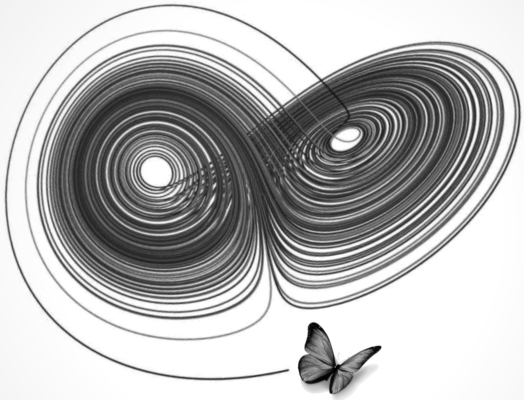
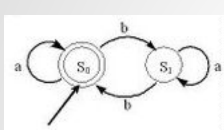
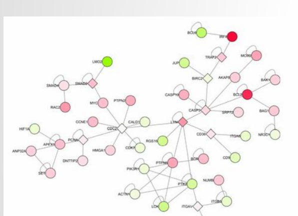
Biology and Dynamical Systems



Week 2: Modeling biological systems with differential equations

Mathematical modeling of biological systems can mean many different things



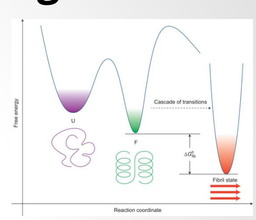


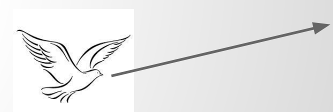
Discrete stochastic

Continuous stochastic

Discrete deterministic

Continuous deterministic





In this introductory class, we will focus on the simplest and most widely used type of continuous deterministic model: differential equations (diff E Q)

Today’s aim is to show what differential equations are and how to create them from reaction diagrams

Outline:

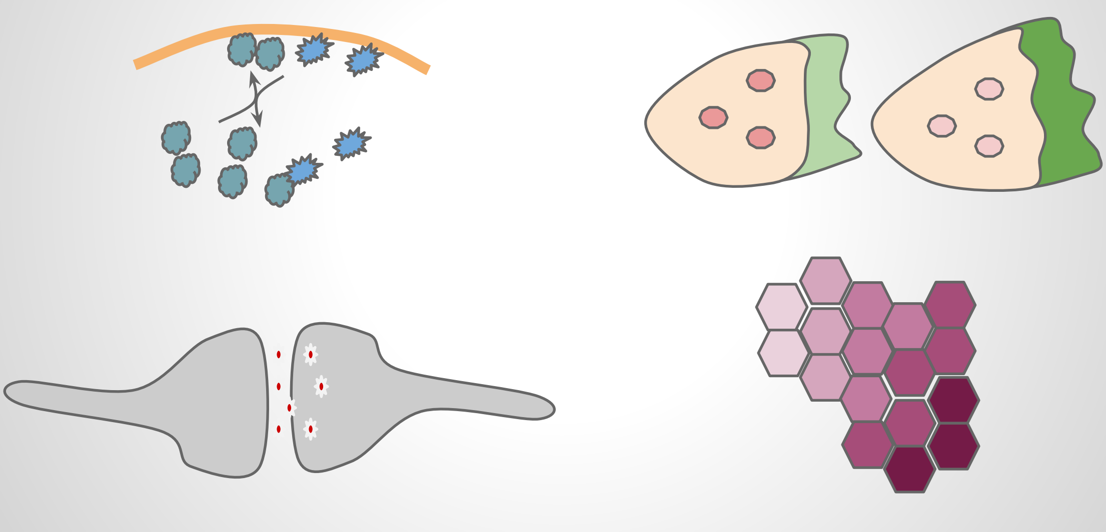
Describing rates of change

How to interpret reaction diagrams

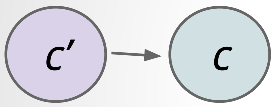
Converting reaction diagrams to differential equations

Examples

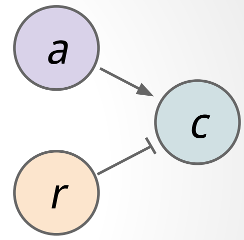
Biological models frequently aim to describe how proteins levels change over time (and space)



We use reaction diagrams to illustrate how protein levels change based on other cellular components



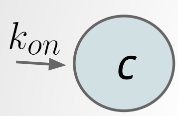
c′ is converted into c



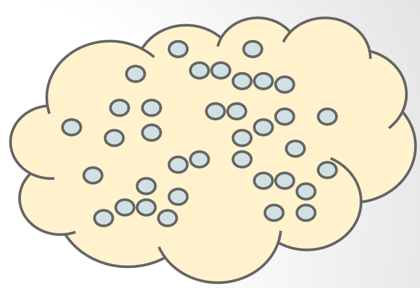
c is upregulated by a
c is downregulated by r

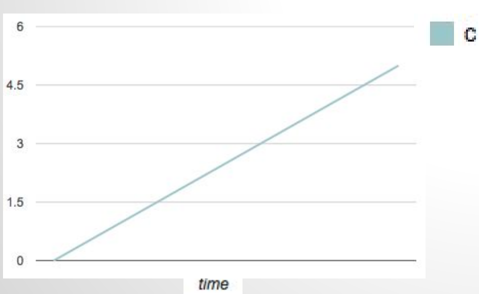
Differential equations formalize these relationships by defining protein level *rates of change*.

Differential equations state how protein levels change at every moment in time



Example: synthesis of a gene





$$\frac{dc}{dt} = k_{on}$$

change in concentration → over small time period →

$$\frac{dc}{dt} = k_{on}$$

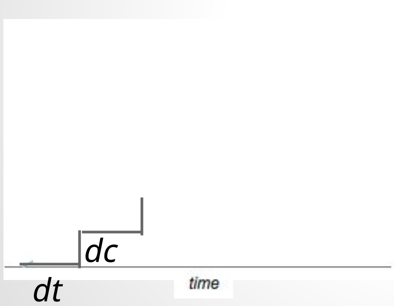
← is a constant

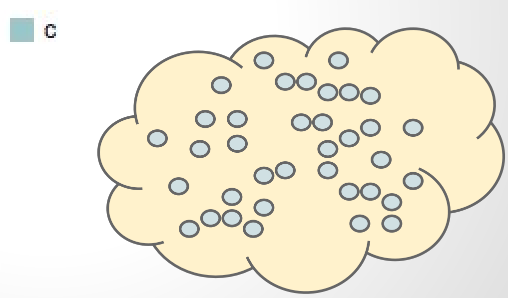
change in concentration → over small time period →

$$\frac{dc}{dt} = k_{on}$$

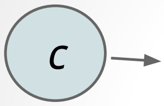
← is a constant (in c/second)

$t =$

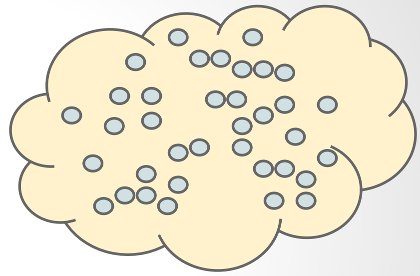




In most situations, a protein's rate of change will depend on how much is around



Example: degradation of a gene



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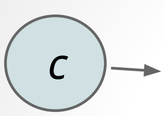

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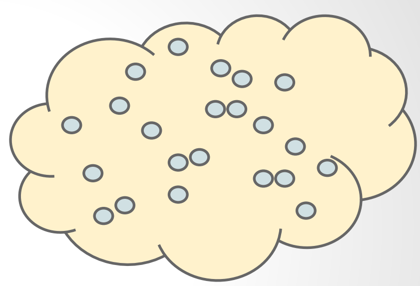

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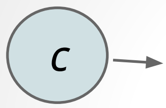
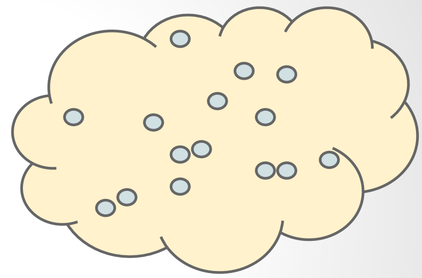


Example: degradation of a gene



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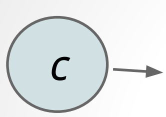
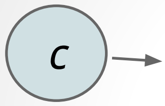
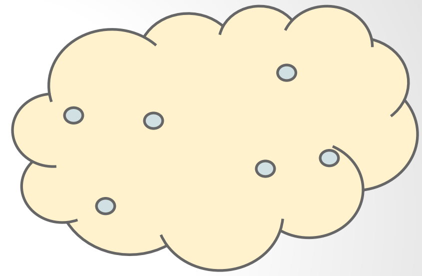


Example: degradation of a gene

In most situations, a protein’s rate of change will depend on how much is around



Example: degradation of a gene



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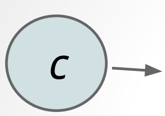
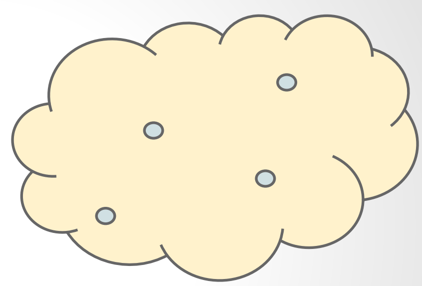


Example: degradation of a gene



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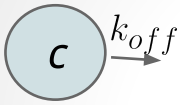
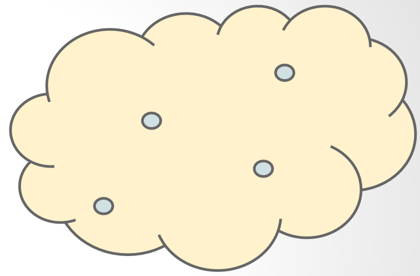
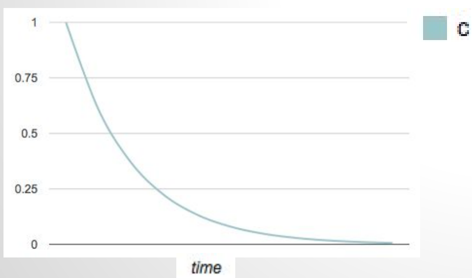


Example: degradation of a gene



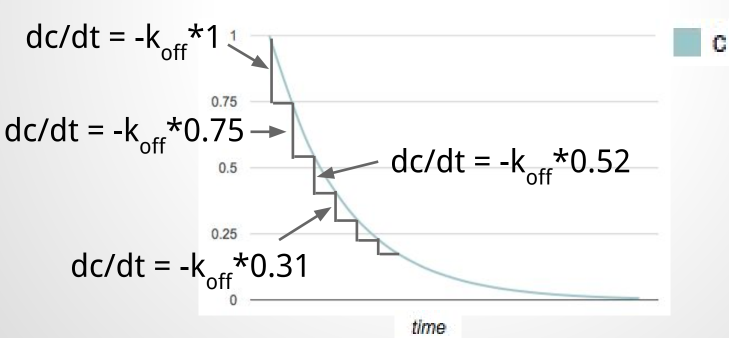


Note that rate depends on c

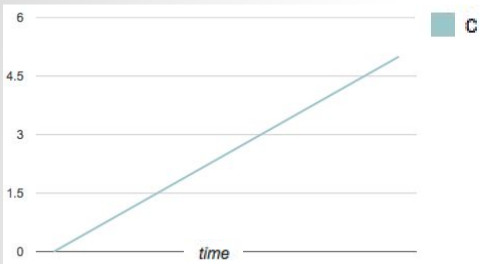
$$\frac{dc}{dt} = -k_{off}c$$

As the amount of *c* gets smaller, the rate of change gets smaller too

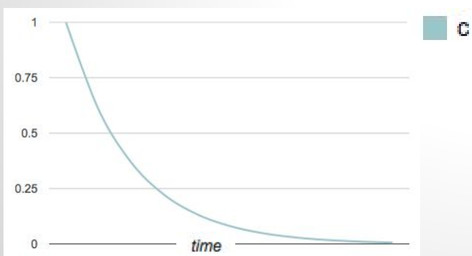
$$\frac{dc}{dt} = -k_{off}c$$



Discussion break:
What do k_on and k_off do?

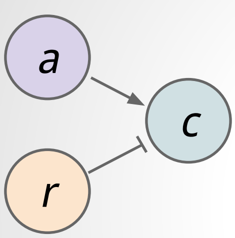


$$\frac{dc}{dt} = k_{on}$$

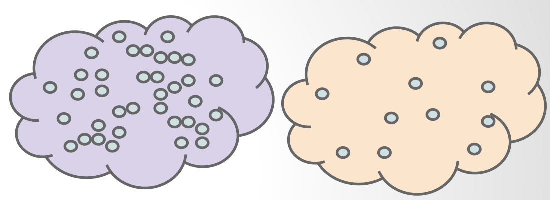


$$\frac{dc}{dt} = -k_{off}c$$

Most of the value in these models comes from describing how one factor affects another

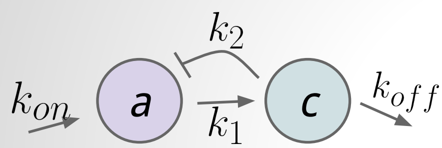


Example: gene regulation



$$\frac{dc}{dt} = a \cdot k_{on} - r \cdot k_{off}c$$

But the real beauty of diff eq is seeing how all the parts of the whole system work together



Example: gene feedback

$$\frac{dc}{dt} = k_1 a - k_{off} c = 0$$

$$\frac{da}{dt} = k_{on} - k_2 c \cdot a = 0$$

Next week we'll learn more about how to analyze these complex systems

Discussion: Let's convert a few reaction diagrams into differential equations (40 pts)

1. Break into groups
2. Each group will get a reaction diagram with a description of a system
3. Work for a few minutes to try to jot down a differential equation (10 pts)
4. We'll regroup after 15-20 minutes and go through each one
5. 10 pts if your group gets it right
6. 10 pts for explaining your reasoning
7. 10 pts for giving good feedback to others

Project: Convert your biological system into a differential equation

1. Continue with the system you wrote about in the last project assignment
2. Write a diff equation describing how your system changes in time (20 pts)
3. Write a few paragraphs on the logic that went into your equation (15pts)
4. Using program of choice, plot a fake result of a possible simulation (5pts)

Reading: Read the one-page handout and be ready for a simple quiz at the start of next week

1. There will be a 5 minute, 4 question (10 pts each) quiz at the start of next class.
2. It is just designed to determine if you read the handout.
3. You probably won't even have to think if you do the reading.