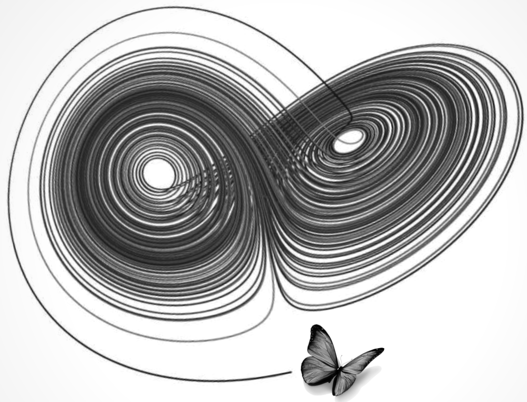
Biology and Dynamical Systems



Week 3: Visualizing equations with graphs

Today's aim is to show what we can do to visualize our equations so we can think about them concretely

Outline:

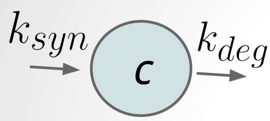
What to do when equations get tricky

Graphically visualizing rates of change

Visualizing the meaning of equilibrium

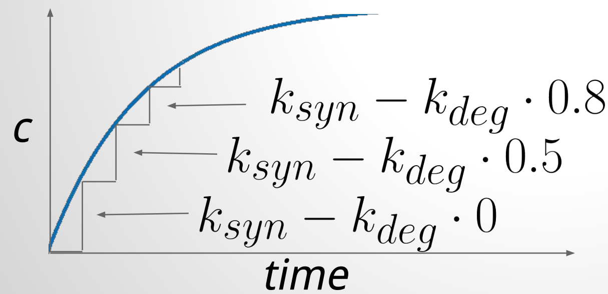
More complicated examples

What happens if we look at a protein that's being synthesized and degraded



synthesis and degradation of gene

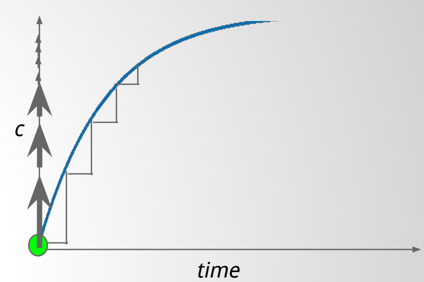
$$\frac{dc}{dt} = k_{syn} - k_{deg}c$$

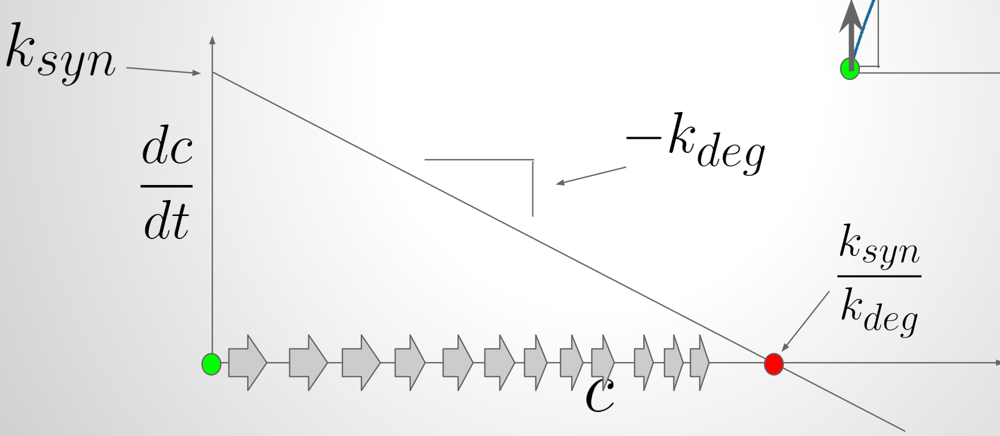


Does it ever stop? When?

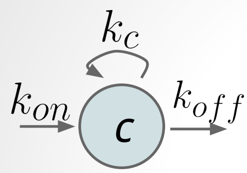
We graph dc/dt to see the protocol for how *c* changes.

$$\frac{dc}{dt} = k_{syn} - k_{deg}c$$



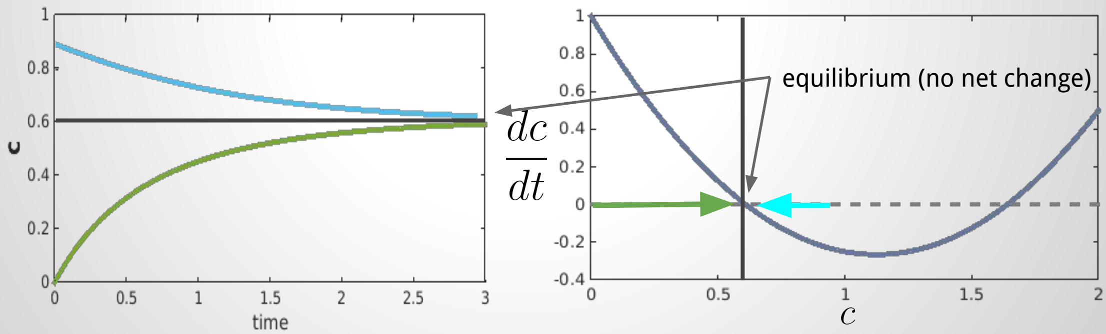


This same graphical presentation can be used to analyze more and more complicated systems

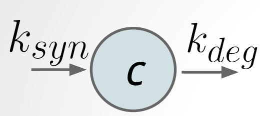


Example: gene auto-regulation

$$\frac{dc}{dt} = k_{on} + k_c c^2 - k_{off} c$$

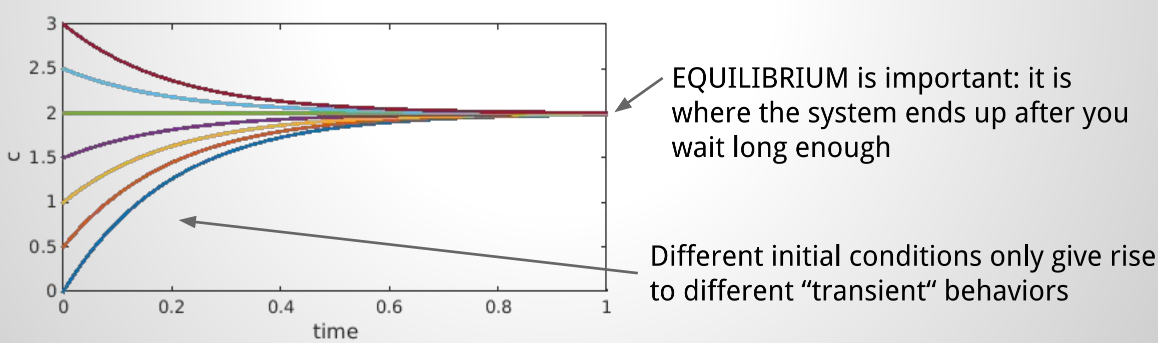


We can find equilibrium points by looking for concentrations at which the net change equals zero

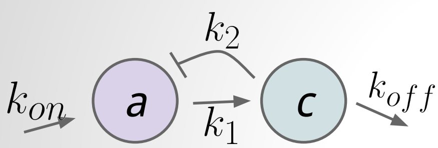


Example: gene auto-regulation

$$\frac{dc}{dt} = k_{syn} - k_{deg} \cdot c = 0$$



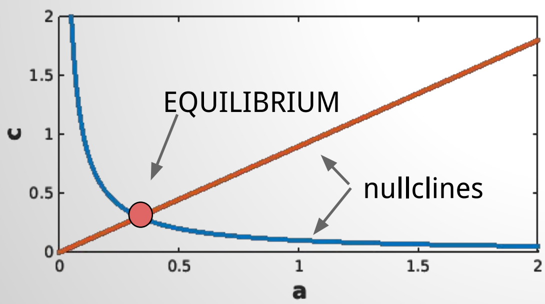
Graphing equilibria makes it much easier to understand two-component systems



Example: gene feedback

$$0 = k_1 a^* - k_{off} c^*$$

$$0 = k_{on} - k_2 c^* \cdot a^*$$



$$\boxed{c^* = \frac{k_1 a^*}{k_{off}}}$$

$$\boxed{c^* = \frac{k_{on}}{k_2 a^*}}$$

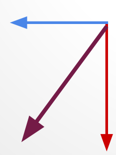
The "protocol" describes how *c* and *a* change depending on their current values

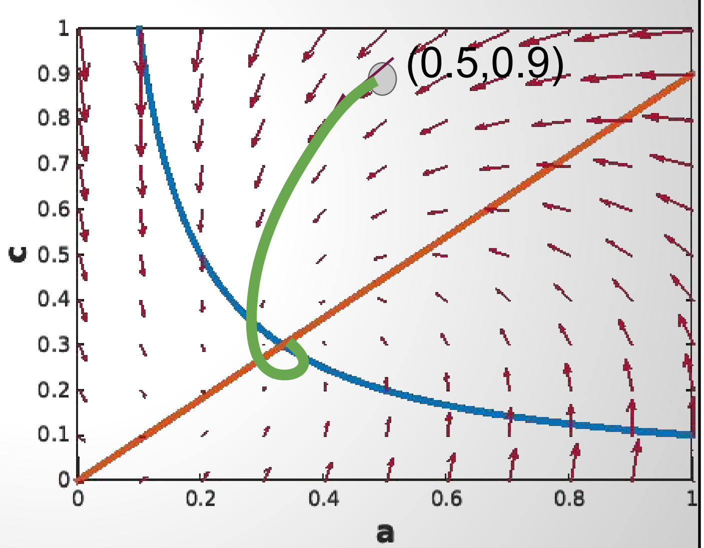
$$\frac{dc}{dt} = \overset{9}{k_1} a - \overset{10}{k_{off}} c$$

$$\frac{da}{dt} = \overset{1}{k_{on}} - \overset{10}{k_2} c \cdot a$$

$$\boxed{\frac{dc}{dt} = -4.5}$$

$$\boxed{\frac{da}{dt} = -3.5}$$





Question: What would happen if we multipied all of the constants (k) by 10?

$$\frac{dc}{dt} = \overset{9}{k_1} a - \overset{10}{k_{off}} c \qquad \text{vs} \qquad \frac{dc}{dt} = \overset{90}{k_1} a - \overset{100}{k_{off}} c$$

$$\frac{da}{dt} = \overset{1}{k_{on}} - \overset{10}{k_2} c \cdot a \qquad \qquad \frac{da}{dt} = \overset{10}{k_{on}} - \overset{100}{k_2} c \cdot a$$

Discussion: Let's convert a few reaction diagrams into differential equations (40 pts)

1. Break into groups
2. Each group will get a differential equation with a description of a system
3. Work for a few minutes to plot the nullclines and the arrows (10 pts)
4. We'll regroup after 15-20 minutes and go through each one
5. 10 pts if your group get both the nullcline and the arrows plotted
6. 10 pts for explaining your reasoning
7. 10 pts for giving good feedback to others

Project: Simplify and graph your system

1. Continue with the system you wrote about in the last project assignment
2. Try to simplify your equations to have two components (5 pts)
   a. You can assume that everything else is magically held constant
3. Write the nullcline equations and graph them for the system(10pts)
4. Find any relevant equilibrium points (5 pts)
5. Draw the “protocol” arrows on your graph and trace a sample trajectory
   a. 5 pts for hand drawn, 5 pts for MATLAB plots
6. Write a few paragraphs describing your logic (10pts)

Reading: Read the one-page handout and be ready for a simple quiz at the start of next week

1. There will be a 5 minute, 4 question (10 pts each) quiz at the start of next class.
2. It is just designed to determine if you read the handout.
3. You probably won’t even have to think if you do the reading.