

Grammar

Non-terminals:

```
Expr → nr | Expr iop Expr | var | ( E )
Cond → bl | Expr bop Expr
Stmt → ( ) | Stmt ; Stmt | var = Expr
      | if Cond then Body else Body
      | while Cond do Body
Body → Stmt
      | { Stmt }
```

Terminals:

```
iop → + | - | * | /
bop → == | != | < | > | <= | >=
var → ⟨ string ⟩
nr  → ⟨ int ⟩ | ⟨ float ⟩
bl  → true | false
```

Rules

Arithmetic expressions (E):

$$\begin{aligned}\langle n\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, n\mathbf{s}, \mathbf{m} \rangle \\ \langle v\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, m(v), \mathbf{s}, \mathbf{m} \rangle \\ \langle (E_1 \text{ iop } E_2)\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle E_1 \ E_2 \text{ iop } \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle \\ \langle \text{iop } \mathbf{c}, n_2 n_1 \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, n\mathbf{s}, \mathbf{m} \rangle \text{ where } n = n_1 \underline{\text{iop}} n_2\end{aligned}$$

Boolean conditions (C):

$$\begin{aligned}\langle b\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle b, n\mathbf{s}, \mathbf{m} \rangle \\ \langle (E_1 \text{ bop } E_2)\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle E_1 \ E_2 \text{ bop } \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle \\ \langle \text{bop } \mathbf{c}, n_2 n_1 \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, b\mathbf{s}, \mathbf{m} \rangle \text{ where } b = n_1 \underline{\text{bop}} n_2\end{aligned}$$

Statements (S):

$$\begin{aligned}\langle ()\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle \\ \langle (S_1; S_2)\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle S_1 S_2 \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle \\ \langle v = E\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle E \text{ assign } \mathbf{c}, v\mathbf{s}, \mathbf{m} \rangle \\ \langle \text{assign } \mathbf{c}, n\mathbf{v}\mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, \mathbf{s}, \mathbf{m}[v = n] \rangle\end{aligned}$$

Branching (if):

$$\begin{aligned}\langle (if \ C \text{ then } S_t \text{ else } S_f)\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle C \text{ branch } \mathbf{c}, S_t S_f \mathbf{s}, \mathbf{m} \rangle \\ \langle \text{branch } \mathbf{c}, \text{true } S_t S_f \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle S_t \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle \\ \langle \text{branch } \mathbf{c}, \text{false } S_t S_f \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle S_f \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle\end{aligned}$$

Looping ($while$):

$$\begin{aligned}\langle (while \ C \text{ do } S)\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle C \text{ loop } \mathbf{c}, CS\mathbf{s}, \mathbf{m} \rangle \\ \langle \text{loop } \mathbf{c}, \text{false } CS\mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle \mathbf{c}, \mathbf{s}, \mathbf{m} \rangle \\ \langle \text{loop } \mathbf{c}, \text{true } CS\mathbf{s}, \mathbf{m} \rangle &\rightarrow \langle S(while \ C \text{ do } S)\mathbf{c}, \mathbf{s}, \mathbf{m} \rangle\end{aligned}$$