side still has cross terms. To remedy this, in the case where an external stress is applied, I need to manually decouple filaments that lie along the  $x = 0$  boundary of the domain. This is carried out in `activenet_mass_sp.m`, but it is not implemented in `activenet_mass.m` because that code only runs in the case where there is internal activity of the filaments and no external constraints.

Additionally, in `activenet_mass_sp.m`, I utilize a sparse matrix for reasons that I honestly don't exactly remember. I assume that I found in that condition I could get some kind of marginal speedup by using a sparse matrix instead of the full matrix.

### *A.1.6 Line intersection helper Functions*

There are a series of helper functions that are called to aid in the calculations of filament intersections. You can analyze the code directly for more detail but briefly, the lineSegmentIntersect.m implementation manually computes the intersection between all pairs of segments while lineSegmentGrid.m first bins lines into grids before testing intersection just on those in the bin. I saw a significant improvement when I moved to lineSegmentGrid even though asymptotically they both perform worst case  $O(n^2)$ , which I believe is due to the uniformity of the line segment distribution. Some mathematician might be able to prove that. It is interesting to note that the classical best-case line scanning algorithm (i.e. sweep line) is  $O(n \log n)$ .

### *A.1.7 Visualization code*

In the analysis package, I have included some code called netplot\_str.m to aid with visualizing the output of the simulations. This code opens the output file and an accompanying script file to get the parameters passed to the function. The code then goes on to render all the lines in a MATLAB plot. It could be modified easily to render the output in whatever format you may need. However, it is important to note that you need the input parameters to correctly associate output nodes to their correct filament.

## **A.2 Finding Source Code Online**

Source code and up-to-date documentation are available online for the projects described in this dissertation.

- <https://github.com/wmcfadden/smPress>
- <https://github.com/wmcfadden/activnet>
- <https://github.com/munrolab/pulse-reaction-dynamics>

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