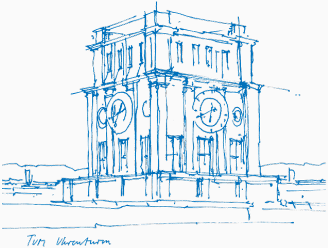
Computer Course Linear Programming

Introduction to Gurobipy



Stefan Kober

28-29 October 2020

Technical University of Munich

Organizational Things

What to expect

What this course offers:

- praxis-oriented introduction to python and gurobipy
- lots of examples
- preparation for further lectures, case studies and theses

What this course does not offer:

- detailed installation instructions
- the time needed to become an expert in python and gurobipy

Schedule

- Wednesday:
  - Introduction to Python
  - Introduction to Gurobi
- Thursday:
  - Features Python (advanced input and output methods)
  - Features Gurobi (advanced variable types and output interpretation)

Schedule

10:15	first slot
11:45	lunch break
13:15	second slot
14:45	coffee break
15:15	third slot

Work in teams!

Outlook

Structure of Gurobi

Basics

Linear Programming

Modelling

Output Interpretation

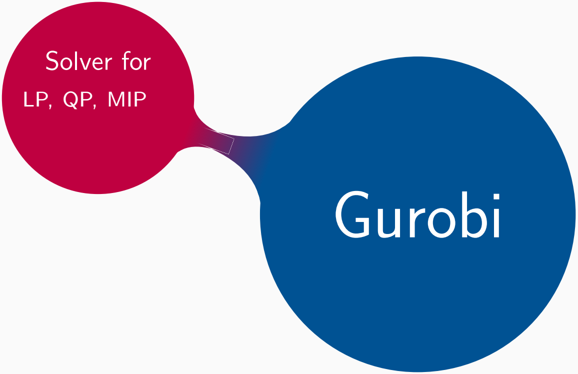
Advanced Input Methods

Advanced Gurobi Datatypes

Visualization

Structure of Gurobi

What is Gurobi?



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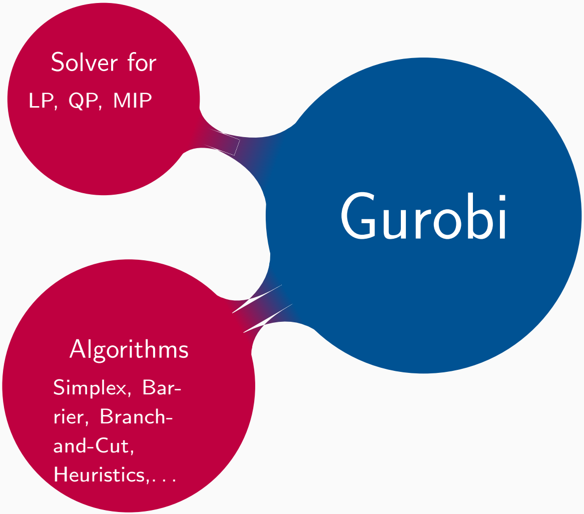

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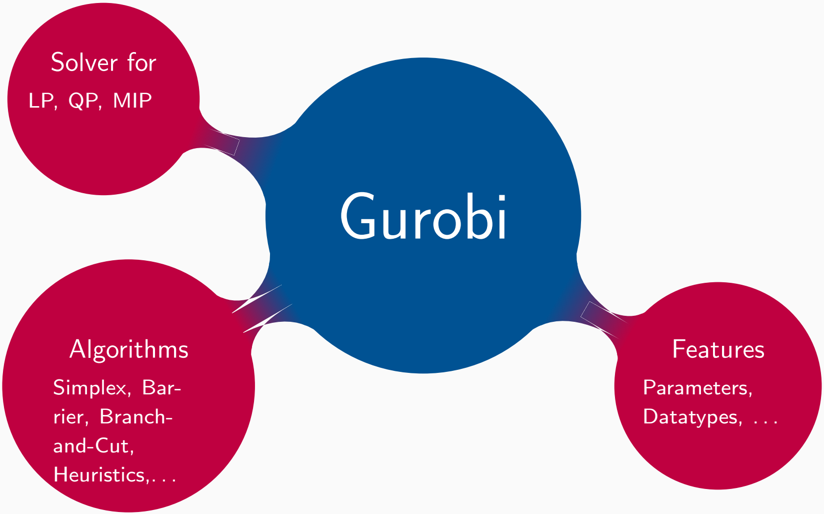

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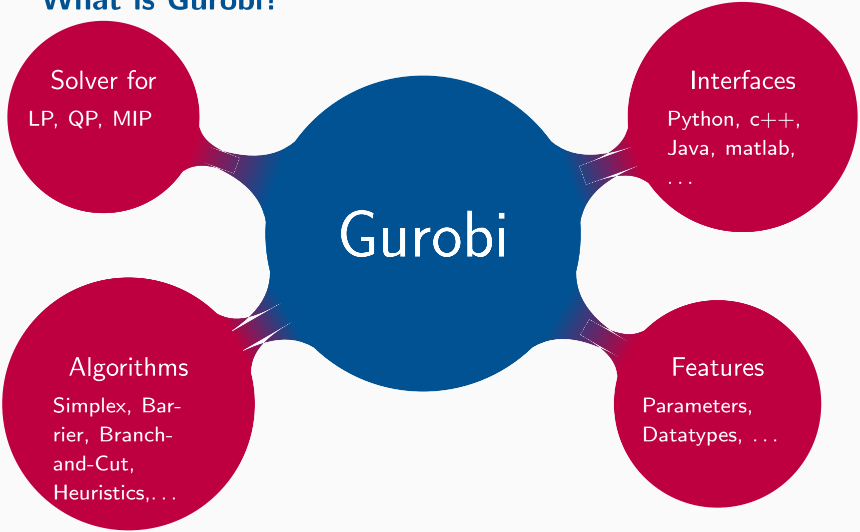
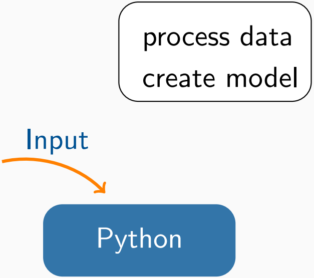


How can we use Gurobi?

Input

Python

How can we use Gurobi?



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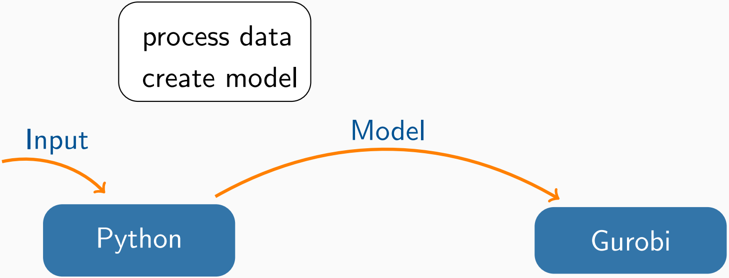

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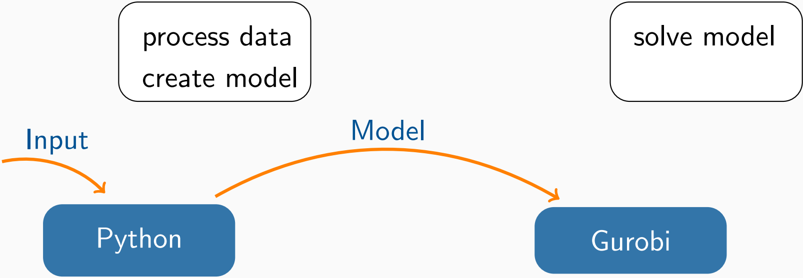

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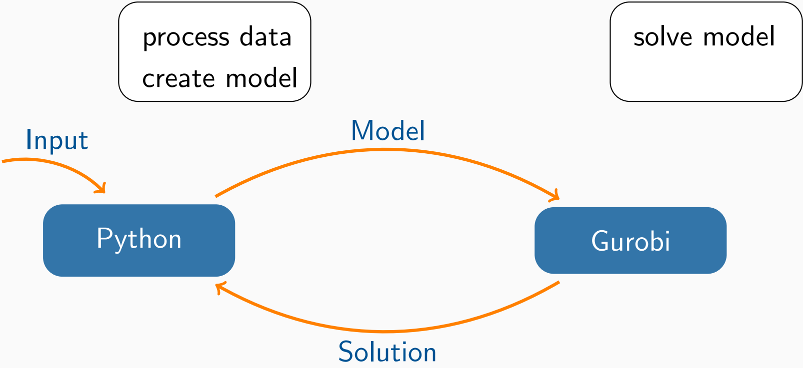

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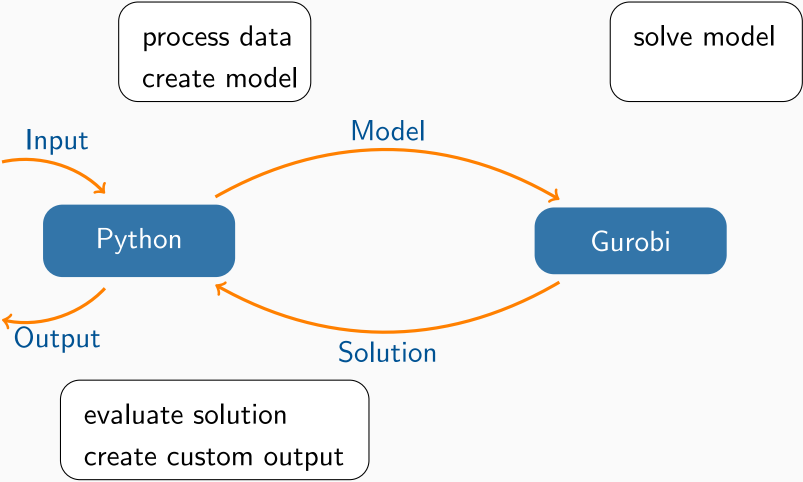

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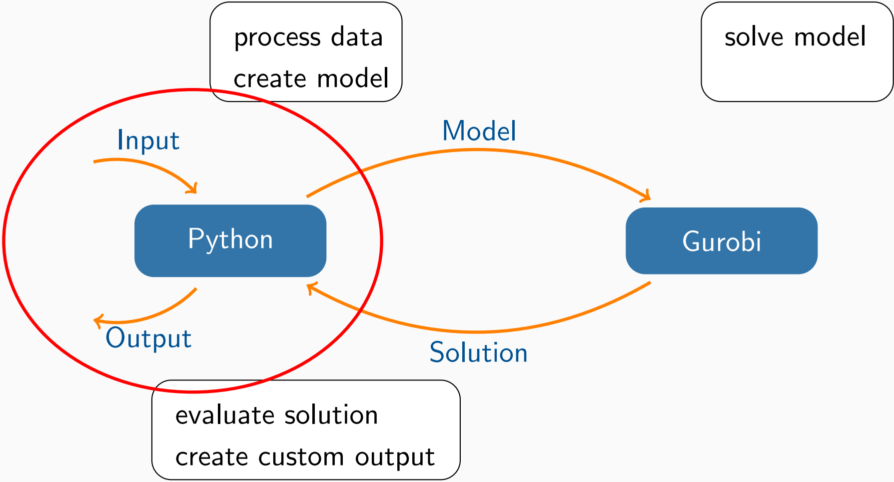


Credits

The materials used in this course have been developed and improved by

- Melanie Herzog
- Anja Kirschbaum
- Fabian Klemm
- Michael Ritter
- Matthias Silbernagel
- Paul Stursberg
- Stefan Kober

Basics

Python

- open source
- most popular programming language
- object-oriented, procedural, functional
- interactive
- easy to learn

Advantages

- high-level
  - direct interpretation of objects
  - readable and accessible
- many useful libraries (graphs, visualization, computations, data management,...)

Limits

- slow running times
- somewhat restricted
- possibly not best choice for large object oriented project

Basic Knowledge

- Datatypes
  - integer, float, string
  - list, tuple, dict, set
- Indentation
- Output
  - print
  - formatted print
- Imports

Linear Programming

What is a linear program?

$$\begin{aligned} \min c^\top x \quad & \text{s.t.} \\ Ax \leq b \end{aligned}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 7 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \quad c = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

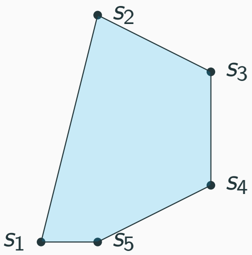
What is a linear program?

$$\begin{aligned} \min c^\top x \quad \text{s.t.} \\ Ax \leq b \end{aligned}$$

- set of variables x
- set of linear constraints $Ax \leq b$
- linear objective function $\min c^\top x$

What is a linear program?

$$\min c^\top x \quad \text{s.t.}$$
$$Ax \leq b$$



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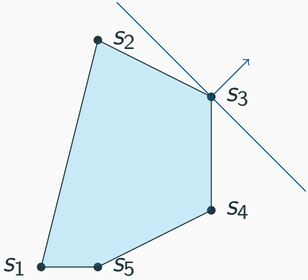
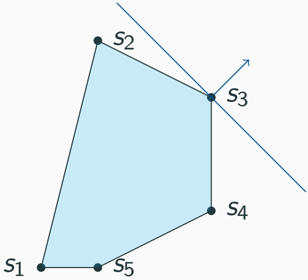


How to solve a linear program?

The Simplex Algorithm

- Find a feasible solution
- Travel along improving edges
- Terminate at optimal solution



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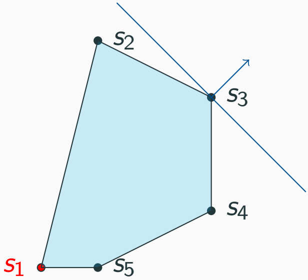

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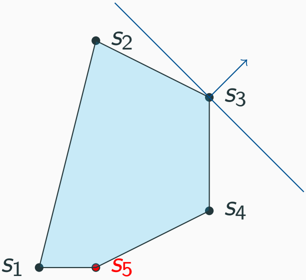

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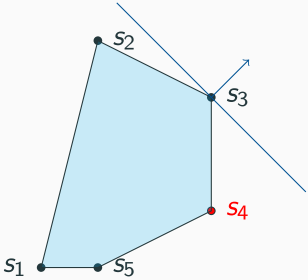

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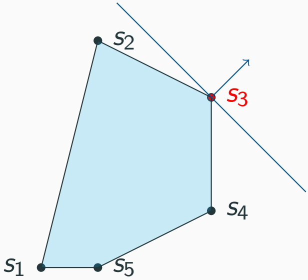

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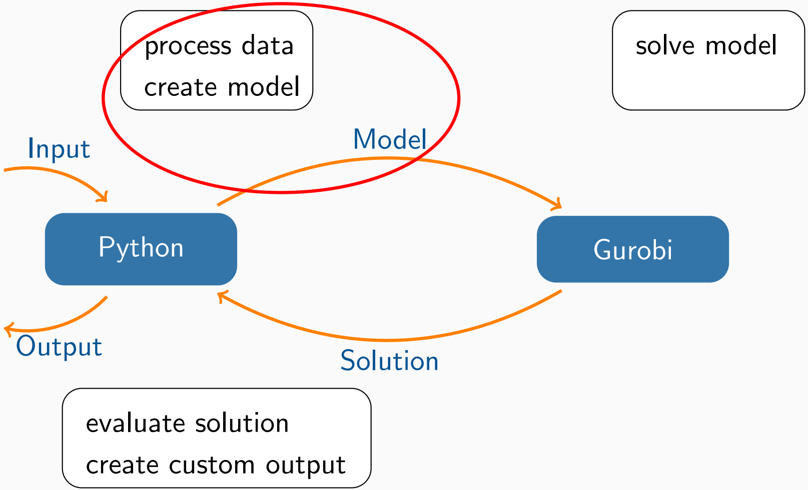
Good News:
Gurobi does that for us

Modelling

Modelling


process data
create model
solve model
Input
Model
Python
Gurobi
Output
Solution
evaluate solution
create custom output

Problem: Crude Oil Refinement 1

10l crude oil

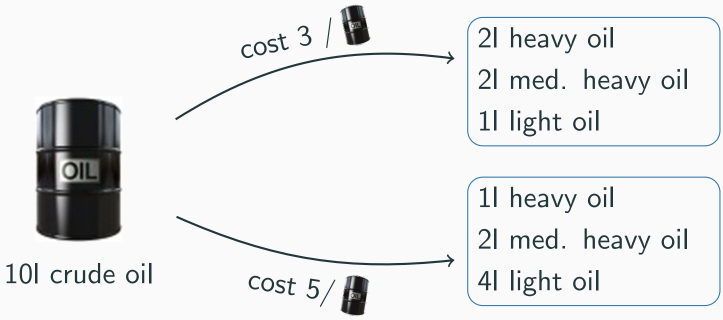
Problem: Crude Oil Refinement 1

cost 3 / 

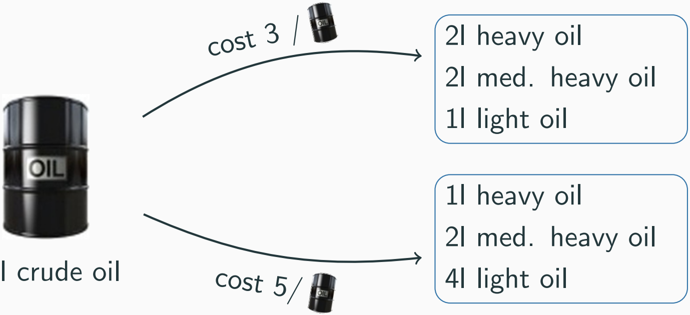
2l heavy oil
2l med. heavy oil
1l light oil

10l crude oil

Problem: Crude Oil Refinement 1

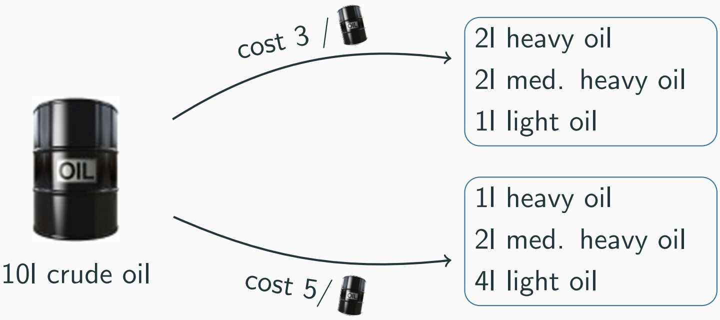


Problem: Crude Oil Refinement 1



demand: 3l heavy oil, 5l med. heavy oil, 4l light oil

Problem: Crude Oil Refinement 1



demand: 3l heavy oil, 5l med. heavy oil, 4l light oil

objective: minimize cost

LP Model Problem 1

$$\min 3x_1 + 5x_2$$

s.t.

$$\begin{aligned} 2x_1 + 1x_2 &\geq 3 \\ 2x_1 + 2x_2 &\geq 5 \\ 1x_1 + 4x_2 &\geq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

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Initialize gurobipy and create set of variables x

```
from gurobipy import *

# Create a new model
m = Model()

# Create variables
x = m.addVar(vtype=GRB.CONTINUOUS)
y = m.addVar(vtype=GRB.CONTINUOUS)
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```

Variable types

- GRB.CONTINUOUS $(-\infty, \infty)$
- GRB.BINARY $\{0, 1\}$
- GRB.INTEGER $\{0, 1, 2, \ldots\}$
- GRB.SEMICONT $\{0\} \cup (a, b)$
- GRB.SEMIINT $\{0\} \cup (a, b) \cap \mathbb{Z}$

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```

Add Variables

```
addVar(lb=0,ub=GRB.INFINITY,obj=0.0,
          vtype=GRB.CONTINUOUS,name="")
```

- *lb*, *ub*: variable lower and upper bound
- *obj*: coefficient of the linear objective function
- *vtype*: variable type
- *name*: name for further referencing

Add Variables

```
addVars(indices,lb=0,ub=GRB.INFINITY,obj=0.0,
        vtype=GRB.CONTINUOUS,name="")
```

- *lb*, *ub*: variable lower and upper bound
- *obj*: coefficient of the linear objective function
- *vtype*: variable type
- *name*: name for further referencing
- *indices*: integer, range, list or dictionary used to generate set of variables

Create set of linear constraints $Ax \geq b$

```
# Add constraints
c1 = m.addConstr(2*x+y>=3)
c2 = m.addConstr(2*x+2*y>=5)
c3 = m.addConstr(x+4*y>=4)
c4 = m.addConstr(x>=0)
c5 = m.addConstr(y>=0)
```

Create set of linear constraints $Ax \geq b$

```
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```

Add Constraints

Basic form:

```
m.addConstr(LinExpr>=a)
```

Add Constraints

Basic form:

```
m.addConstr(LinExpr>=a)
```

Linear expressions can be created by:

- $le = 2 * x + 3 * y$
- $le = x.prod([2, 3])$
- $le = x.sum()$
- $le = quicksum([2 * x, 3 * y])$

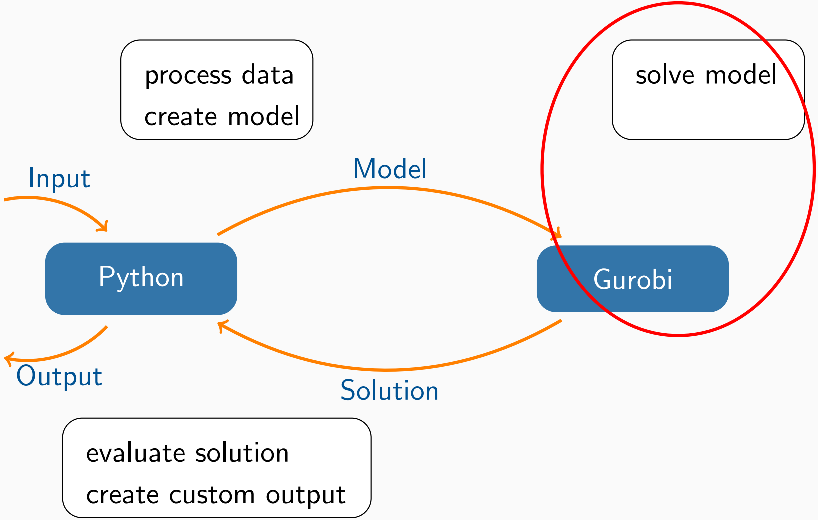
Set linear objective function $\min c^\top x$ and optimize the model

```
# Set objective function
m.setObjective(3*x+5*y,GRB.MINIMIZE)

# Optimize model
m.optimize()
```

Output Interpretation

Gurobi - LP solver



Presolve

```
Optimize a model with 1500 rows,
   2250000 columns and 4497000 nonzeros
Coefficient statistics:
  Matrix range      [1e+00, 1e+00]
  Objective range   [1e+00, 1e+02]
  Bounds range      [0e+00, 0e+00]
  RHS range         [1e+00, 2e+01]
Presolve removed 0 rows and 1611 columns
Presolve time: 4.37s
Presolved: 1500 rows, 2248389 columns, 4496778 nonzeros
```

Presolve - Example

$$
\begin{aligned}
x_1 + x_2 + x_3 &\geq 15 \\
x_1 &\leq 7 \\
x_2 &\leq 3 \\
x_3 &\leq 5
\end{aligned}
$$

Presolve - Example

$$
\begin{aligned}
x_1 + x_2 + x_3 &\geq 15 \\
x_1 &\leq 7 \\
x_2 &\leq 3 \\
x_3 &\leq 5
\end{aligned}
$$

delete constraint and variables in order to reduce the problem

LP methods

- 3 different methods
  - primal/dual simplex
    - robust
    - easy to restart after model modification
  - barrier
    - can be run on multiple cores
- concurrent optimization

Concurrent Optimization

- run simplex *and* barrier at the same time
- first one to finish reports solution
- use multiple cores
- fastest choice for general model

Primal Simplex

Iteration	Objective	Primal Inf.	Dual Inf.	Time
0	-1.2573140e+02	2.968000e+03	5.593346e+11	7s
3297	1.2166285e+05	0.000000e+00	6.532640e+07	10s
4619	8.3232592e+04	0.000000e+00	1.162234e+08	15s
7000	5.2154173e+04	0.000000e+00	4.234078e+06	25s
9189	3.7601369e+04	0.000000e+00	1.177763e+07	35s
10706	3.0986489e+04	0.000000e+00	2.645246e+06	45s
12019	2.7488411e+04	0.000000e+00	4.758955e+06	56s
13844	2.3646778e+04	0.000000e+00	3.840044e+05	65s
15403	2.1713789e+04	0.000000e+00	3.055039e+04	75s
17347	2.1105977e+04	0.000000e+00	1.777696e+01	85s
17419	2.1108270e+04	0.000000e+00	0.000000e+00	91s

Solved in 17419 iterations and 90.82 seconds
Optimal objective 2.110826998e+04

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Primal Dual Relations

Primal:

$$\begin{aligned} \min c^\top x \quad & \text{s.t.} \\ Ax \leq b & \end{aligned}$$

Dual:

$$\begin{aligned} \max b^\top y \quad & \text{s.t.} \\ A^\top y = c & \\ y \geq 0 & \end{aligned}$$

Primal Dual Relations

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$$\min c^\top x \quad \text{s.t.}$$
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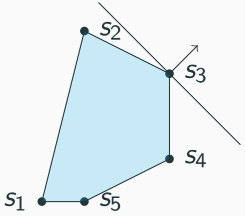

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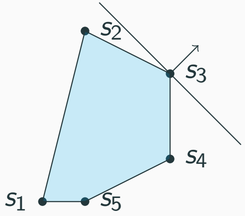

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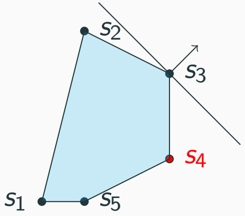

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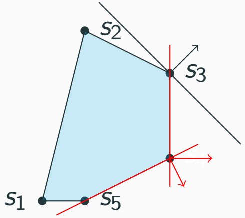

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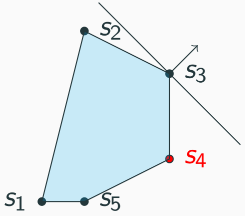

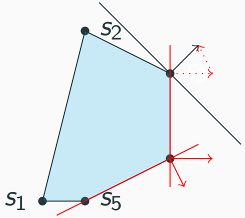

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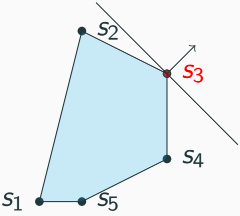

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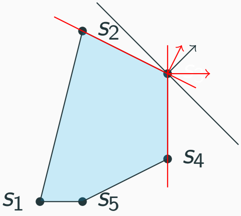

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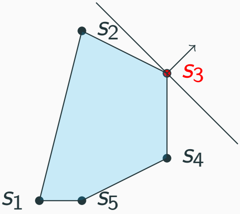

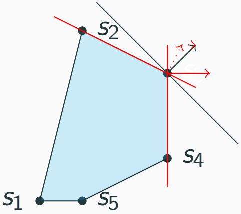

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$$c^\top \bar{x} = \bar{y}^\top A\bar{x} =^{(*)} \bar{y}^\top b$$

Dual Simplex

- simplex on dual problem
- obtain primal solution as discussed before
- dominant algorithm used in MIP solving

Barrier Method

Idea:

$$\min c^\top x \quad \text{s.t.}$$
$$Ax \leq b$$

Barrier Method

Idea:

$$\min f(x) \quad \text{s.t.}$$
$$c_i(x) \leq 0$$

with f and c_i linear

Barrier Method

Idea:

$$\min f(x) \quad \text{s.t.}$$
$$c_i(x) \leq 0$$

with f and c_i linear

$$\Downarrow$$

$$\min f(x) - \mu \sum_i log(c_i(x))$$

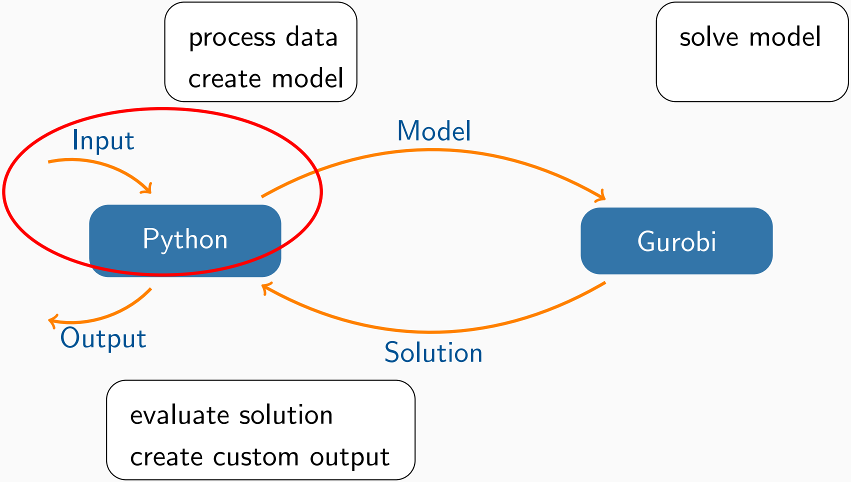
Barrier Method

$$\min f(x) - \mu \sum_i log(c_i(x))$$

- $\mu \to 0$ and solve as nonlinear optimization problem
- also known as Interior Point Method
- few, but expensive calculations
- not clear whether *warm start* is doable

Advanced Input Methods

Advanced Input



Reading Data from Files

Advantages:

- separation between data and code
- faster replacement and sharing of data
- more flexible code
- clearer code

File Formats

- excel (pandas, xlrd)
- csv (csv)
- json (json)
- basically any text-file of some custom format (write parser)

The JSON File Format


```
{
    "oil_types":[
        "heavy","medium","light"
    ],
    "processes":[0,1],
    "production":{
        "heavy":[2,1],"medium":[2,2],"light":[1,4]
    },
    "demand":{
        "heavy":3,"medium":5,"light":4
    },
    "process_cost":[3,5]
}
```

Data Types

- Boolean
- Number (integer or floating point)
- String
- Array [] (ordered list of elements of arbitrary type)
- Object {} (unordered collection of name-value pairs)

Data Types

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Translates straight-forward to Python!

Reading JSON files in Python

```
import json

with open("data.json") as json_file:
data = json.load(json_file)
```

Read text files

- generally, input files not given as *json* or *csv* or *xslx*
- any custom format
- solution: read file line by line according to its syntax and create list/dictionary/object

Reading general textfiles in Python

```
filename = 'data.txt'
with open(filename, "r") as file:
    line = file.readline()
    while line:
        print(line.split("separator"))
```

Write text files

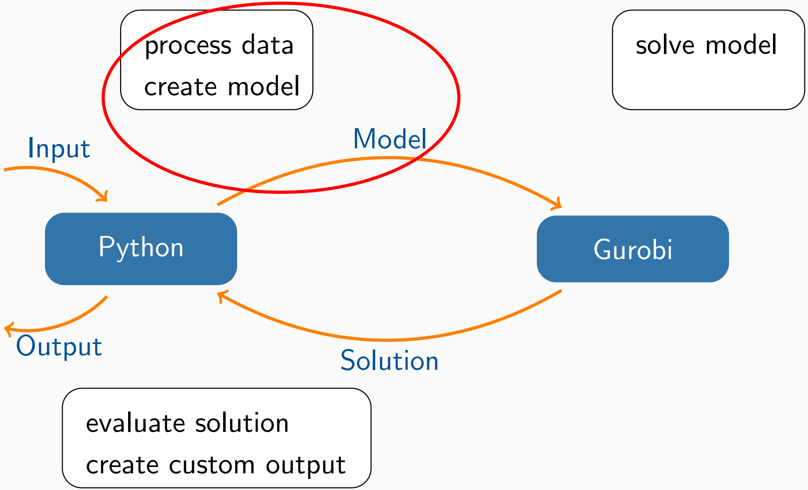
- generate different datasets to have stable collection of example data
- straightforward in python (very similar to *print*())

Writing textfiles in Python

```
filename = 'data.txt'
with open(filename, "w") as file:
    file.write("sometext\n")
    file.close()
```

Advanced Gurobi Datatypes

Advanced Datatypes



Tuplelist

- subclass of python list
- list of tuples of same size
- easy notation to find specific subsets
- same is doable with list comprehension *but* tuplelist is faster

Tuplelist

Creation
`l = tuplelist([(1, 2), (1, 3), (2, 3), (2, 4)])`

Tuplelist

Creation

```
l = tuplelist([(1, 2), (1, 3), (2, 3), (2, 4)])
```

Queries ('∗' is wildcard character)

```
l.select(1, '*')
l.select('*', [2, 4])
l.select('*', '*')
```

Tupledict

- subclass of python dictionary
- dictionary with tuplelist as keys
- usually used for variables of complex systems
- easy access via *select*
- easy constraint generation via *sum* and *prod*

Tupledict

Creation

```
l = tuplelist([(1, 2), (1, 3), (2, 3), (2, 4)])
d=model.addVars(l)
```

Tupledict

Creation

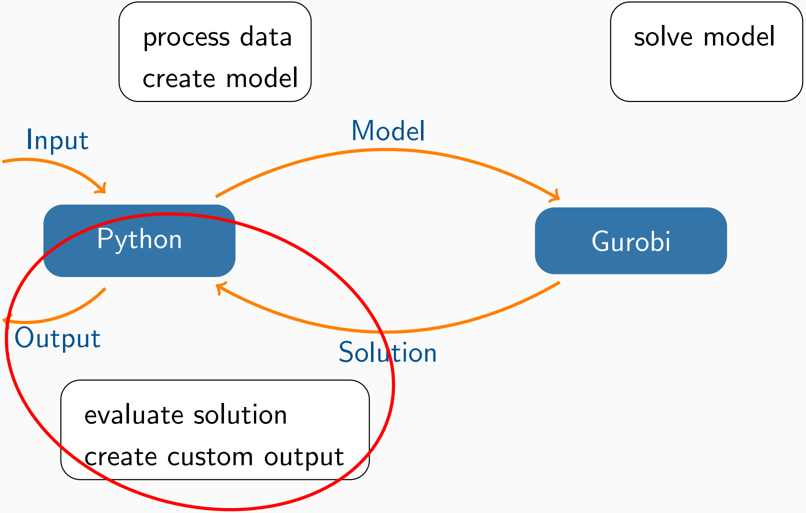
```
l = tuplelist([(1, 2), (1, 3), (2, 3), (2, 4)])
d=model.addVars(l)
```

Queries and creation of expressions

```
d.select(1, '*')
d.sum(1, '*')
coeff=[2,5]
d.prod(coeff, 1, '*')
```

Visualization

Result Visualization



Visualization of Graphs

- natural connection between many LPs and graphs
- visualizing graphs improves understanding
- matplotlib

Visualizing graphs

```
import networkx as nx
from random import randint
n = 10
position = [[randint(0,100),randint(0,100)]
    for i in range(n)]
edges = [(randint(0,n-1),randint(0,n-1))
    for i in range(3*n)]
G = nx.Graph()
G.add_nodes_from([(i, {'x': coord[0], 'y': coord[1]})
    for i, coord in enumerate(position)])
G.add_edges_from([(e[0], e[1]) for e in edges])
nx.draw_networkx_nodes(G, position, node_color="black")
nx.draw_networkx_edges(G, position,edge_color="black")
```