NPTEL MOOC,JAN-FEB 2015
Week 8, Module 2

DESIGN AND ANALYSIS OF ALGORITHMS

LP modelling: Production planning

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Linear programming

- Optimization problem where constraints and quantity to be optimized are linear functions
    - Constraints: $ax + by + \ldots. \leq K$, $ax + by + \ldots. \geq K$
    - Quantity (objective function): $ax + by + \ldots$

Solving linear programs

Simplex Algorithm

- Start at any vertex, evaluate objective function
- If an adjacent vertex has a better value, move
- If current vertex is better than all neighbours, stop
- Can be exponential, but efficient in practice
- Theoretically efficient algorithms exist

LP duality

- Can always construct a combination of constraints that tightly captures upper bound on objective function
- Dual LP problem
  - Minimize linear combination of constraints
  - Variables are the multipliers
  - Optimum solution solves both original (primal) and dual LP

Production planning

Handwoven carpets

- 30 employees, each produces 20 carpets a month, salary Rs 20,000
  - Labour cost is Rs 1000 per carpet
- Monthly demand is seasonal
  - Ranges from 440 to 920
  - $d_1 \ldots d_{12}$ from January to December

Coping with varying demand

- Overtime
  - Pay 80% extra, overtime limit 30% per worker
- Hiring and firing
  - Costs Rs 3200 and Rs 4000 per worker
- Store surplus
  - Costs Rs 80 per carpet per month

Formulate a linear program

* w_i : workers in month i, $w_0 = 30$
* x_i : carpets made in month i
* o_i : carpets made in overtime in month i
* h_i : number of workers hired at start of month i
* f_i : number of workers fired at start of month i
* s_i : surplus carpets after month i, $s_0 = 0$

72 variables, plus w_0, s_0

Constraints

- All variables are nonnegative
  - $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$, for i in 1..12
- Carpets made = regular production + overtime
  - $x_i = 20w_i + o_i$
- Number of workers match hiring/firing numbers
  - $w_i = w_{i-1} + h_i - f_i$

Constraints …

- Number of stored carpets connected to earlier stock, production, demand
  - $s_i = s_{i-1} + x_i - d_i$
- Overtime production is at most 6 carpets per worker (30% of regular production)
  - $o_i \leq 6w_i$

Objective function

- Minimize the cost
  - $20000\,(w_1 + w_2 + \ldots + w_{12}) +$
    $3200\,(h_1 + h_2 + \ldots + h_{12}) +$
    $4000\,(f_1 + f_2 + \ldots + f_{12}) +$
    $80\,(s_1 + s_2 + \ldots + s_{12}) +$
    $1800\,(o_1 + o_2 + \ldots + o_{12})$

Solve

- Run Simplex and find a solution
- Are we done?
- Optimum may have fractional values
    - Hire 10.6 workers in March

Fractional solutions

- Hire 10.6 workers in March
    - Round off to 10 or 11 and reevaluate total cost
    - Values are “large”, rounding does not affect quality of solution that much
    - Values are “small”, rounding requires more care
- Insisting on integer solutions makes the problem computational intractable!
    - Integer Linear Programming