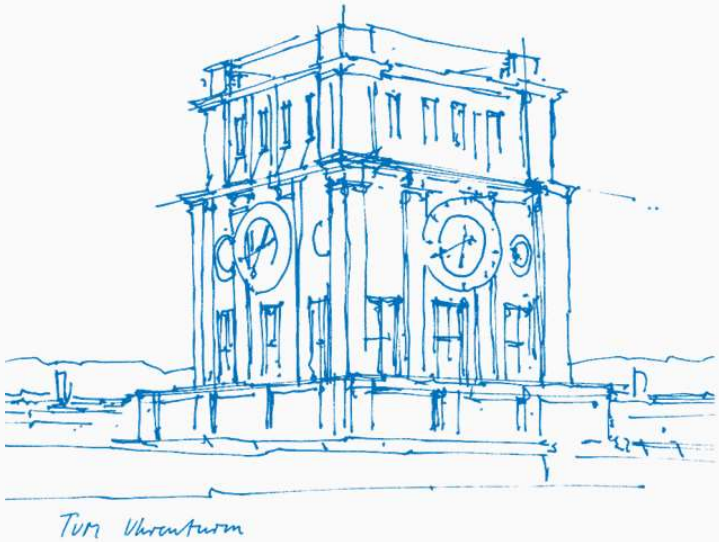
Computer Course Linear Programming

Introduction to Gurobipy



Stefan Kober

28-29 October 2020

Technical University of Munich

Organizational Things

What to expect

What this course offers:

- praxis-oriented introduction to python and gurobipy
- lots of examples
- preparation for further lectures, case studies and theses

What this course does not offer:

- detailed installation instructions
- the time needed to become an expert in python and gurobipy

Schedule

- Wednesday:
  - Introduction to Python
  - Introduction to Gurobi
- Thursday:
  - Features Python (advanced input and output methods)
  - Features Gurobi (advanced variable types and output interpretation)

Schedule

10:15 first slot

11:45 lunch break

13:15 second slot

14:45 coffee break

15:15 third slot

Work in teams!

Outlook

Structure of Gurobi

Basics

Linear Programming

Modelling

Output Interpretation

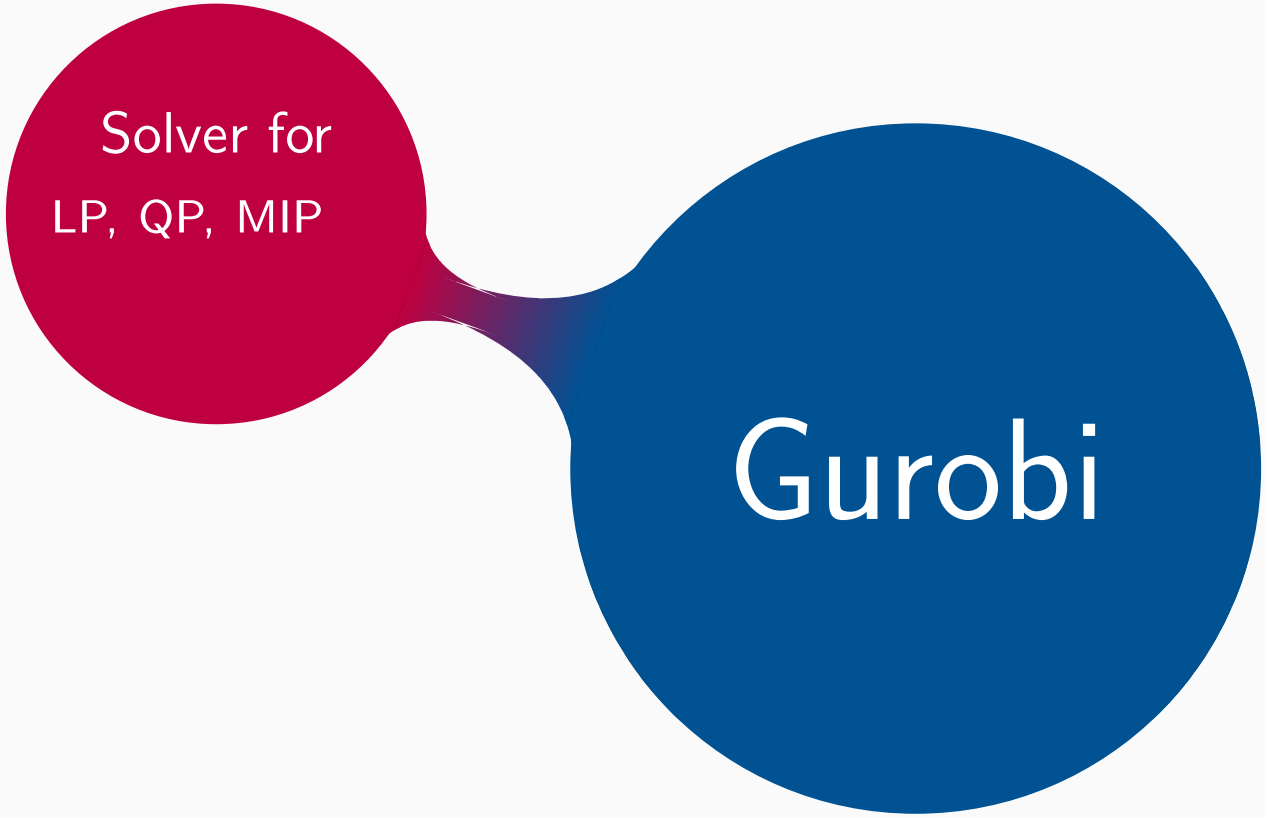
Advanced Input Methods

Advanced Gurobi Datatypes

Visualization

Structure of Gurobi

What is Gurobi?



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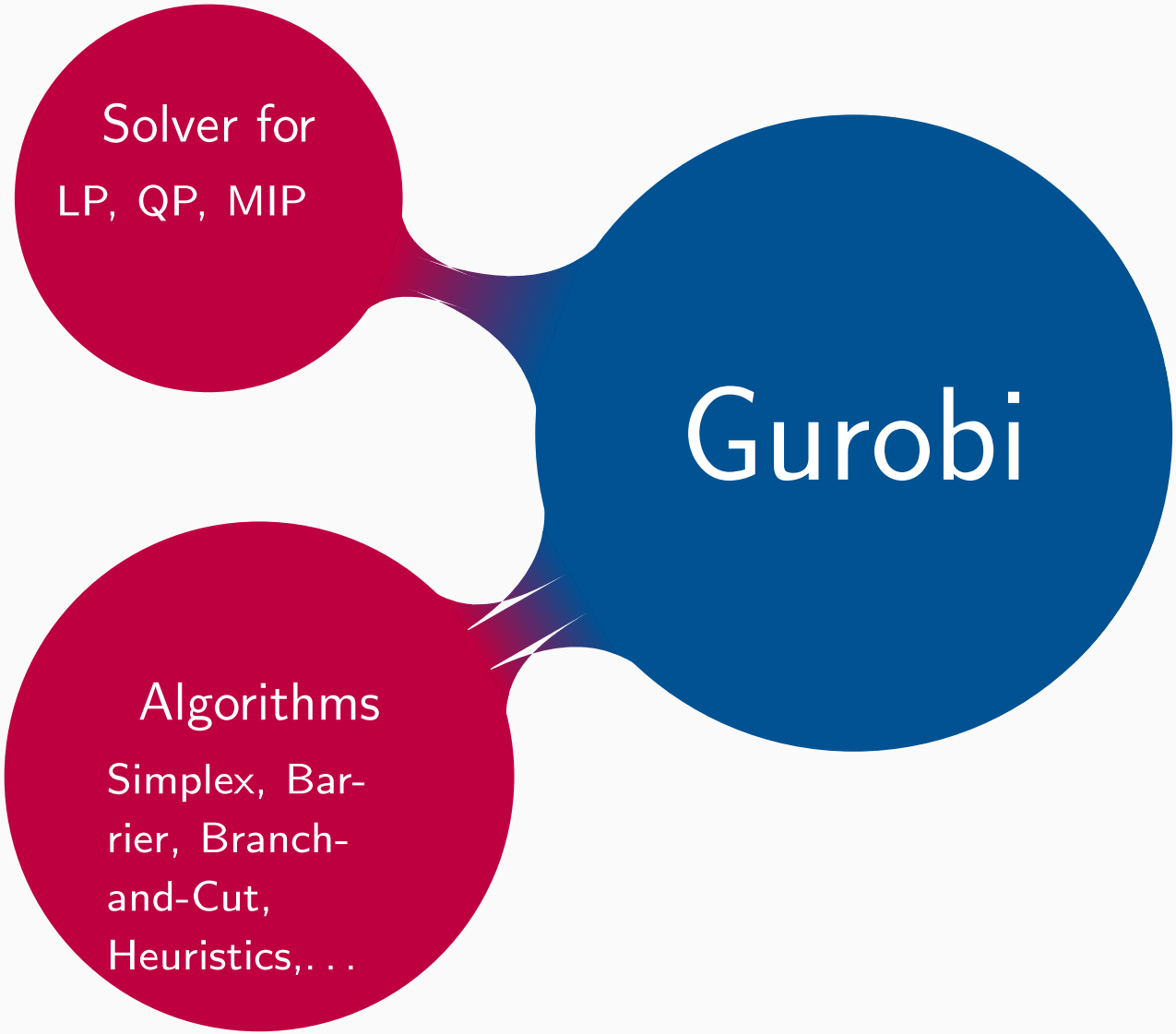

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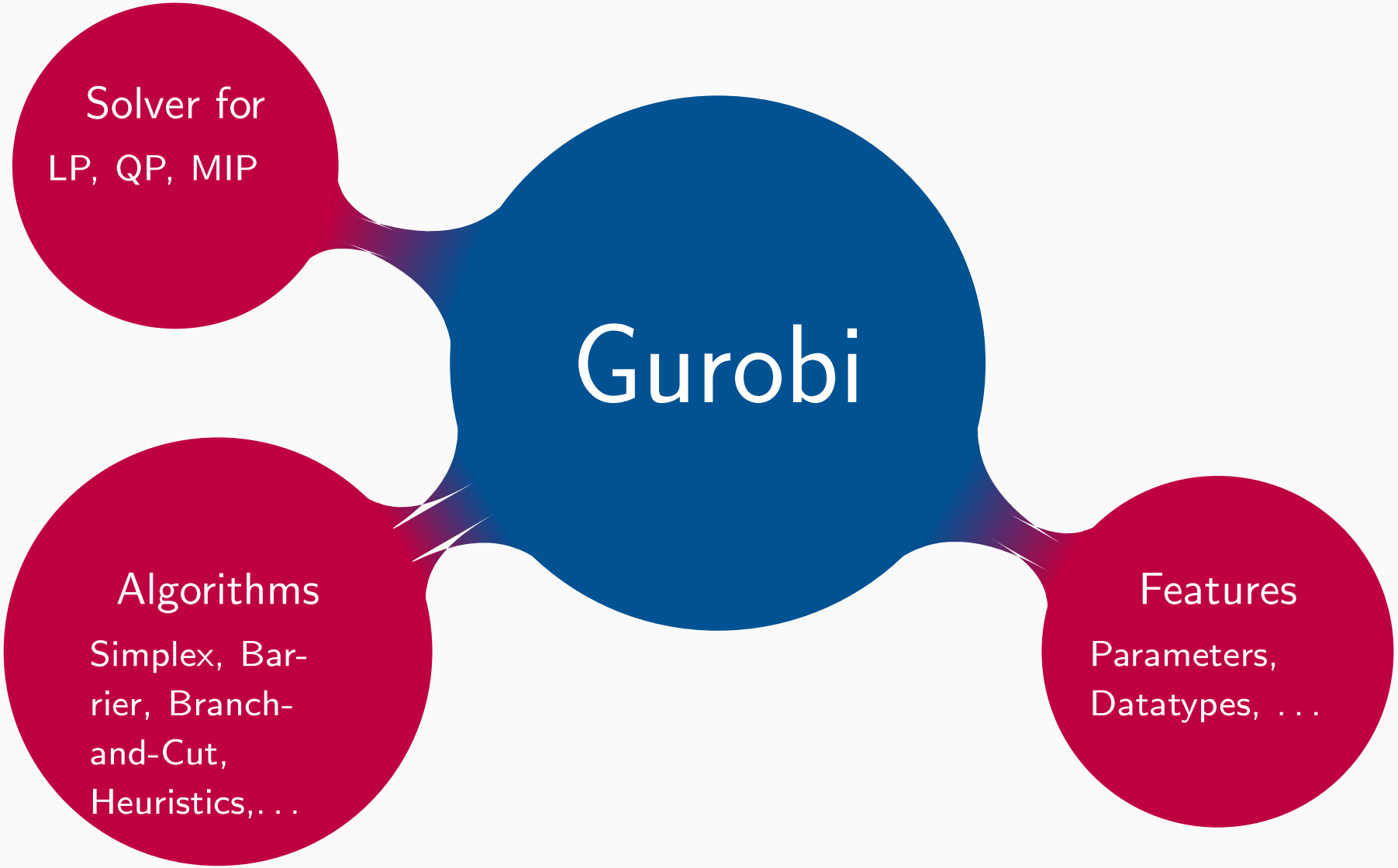

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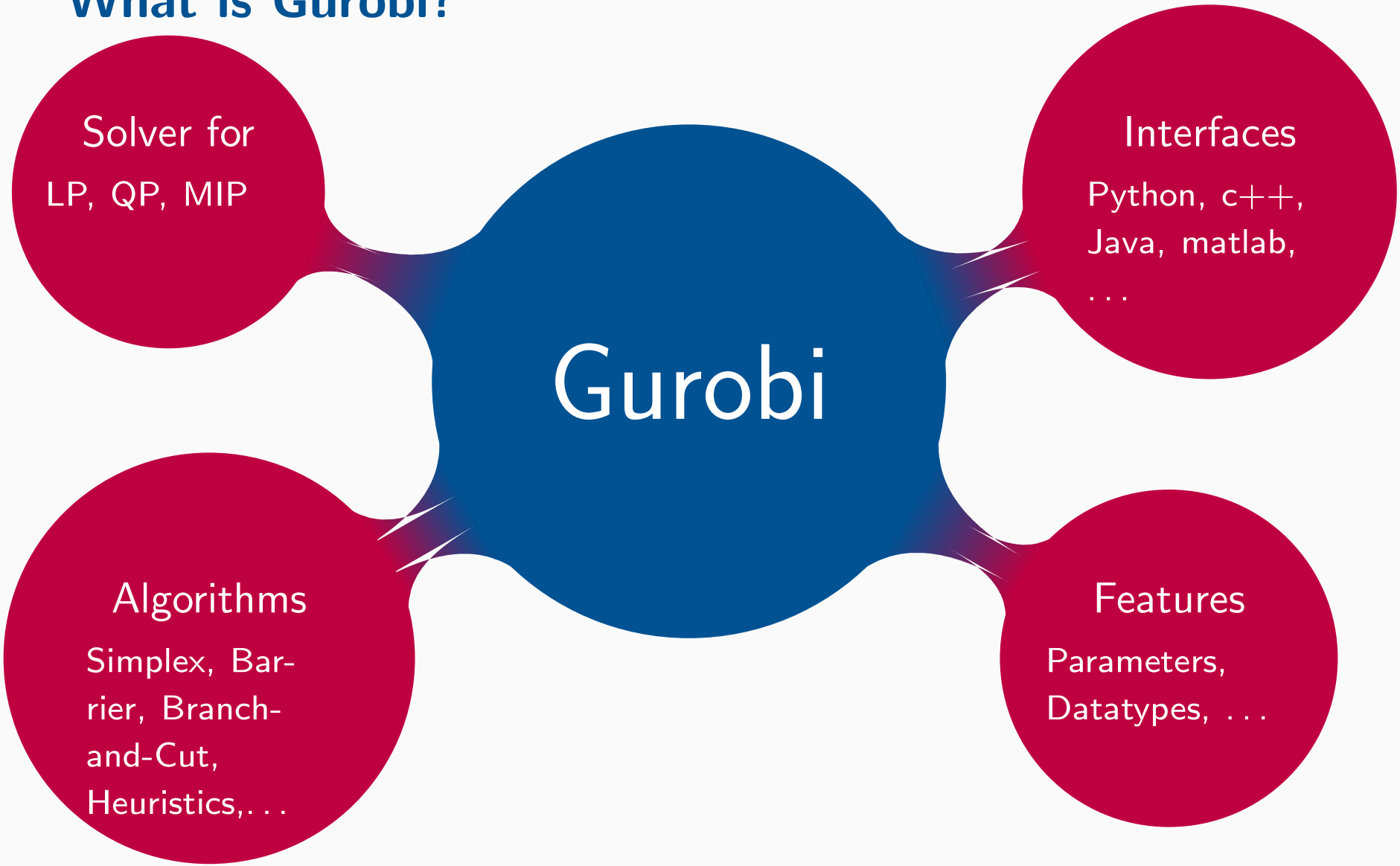
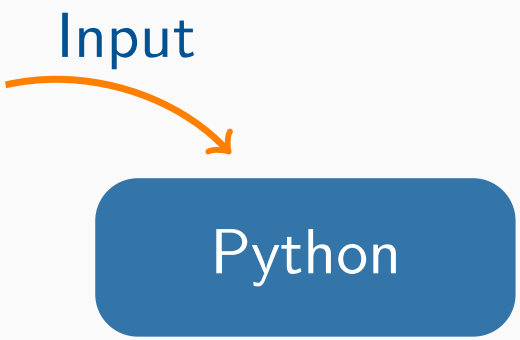


How can we use Gurobi?



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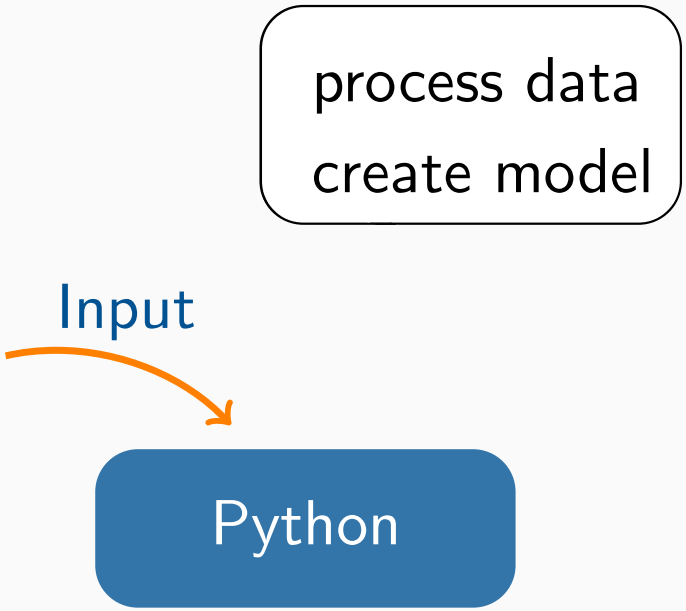

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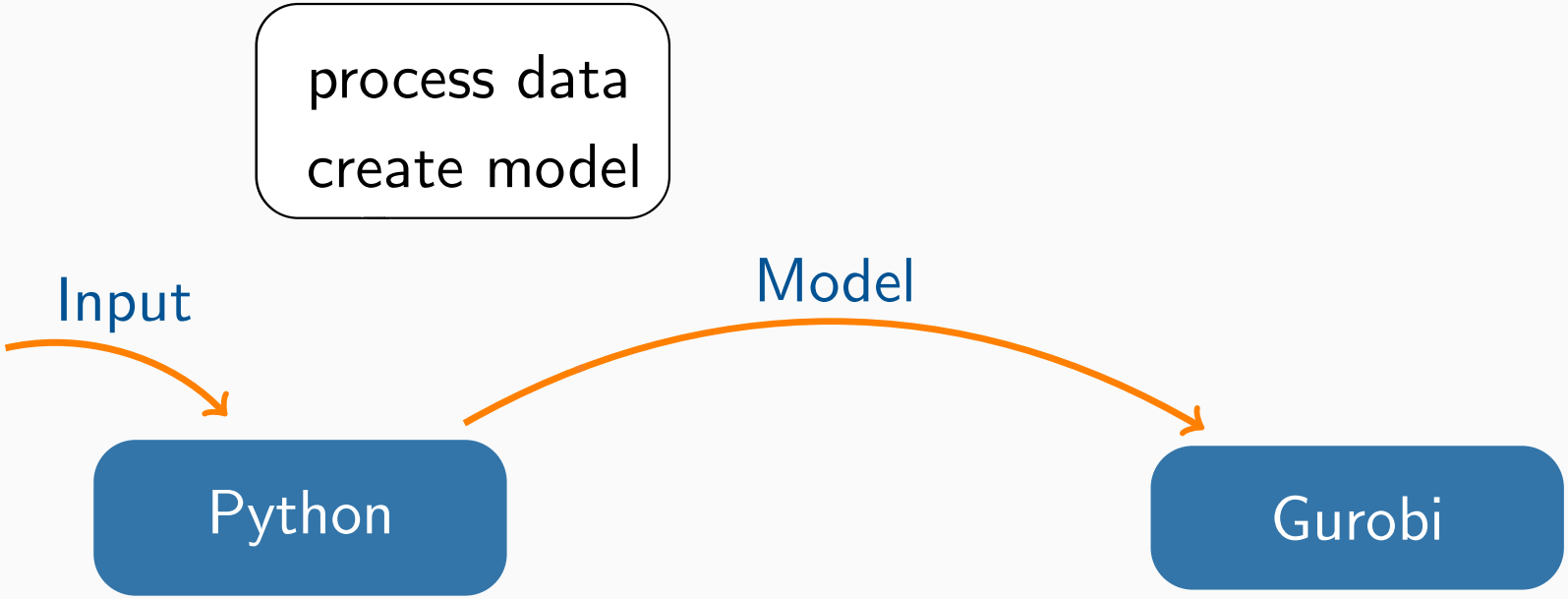

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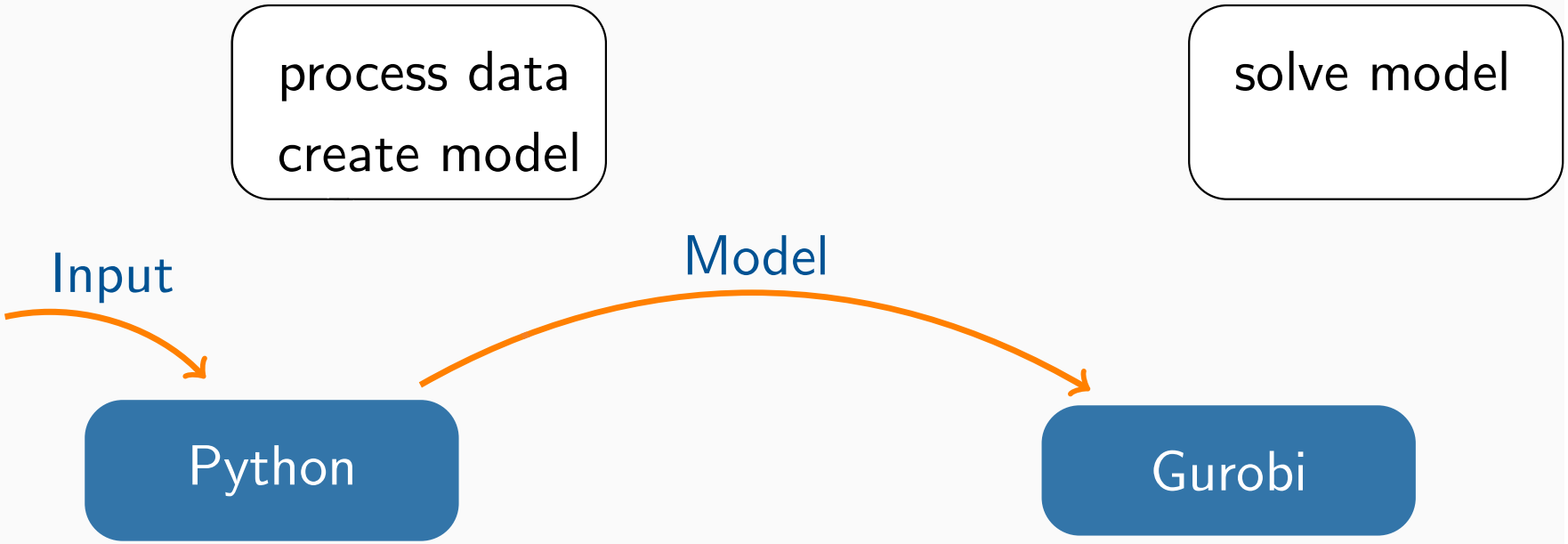

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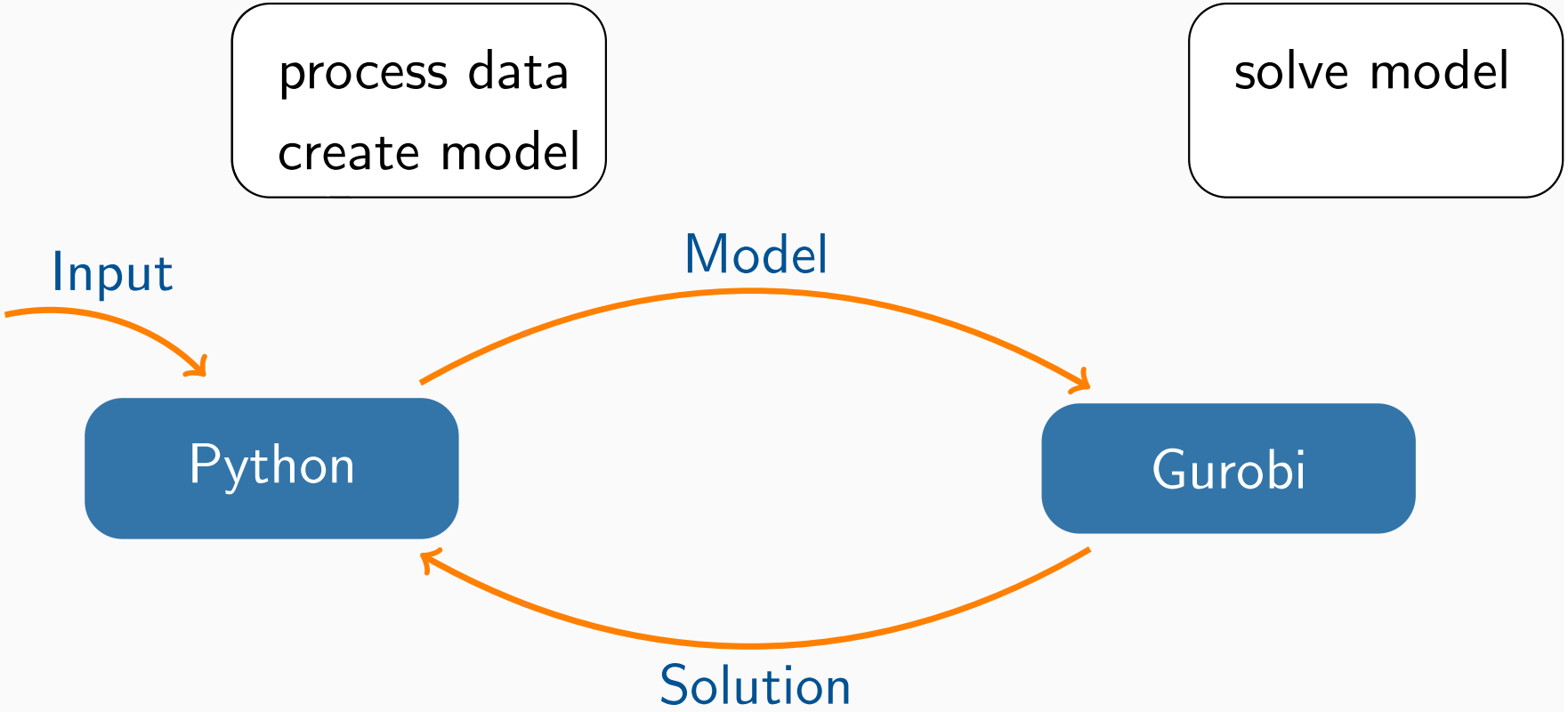

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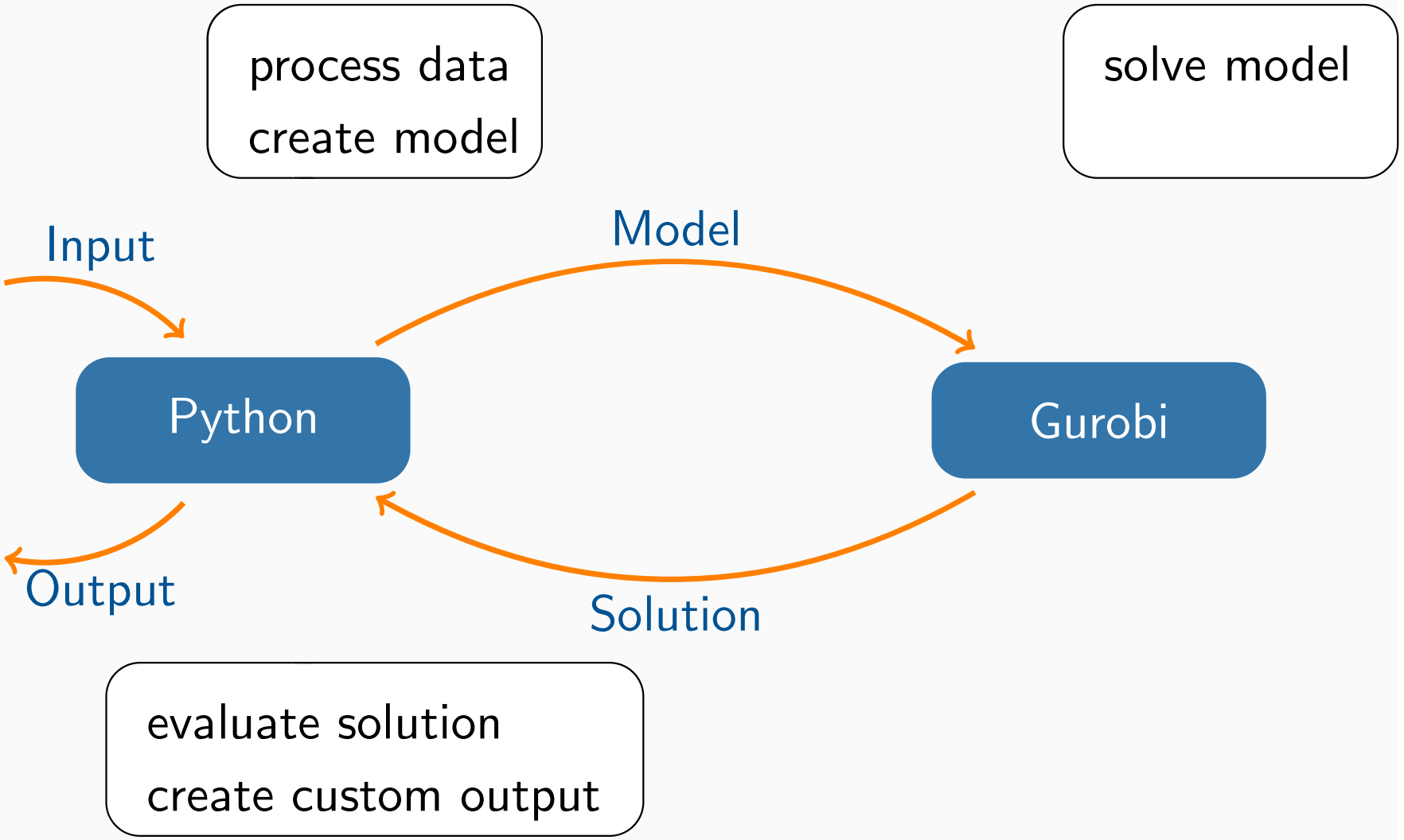

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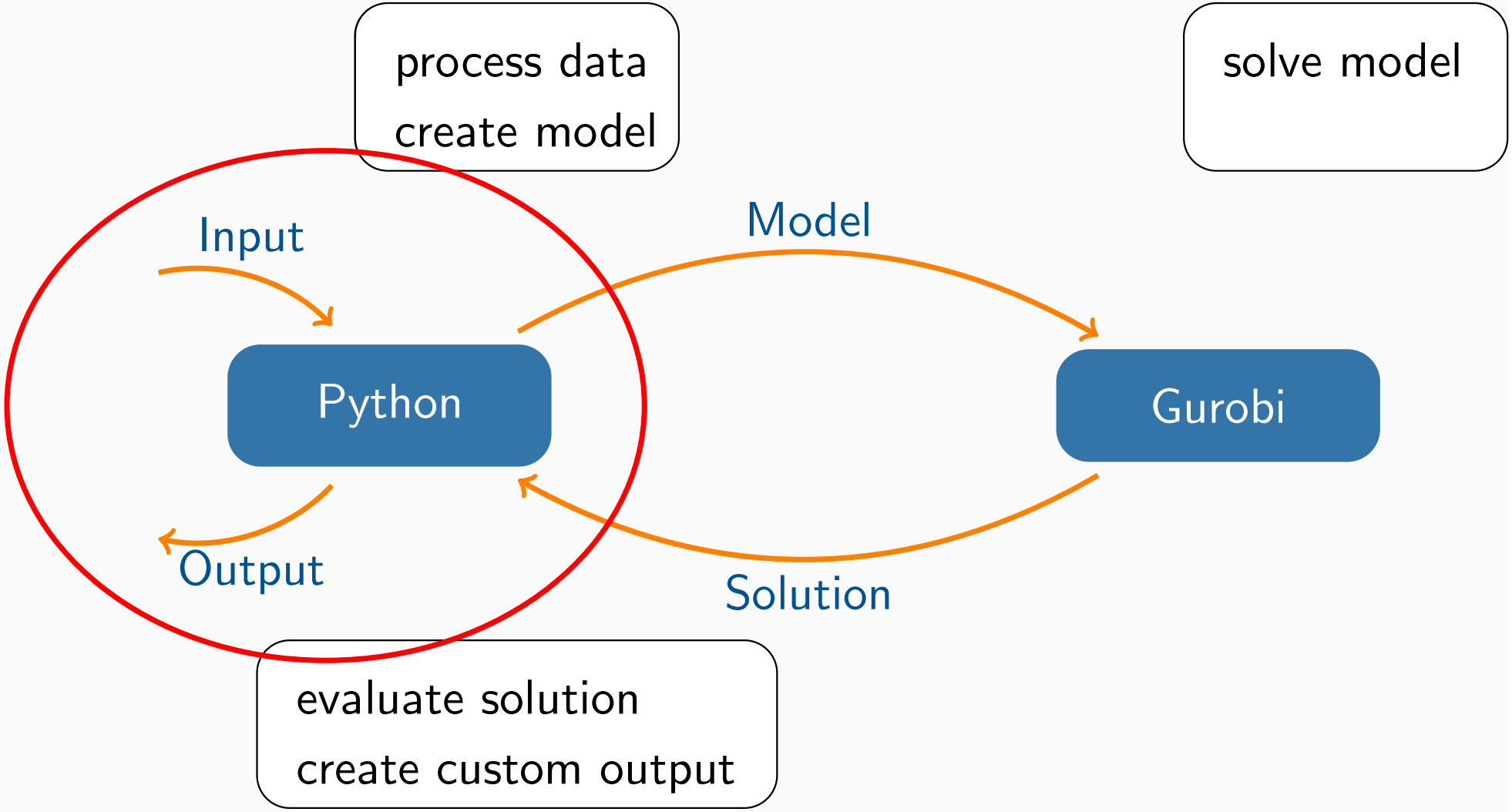

Credits

The materials used in this course have been developed and improved by

- Melanie Herzog
- Anja Kirschbaum
- Fabian Klemm
- Michael Ritter
- Matthias Silbernagel
- Paul Stursberg
- Stefan Kober

Basics

Python

- open source
- most popular programming language
- object-oriented, procedural, functional
- interactive
- easy to learn

Advantages

- high-level
  - direct interpretation of objects
  - readable and accessible
- many useful libraries (graphs, visualization, computations, data management,...)

Limits

- slow running times
- somewhat restricted
- possibly not best choice for large object oriented project

Basic Knowledge

- Datatypes
  - integer, float, string
  - list, tuple, dict, set
- Indentation
- Output
  - print
  - formatted print
- Imports

Linear Programming

What is a linear program?

$$\min c^\top x \quad \text{s.t.}$$
$$Ax \leq b$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 7 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \quad c = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

What is a linear program?

$$\min c^\top x \quad \text{s.t.}$$
$$Ax \leq b$$

- set of variables x
- set of linear constraints $Ax \leq b$
- linear objective function $\min c^\top x$

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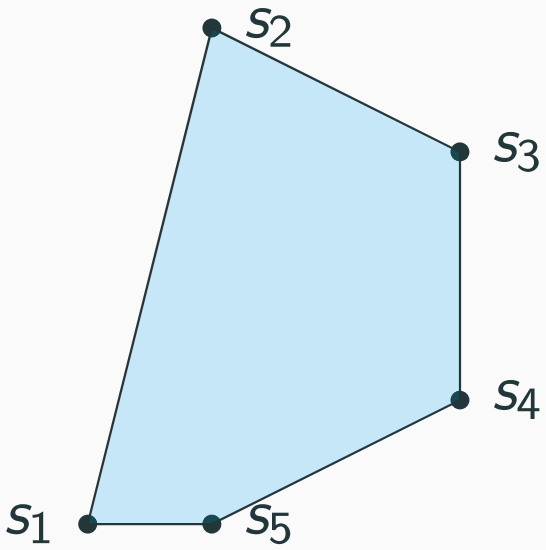

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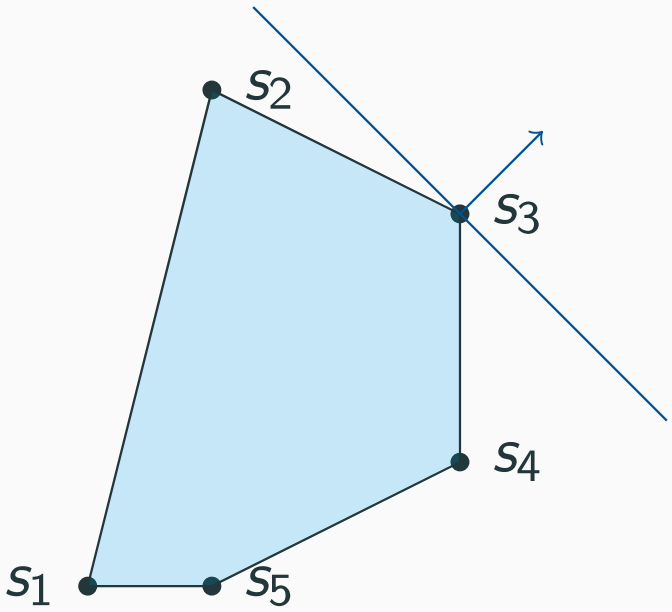
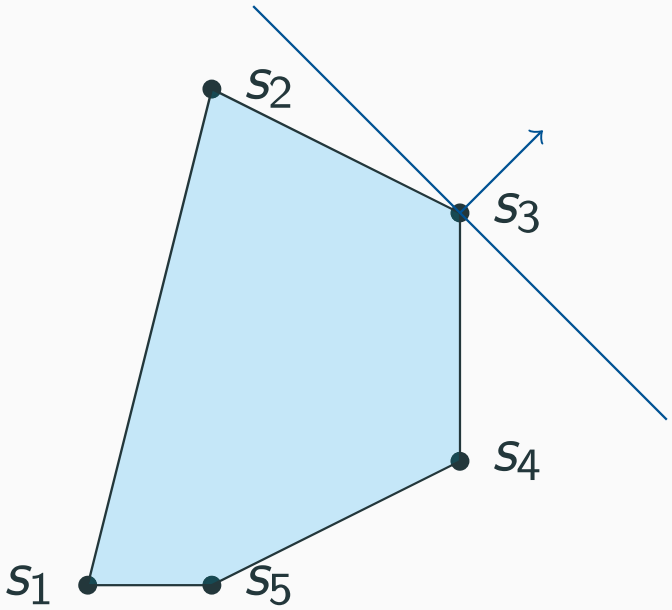


How to solve a linear program?

The Simplex Algorithm

- Find a feasible solution
- Travel along improving edges
- Terminate at optimal solution



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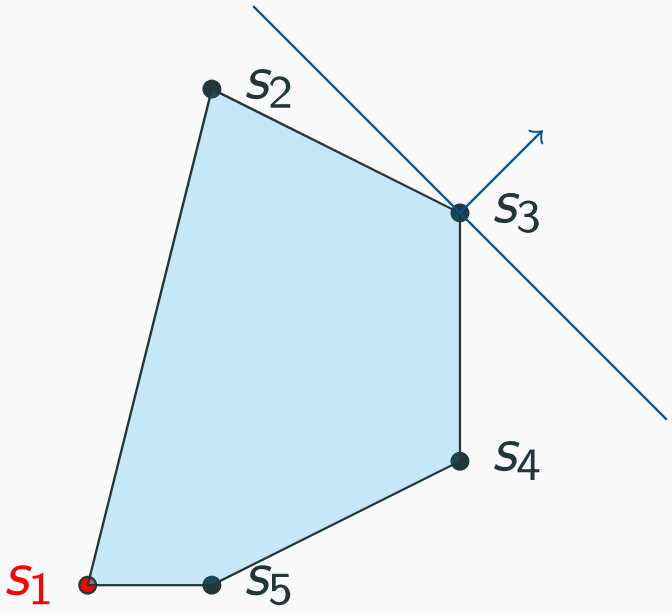

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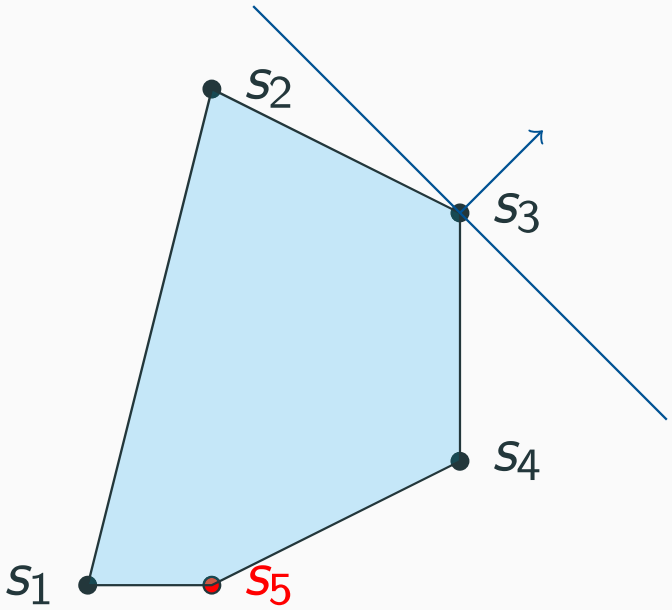

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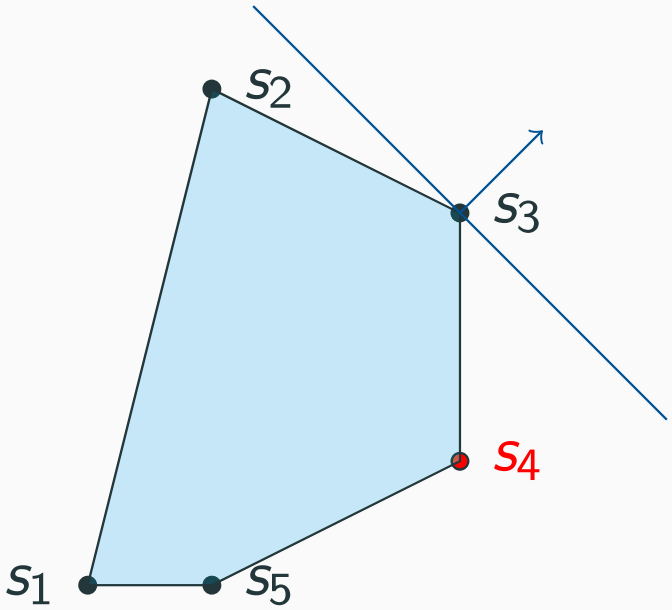

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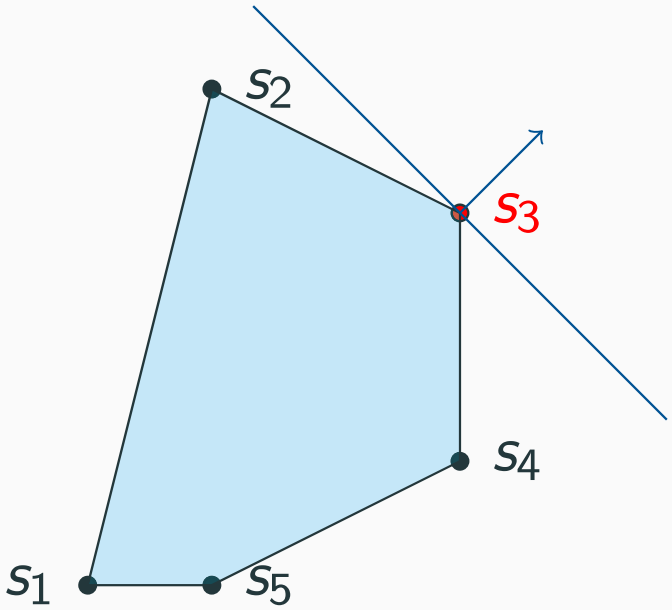

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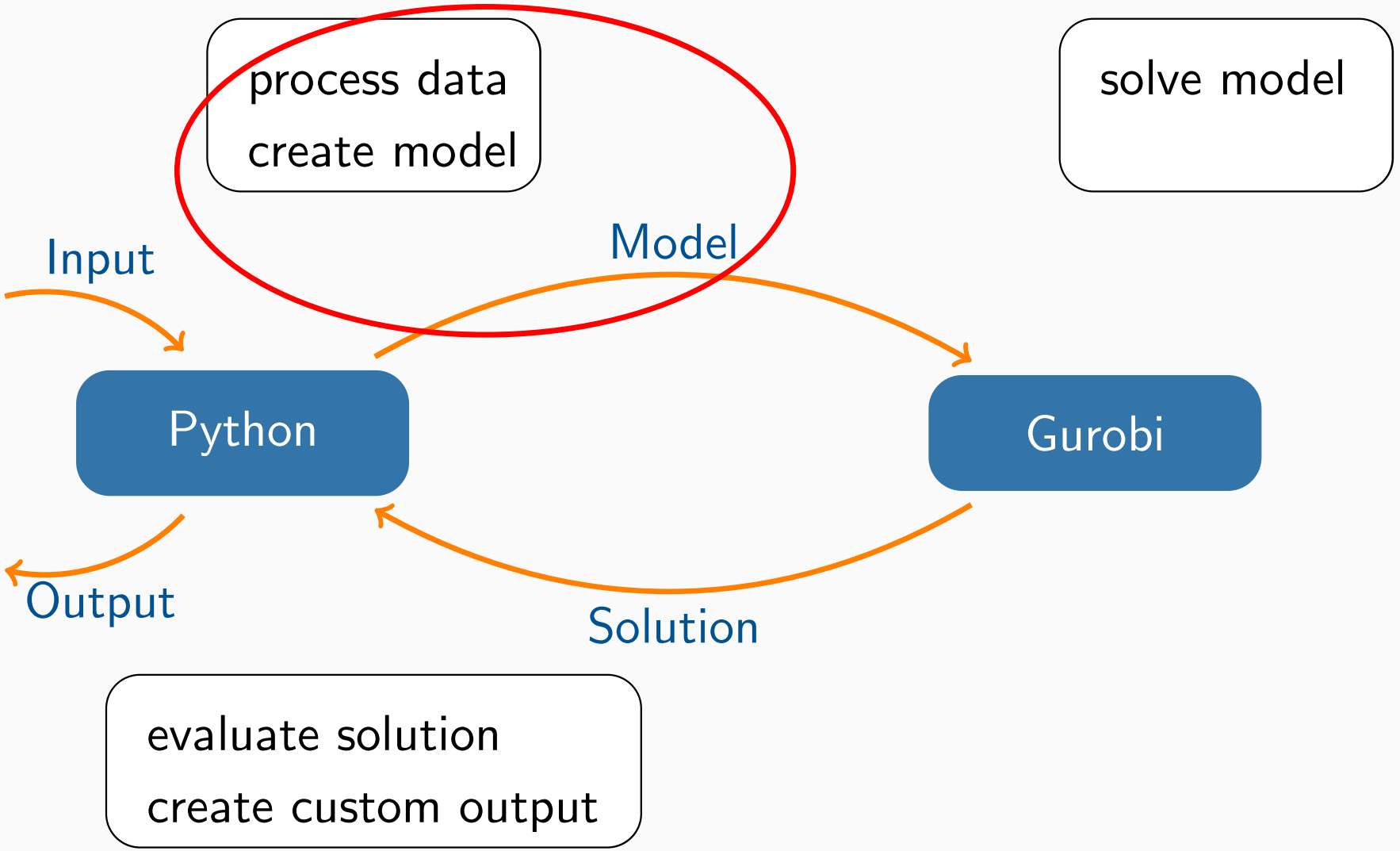
Good News:
Gurobi does that for us

Modelling

Modelling


process data
create model
solve model
Input
Model
Python
Gurobi
Output
Solution
evaluate solution
create custom output

Problem: Crude Oil Refinement 1

10l crude oil

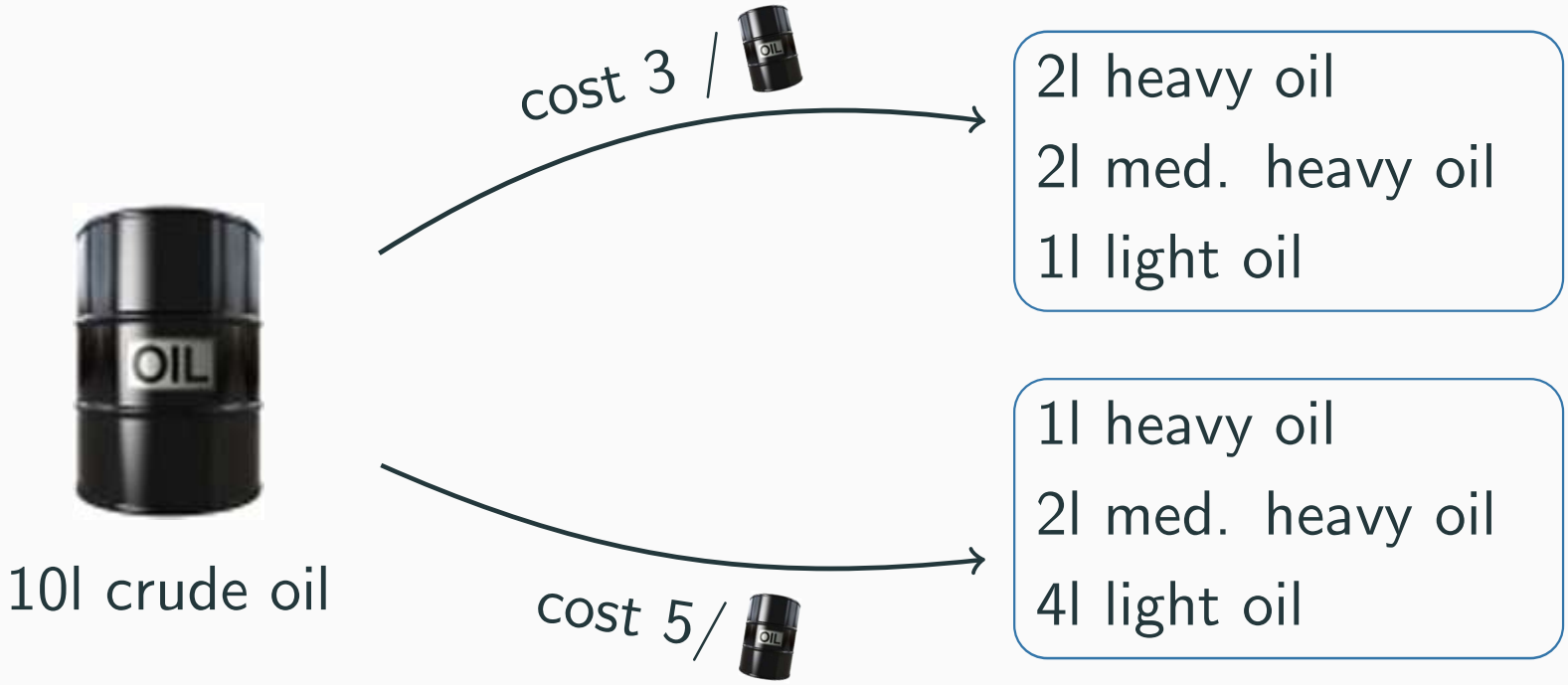
Problem: Crude Oil Refinement 1

10l crude oil

cost 3 /

2l heavy oil
2l med. heavy oil
1l light oil

Problem: Crude Oil Refinement 1



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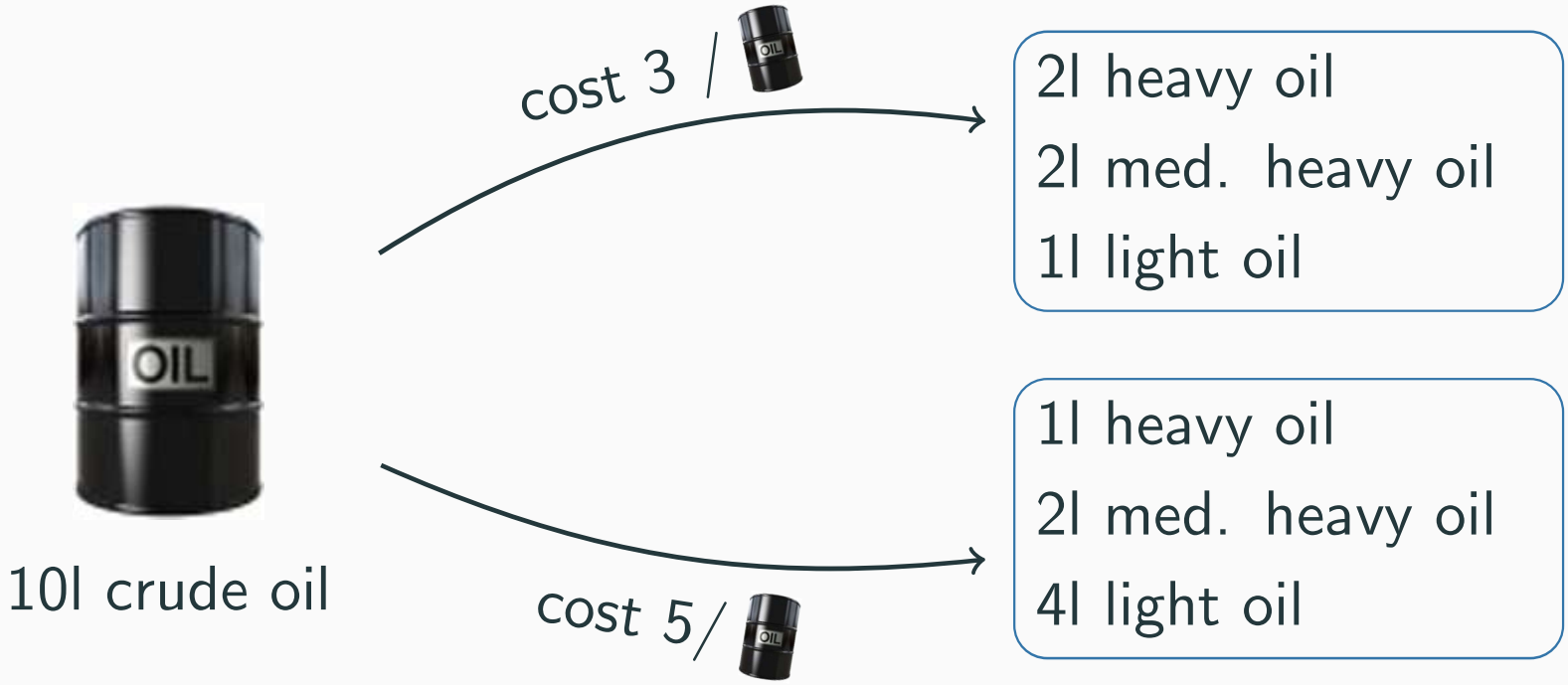

demand: 3l heavy oil, 5l med. heavy oil, 4l light oil

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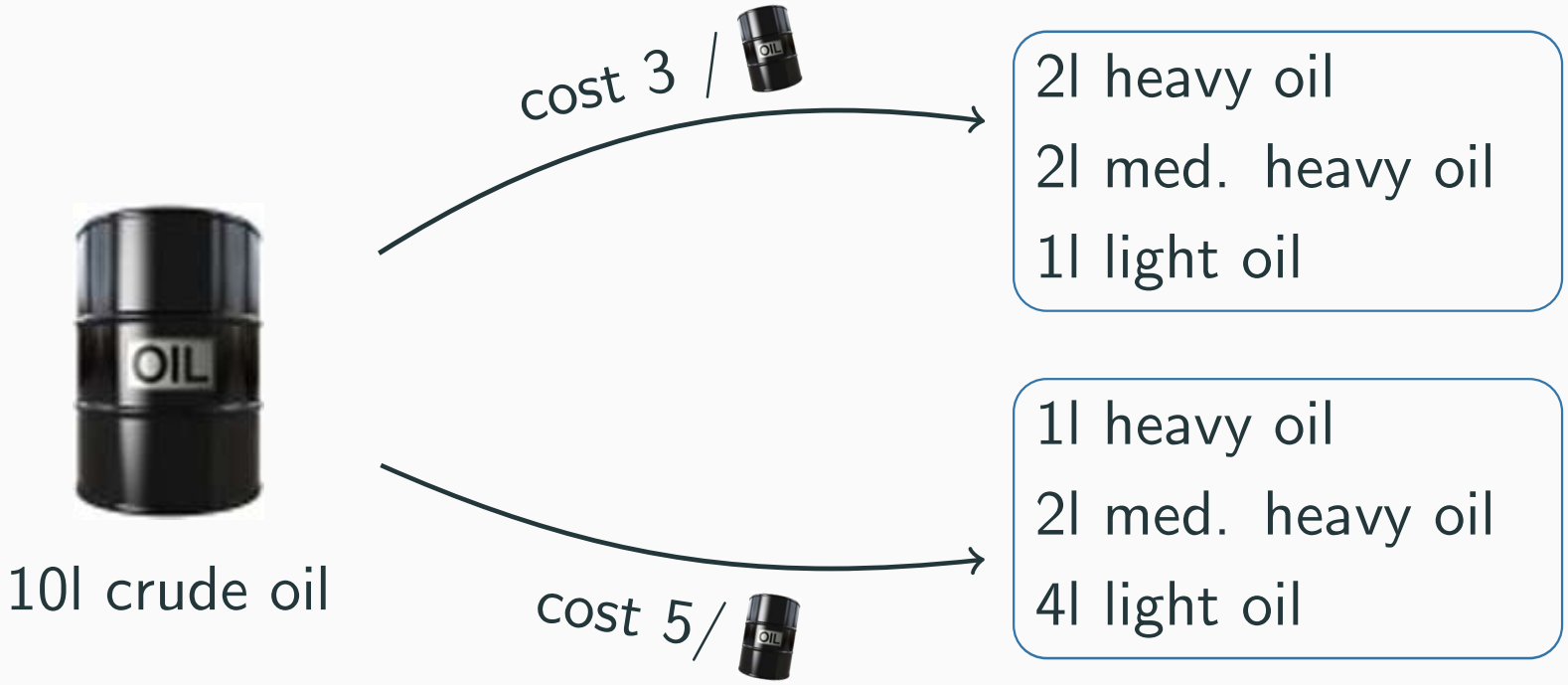

demand: 3l heavy oil, 5l med. heavy oil, 4l light oil

objective: minimize cost

LP Model Problem 1

$$\min 3x_1 + 5x_2$$

s.t.

$$2x_1 + 1x_2 \geq 3$$
$$2x_1 + 2x_2 \geq 5$$
$$1x_1 + 4x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

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LP Model Problem 1

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s.t.

$$\begin{aligned} 2x_1 + 1x_2 &\geq 3 \\ 2x_1 + 2x_2 &\geq 5 \\ 1x_1 + 4x_2 &\geq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Initialize gurobipy and create set of variables x

```
from gurobipy import *

# Create a new model
m = Model()

# Create variables
x = m.addVar(vtype=GRB.CONTINUOUS)
y = m.addVar(vtype=GRB.CONTINUOUS)
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Variable types

- GRB.CONTINUOUS $(-\infty, \infty)$
- GRB.BINARY $\{0, 1\}$
- GRB.INTEGER $\{0, 1, 2, \ldots\}$
- GRB.SEMICONT $\{0\} \cup (a, b)$
- GRB.SEMIINT $\{0\} \cup (a, b) \cap \mathbb{Z}$

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```

Add Variables

```
addVar(lb=0,ub=GRB.INFINITY,obj=0.0,
       vtype=GRB.CONTINUOUS,name="")
```

- *lb*, *ub*: variable lower and upper bound
- *obj*: coefficient of the linear objective function
- *vtype*: variable type
- *name*: name for further referencing

Add Variables

```
addVars(indices, lb=0, ub=GRB.INFINITY, obj=0.0,
        vtype=GRB.CONTINUOUS, name="")
```

- *lb*, *ub*: variable lower and upper bound
- *obj*: coefficient of the linear objective function
- *vtype*: variable type
- *name*: name for further referencing
- *indices*: integer, range, list or dictionary used to generate set of variables

Create set of linear constraints $Ax \geq b$

```
# Add constraints
c1 = m.addConstr(2*x+y>=3)
c2 = m.addConstr(2*x+2*y>=5)
c3 = m.addConstr(x+4*y>=4)
c4 = m.addConstr(x>=0)
c5 = m.addConstr(y>=0)
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Add Constraints

Basic form:

```
m.addConstr(LinExpr>=a)
```

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```
m.addConstr(LinExpr>=a)
```

Linear expressions can be created by:

- $le = 2 * x + 3 * y$
- $le = x.prod([2, 3])$
- $le = x.sum()$
- $le = quicksum([2 * x, 3 * y])$

Set linear objective function $\min c^\top x$ and optimize the model

```
# Set objective function
m.setObjective(3*x+5*y,GRB.MINIMIZE)

# Optimize model
m.optimize()
```