The background features a large, semi-transparent watermark of the NPTEL logo. It consists of a circular emblem with a stylized flower or star in the center, surrounded by a ring of colored segments. Below the emblem, the word "NPTEL" is written in large, bold, sans-serif capital letters.

NPTEL MOOC, JAN-FEB 2015
Week 8, Module 5

DESIGN AND ANALYSIS OF ALGORITHMS

Reductions

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Bipartite Matching

- * Each instructor is willing to teach a set of courses
- * Find an allocation so that
 - * Each course is taught by a single instructor
 - * Each instructor teaches only one course, which he/she is willing to teach

Teachers

Courses

Abbas

Math

Chitra

History

Madan

Biology

Sunita

Economics



Bipartite Matching

- * V partitioned into V_0, V_1
- * All edges from V_0 to V_1
- * **Matching**: subset of edges so that no two of them share an endpoint
- * Find largest matching
 - * If possible, a **perfect matching**, all nodes covered

Teachers

Courses

Abbas

Math

Chitra

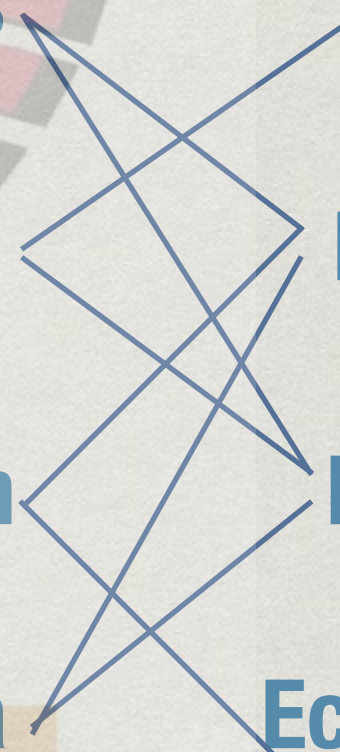
History

Madan

Biology

Sunita

Economics

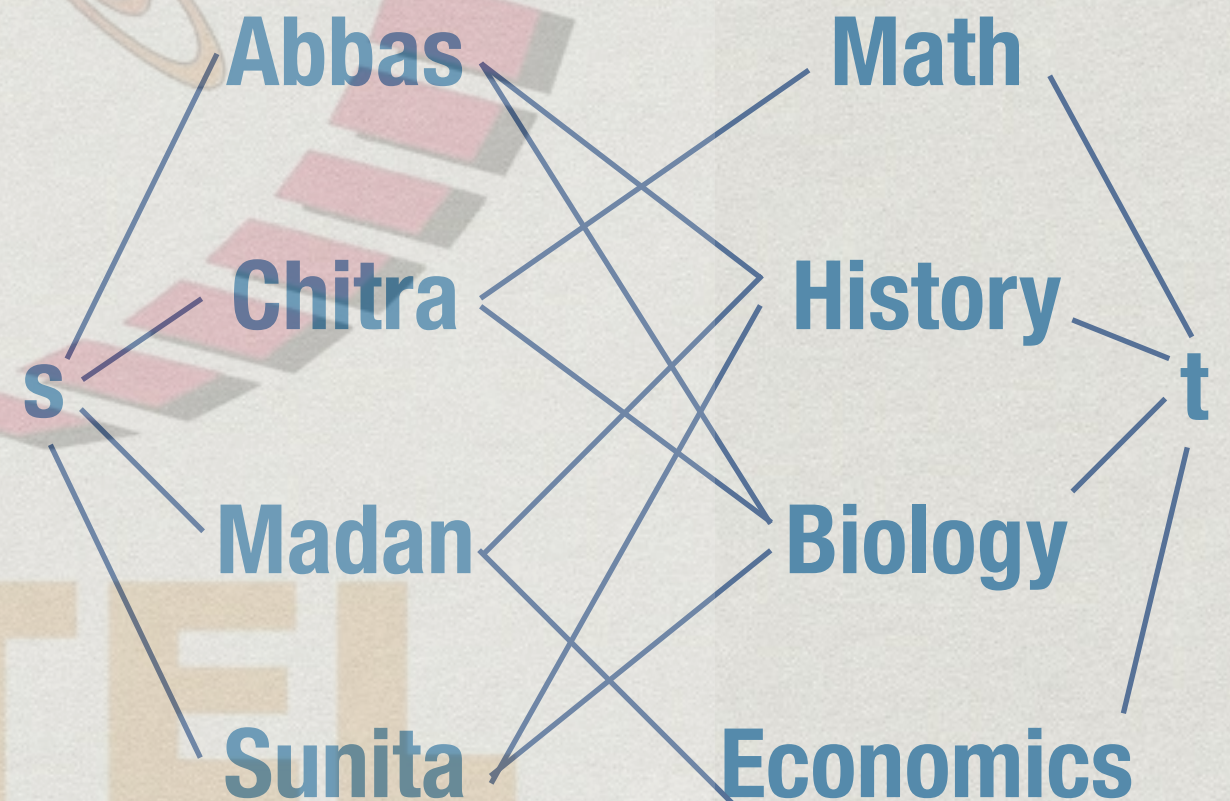


Bipartite Matching

- * Add a source and sink
- * All edge capacities are 1
- * Find a maximum flow from s to t!

Teachers

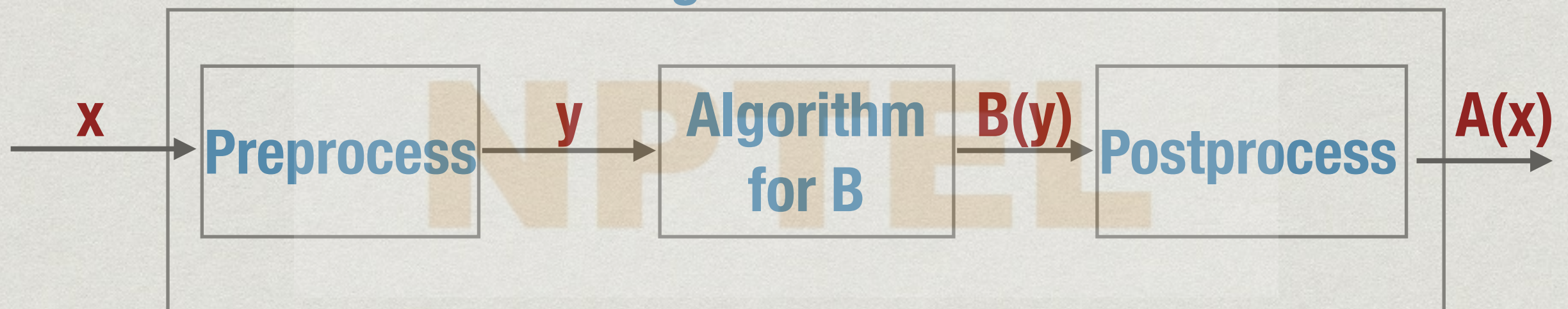
Courses



Reductions

- * We want to solve problem A
- * We know how to solve problem B
- * Convert input for A into input for B
- * Interpret output of B as output of A

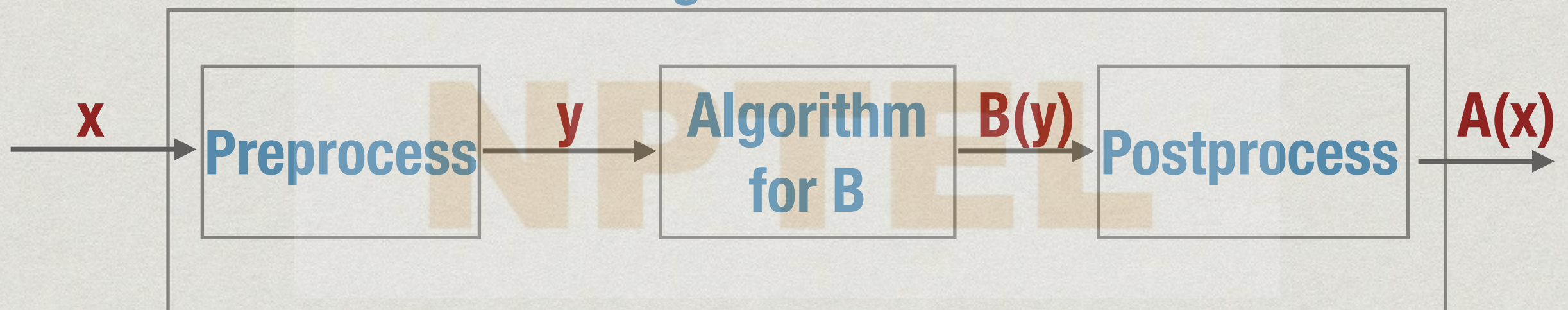
Algorithm for A



Reductions

- * A **reduces to** B
- * Can transfer efficient solution from B to A
 - * But preprocessing and postprocessing must also be efficient!
- * Typically, both should be polynomial time

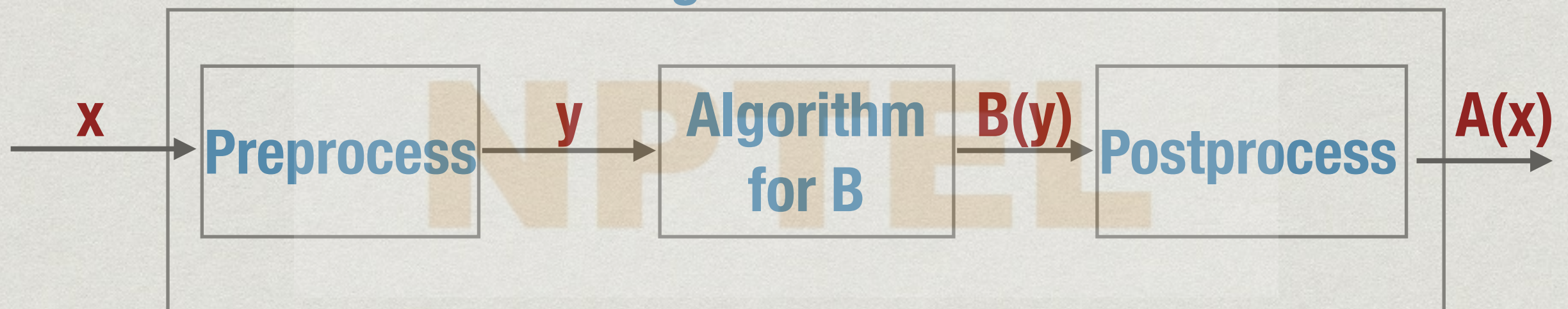
Algorithm for A



Reductions

- * Bipartite matching reduces to max flow
- * Max flow reduces to LP
- * Number of variables, constraints is linear in the size of the graph

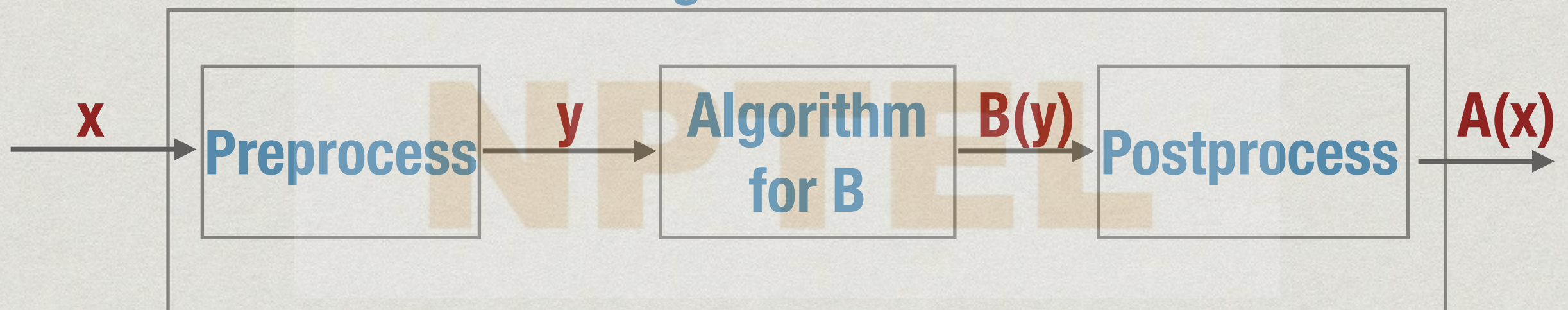
Algorithm for A



Reductions

- * Reverse interpretation is also useful
- * If A is known to be intractable and A reduces to B , then B must also be intractable
- * Otherwise, solution for B will yield solution for A

Algorithm for A



Big hammers

- * LP and network flows are powerful tools
- * Many algorithmic problems can be reduced to them
- * Efficient, off-the-shelf implementations are available
- * Useful to understand what can (and cannot) be modelled in terms of LP and flows