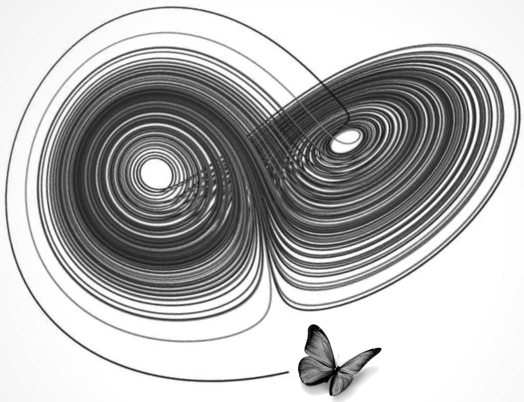
Biology and Dynamical Systems

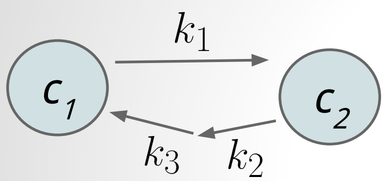


Week 5: Analyzing equations and understanding simulation output

Today's aim is to show how our models are a way to provide clarity on the complex systems

1. We've seen that raw simulation output can be unwieldy
2. Looking for qualitative differences in simulations
3. Reducing the number of extraneous constants
4. Finding a phase diagram

How many parameters do we have to vary to get a good idea of how our model works?



$$\frac{dc_1}{dt} = k_3 C - k_3 c_2 - (k_3 + k_1) c_1$$

$$\frac{dc_2}{dt} = k_1 c_1 - k_2 c_2$$

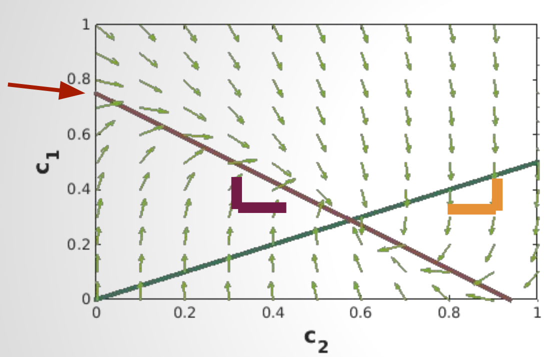
What happens if you double k_3 or k_1?

What about doubling C and halving k_3?

It is nice to know that the computer understands the problem. But I would like to understand it too.

--Eugene Wigner

Constants set quantitative values of a model, but it's often better to look for qualitative differences



$$c_1^* = \frac{k_3 C}{k_1 + k_3} - \frac{k_3 c_2^*}{k_1 + k_3}$$

$$c_1^* = \frac{k_2 c_2^*}{k_1}$$

We can get the same equilibrium value for different parameter values

Can equilibrium become impossible? How?

Limiting the number of parameters

$$\frac{dc_1}{dt} = k_3C - k_3c_2 - (k_3 + k_1)c_1$$
$$\frac{dc_2}{dt} = k_1c_1 - k_2c_2$$

```
function dc = myode(t,c,C,k1,k2,k3)
    c1 = c(1);
    c2 = c(2);
    dc1 = k3*C - k3*c2 - (k3+k1)*c1;
    dc2 = k1*c1 - k2*c2;
    dc = [dc1;dc2];
end
```

At first glance, it appears that we have to understand how the 4 parameters come together and how each one affects the others. But based on the form of the equation we know that we really only care about 2 parameters

Using variable substitution to simplify all of those extraneous constants

$$\frac{dc_1}{dt} = k_3C - k_3c_2 - (k_3 + k_1)c_1 \qquad \frac{dc_2}{dt} = k_1c_1 - k_2c_2$$

substitute variables so that we hide 1 constant for each variable

$$\tau = (k_1 + k_3)t \qquad \rho_1 = \frac{k_1 + k_3}{k_3C}c_1 \qquad \rho_2 = \frac{(k_1 + k_3)^2}{k_1k_3C}c_2$$

rename the remaining factors so that everything looks pretty

$$\alpha = \frac{k_1k_3}{(k_1 + k_3)^2} \qquad \beta = \frac{k_2}{k_1 + k_3}$$

By making those substitutions we can reduce our system to depend on just 2 parameters

$$\frac{dc_1}{dt} = k_3 C - k_3 c_2 - (k_3 + k_1) c_1 \qquad \frac{dc_2}{dt} = k_1 c_1 - k_2 c_2$$

$$\frac{d\rho_1}{dt} = 1 - \alpha\rho_2 - \rho_1 \qquad \frac{d\rho_2}{dt} = \rho_1 - \beta\rho_2$$

$$\rho_1^* = 1 - \alpha\rho_2^* \qquad \rho_1^* = \beta\rho_2^*$$

$$c_1^* = \frac{k_3 C}{k_1 + k_3} - \frac{k_3 c_2^*}{k_1 + k_3} \qquad c_1^* = \frac{k_2 c_2^*}{k_1}$$

Saving time and complexity

We went from needing to understand

6 parameters and 3 variables ----------------------->

To only needing 2 parameters and 2 variables

$$\frac{dc_1}{dt} = k_3 c_3 - k_1 c_1$$

$$\frac{dc_2}{dt} = k_1 c_1 - k_2 c_2$$

$$\frac{dc_3}{dt} = k_2 c_2 - k_3 c_3$$

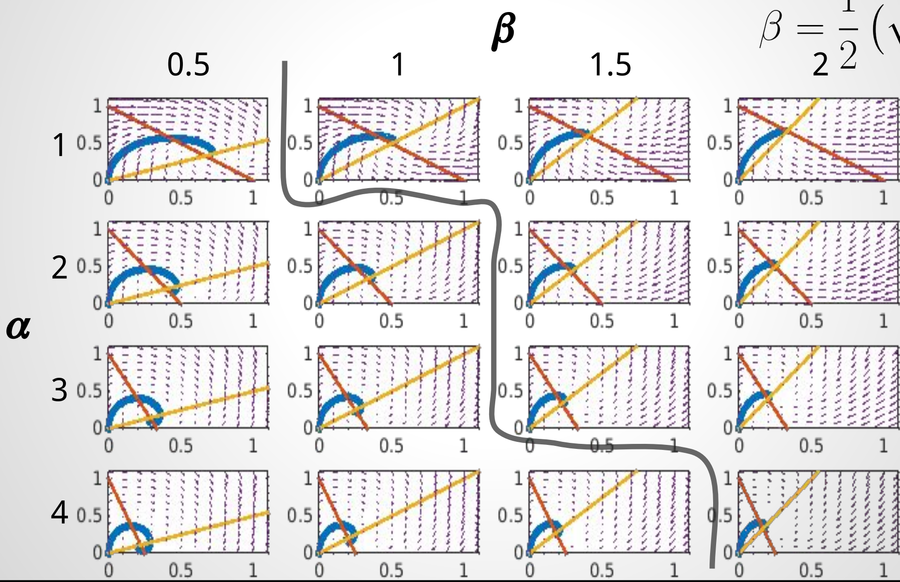
$$\frac{d\rho_1}{dt} = 1 - \alpha\rho_2 - \rho_1$$

$$\frac{d\rho_2}{dt} = \rho_1 - \beta\rho_2$$

```
function dc = myode(t,p,a,b)
        p1 = p(1);
        p2 = p(2);
        dp1 = 1 - a*p2 - p1;
        dp2 =p1 - b*p2;
        dp = [dp1;dp2];
end
```

With 2 variables we can see how the qualitative behavior changes depending on the value of α and β

$$\beta = \frac{1}{2}\left(\sqrt{4\alpha - 3} - 1\right)$$



You might be asking, "Was there some sleight of hand? How can you make the problem easier like that?"

Basically, what we did was pick the units of concentration and the units of time in a special way such that our parameters canceled out of the equations. Picking very special units doesn't matter because we can always get to new units by multiplying everything by the same amount.

For example, if I say some reaction takes 2 hours and then we change units so that it takes 72000 seconds, that doesn't mean the dynamical system changes in any way. All the numbers get bigger by the same amount.

Discussion: Let’s simplify some equations (40 pts)

1. Break into groups
2. Each group will get a differential equation with a description of a system
3. Reduce the number of extra constants (5 pts)
4. Implement a simulation with the simplified equation (5pts)
5. Diagram how the behavior changes as the constants are varied (10 pts)
6. We’ll regroup after 15-20 minutes and go through each one
7. 10 pts for explaining your reasoning
8. 10 pts for giving good feedback to others

Project: Diagram the behaviors of your system

1. Continue with the system you wrote about in the last project assignment
2. Try to simplify your equations again (10 pts)
    a. Try to use tricks like the ones we came up with today
3. Modify your simulation to reflect your simplification (5 pts)
4. Systematically vary parameters to find qualitative changes (10pts)
5. Describe the meaning of each of your simplified parameters (15 pts)
    a. explain the role that each constant plays in setting the dynamics

Reading: Read the one-page handout and be ready for a simple quiz at the start of next week

1. There will be a 5 minute, 4 question (10 pts each) quiz at the start of next class.
2. It is just designed to determine if you read the handout.
3. You probably won’t even have to think if you do the reading.