Spacecraft Attitude Dynamics

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Final assignment: Attitude dynamics simulation

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1. Symbols

Symbol	Meaning	Unit Measure
a	Orbit semi-major axis	[km]
ARW	Angular Random Walk	[° $h^{-1/2}$]
c	Light speed	[m s^{-1}]
DCM	Direction cosines matrix	[rad]
e	Orbit eccentricity	[-]
F_e	Direct solar radiation	[W m^{-2}]
G	Gravity constant	[m^3 kg^{-1} s^2]
h	Angular momentum	[Nm/s]
i	Orbit inclination	[rad]
I	Matrix of inertia	[kg m^2]
I_i	Moment of inertia along the i-axis	[kg m^2]
LVLH	Local Vertical Local Horizon	[-]
m_t	Earth mass	[kg]
μ	Earth planetary constant	[km^3 s^{-2}]
n	Orbital angular velocity	[rad s^{-1}]
R	Earth's radius	[m]
r	Distance from earth	[m]
RRW	Bias Instability	[° $h^{-3/2}$]
ρ_s	Coefficient of specular reflection	[-]
ρ_d	Coefficient of diffusive reflection	[-]
T	Orbital Period	[s]
ω	Angular velocity	[rad s^{-1}]

2. Introduction

The aim of this project is simulating the behaviour of a spacecraft orbiting around the earth.

In this model, the spacecraft is able to retrieve its attitude and angular velocity and, due to an onboard control system, it can point in the desire direction, despite the presence of disturbance torques coming from the environment.

3. Project specifications

The specifications of the project are summarized in the following table. No changes were added during the implementation of the model.

Spacecraft	Attitude Parameters	Sensors	Actuators
12U CubeSat	Direction Cosines	Gyroscopes	3 Reaction wheels

3.1 Spacecraft specifications

A 12U Cubesat is a rectangular prism with the following specifications[1]

Length*	Width*	Height*	Mass**	Ix	Iy	Iz
20	20	34.05	9.6	0.125	0.125	0.064

*Measured in cm

**Measured in kg

3.2 Orbit specifications

The orbit parameters relevant for the purpose of this project are

μ	a	e	i
$3.99 * 10^5$	19000	0.579	0.175

From these, it's possible to compute the orbital angular velocity and the period of the orbit

$$n = \sqrt{\frac{\mu}{a^3}}$$

$$T = \frac{2\pi}{n}$$

4. Mission description

The mission is composed of three different phases:

1. De-Tumbling
2. Slew maneuver
3. Inertial pointing

4.1 De-tumbling

Before proceeding with any kind of operations, the spacecraft angular velocity needs to go as close as possible to zero. This is the only goal at this stage of the mission, therefore the attitude of the spacecraft has no relevance whatsoever.

4.2 Slew maneuver

The second phase of the mission consists of following the LVLH frame, which is such that:

- The z-axis is pointing in the same direction of the angular momentum
- The x-axis is in the radial direction
- The y-axis completes the right handed frame

Therefore, the x and y axes have to lay on the orbital plane for the rest of this phase, which in turn means that the spacecraft has to rotate only around the z-axis.

The angular velocity and attitude target are then

$$\omega = \begin{Bmatrix} 0 \\ 0 \\ n \end{Bmatrix}$$

$$A = \begin{Bmatrix} cos(nt) & sin(nt) & 0 \\ -sin(nt) & cos(nt) & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

4.3 Inertial pointing

During the final phase of the mission, the body frame should chase the sun centered inertial one, that is characterized by

- Zero angular velocity
- Identity matrix as direction cosines matrix

It should be noted that the origins of inertial and body frames have been considered as coincident.

5. ADCS architecture[2]

As the Fig. 5.1 shows it is possible to distinguish six different blocks:

1. Dynamics
2. Kinematics
3. Environment
4. Sensors
5. Attitude Determination
6. Actuators

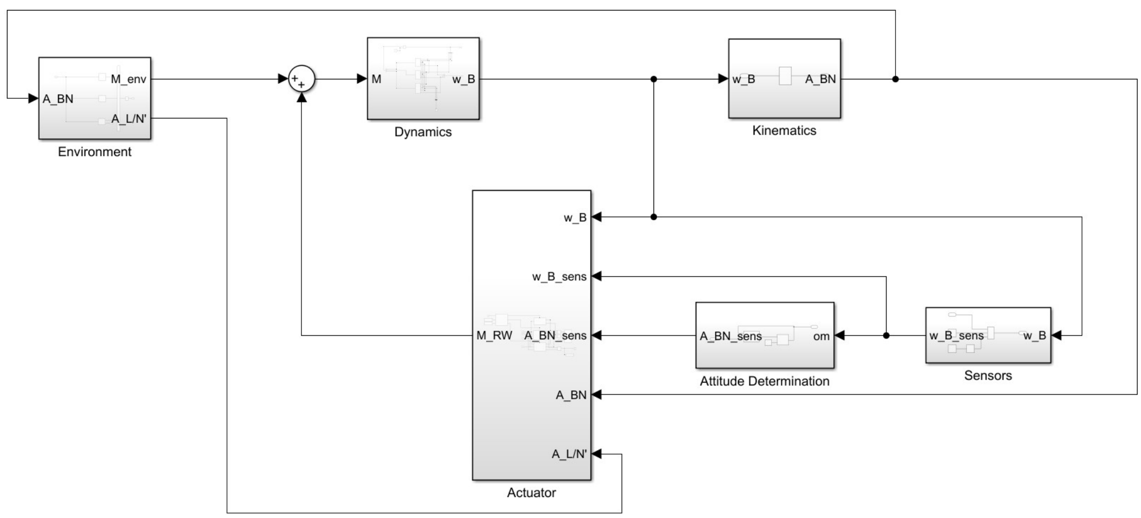

Fig. 5.1 ACDS Architecture

5.1 Dynamics

To compute the dynamics of the spacecraft, i.e. its angular velocity in body frame, it has been used a set of Euler equations, defined as:

$$I\frac{d\omega}{dt} = I\omega \wedge \omega + \mathbf{M}$$

Where **M** is the sum of the torque coming from the environment and the one generated by the actuators.

5.2 Kinematics

Knowing the spacecraft angualr velocity ω, the direction cosines matrix can be computed by integrating the following formula:

$$\frac{dA(t)}{dt} = -[\omega\wedge]A(t)$$

Where

$$[\omega\wedge] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

At each time step the DCM has been orthogonalized

$$A_{k+1}(t) = \frac{3}{2} * A_k(t) - \frac{1}{2} A_k(t) * A_k(t)^T * A_k(t)$$

5.3 Environment

The environment block computes the disturbance torque caused by factors that are external to the spacecraft itself.

On a preliminary analysis, four sources of disturbance were considered:

1. Magnetic field of the earth
2. Solar radiation pressure
3. Gravity gradient of the earth
4. Air drag of earth atmosphere

5.3.1. Magnetic field

The disturbance torque coming from the earth magnetic field can be computed as follows:

$$\mathbf{M} = \mathbf{m} \ \wedge \ \mathbf{b}$$

Where

- $\mathbf{b_N}$ is earth magnetic field (in inertial frame), modeled as a simple dipole
- **m** is the parasitic magnetic torque

$$\mathbf{b_N} = \frac{R^3 H_0}{r^3} [3(\hat{m} \cdot \hat{r})\hat{r} - \hat{m}]$$

$$H_0^* = ((g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2)^{1/2}$$

$$\hat{m} = \begin{bmatrix} sin 11.5^\circ cos(\omega_\oplus t) \\ sin 11.5^\circ sin(\omega_\oplus t) \\ cos 11.5^\circ \end{bmatrix}$$

$$\mathbf{m} = [0.01 \ \ 0.05 \ \ 0.01]^T \ Am^2$$

*The coefficient used to compute H_0 are taken from the IGRF 2000 model.

5.3.2 Solar radiation pressure

The solar panels have been modeled as two squares with a length of 570 mm and zero thickness. The values have been selected considering as reference the Sparkwing products.[2]

The solar radiation that hits the surfaces of a spacecraft generates a pressure, which in turn leads to a torque:

$$\mathbf{F_i} = -PA_i(\hat{S}_b \cdot \hat{N}_{bi})[(1-\rho_s)\hat{S}_b + (2\rho_s(\hat{S}_b \cdot \hat{N}_{bi}) + \frac{2}{3}\rho_d)\hat{N}_{bi}]$$

$$\mathbf{T_{SRP}} = \begin{cases} \sum_{i=1}^{n} \mathbf{r_i} \wedge \mathbf{F_i} & \hat{S}_b \cdot \hat{N}_b > 0 \\ 0 & \hat{S}_b \cdot \hat{N}_b < 0 \end{cases}$$

Where

- $\hat{S}_b$ is the direction of the sun with respect to the spacecraft in body frame
- $\hat{N}_{bi}$ is the unit vector normal to the surface i of the spacecraft in body frame
- P is the average pressure due to radiation

The formula to compute P is

$$P = \frac{F_e}{c}$$

Although F_e should consider, not only the direct solar radiation, but also the radiation coming from the earth, this contribution has been neglected since it's at least two order of magnitude smaller than the direct solar radiation.

Therefore

$$F_e = 1358 \ \frac{W}{m^2}$$

5.3.3 Gravity gradient

The torque generated by the earth gravity gradient can be computed as follows:

$$\mathbf{M} = \frac{3Gm_t}{r^3} \begin{Bmatrix} (I_z - I_y)c_2c_3 \\ (I_x - I_z)c_1c_3 \\ (I_y - I_x)c_1c_2 \end{Bmatrix}$$

Where

$$\mathbf{c} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = A_{B/L} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Where $A_{B/L}$ is the DCM that represents the orientation of the body frame with respect to the LVLH reference frame.

The gravity gradient has been considered as a source of disturbance even though the torque generated is one order of magnitude smaller than the ones due to the magnetic field and the solar radiation pressure.

5.3.4 Air drag

The formula to compute the air drag torque is

$$\mathbf{T} = \frac{1}{2}\rho\, v^2 A_s\, C_D\, r_i$$

where

- ρ is the air density
- v is the relative speed
- A_i is the exposed surface
- C_D is the drag coefficient
- r_i is the distance between the center of pressure and the center of mass

It can be showed that at the perigee, where the atmosphere is at its thickest, the torque due the air drag is several order of magnitude smaller than the others, it was therefore neglected in the whole simulation.

5.4 Sensors and Attitude Determination

The sensors used are three MEMS gyroscopes, each of them was supposed to be aligned with one of the three body axes.

The specifications of the gyros have been taken using as a reference the STIM300 of Safran.[3]

ARW	RRW	Frequency*
0.15	0.0003	100

The gyros have been modelled as devices that introduce two white noises, such that

$$\omega^{\mathbf{M}} = \omega + \mathbf{n} + \mathbf{b}$$

where **n** and $\frac{d\mathbf{b}}{dt}$ are Gaussian white noises with a standard deviation which is respectively the angular random walk and the bias instability.

The Spacecraft attitude can be retrieved by building the DCM using the output of the sensors block as the angular velocity.

Both the measured angular velocity and the estimated DCM matrix were used into the control block, whose discussion is detailed in the following paragraph.

*Measured in Hertz

5.5 Actuators

The actuators used are three reaction wheels, each of them was supposed to be aligned with one of the three body axes.

The specifications of the wheels have been taken using as a reference the RW400 of AAC Clyde Space.[4]

Maximum Torque	Mass	Radius
+/- 8 mNm	197 g	27 mm

6. Control Law[2]

The dynamics of the reaction wheels was modeled as follows:

$$\dot{\mathbf{h}}_r = A^*[-M_c^{id} - \omega \wedge I\omega - \omega \wedge Ah_r]$$

$$M_c^{real} = -A\dot{\mathbf{h}}_{\mathbf{r}} - \omega \wedge Ah_r$$

Where

- $\mathbf{h}_r$ is the angular momentum of the reaction wheels
- A is the configuration of matrix of the reaction wheels, which is equal to the identity matrix, since the reaction wheels were supposed to be aligned with the body frame.
- $\mathbf{M_c}$ is instead the control torque generated by the wheels.

In order to compute the ideal torque, $\mathbf{M_c^{id}}$ a nonlinear attitude control method has been used.

$$I\frac{d\omega}{dt} = I\omega \ \wedge \ \omega + \mathbf{u}$$

Where $\mathbf{u}$ is the ideal torque.

To guarantee the stability of the control-law different Lyapunov functions have been defined, for all the maneuvers. Recalling Lyapunov's theorem, the function V has to be such that

$$V(\mathbf{x}) > 0 \ \forall \mathbf{x} \neq \mathbf{x}^* in \ D \ and \ V(\mathbf{x}^*) = 0$$
$$\dot{V}(\mathbf{x}) < 0 \ \forall \mathbf{x} \neq \mathbf{x}^* in \ D \ and \ \dot{V}(\mathbf{x}^*) = 0$$

Where D is a neighbourhood of the equilibrium configuration $\mathbf{x}^*$. If we define $s(\mathbf{x}, \mathbf{u})$ as

$$\frac{dV}{dt} = s(\mathbf{x}, \mathbf{u})$$

We can compute $\mathbf{u}$ such that

$$s(\mathbf{x}, \mathbf{u}) < 0$$

Since each phase of the mission has a different target, three different functions have been computed.

6.1 De-tumbling

The aim of this phase is to reach zero angular velocity, the control law becomes then

$$V(\omega) = \frac{1}{2}\omega^{\mathbf{T}} I \omega$$
$$\mathbf{u} = -k_1 \omega$$

Where $\boldsymbol{\omega}$ is the output of the sensors block.

The discussion about the value of the parameter k_1 is left to the dedicated paragraph.

6.2 Slew maneuver

Here, the spacecraft has to reach not only a certain angular velocity but also a particular attitude. The Lyapunov function should include then a term such that the RWs generate a null torque when both the velocity and the DCM reach the desired value.

Calling A_d and $\boldsymbol{\omega_d}$ the target orientation and velocity, whose values have been discussed in the paragraph 4.

$$A_e = A A_d^T$$

$$\omega_{\mathbf{e}} = \omega - A_e \omega_{\mathbf{d}}$$

It can be easily noticed that A_e is equal to the identity matrix once the body frame and the LVLH frame overlap, therefore

$$V = \frac{1}{2}\omega_{\mathbf{e}}^{\mathbf{T}}\, I\, \omega_{\mathbf{e}} + 2\, k_2\, tr(I_3 - A_e)$$

$$\mathbf{u} = -k_1 \omega_{\mathbf{e}} \; - k_2 (A_e^T - A_e)^V \; + \omega \wedge I\omega$$

The discussion about the value of the parameters k_1 and k_2 is left to the dedicated paragraph.

6.3 Inertial pointing

The inertial pointing maneuver can be seen as a slew maneuver in which the target velocity is null and the target DCM is the identity matrix.

$$A_e = A$$

$$\omega_{\mathbf{e}} = \omega$$

$$V = \frac{1}{2}\omega_{\mathbf{e}}^{\mathbf{T}}\, I\, \omega_{\mathbf{e}} + \; k_2\, tr(I_3 - A_e)$$

$$\mathbf{u} = -k_1\omega_{\mathbf{e}} \; - k_2(A_e^T - A_e)^V$$

The discussion about the value of the parameters k_1 and k_2 is left to the dedicated paragraph.

6.4 Control coefficients tuning

The coefficients used to compute the ideal control torque, k_1 and k_2, have been defined from an initial value and then tuned to reach convergence sooner and to have a torque below the maximum guaranteed by the reaction wheels.

The initial values were taken from a linear system dynamics of second order:

$$\begin{aligned} k_1 &= I_z\omega^2 \\ k_2 &= 2I_z\xi\omega \end{aligned}$$

Where ω is the closed loop natural frequency and ξ the damping. A damping of 0.7 and a natural frequency of $20n$ were assumed, with n that is the orbital frequency.

After the tuning process, the final values are

$$k_1 = 0.09 \quad k_2 = 0.0085$$

Since both of the coefficients are positive, the stability of the control law is guaranteed.

7. Results

In order to evaluate the validity of the model, it has been simulated the behaviour of the spacecraft over a time span of one orbital period, with an intial velocity of

$$\omega = \begin{Bmatrix} n \\ n \\ n \end{Bmatrix}$$

In the following paragraphs there were anlyzed the results obtained in terms of angular velocity and attitude, consideting both the controlled and uncontrolled motions.

7.1 Angular velocity

The Fig. 7.1.1 shows an overview of the angular velocity throughout the all mission. The Fig. 7.1.2, 7.1.3 and 7.1.4, instead focus on the initial phases of each stage of the missions.

At each target transitions, the velocity reaches high peaks of oscillations and then, after a settling time, it goes to the desired value.

It has to be noticed that, in the de-tumbling maneuver, the settling time could have been bigger if the initial velocity would have been considerably higher.

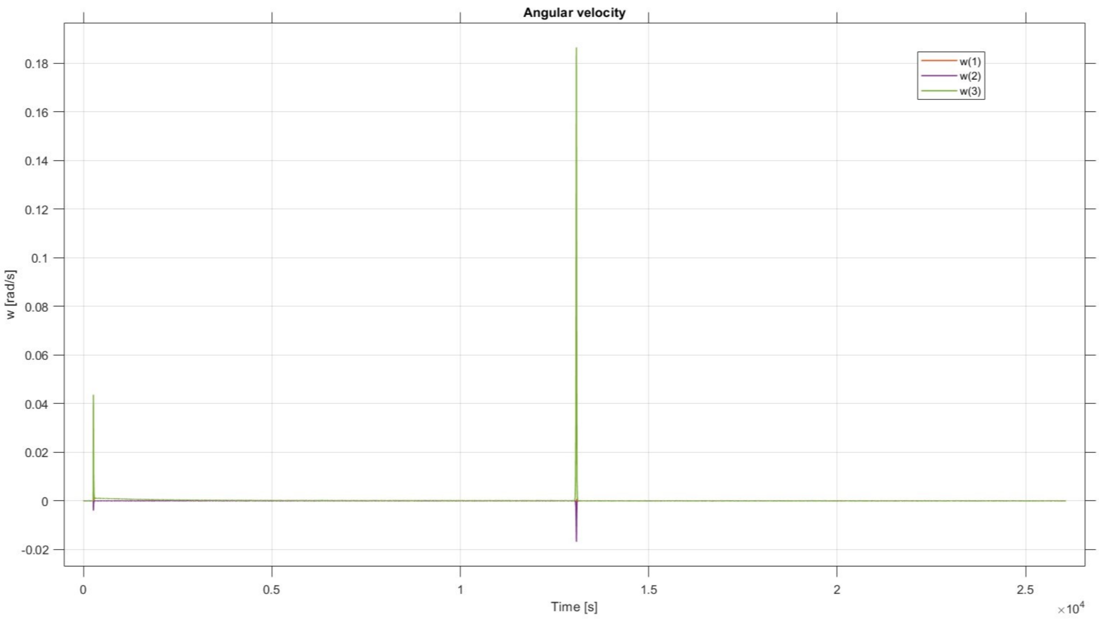

Fig. 7.1.1 Overall mission

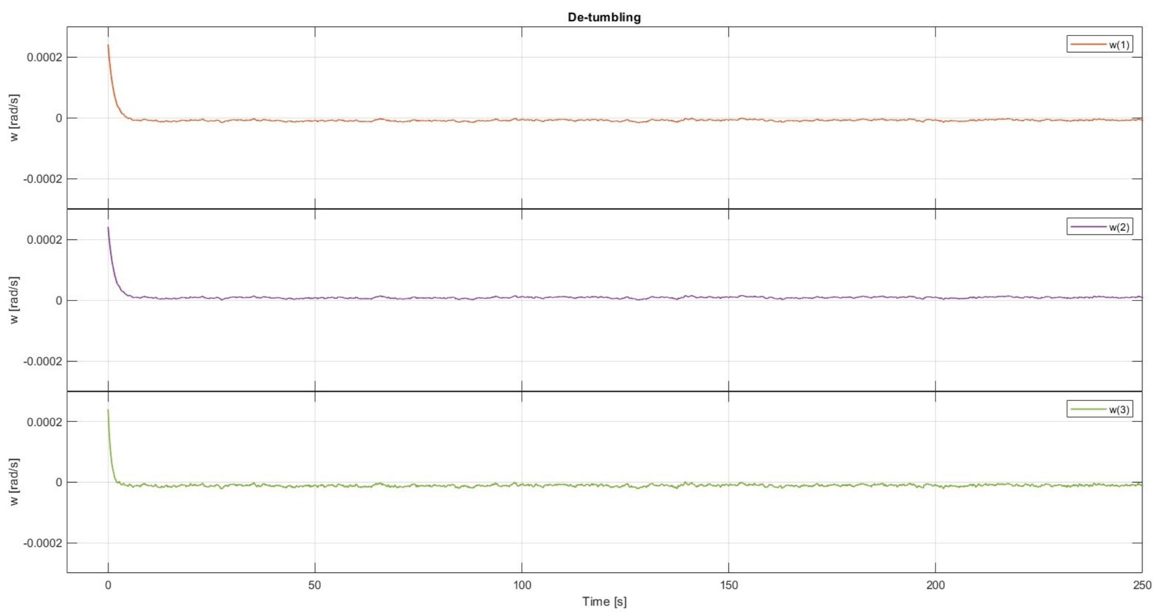

Fig. 7.1.2 De-tumbling

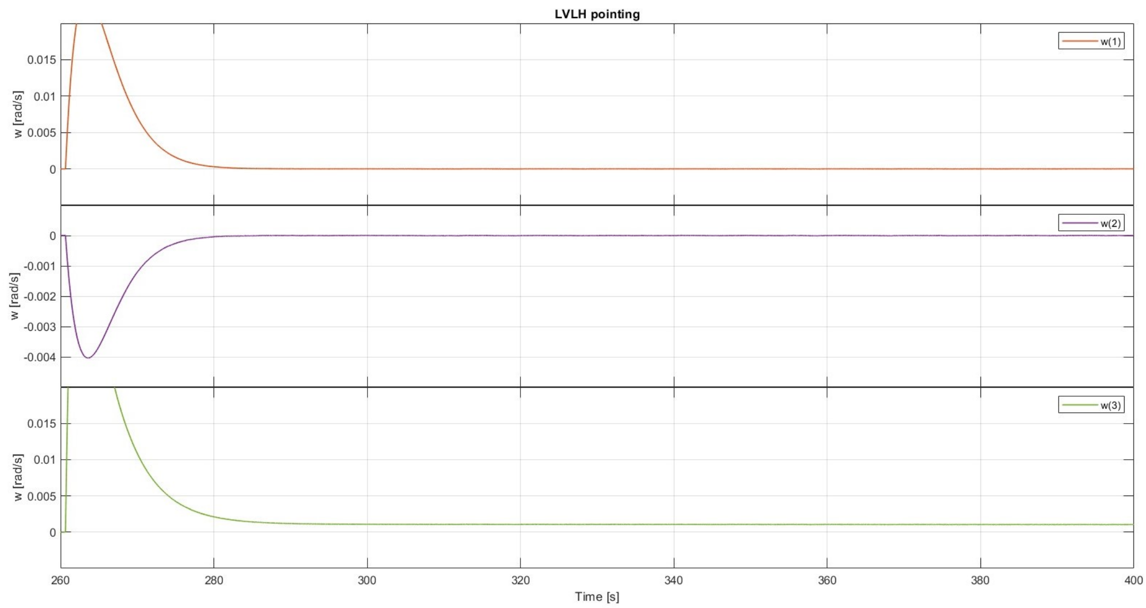

Fig. 7.1.3 LVLH pointing

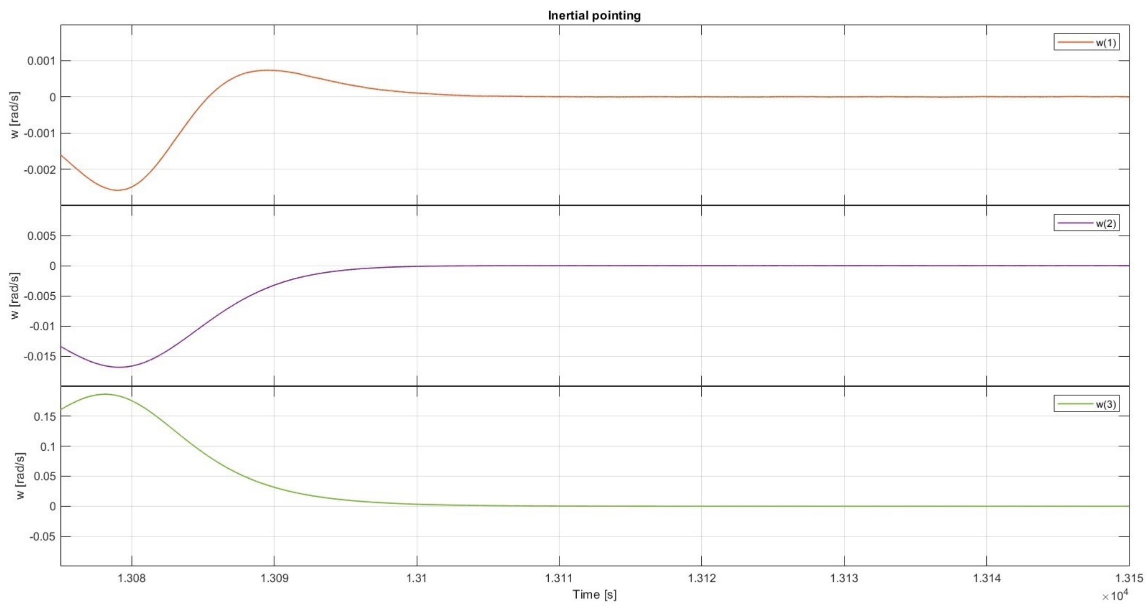

Fig. 7.1.4 inertial pointing

7.2 Pointing error

The pointing error has been defined as the vector of the angles between the axes of the actual spacecraft DCM and the target ones.

Since the DCM rows represent the projection of the body axes onto the inertial frame, the pointing error can be computed as follows

$$A = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}, \ A_t = \begin{Bmatrix} X_t \\ Y_t \\ Z_t \end{Bmatrix}$$

$$\theta = acos(X \cdot X_t)$$
$$\gamma = acos(Y \cdot Y_t)$$
$$\delta = acos(Z \cdot Z_t)$$

$$err = \begin{Bmatrix} \theta \\ \gamma \\ \delta \end{Bmatrix}$$

Since the three phases of the mission have different DCM target, at each transition there's a peak in the error angles. For the rest of the time the angles stay, as expected, in the proximity of zero.

The pointing error during the de-tumbling phase was considered null since the spacecraft doesn't need to point in any particular direction.

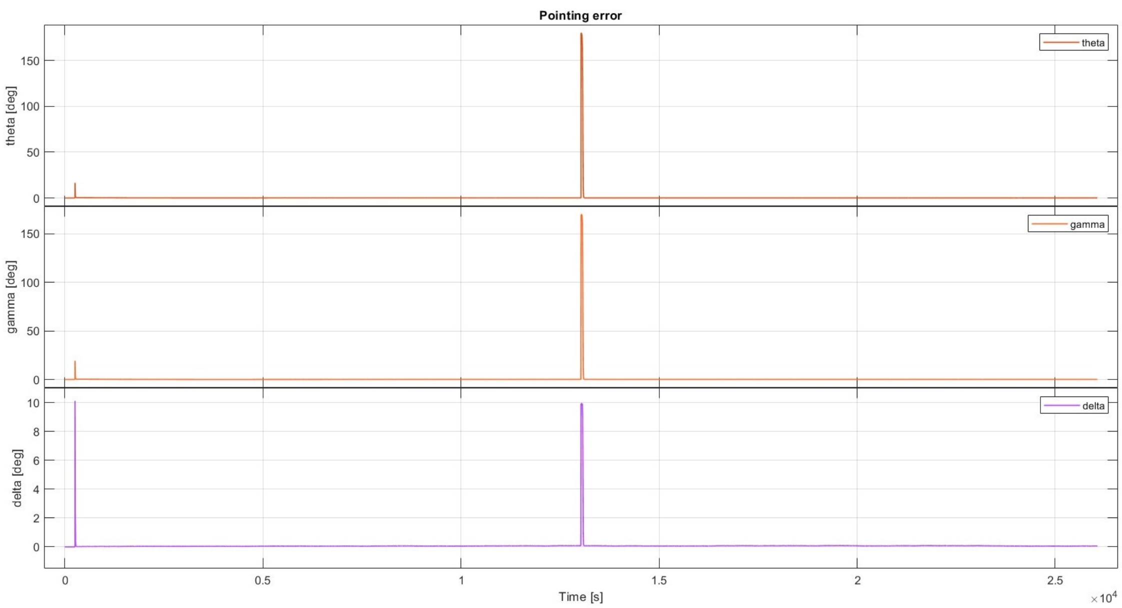

Fig. 7.2.1 Pointing error

7.3 Reaction wheels' torque

Fig. 7.3.1 shows, as it could be easily predicted, that the control torque needed is much higher in correspondance of the change of targets, while it oscillates around zero for the rest of the mission.

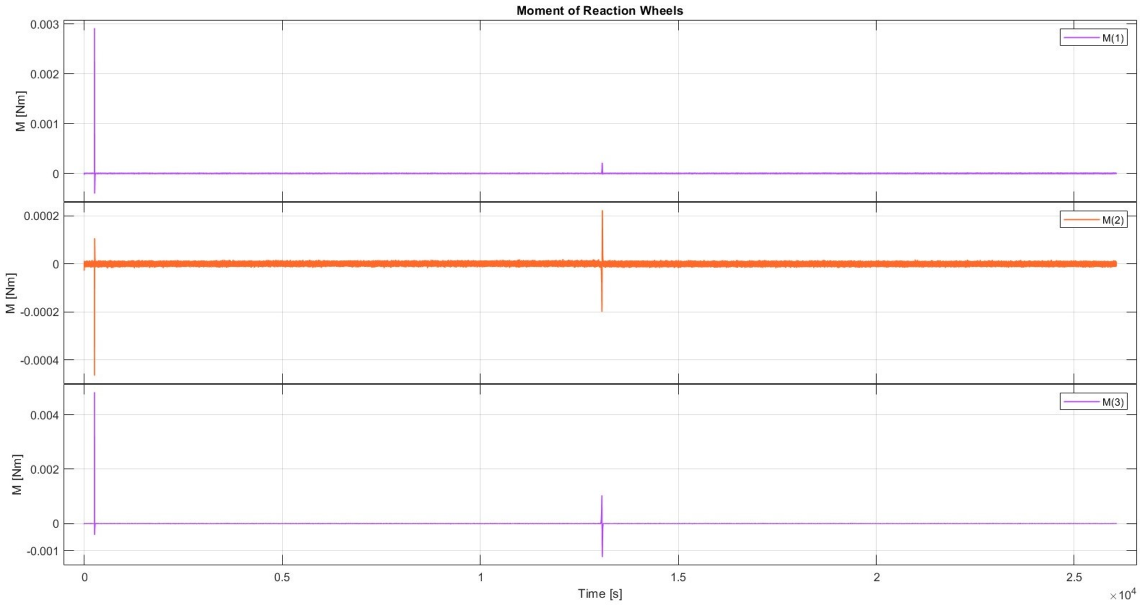

Fig. 7.3.1 RWs torque

7.4 Uncontrolled case

The Fig. 7.4.1 and 7.4.2 shows the change over time of angular velocity and the pointing error of the spacecraft, showing the relevance of the control system inside the spacecraft.

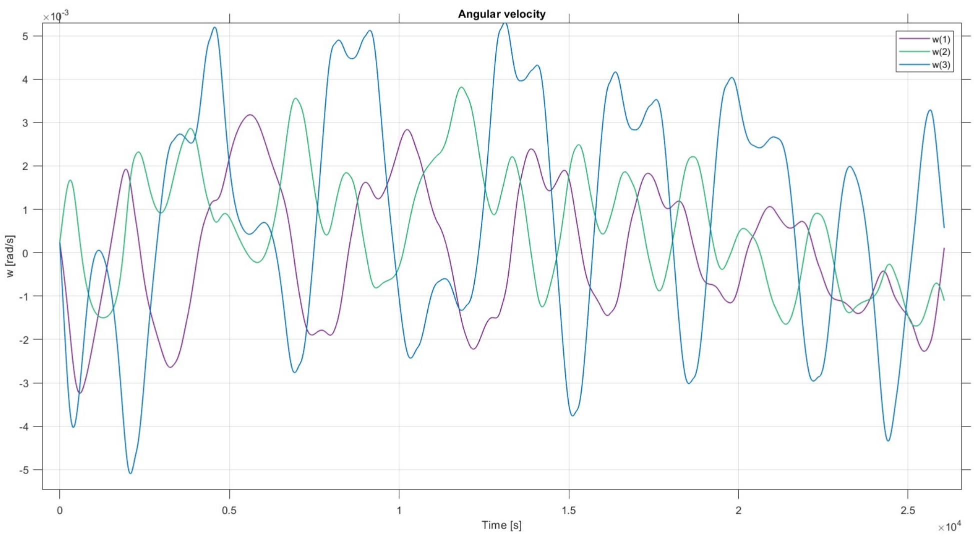

Fig. 7.4.1 Unctrolled spacecraft angular velocity

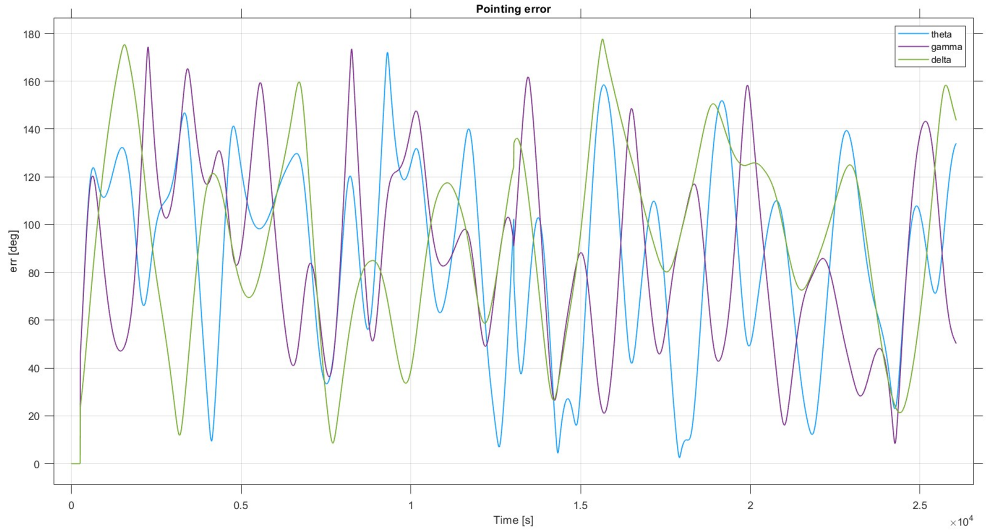

Fig. 7.4.2 Unctrolled pointing error

7.5 Statistical Analysis

A further analysis of the data obtained in the two cases, i.e. controlled and uncontrolled, can be done by generating gaussian distributions, exploiting the Matlab avg, std and makedist functions.

The following figures confirm that in the control case, the pointing error is considerably small for almost the whole mission.

In the unctrol situation, instead, the average error is around 90°, with a standard deviation close to 39°, that is to say it's less likely to have an attitude close to the mean one, which is characterized by an error much larger than zero.

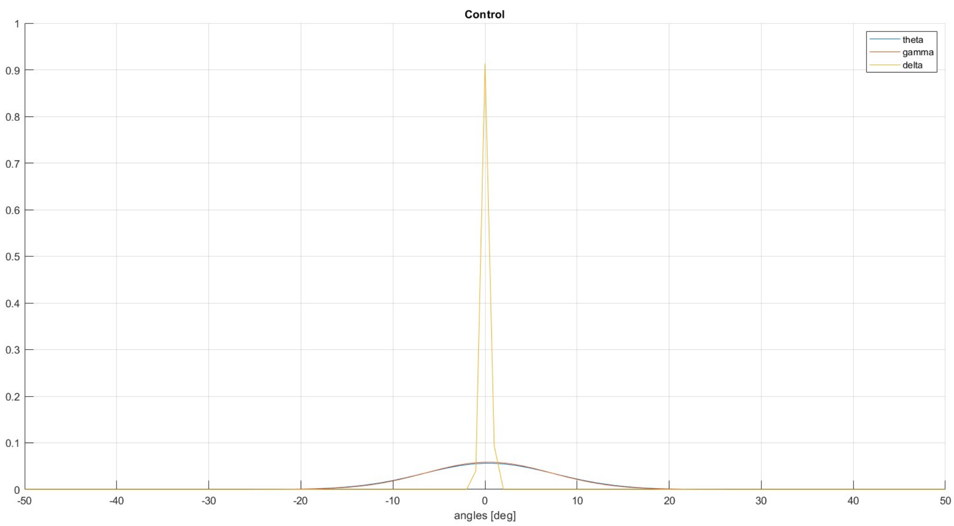

Fig. 7.5.1 Controlled case

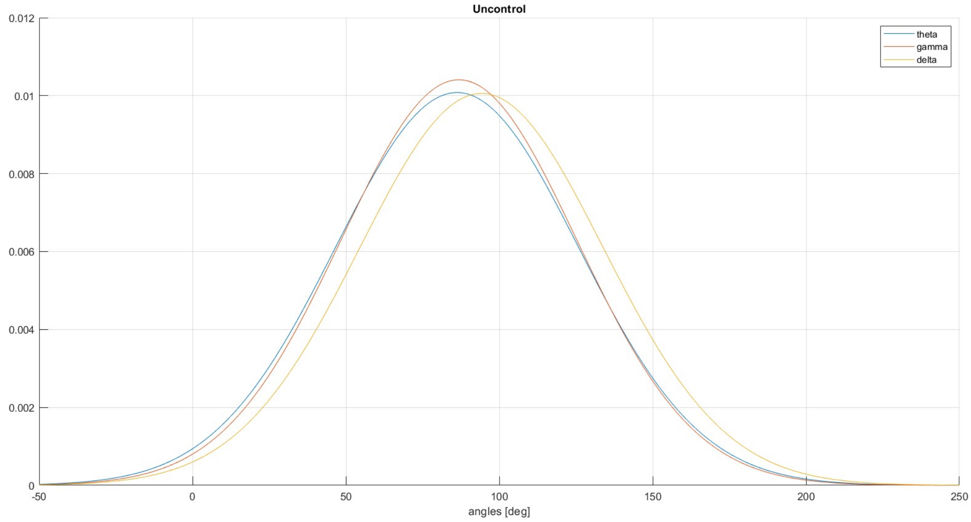

Fig. 7.5.2 Unctrolled case

8. References

[1] https://www.nanosats.eu/cubesat

[2] Course material

[3] https://sparkwing.space/#downloads

[4] https://www.sensonor.com/products/inertial-measurement-units/stim300/

[5] https://www.aac-clyde.space/wp-content/uploads/2021/11/RW400.pdf