NPTEL MOOC,JAN-FEB 2015
Week 8, Module 3

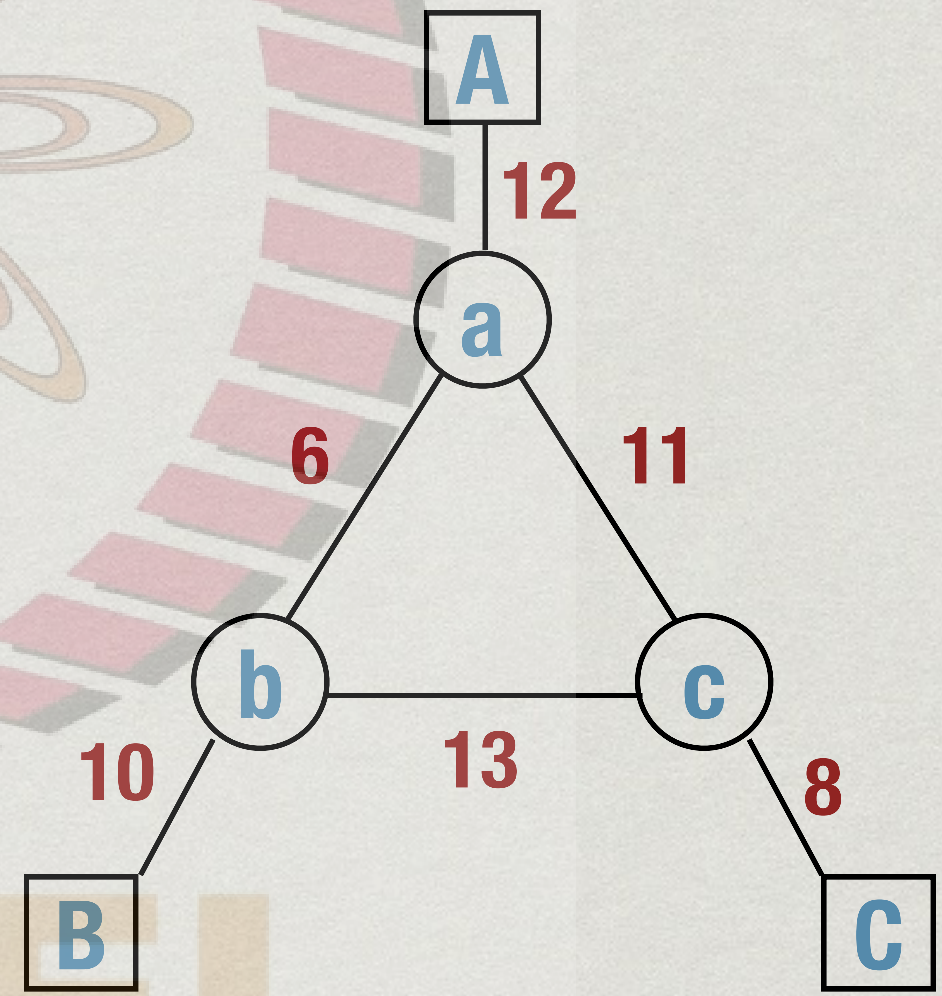
DESIGN AND ANALYSIS OF ALGORITHMS

LP modelling: Bandwidth allocation

MADHAVAN MUKUND, CHENNAI MATHEMATICAL INSTITUTE
http://www.cmi.ac.in/~madhavan

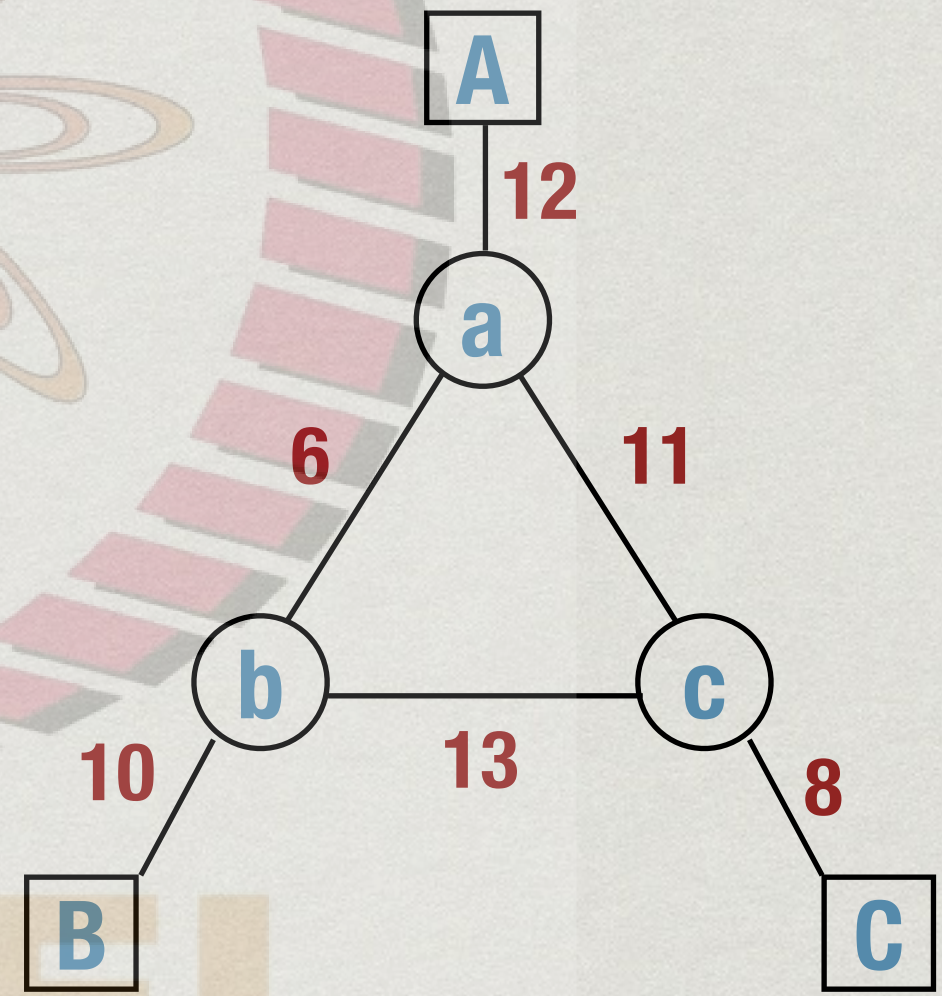
Network bandwidth

- 3 users, A, B, C to be connected to each other
- Each connection A-B, B-C, A-C should have at least 2 Mbps of bandwidth
- Direct and/or indirect connections allowed: A-a-b-B, A-a-c-b-B
- Each link has capacity constraint (in Mbps)



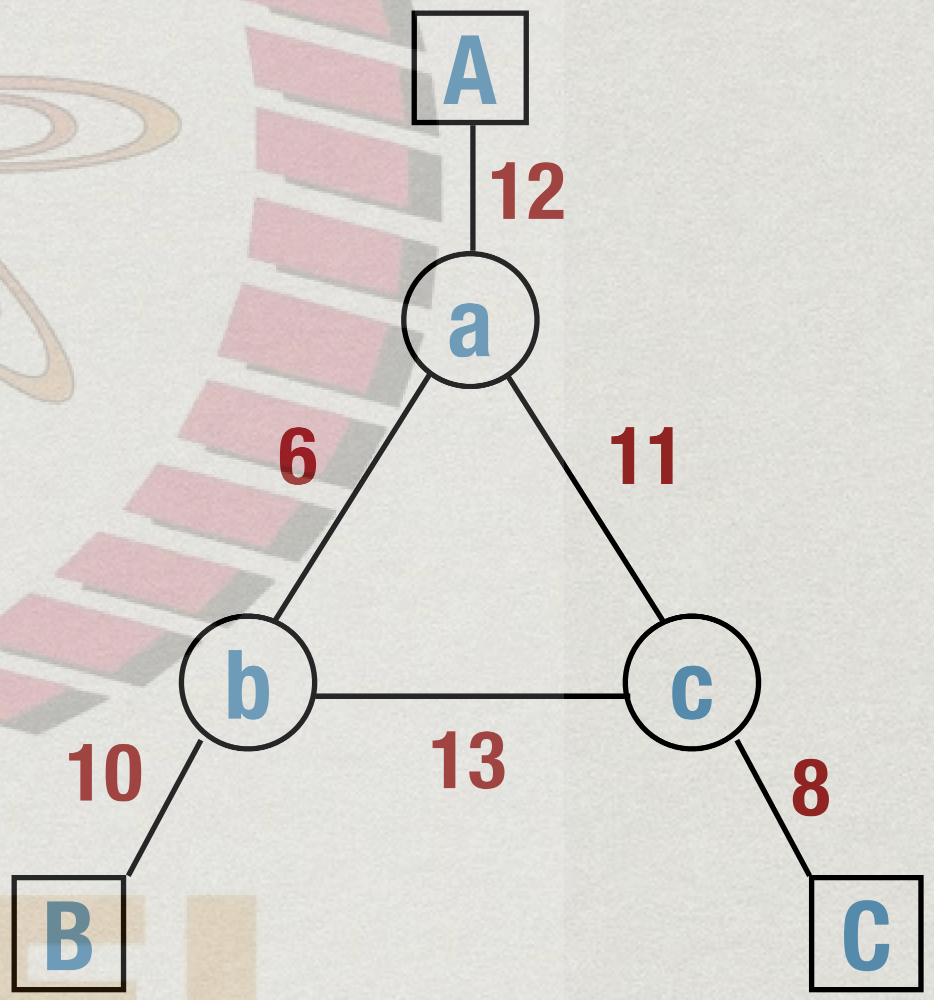
Network bandwidth

- Each connection earns a revenue, per Mbps
  - A-B, Rs 300/Mbps
  - B-C, Rs 200/Mbps
  - A-C, Rs 400/Mbps
- Allocate bandwidth to maximize revenue



Linear program

- x_{AB} — bandwidth via short connection A-a-b-B
- y_{AB} — bandwidth via long connection A-a-c-b-B
- Likewise, x_{BC}, y_{BC} and x_{AC}, y_{AC}



Constraints

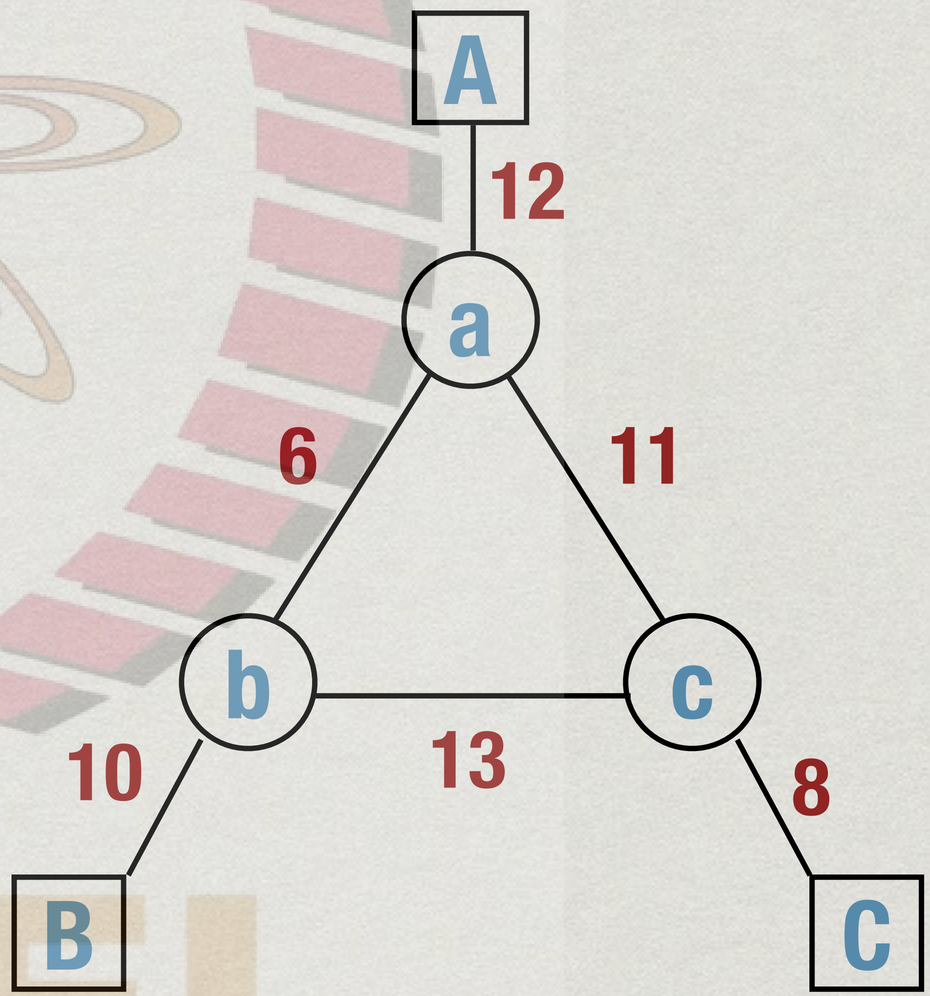
- x_{AB}, y_{AB} both flow though edge b-B, as do x_{BC}, y_{BC}

  $x_{AB} + y_{AB} + x_{BC} + y_{BC} \leq 10$

- Likewise

  $x_{AB} + y_{AB} + x_{AC} + y_{AC} \leq 12$

  $x_{AC} + y_{AC} + x_{BC} + y_{BC} \leq 8$



More constraints …

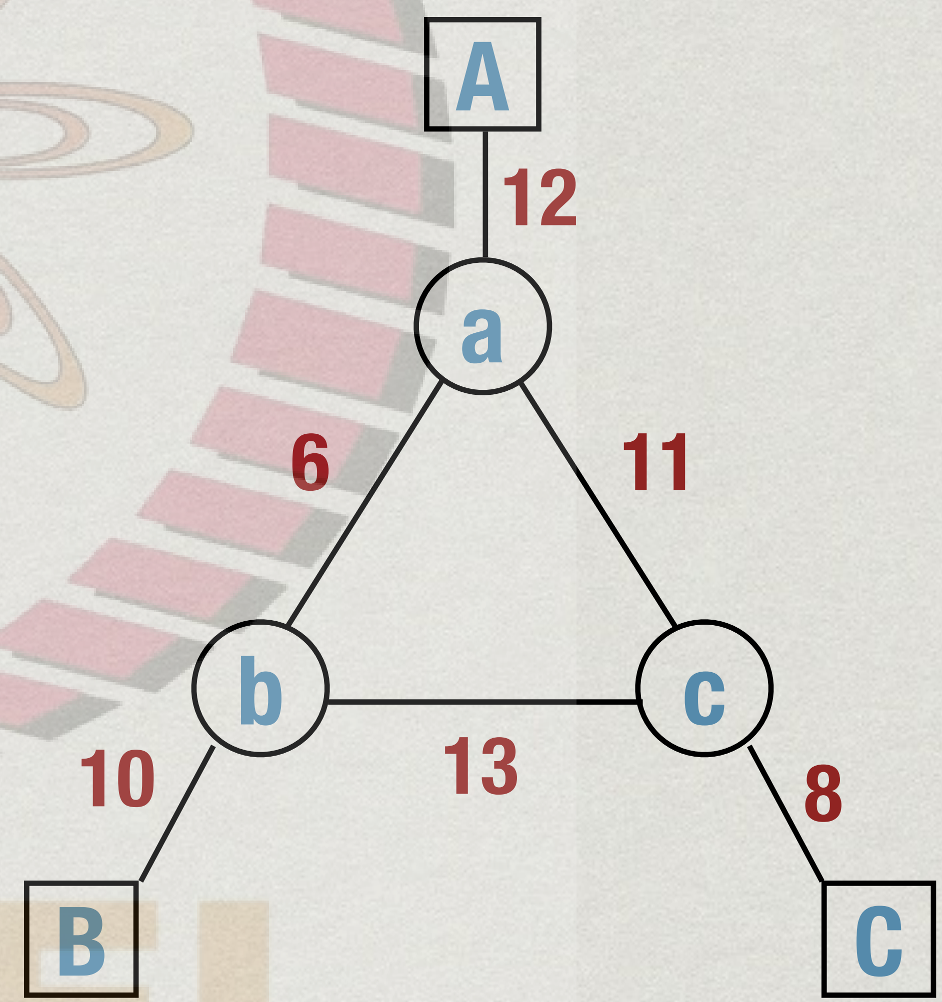
- x_{AB}, y_{BC}, y_{AC} all flow though edge a-b

  $x_{AB} + y_{BC} + y_{AC} \leq 6$

- Likewise

  $y_{AB} + x_{BC} + y_{AC} \leq 13$

  $y_{AB} + y_{BC} + x_{AC} \leq 11$



Minimum bandwidth

- Each pair connected by at least 2 Mbps

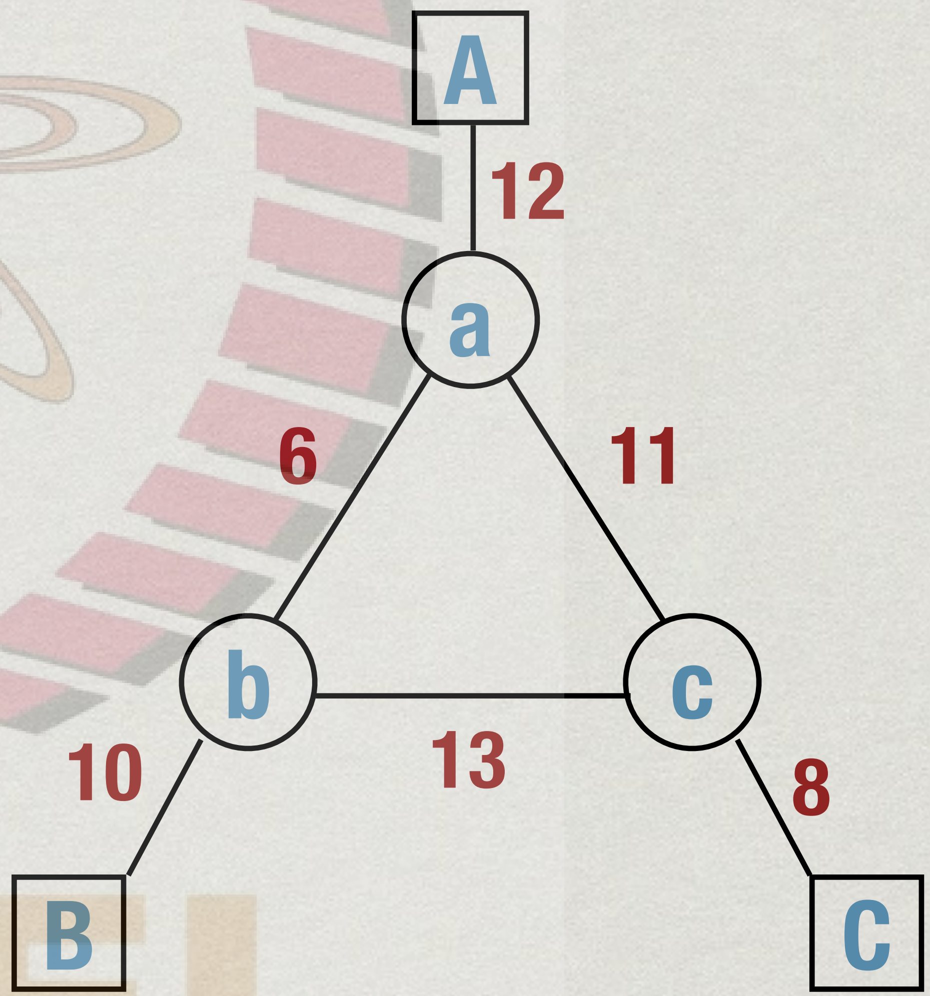
  $x_{AB} + y_{AB} \geq 2$

  $x_{BC} + y_{BC} \geq 2$

  $x_{AC} + y_{AC} \geq 2$

- All routes are non-negative

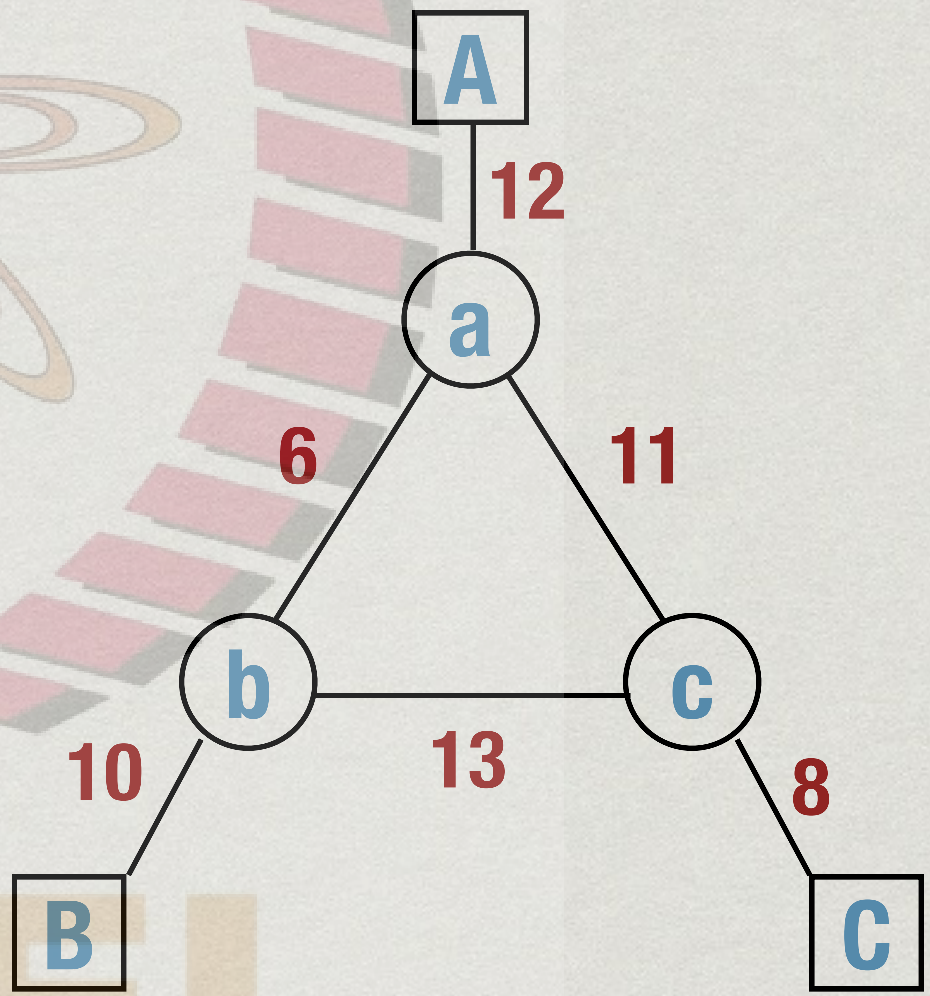
  $x_{AB}, y_{AB}, x_{BC}, y_{BC}, x_{AC}, y_{AC} \geq 0$



Objective function

- Revenue
  - A-B, Rs 300/Mbps
  - B-C, Rs 200/Mbps
  - A-C, Rs 400/Mbps
- Maximize

  $300(x_{AB} + y_{AB}) +$
  $200(x_{BC} + y_{BC}) +$
  $400(x_{AC} + y_{AC})$



Solution

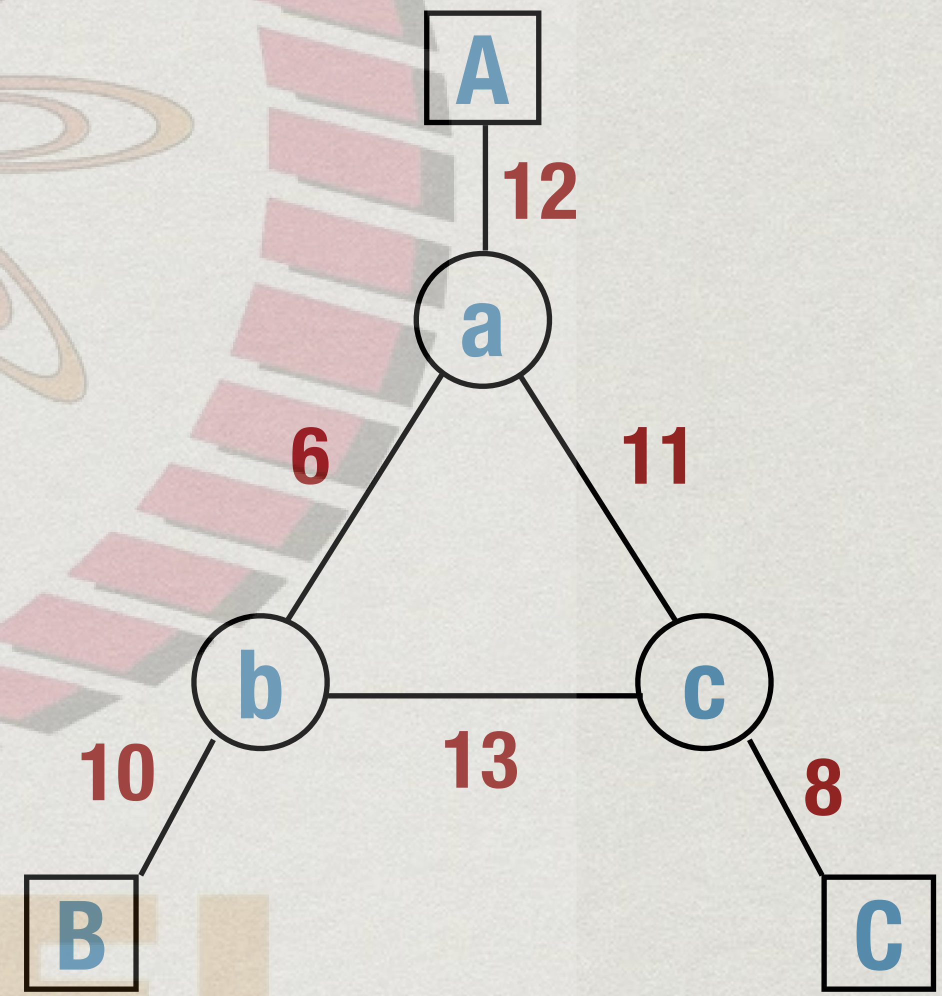
- Simplex yields

  $x_{AB} = 0, y_{AB} = 7$

  $x_{BC} = 1.5, y_{BC} = 1.5$

  $x_{AC} = 0.5, y_{AC} = 4.5$

- Fractional, but OK
- All edges full capacity, except a-c



Note about the model

- One variable per path
- Number of paths is exponential
- Modelling strategy does not scale well
- Will look at a better strategy for analyzing such network flows