NPTEL MOOC,JAN-FEB 2015
Week 4, Module 1

DESIGN AND ANALYSIS OF ALGORITHMS

Shortest paths in weighted graphs

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Recall that …

- BFS and DFS are two systematic ways to explore a graph
  - Both take time linear in the size of the graph with adjacency lists
- Recover paths by keeping parent information
- BFS can compute shortest paths, in terms of number of edges
- DFS numbering can reveal many interesting features

Adding edge weights

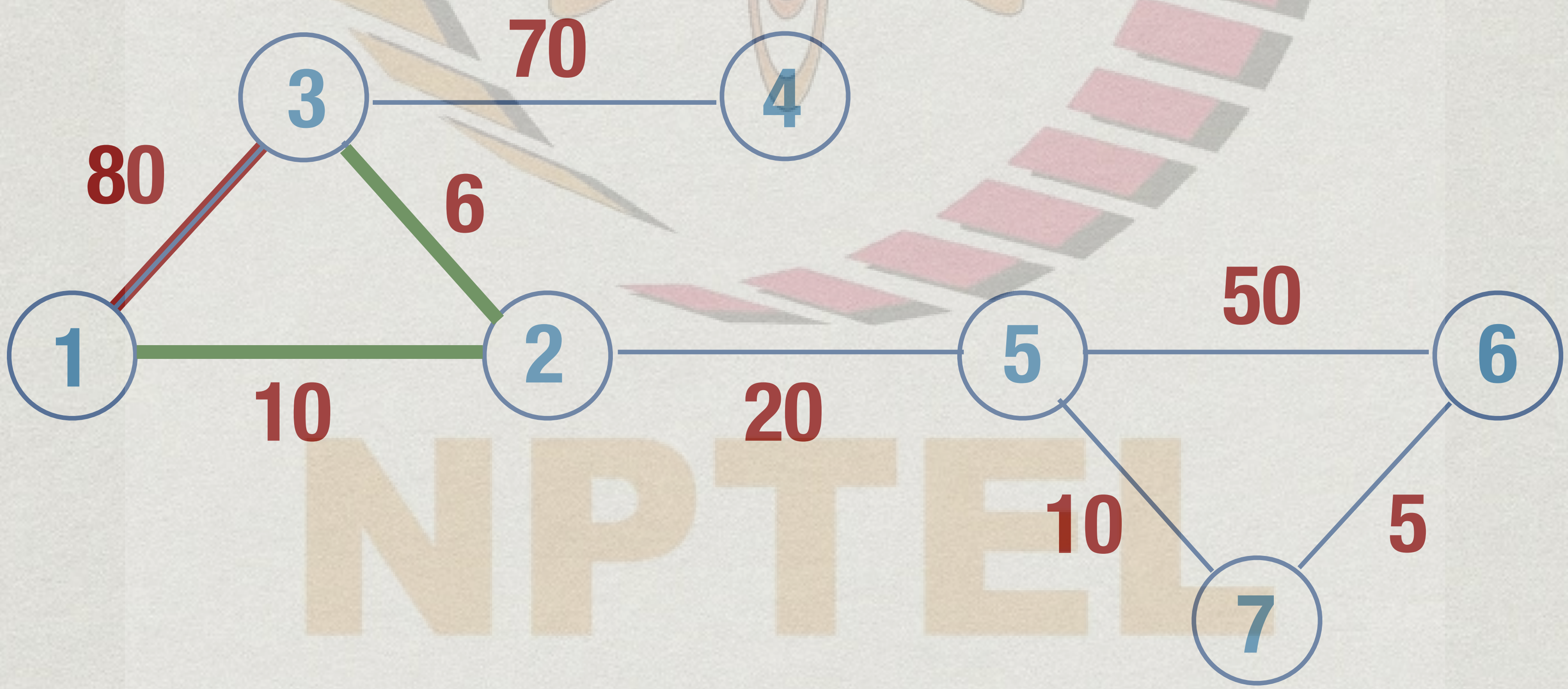
- Label each edge with a number—**cost**
  - Ticket price on a flight sector
  - Tolls on highway segment
  - Distance travelled between two stations
  - Typical time between two locations during peak hour traffic

Shortest paths

- **Weighted graph**
  - $G=(V,E)$ together with
  - **Weight function,** $w : E \rightarrow \text{Reals}$
- Let $e_1=(v_0,v_1)$, $e_2 = (v_1,v_2)$, …, $e_n = (v_{n-1},v_n)$ be a path from v_0 to v_n
- Cost of the path is $w(e_1) + w(e_2) + \ldots + w(e_n)$
- **Shortest path** from v_0 to v_n : minimum cost

Shortest paths …

* BFS finds path with fewest number of edges
* In a weighted graph, need not be the shortest path



Shortest path problems

- **Single source**
  - Find shortest paths from some fixed vertex, say 1, to every other vertex
  - Transport finished product from factory (single source) to all retail outlets
  - Courier company delivers items from distribution centre (single source) to addressees

Shortest path problems

- **All pairs**
  - Find shortest paths between every pair of vertices i and j
  - Railway routes, shortest way to travel between any pair of cities

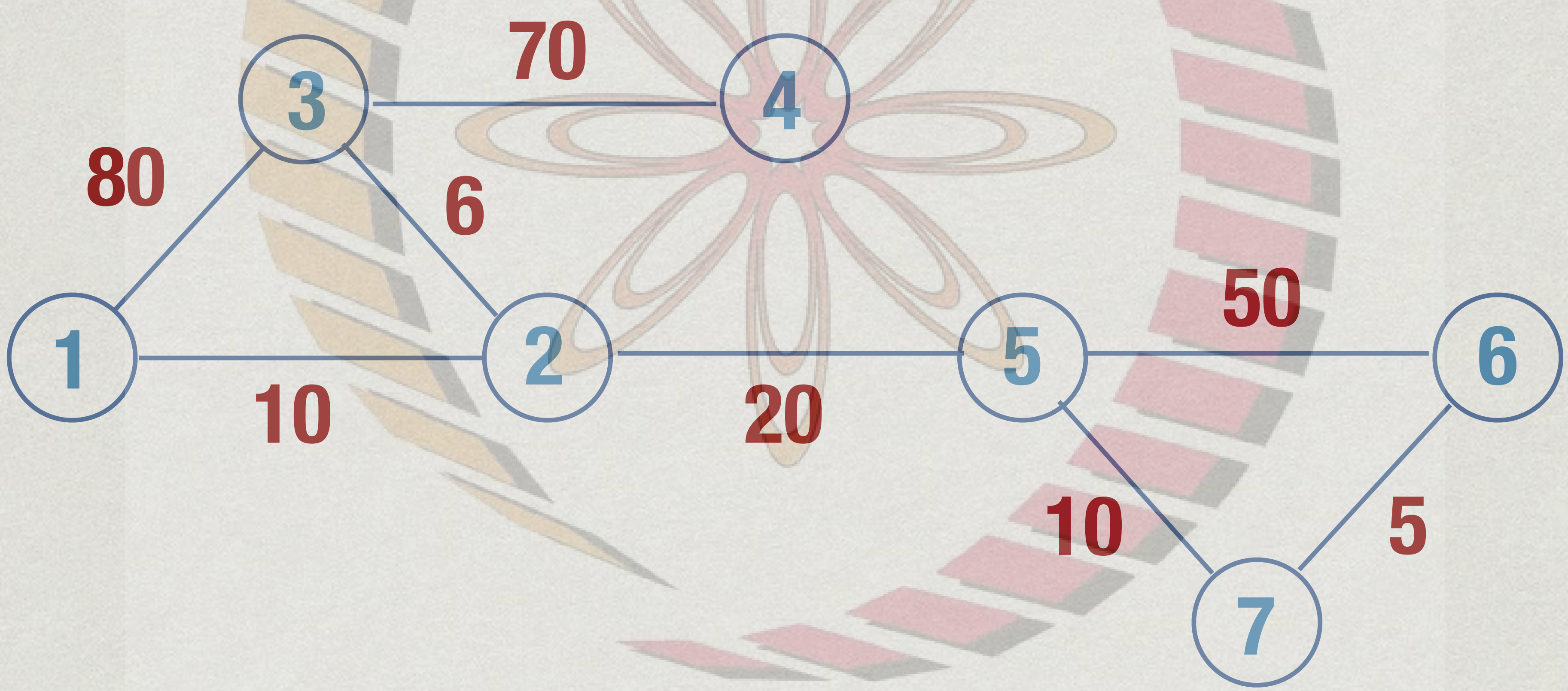
This lecture…

- Single source shortest paths
- For instance, shortest paths from 1 to 2,3,…,7

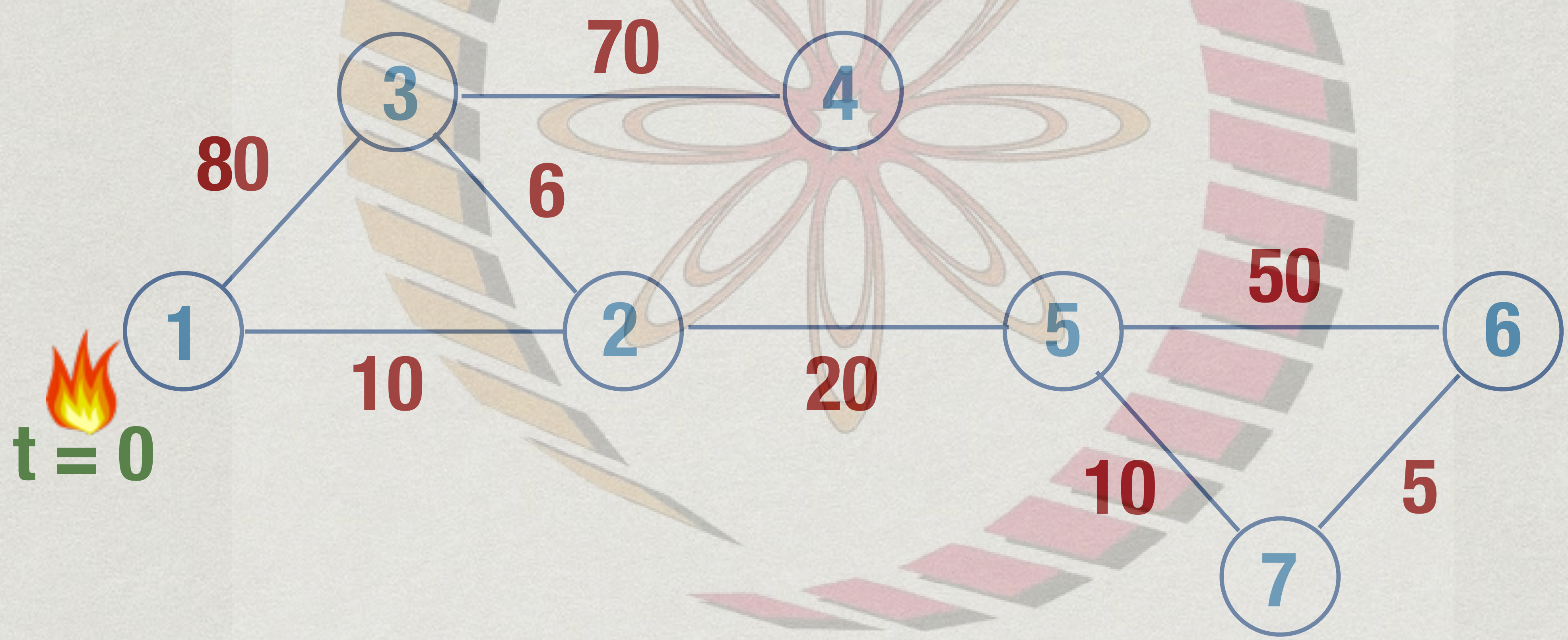
Single source shortest paths

- Imagine vertices are oil depots, edges are pipelines
- Set fire to oil depot at vertex 1
  - Fire travels at uniform speed along each pipeline
- First oil depot to catch fire after 1 is nearest vertex
- Next oil depot is second nearest vertex
- ...

Single source shortest paths



Single source shortest paths



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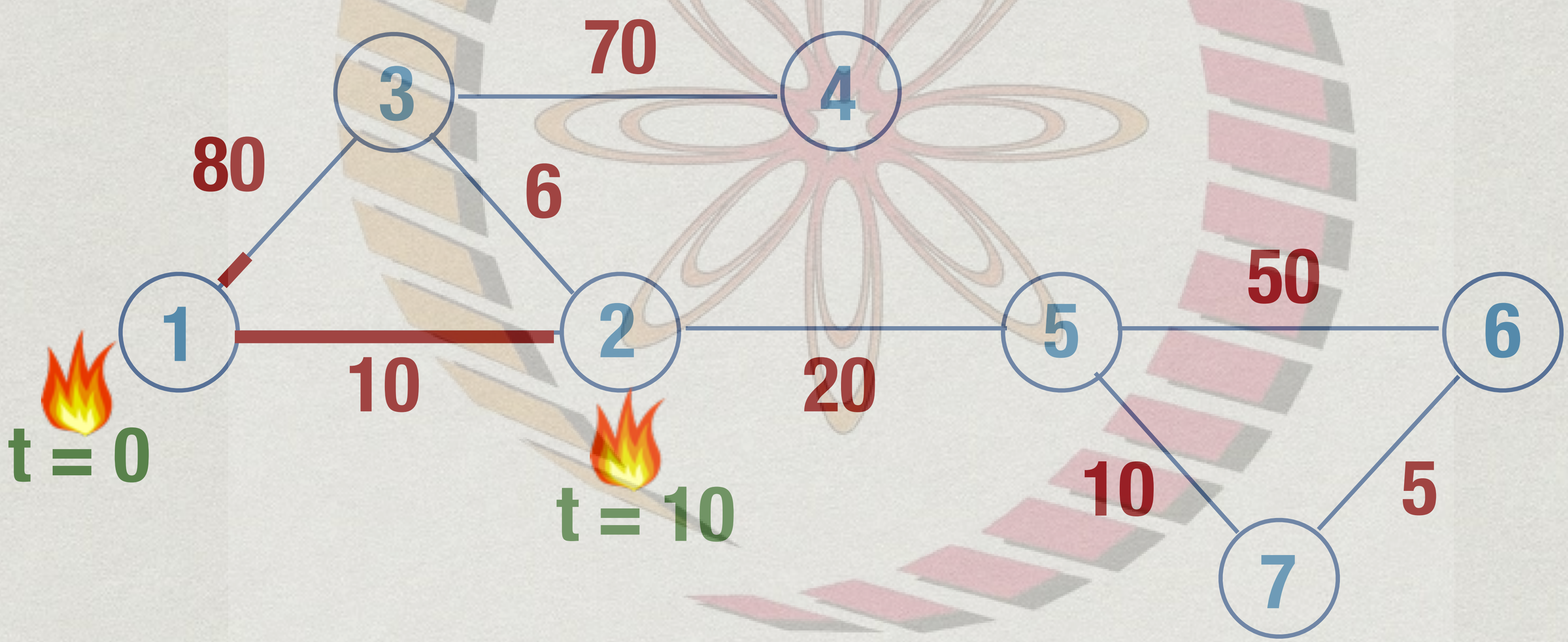

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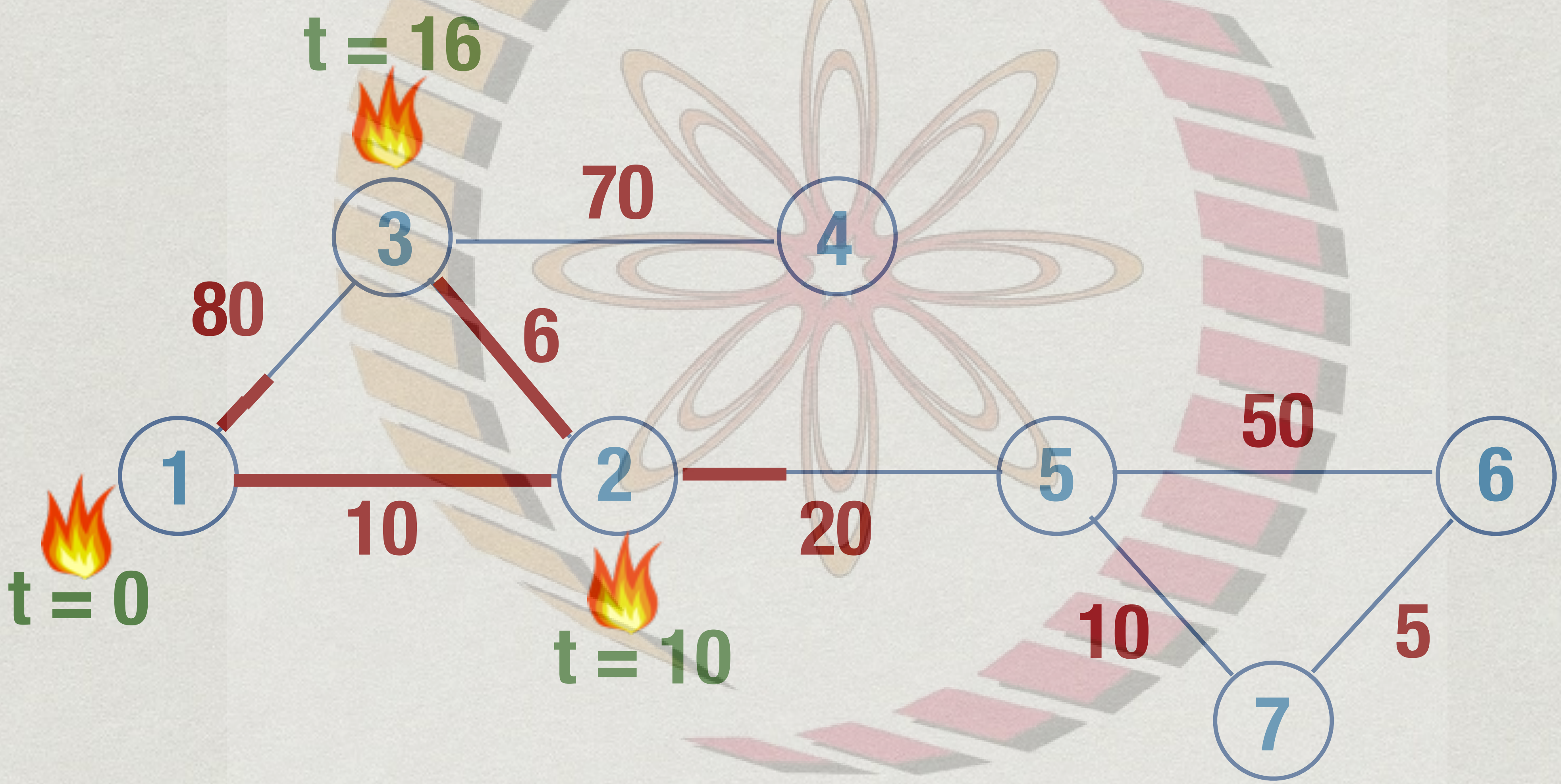

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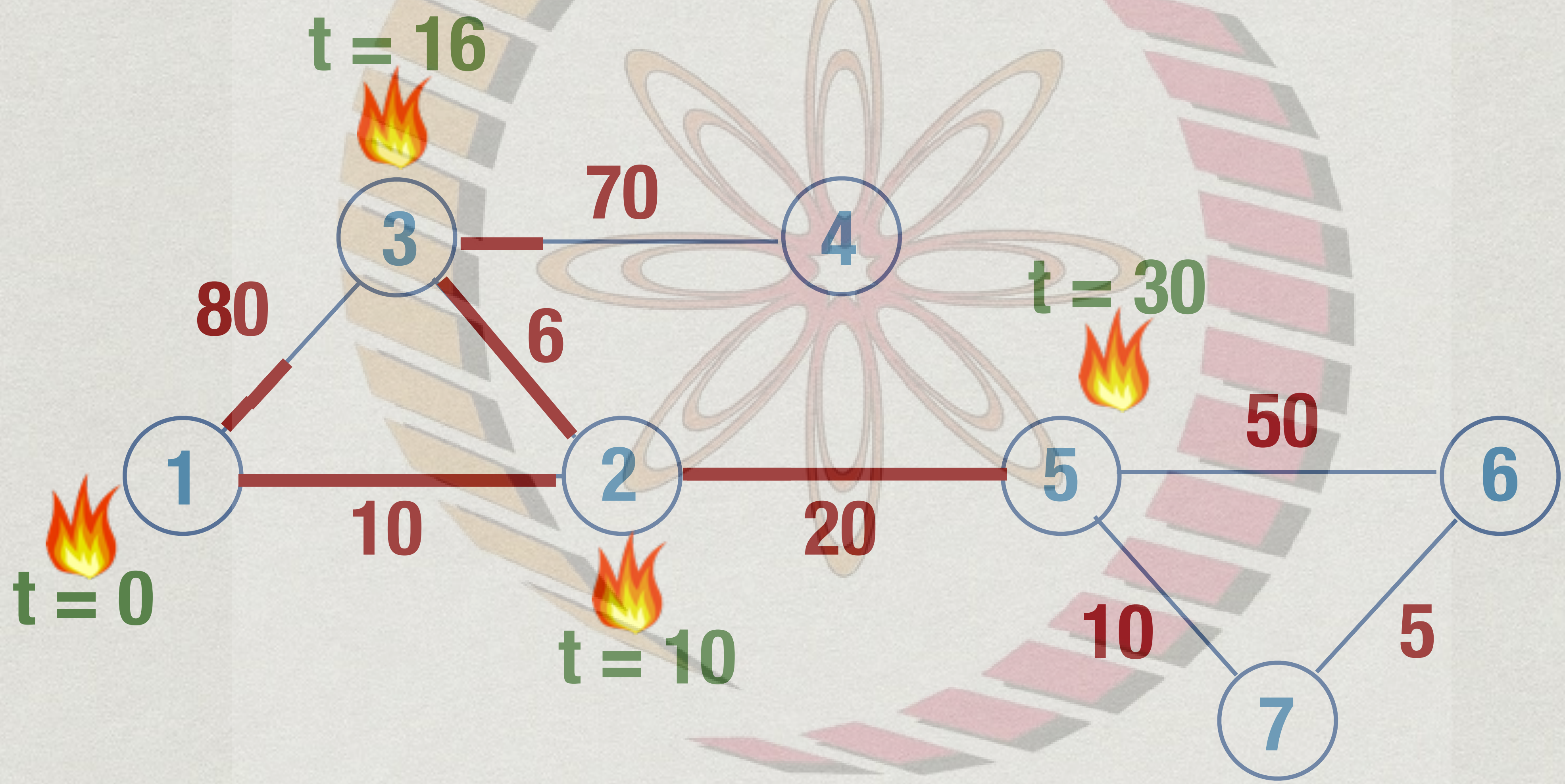

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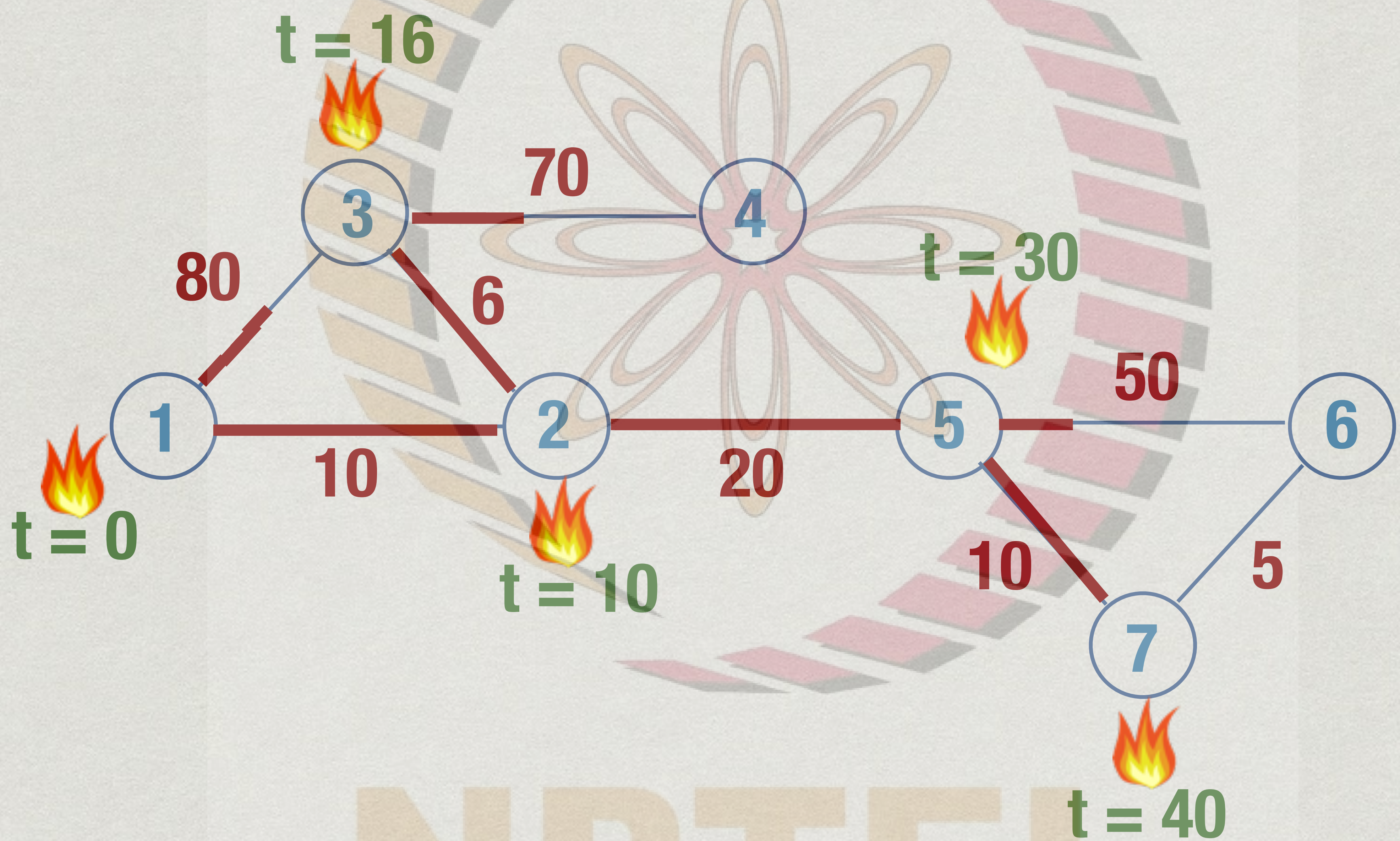

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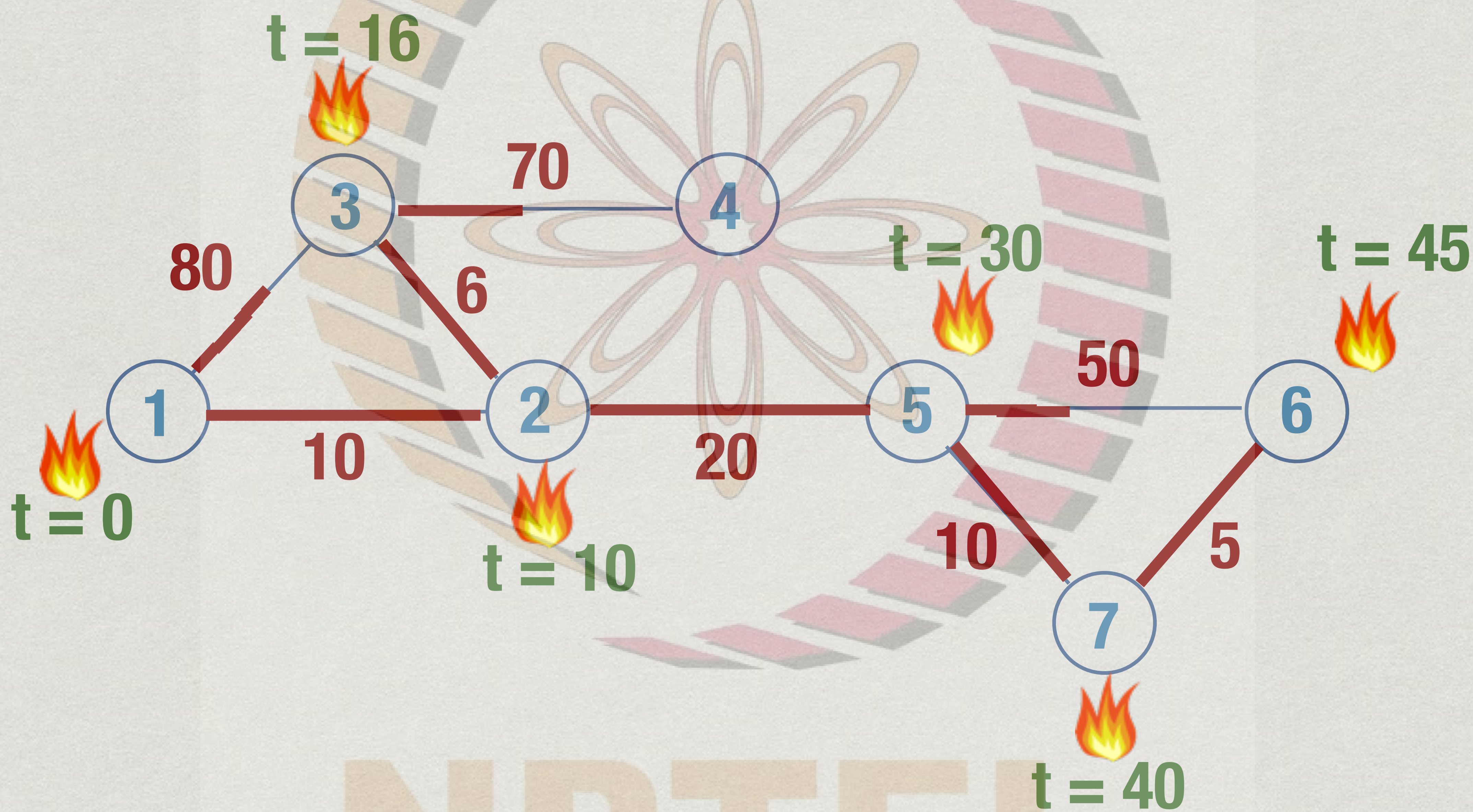

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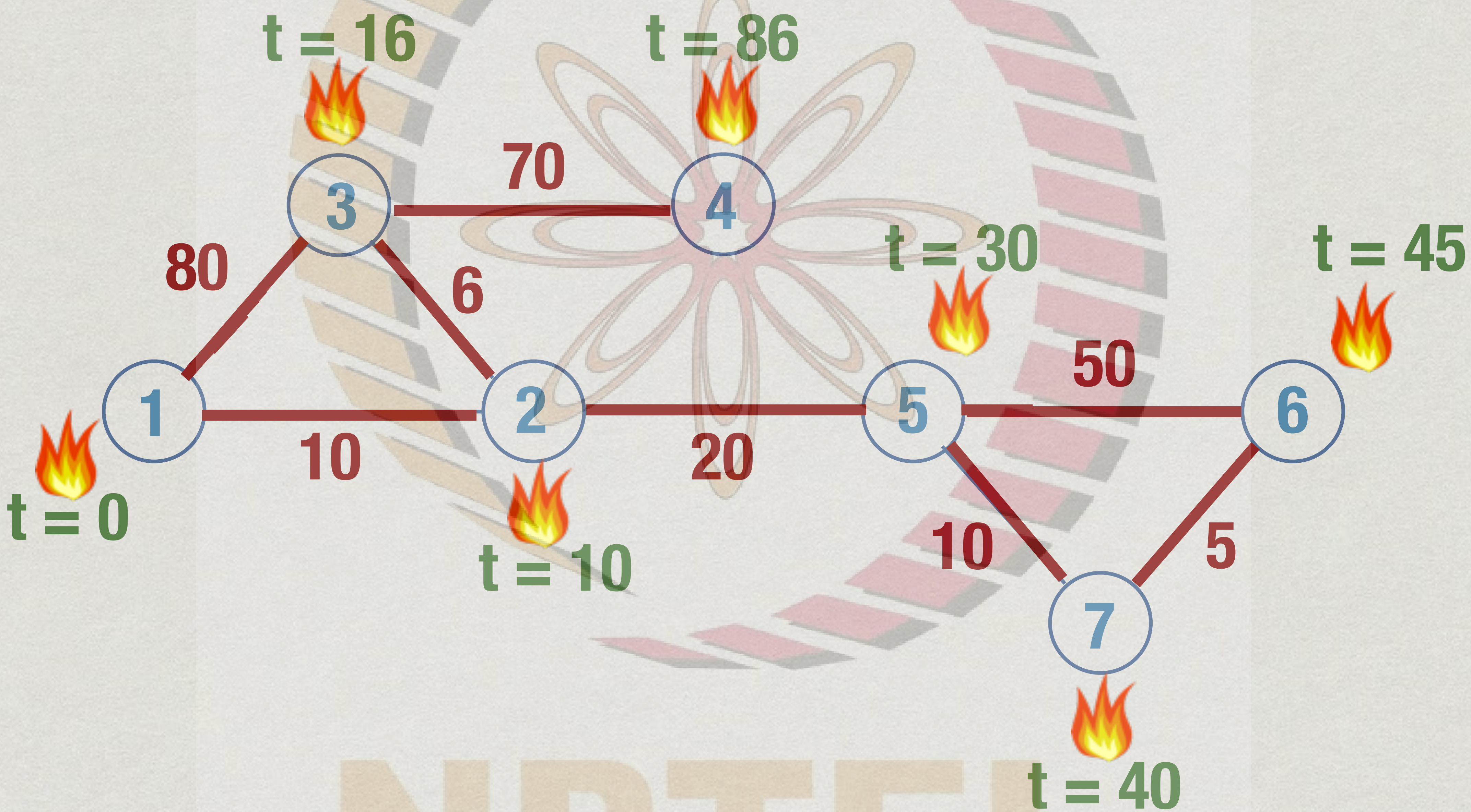
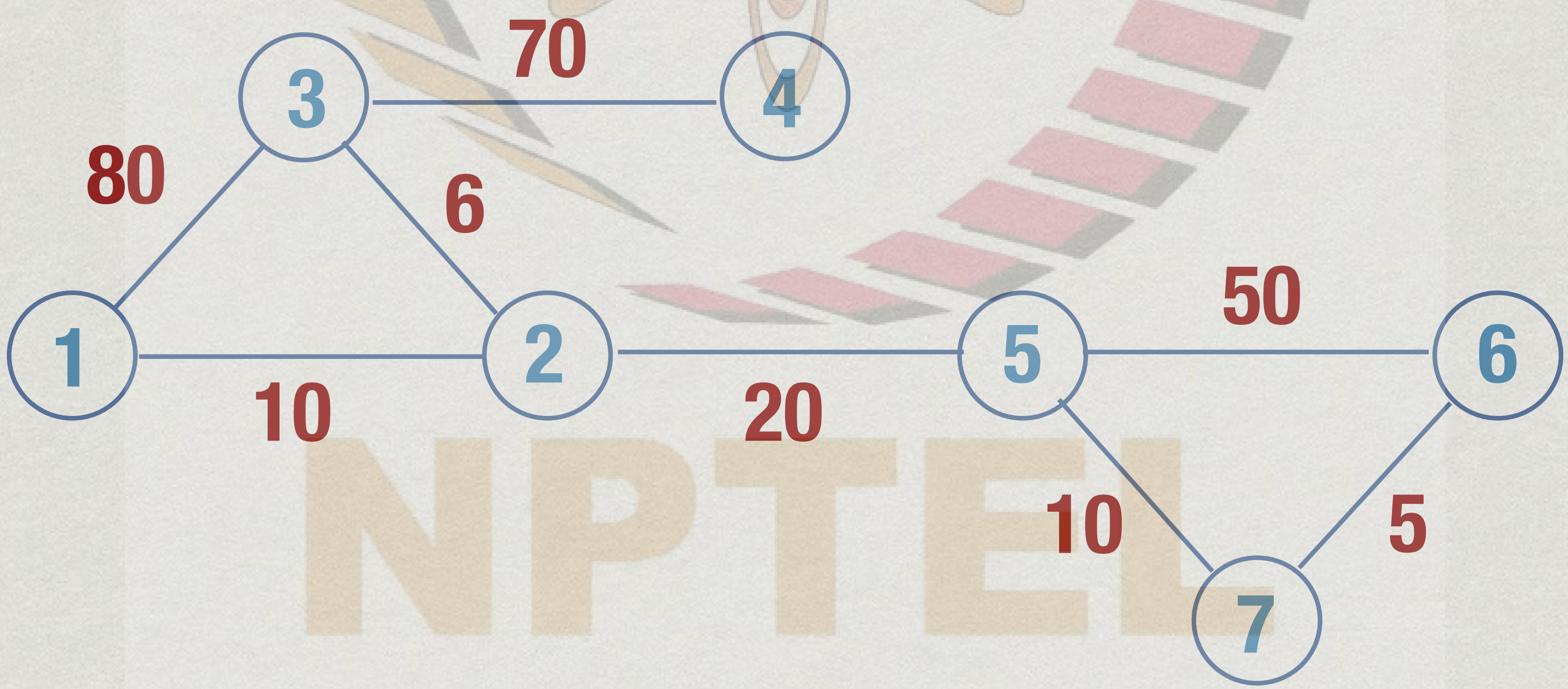


Single source shortest paths

- Compute expected time to burn of each vertex
- Update this each time a new vertex burns



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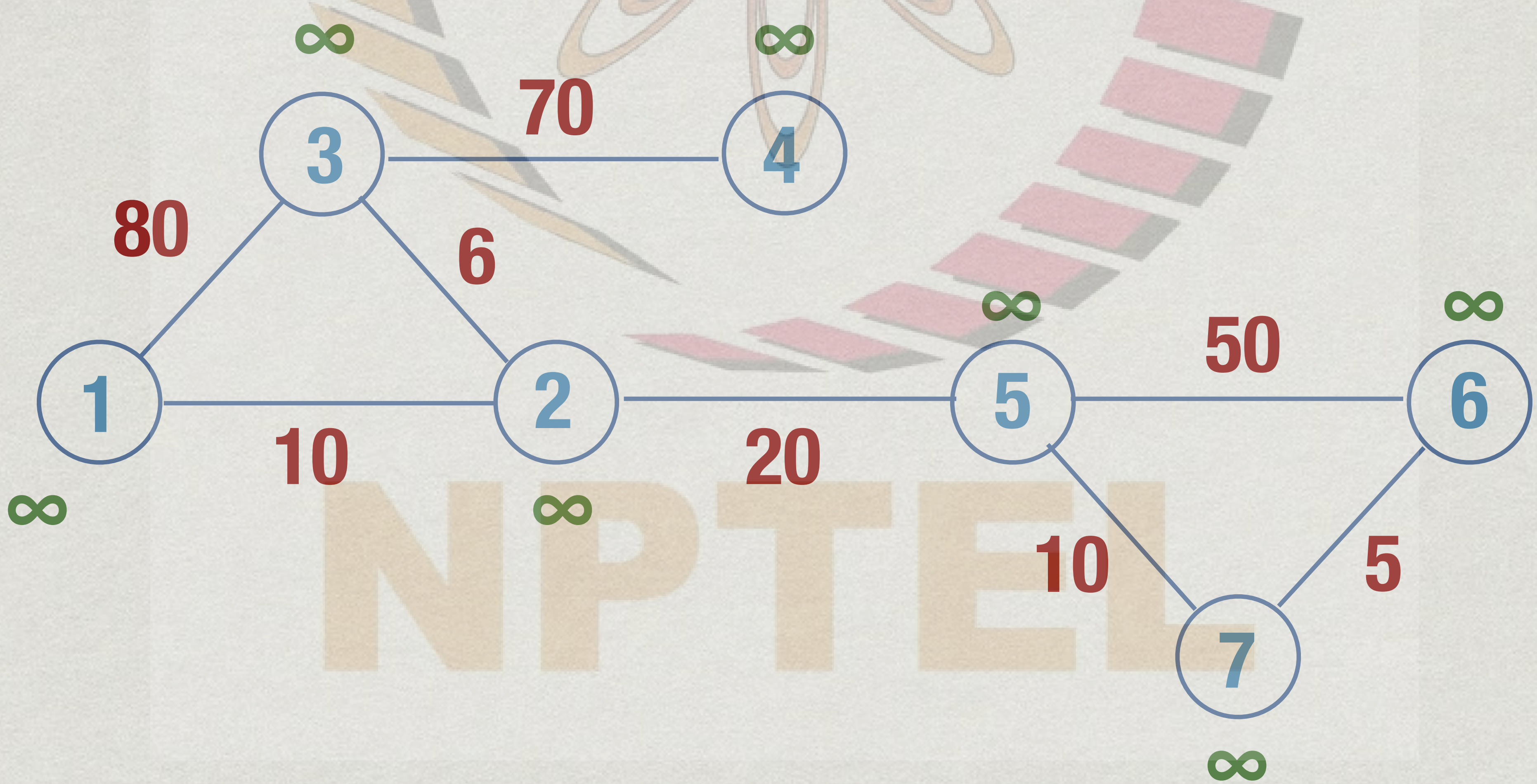

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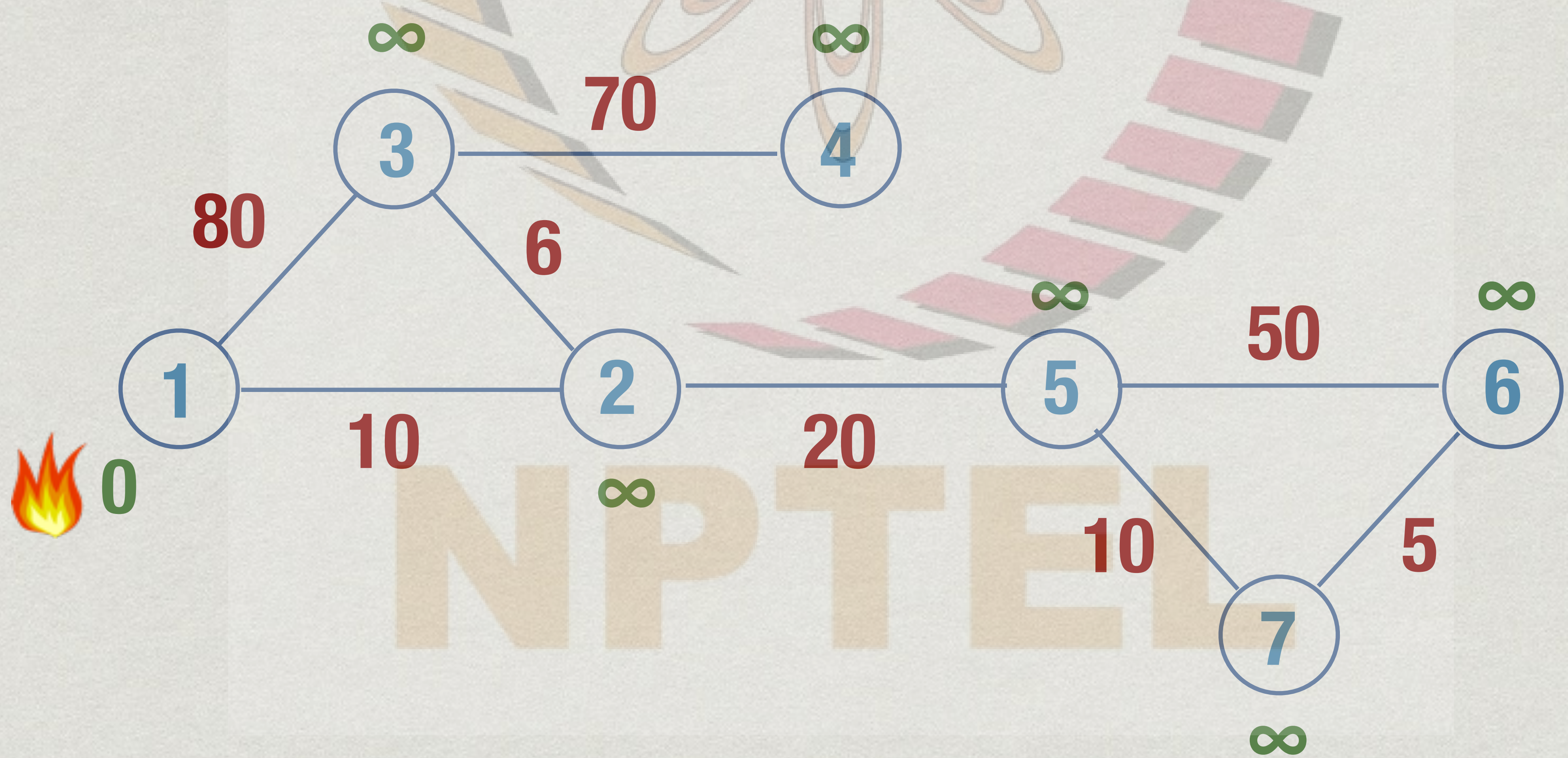

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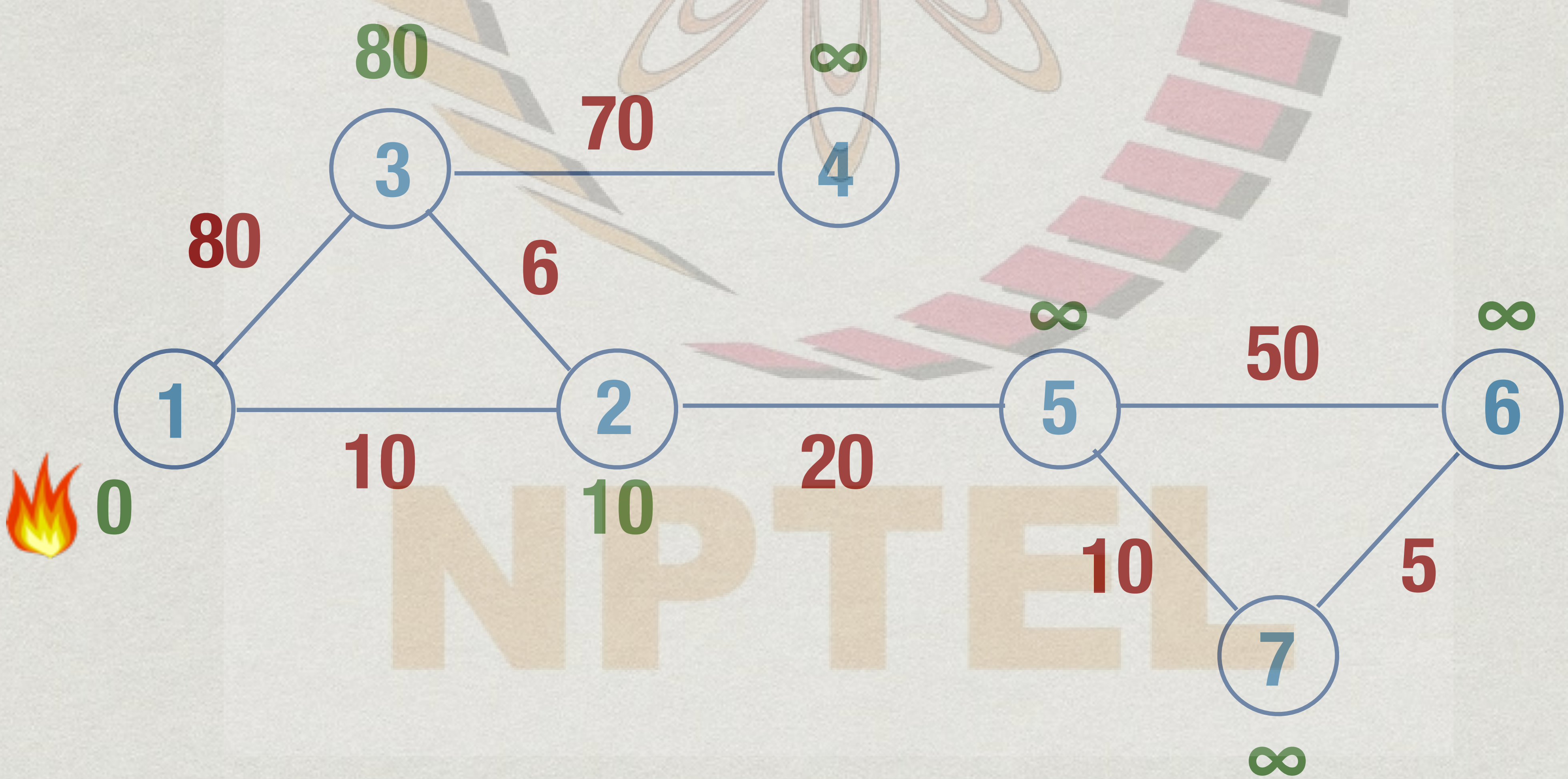

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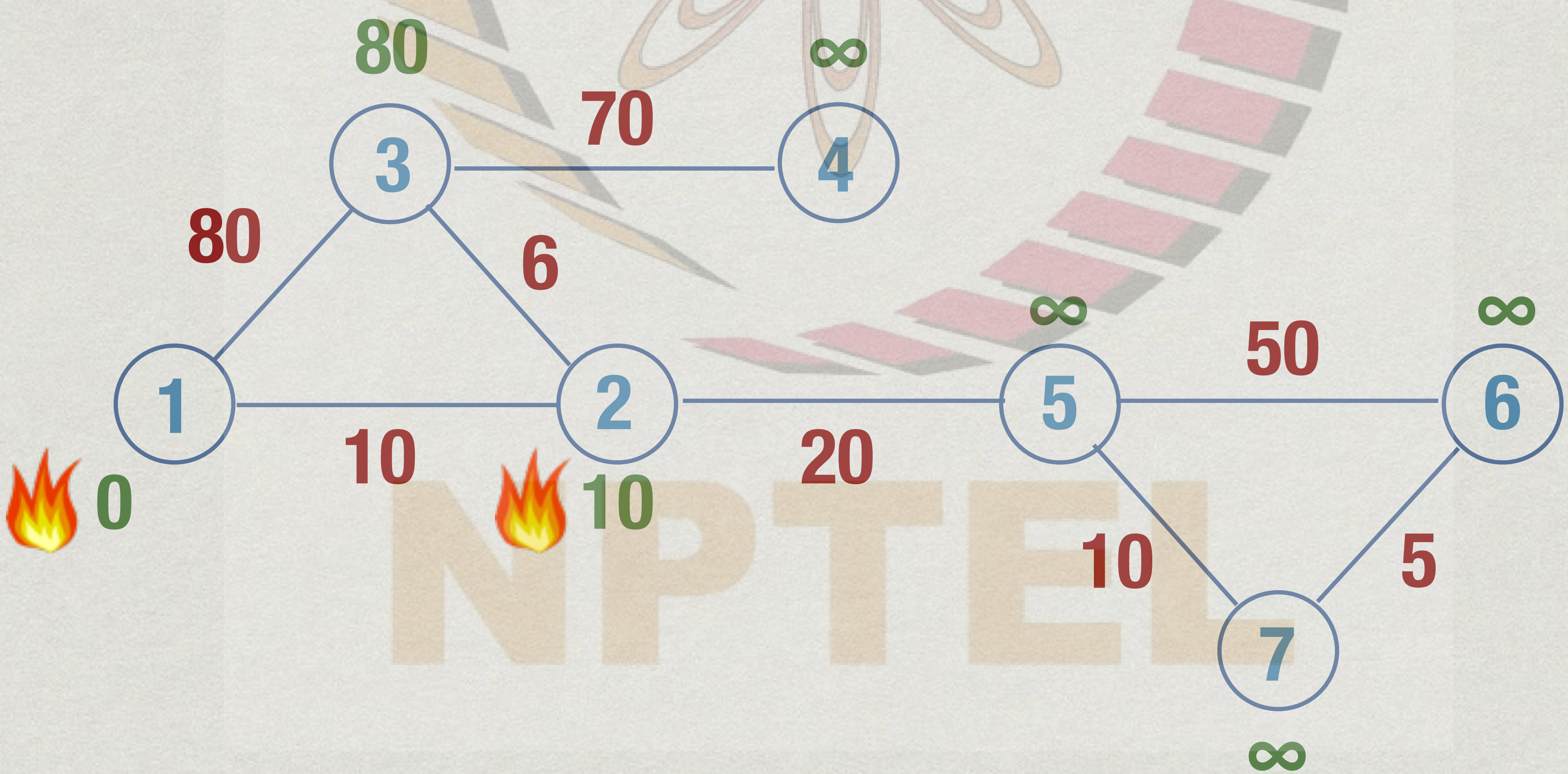

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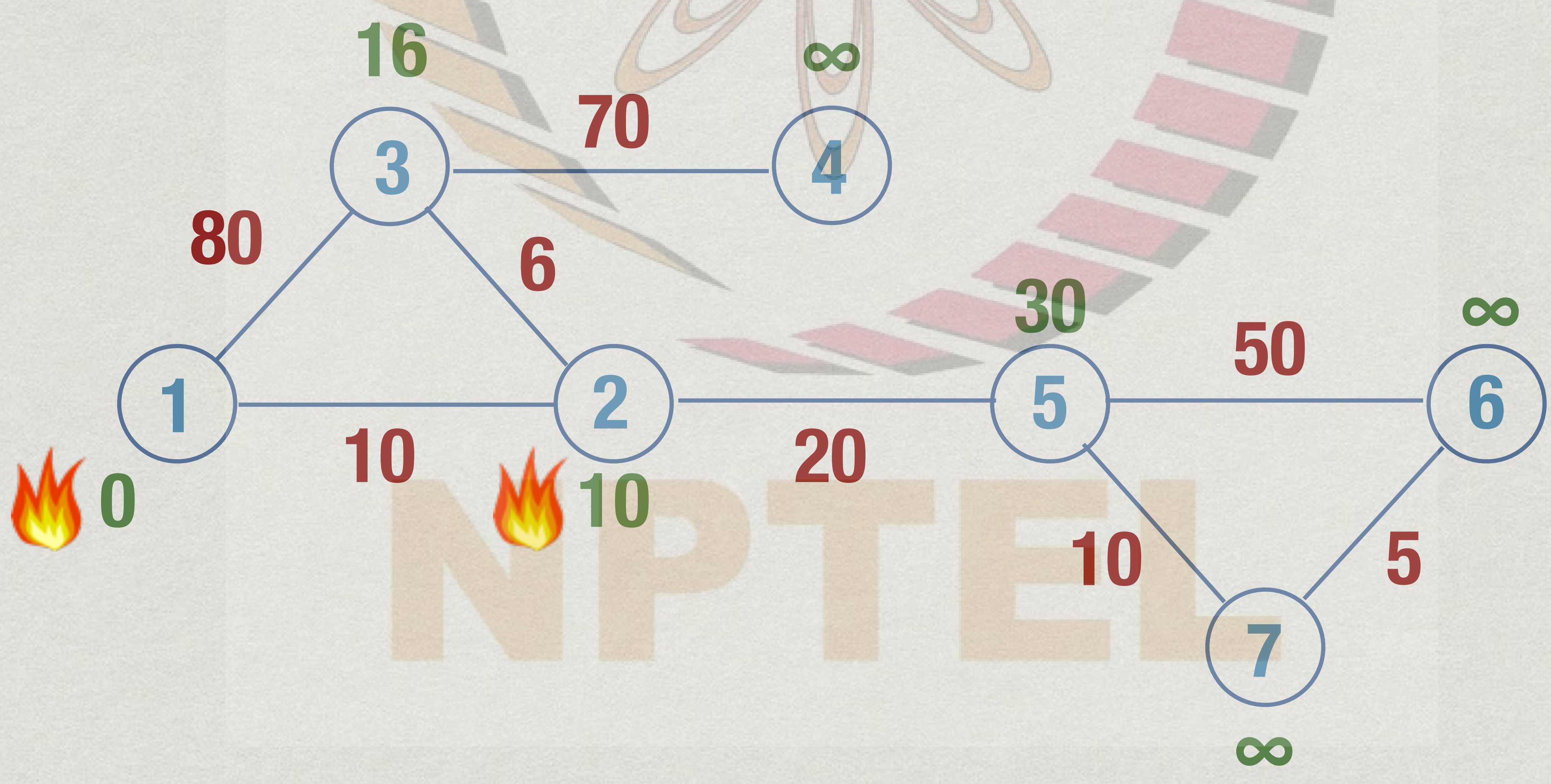

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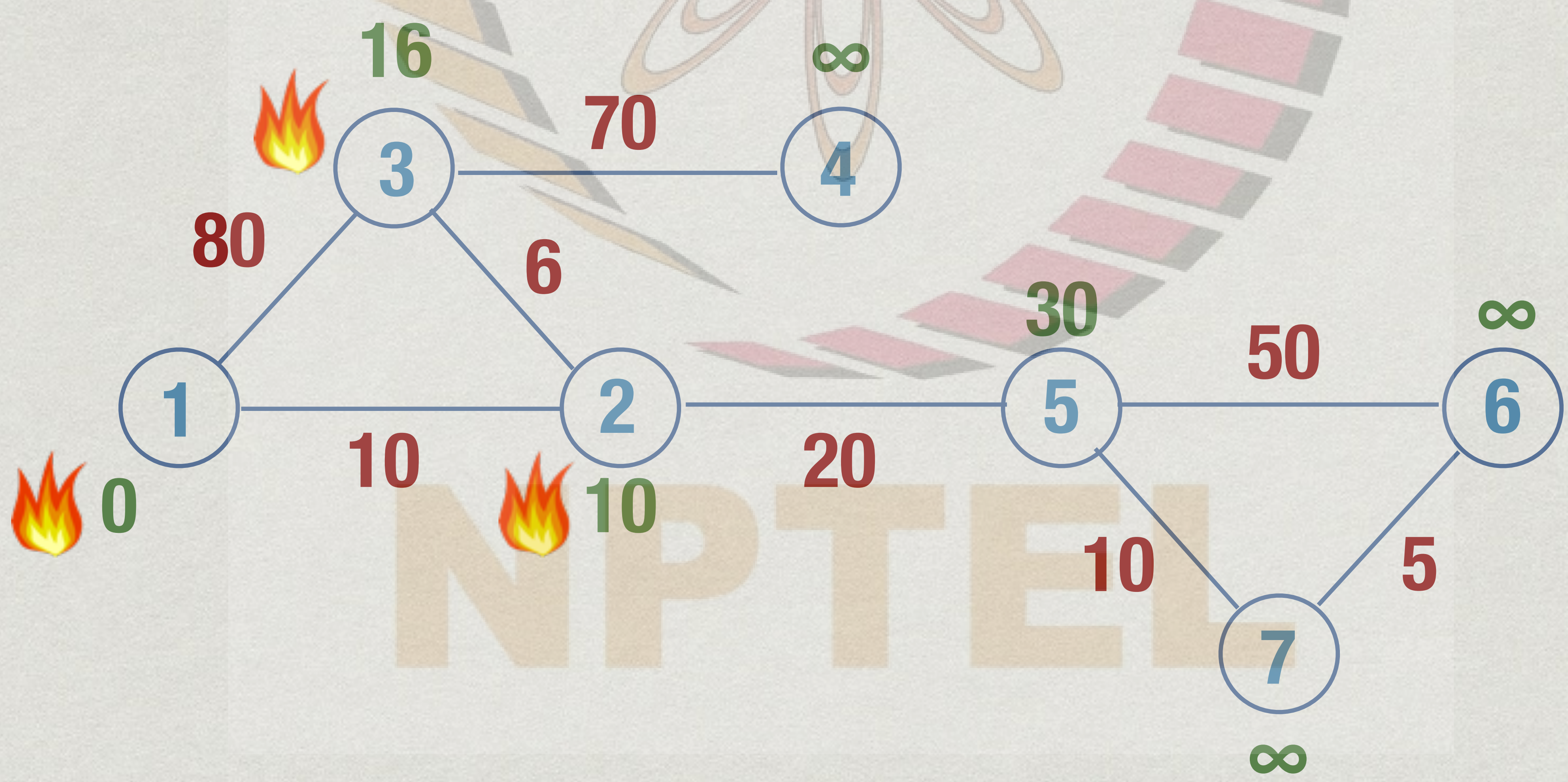

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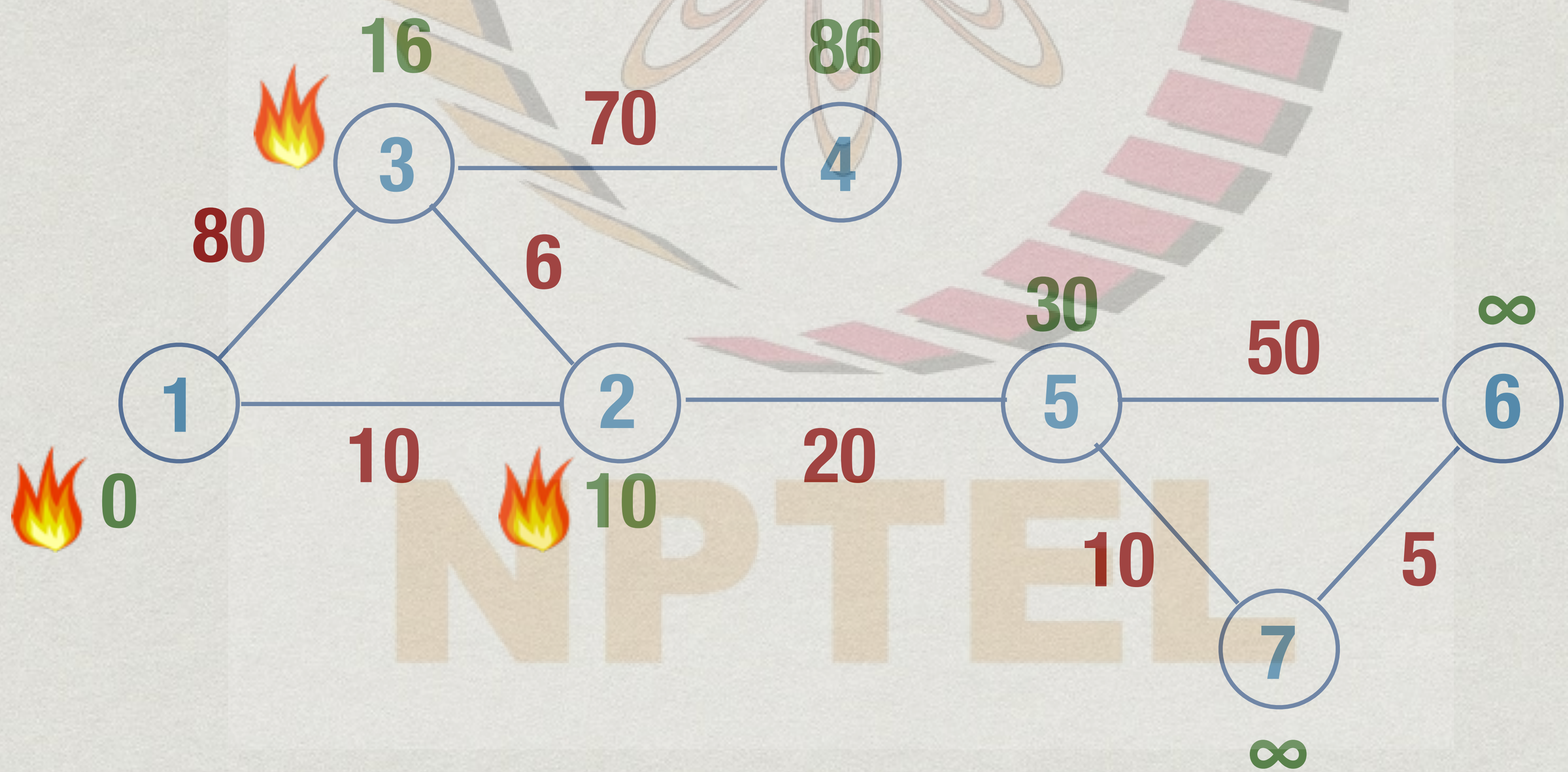

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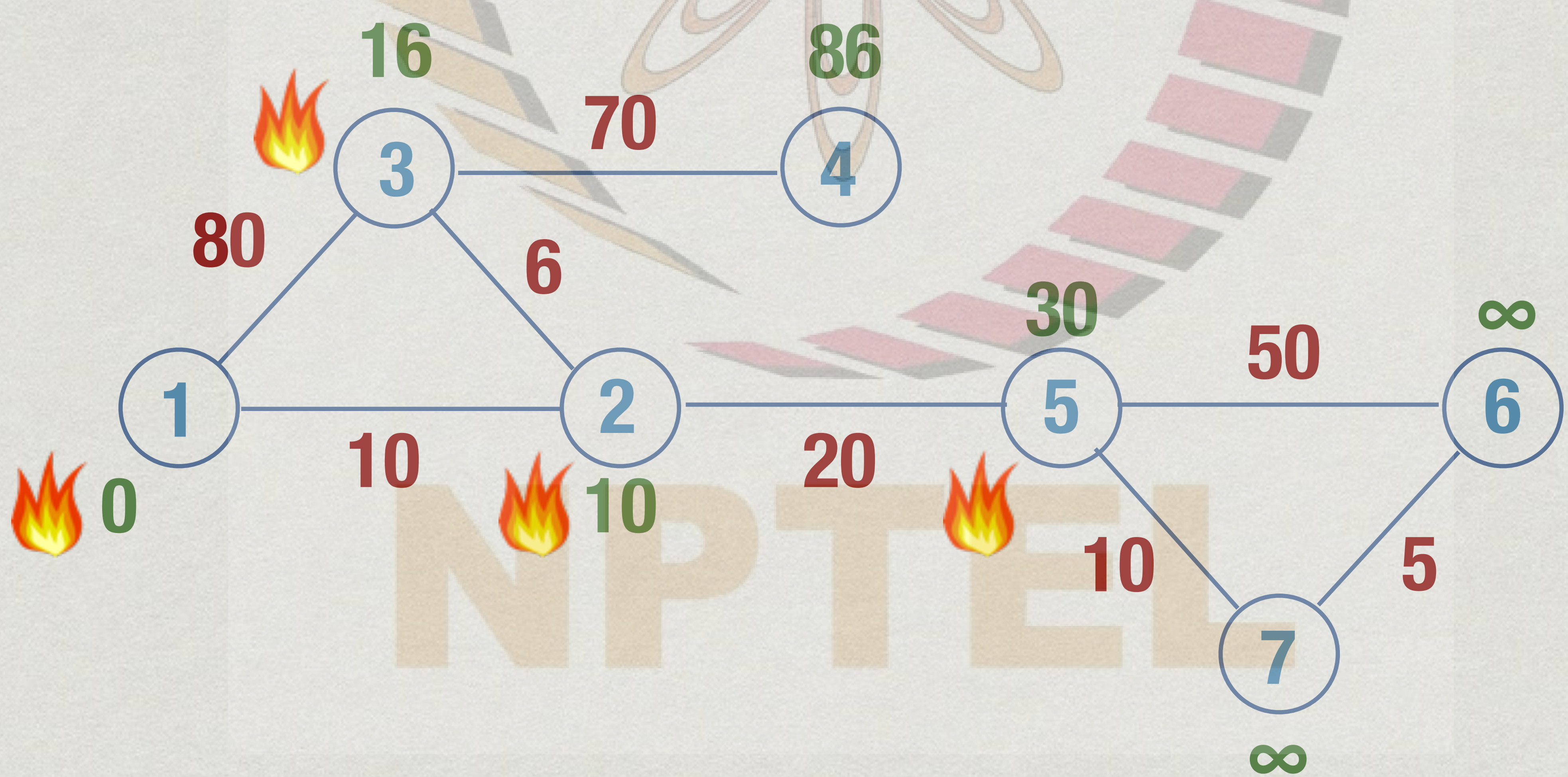

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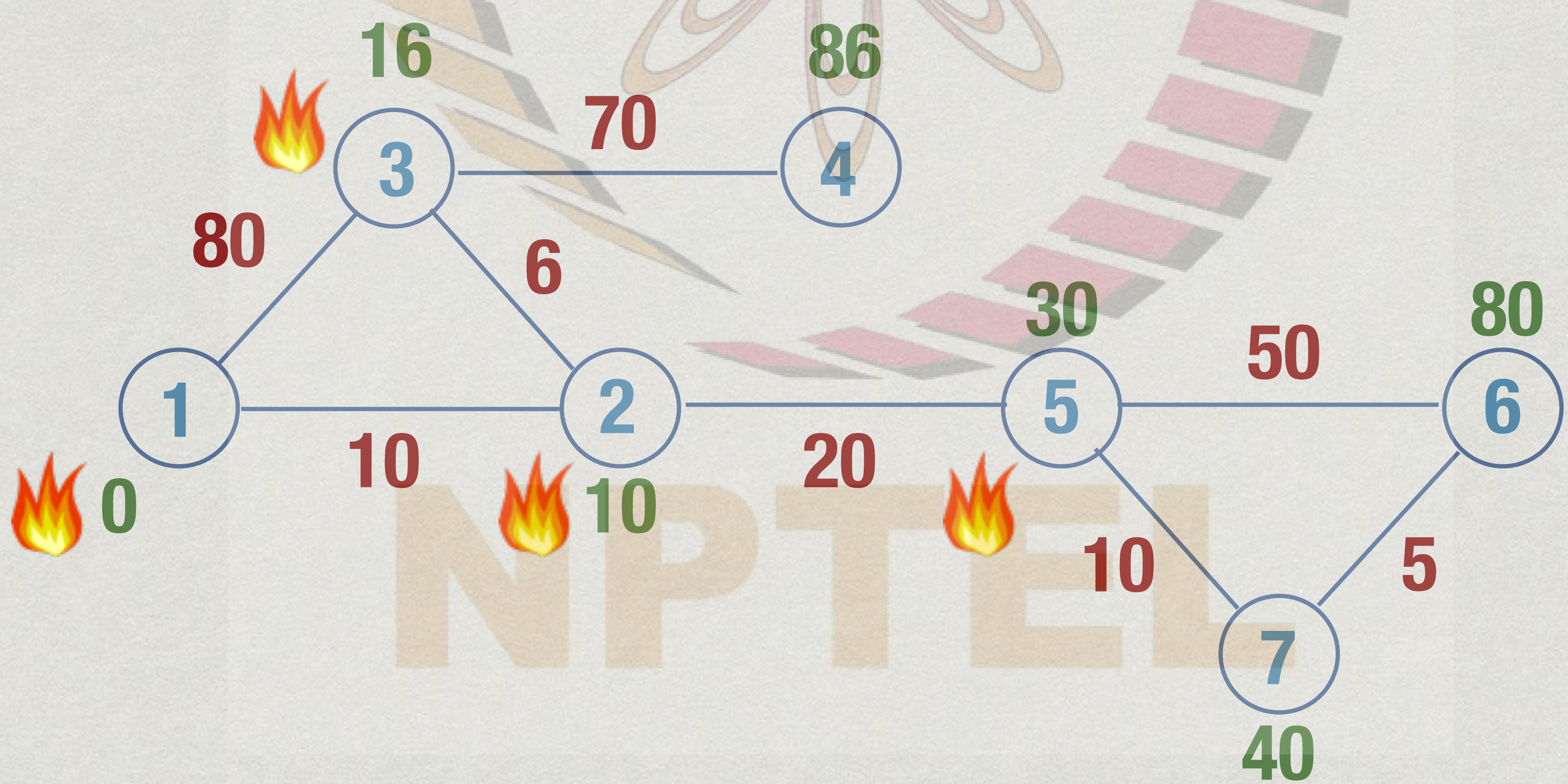

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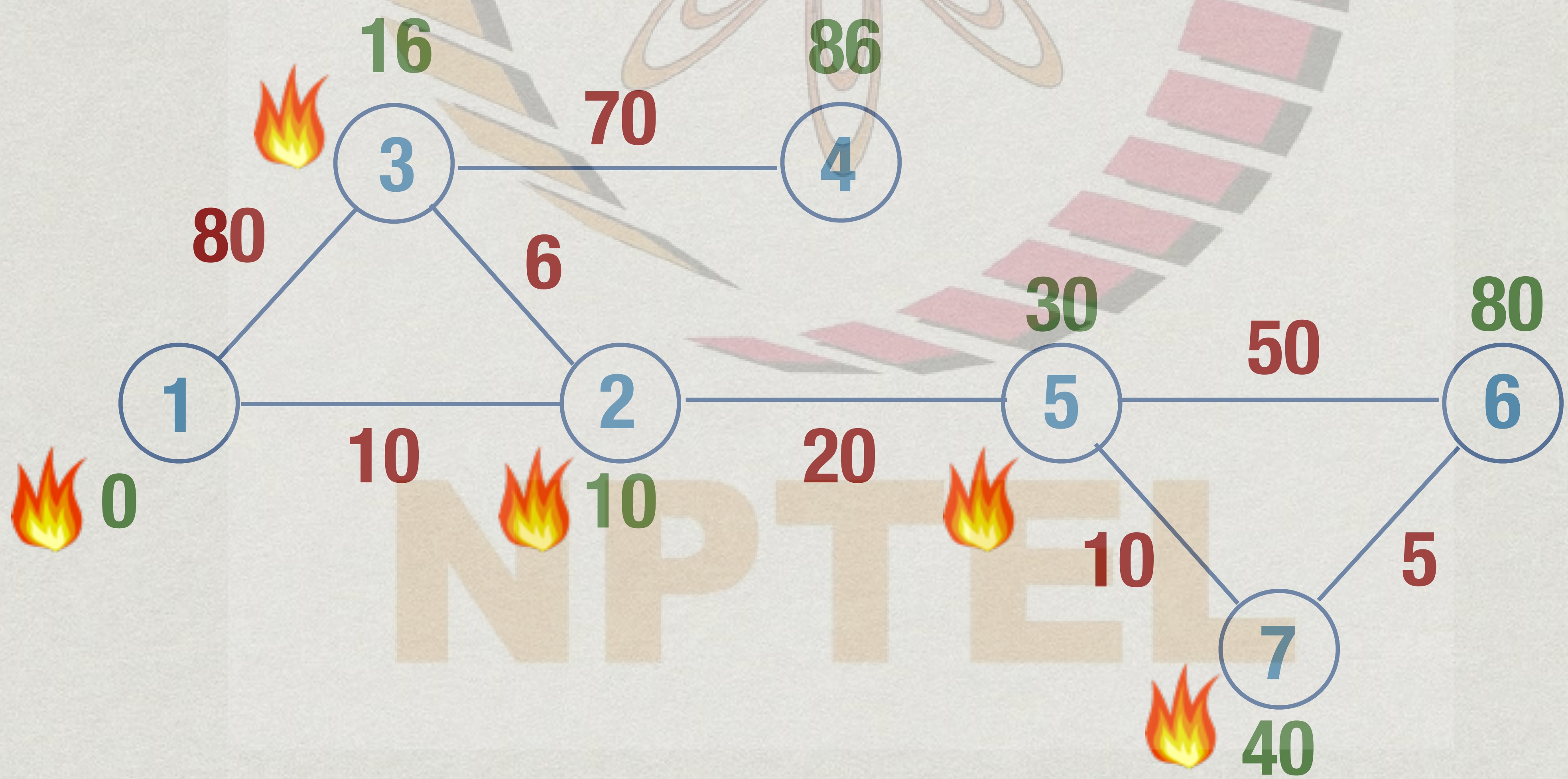

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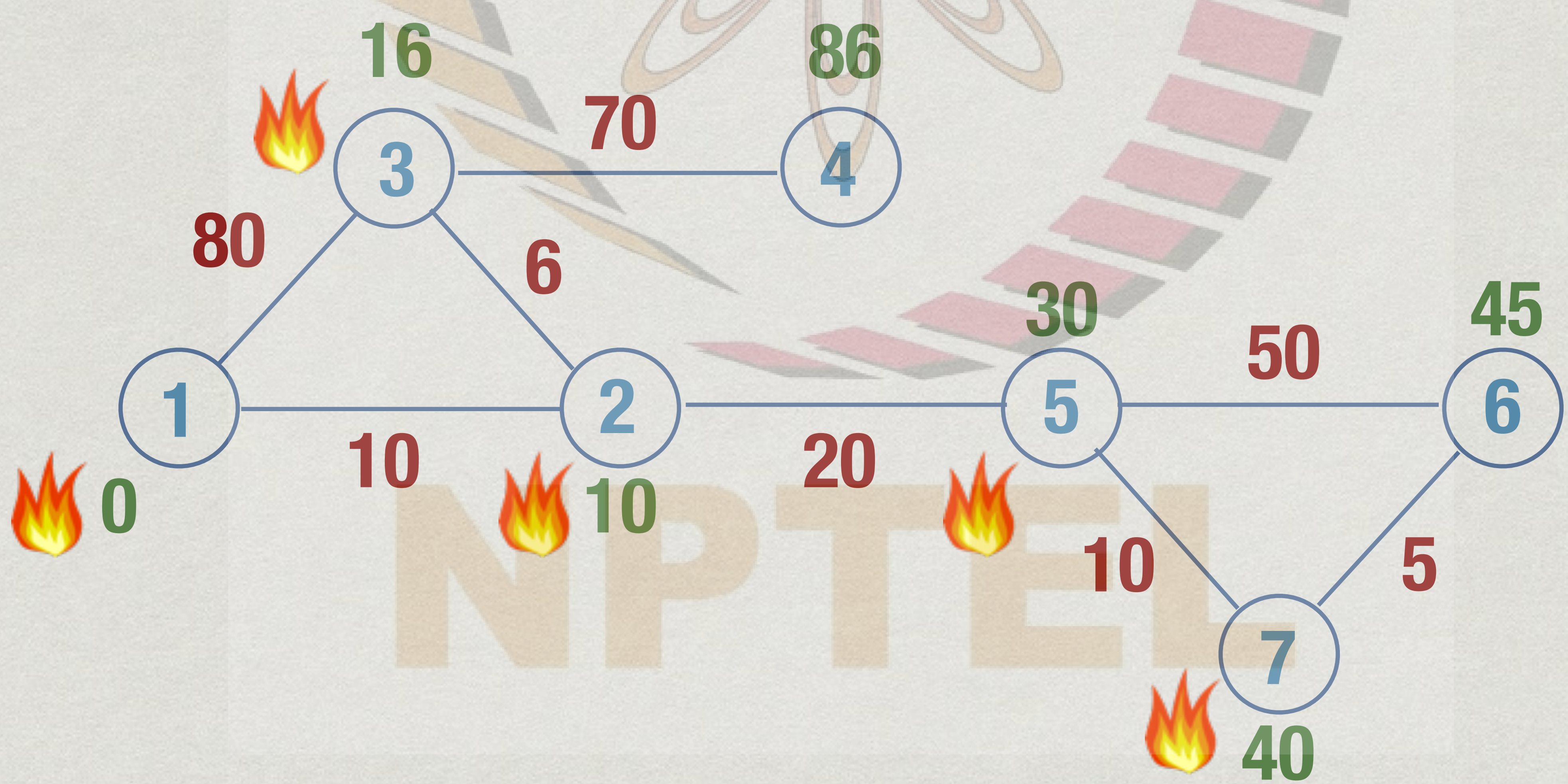

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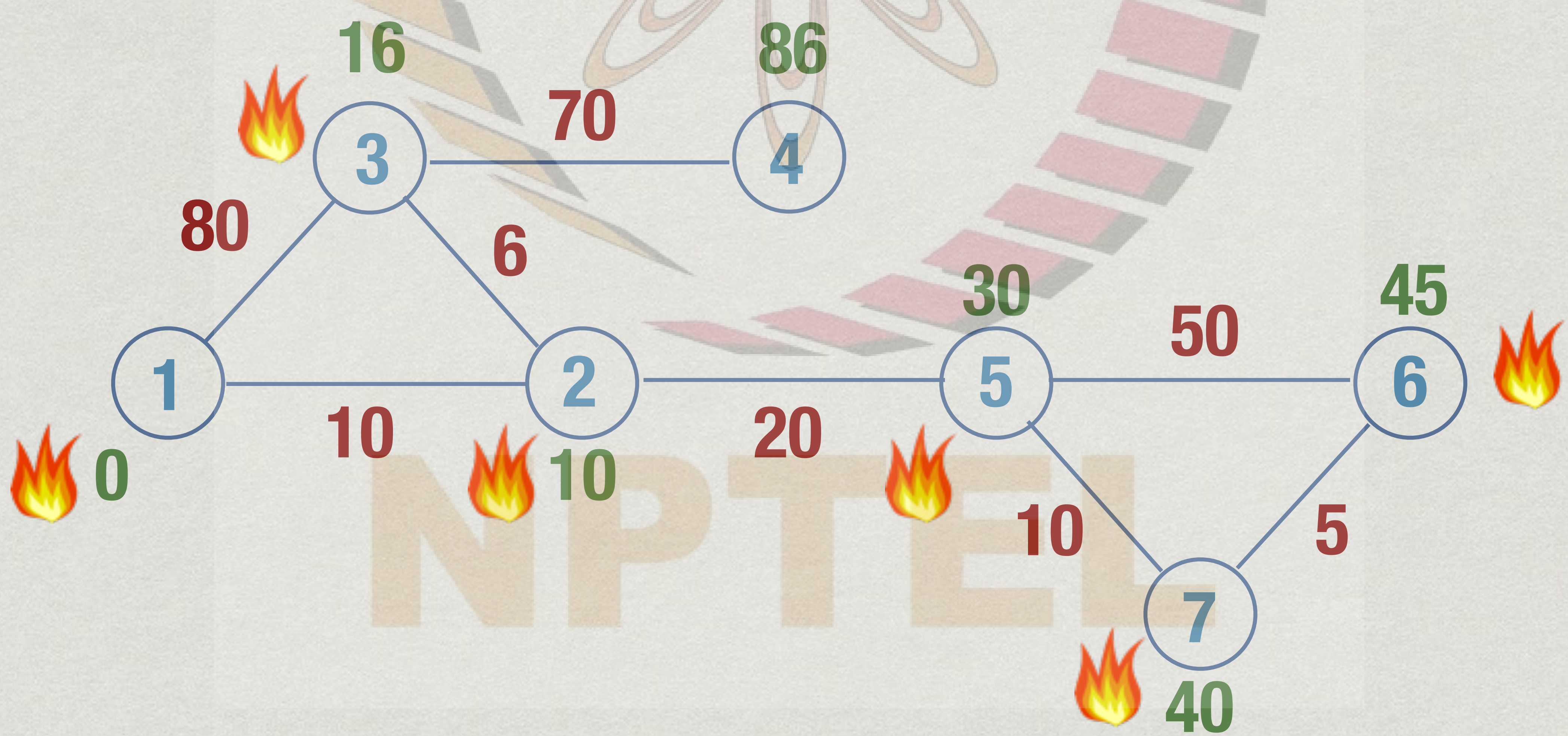

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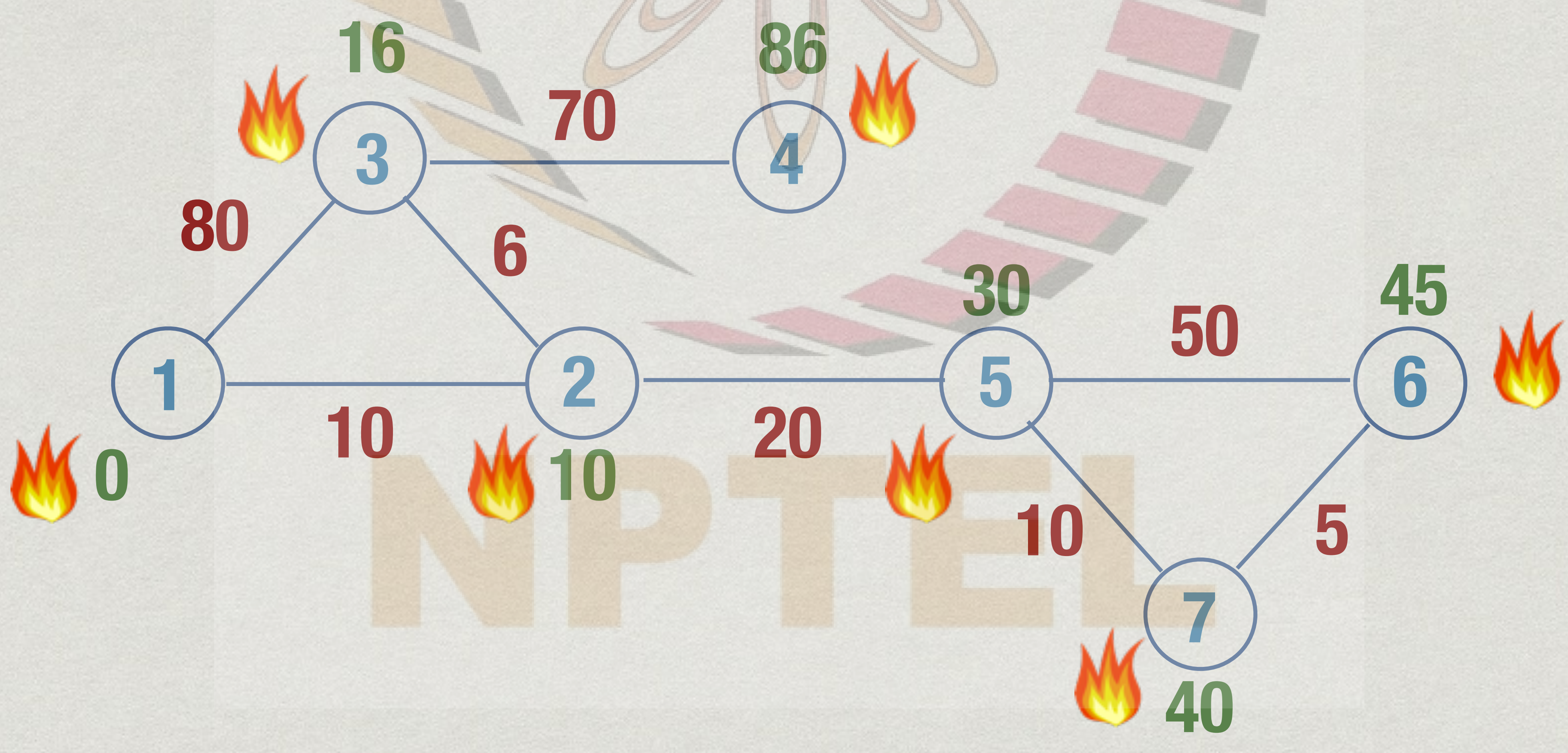

Algorithmically

- Maintain two arrays
  - `BurntVertices[ ]`, initially `False` for all i
  - `ExpectedBurnTime[ ]`, initially ∞ for all i
    - For ∞, use sum of all edge weights + 1
- Set `ExpectedBurnTime[1] = 0`
- Repeat, until all vertices are burnt
  - Find `j` with minimum `ExpectedBurnTime`
  - Set `BurntVertices[j] = True`
  - Recompute `ExpectedBurnTime[k]` for each neighbour `k` of `j`

Dijkstra’s algorithm

```
function ShortestPaths(s){ // assume source is s
  for i = 1 to n
    BV[i] = False; EBT[i] = infinity

  EBT[s] = 0

  for i = 1 to n
    Choose u such that BV[u] == False
                           and EBT[u] is minimum
    BV[u] = True
    for each edge (u,v) with BV[v] == False
      if EBT[v] > EBT[u] + weight(u,v)
        EBT[v] = EBT[u] + weight(u,v)
```

Dijkstra's algorithm

```
function ShortestPaths(s){ // assume source is s
  for i = 1 to n
    Visited[i] = False; Distance[i] = infinity

  Distance[s] = 0

  for i = 1 to n
    Choose u such that Visited[u] == False
                                and Distance[u] is minimum
    Visited[u] = True
    for each edge (u,v) with Visited[v] == False
      if Distance[v] > Distance[u] + weight(u,v)
        Distance[v] = Distance[u] + weight(u,v)
```