Seismicity Statistics using Extreme Value Theory

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1 Introduction

Probabilistic Seismic Hazard Analysis (PSHA) relies fundamentally on earthquake statistics and on rock and soil mechanics. Earthquake statistics provide the magnitude-recurrence relations. Rock and soil mechanics provide the ground motion response to a given earthquake. Measurements, correlations and modelling rely almost exclusively on events in tectonically active regions, most notably California (Gutenberg and Richter 1944; Knopoff, Kagan, and Knopoff 1982; Kaklamanos, Boore, Thompson, and Campbell 2010). In contrast, the earthquakes occurring on the Australian continent are a prime example of intra-plate seismicity. The processes and drivers of intra-plate seismicity are and remain enigmatic. However, it is clear that these processes are very different from those driving the tectonic earthquakes (Stein 2007). Ground motion predictions are empirically derived equations (for an overview, see Kaklamanos, Boore, Thompson, and Campbell 2010), based on data collected over rock and over siliciclastic sediments, largely from observations in the continental Americas. There have been some attempts to redress this situation for Australian soils, but the models are commonly similar to the American ones ((Lam and Wilson 2008; Leonard, Robinson, Allen, Schneider, Clark, Dhu, and Burbidge 2007)). No implementations have been developed specifically for calcareous sediments. A number of separate but linked investigations are reported upon here with the aim to help and address some of the shortcomings currently plaguing PSHA on the Northwest Shelf.

2 Data Sources

The earthquake data used in this study are taken from the publicly accessible Geoscience Australia earthquake catalogue (available on-line from www.ga.gov.au). The catalogue lists the timing, location, and magnitude of earthquakes in Australia going back to 1888. The vexed issue of completeness has been discussed by Leonard 2008 and also Sagar and Leonard 2007. These authors also discuss issues encountered with the value of magnitude as recorded over the years, and in particular changes to the scales and the way in which these changes have been applied. A subset of this catalogue was defined to include events which have occurred in an area encompassing the Northwest Shelf (16°–23°S, 112°–124°E).

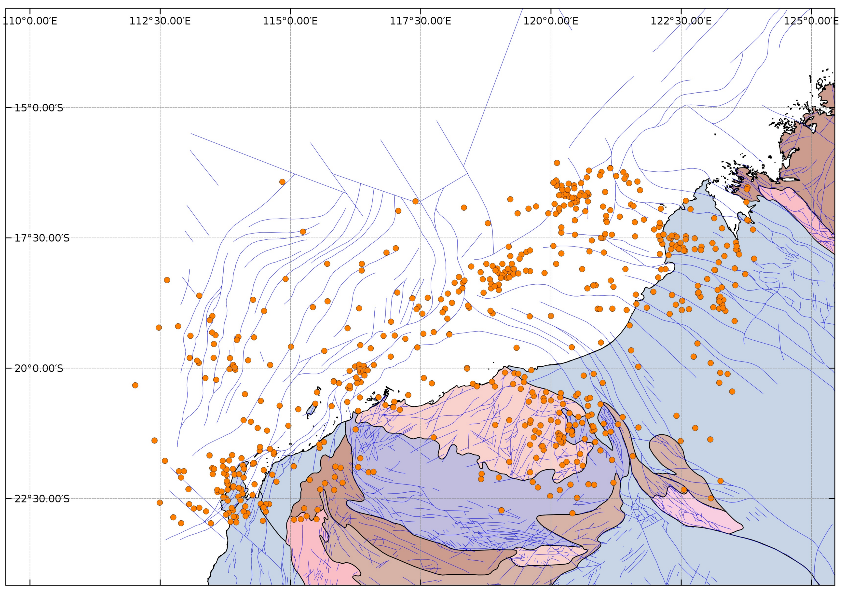


Figure 1: NW Shelf Earthquakes between 1929 and 2017

3 Statistics of Magnitude

One of the important regularities found to govern the complex process of seimic activity is a relationship between size and frequency of earthquakes. The Gutenberg-Richter equation states that

$$N_T(M) = 10^{a-bM} \tag{1}$$

where N is the number of events with magnitude larger than M over a sufficiently long period of time T (Gutenberg and Richter 1944). While this equation has been widely and successfully applied, it suffers from limitations. Gutenberg-Richter plots always show a more or less developed "shoulder" towards the lower end of the magnitude axis, partly due to under reporting of lower magnitude earthquakes. Towards the higher end of the magnitude axis, the scatter of the data points increases considerably, due to the paucity of the largest earthquakes. In many instances, a systematic convexity of the magnitude-frequency line can be seen. These general observations provide a warning that the statistics of a regression analysis will be suspect. A further significant limitation is that the relationship shows no (upper) limit: it allows extrapolations to be made to earthquakes with an (arbitrarily) large magnitude. The finite amount of energy stored in the Earth, and the finite strength of rocks dictate that there must be an upper limit to the magnitude of an earthquake. More or less significant deviations from the Gutenberg-Richter formula have been documented for the largest magnitudes (e.g., Pisarenko and Sornette 2004). Two modifications ("corner" magnitude and maximum magnitude) have been proposed and are often adopted. Their shortcomings have led to the adaptation and adoption of the more appropriate, statistically sound methods from the study of extremal events ((Embrechts, Kluppelberg, and Mikosh 1997)).

3.1 "Corner" magnitude

Kagan 1997; Kagan and Schoenberg 2001 and Vere-Jones, Robinson, and Yang 2001 multiplied the power law distribution of the seismic moments (which corresponds to the Gutenberg-Richter distribution of magnitudes) by an exponential taper. This results in either a Γ or a modified Pareto distribution, with a characteristic moment, or "corner" magnitude equivalent in the corresponding magnitude distribution. The effect is a "soft" truncation of the Gutenberg-Richter law.

3.2 Maximum magnitude

The proposal to set a maximum possible earthquake size $M_{\max}$, a hard truncation of the Gutenberg-Richter law (Consentino, Ficara, and Luzio 1977; Dargahi-Noubary 1983; Main, Irving, Musson, and Reading 1999) plays an important role in seismic risk and seismic hazard studies (Bender and Perkins 1993; Cornell 1994; Kijiko and Graham 1998). It provides a very attractive measure for engineers and insurers alike, as risk and construction standards can be set against a given maximum magnitude. Unfortunately, the estimation or calculation of M_{max} remains unsatisfactory. Its superficial attraction is undermined by undesirable features (Kagan 1993; Pisarenko, Sornette, Sornette, and Rodkin 2008b):

1. $M_{\max}$ is ill-defined, as it does not contain the time scale over which it has been determined, or over which is valid.

2. The cut-off nature of $M_{\max}$ is arbitrary in the sense that the impossibility of $M_{\max} + \epsilon$ for any arbitrarily small value of ϵ has no (physical) justification.

3. $M_{\max}$ is statistically highly unstable.

3.3 Extreme Value Theory

Extreme value theory provides the theory to handle extremes of random phenomena. In contrast to basic statistics, it provides the necessary tools to deal with statistical distribution issues such as skewness, fat tails, rare events and the like (Embrechts, Kluppelberg, and Mikosh 1997). Pisarenko, Sornette, Sornette, and Rodkin 2008a applied with success the Generalised Extreme Value distribution (GEV) and the Generalised Pareto distribution (GPD) in their attempts to characterise the distribution of earthquake magnitudes and go beyond the limitations of the Gutenberg-Richter relation and its various ad-hoc modifications. The Frechet-Fisher-Tipper theorem (Embrechts, Kluppelberg, and Mikosh 1997) leads to the definition of the GEV distribution

$$\Phi(x|\mu,\sigma,\xi) = e^{-(1+\xi(x-\mu)/\sigma)^{-1/\xi)}}, \tag{2}$$

a Pareto (> 0) or a Weibull (< 0) distribution which degenerates for $\xi = 0$ to

$$\Phi(x|\mu,\sigma) = e^{-e^{-(x-\mu)/\sigma}}, \tag{3}$$

(Gumbel distribution) in which μ, σ, ξ are respectively the centering, scale and shape parameters. The theorem provides the statistical and mathematical justification for this definition of the limiting distribution of the maxima of identically independently distributed random variables x as the sample size n goes to infinity. The quantiles Q_q of the GEV distribution $\Phi(x|\mu,\sigma,\xi)$ are

$$Q_q(\tau) = \mu(T) + ((\tau/T\log(1/q))^{\xi} - 1)\sigma(T)/\xi \tag{4}$$

with q the quantile confidence level, τ an arbitrary time interval (in the future) and T the time interval step used to estimate the parameters of the

GEV distribution. The utility and value of the quantiles over $M_{\max}$ as stable and statistically meaningful values was demonstrated and illustrated by Pisarenko, Sornette, Sornette, and Rodkin 2008a; Pisarenko, Sornette, Sornette, and Rodkin 2008b. In practice, the parameters of the GEV distribution can be estimated from data through maximum likelihood calculations.

4 Statistics of Recurrence Rates

The analysis and calculation of recurrence rates or waiting times of seismic events is beset with difficulties equivalent to those encountered in the study of magnitude distribution. It is clear that the obvious extension of the Gutenberg-Richter magnitude-number relation to a magnitude-frequency relation by simply dividing the numbers by the time interval, compounds the statistical problems just discussed by bringing in explicitly the time dimension. Size, number and timing of seismic events are related, but the nature of the relationships is complex and not fully understood. A radically different approach was proposed by Bak, Christensen, Danon, and Scanlon 2002, and further developed and refined by Corral 2005; Corral 2006; Corral 2009. Their application of the theories of self-organised criticality led to a modified Γ distribution, so that

$$p(x) = C\delta x^{\gamma-1} e^{-(x/\alpha)^{\delta}} / \alpha^{\gamma}\Gamma(\gamma/\delta) \tag{5}$$

where $x = \lambda t$ (the average seismic rate multiplied by time).

Saichev and Sornette 2006 criticised this analysis as incompatible and at variance with the data and they promoted the Epidemic-Type Aftershock Sequence as a better approach. The Epidemic-Type Aftershock Sequence (ETAS) was proposed by Kagan and Knopoff 1981 and Ogata 1988, with the statistical and mathematical properties studied and expanded further over the years (see Saichev and Sornette 2007 for an overview). It integrates the Gutenberg-Richter relationship with the Omori relationship of aftershock sequences, and adds a productivity law and a measure of the fractal nature of fault networks.

$$p(x) = (\alpha n\theta\rho^{\theta} x^{-1-\theta} + (1 - n + \alpha n\rho^{\theta} x^{-\theta})^2) e^{(-(1-n)x - \alpha n\theta\rho^{\theta} x^{1-\theta}/(1-\theta))} \tag{6}$$

where $x = \lambda t$ (the average seismic rate multiplied by time), $\alpha = (\lambda_0 c)^{\theta}$ (reflecting the Omori aftershock law), $\rho = \lambda/\lambda_0 = Q(m)(L/L_0)^d$ (a measure of the productivity law), and with n the criticality parameter. The function accounts much better for the observations, as can be expected from a 4-parameter function. The parameters of the function can be fitted to the data through maximum likelihood calculations.

5 Calculations

5.1 Data acquisition

We will need some libraries, so let us declare these first. Then, we read in the data from file. The data provided need a little work. We are interested in

- magnitude
- UTC
- lat and long
- depth

The column names have to be tidied up, and the Date and Time fields have to be converted into UTC format for R. The ordering of the data by ascending UTC will be useful later on. Add a logical field, to flag if an events might be an aftershock.

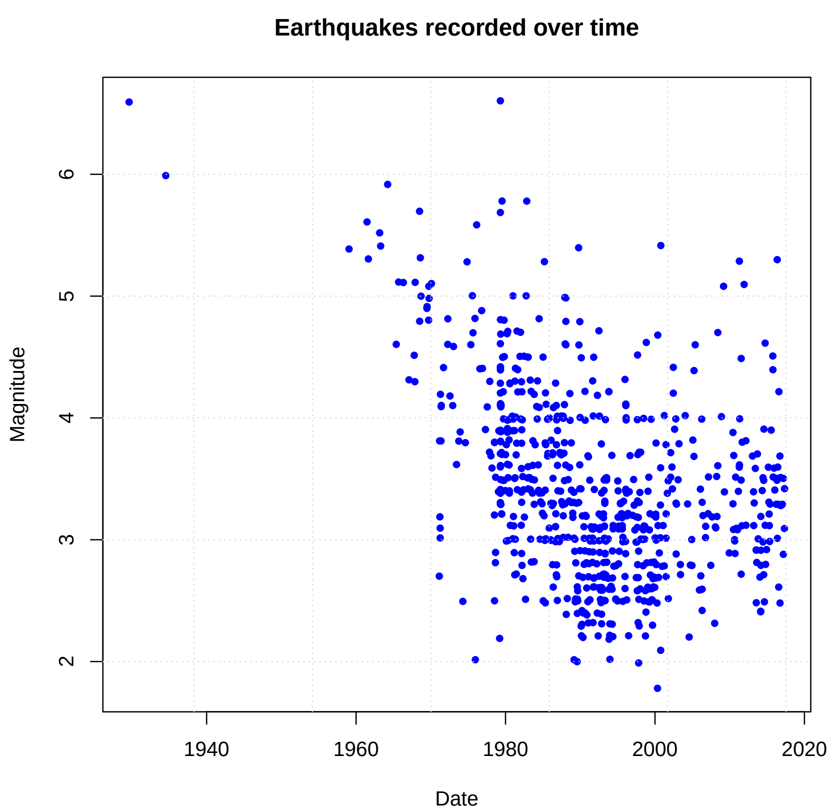
```
library("data.table")
library("dplyr")
library("evd")
library("geosphere")
library("tidyr")

events <- read.table("../data/earthquakes.csv", header = TRUE,
     sep = ",", stringsAsFactors = FALSE)
events <- select(events, -Sydney.Date, -Sydney.Time,
 -Approximate.location, -ORIGIN.ID)
events <- unite(events, UTC, c(UTC.Date, UTC.Time), sep = " ")
events$UTC <- as.POSIXct(events$UTC, tz = "GMT")
events <- arrange(events, UTC)

events <- rename(events,
 Mag = Magnitude,
 Long = Longitude,
 Lat = Latitude,
 Depth = Depth..km.)
events <- mutate(events, Aftershock = FALSE)
```

5.2 Magnitudes and Timing

Before we calculate the various properties we are interested in, it is a good idea to run a few checks and put in place a number of opportunities to test any boundary conditions which may or may not be fulfilled.

First, let us make a simple plot of the data

```
plot(events$UTC,jitter(events$Mag),
     main = "Earthquakes recorded over time",
     xlab = "Date",
     ylab = "Magnitude",
     pch = 20,
     col = "blue")
grid()
```



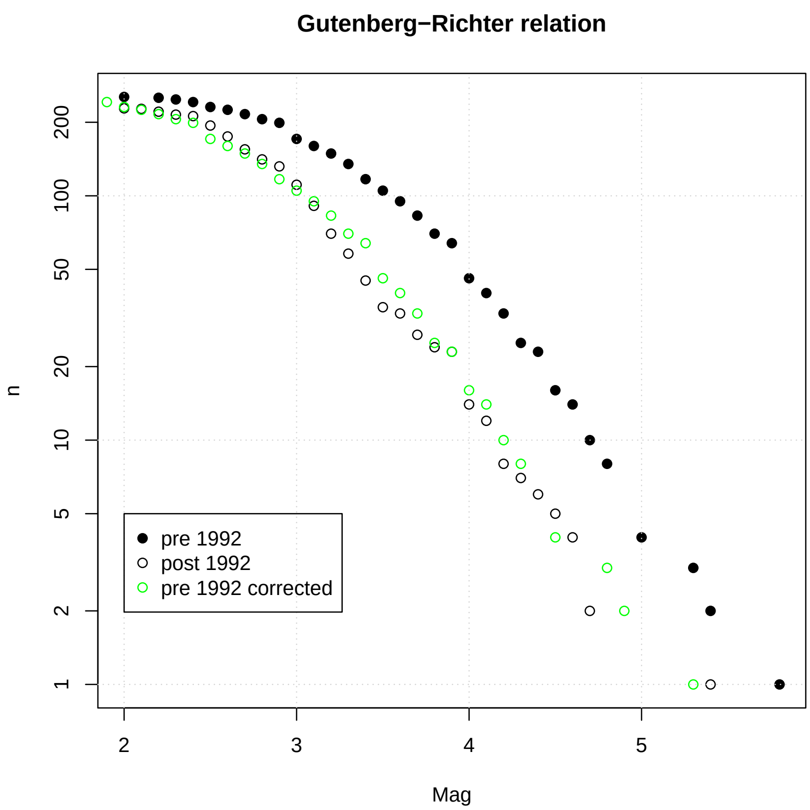
The plot shows that the database is not uniform: far fewer events have been recorded in earlier years.

5.2.1 Catalogue Magnitude

Sagar and Leonard 2007 discussed the magnitude determination and the way this has changed over the years. They illustrated these changes with a figure of the pre- and post-1992 Western Australian earthquake magnitude Cumulative Density graph, including an equivalent graph with the South Australian earthquakes. Let us check if this discrepancy is still the case with the current data file from NW Australia.

When we split the events into pre- and post-1992 groups, and plot their Gutenberg-Richter relation separately, there appears to be a systematic difference between the subsets.

```
GR_pre <- events %>%
    filter(year(UTC) < 1992) %>%
    count(Mag) %>%
    mutate(n = cumsum(n)) %>%
    mutate(n = max(n) - n + 1)

GR_post <- events %>%
    filter(year(UTC) > 1991) %>%
    count(Mag) %>%
    mutate(n = cumsum(n)) %>%
    mutate(n = max(n) - n + 1)

GR_corr<- events %>%
    filter(year(UTC) < 1992) %>%
    mutate(Mag = Mag - 0.5) %>%
    count(Mag) %>%
    mutate(n = cumsum(n)) %>%
    mutate(n = max(n) - n + 1)

plot(GR_pre, log = "y", pch = 19,
     main = "Gutenberg-Richter relation")
points(GR_post)
points(GR_corr, col = "green")
grid()

legend(2, 5,
       legend = c("pre 1992","post 1992","pre 1992 corrected"),
       pch = c(19,21,21),
       col = c("black","black","green"))
```



We can change the magnitude values of all pre-1992 events.

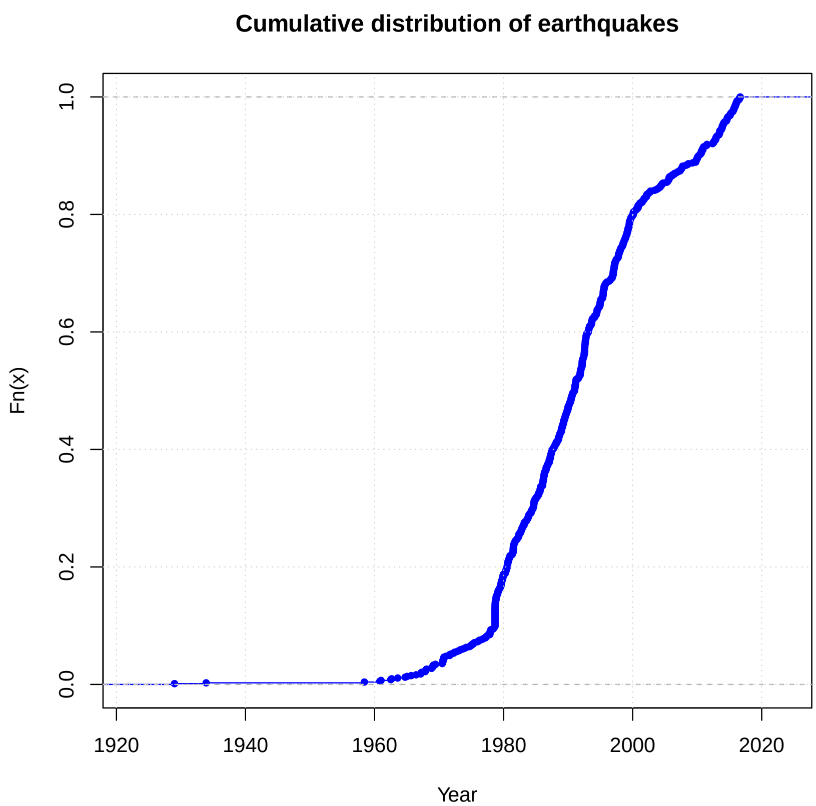
```
events$Mag[(year(events$UTC) < 1992)] <-
    events$Mag[(year(events$UTC) < 1992)] - 0.5
```

5.2.2 Stationary, Poissonian, process

Turning our attention now to the timing of events, we can show the changes hinted at earlier in a more informative, quantitative way through a cumulative density plot

```
events <- mutate(events,
 UTC_d = as.numeric(difftime(UTC,UTC[1],
     unit = "days")))
plot(ecdf(events$UTC_d/365.4 + year(events$UTC[1])),
     main = "Cumulative distribution of earthquakes",
     xlab = "Year",
```

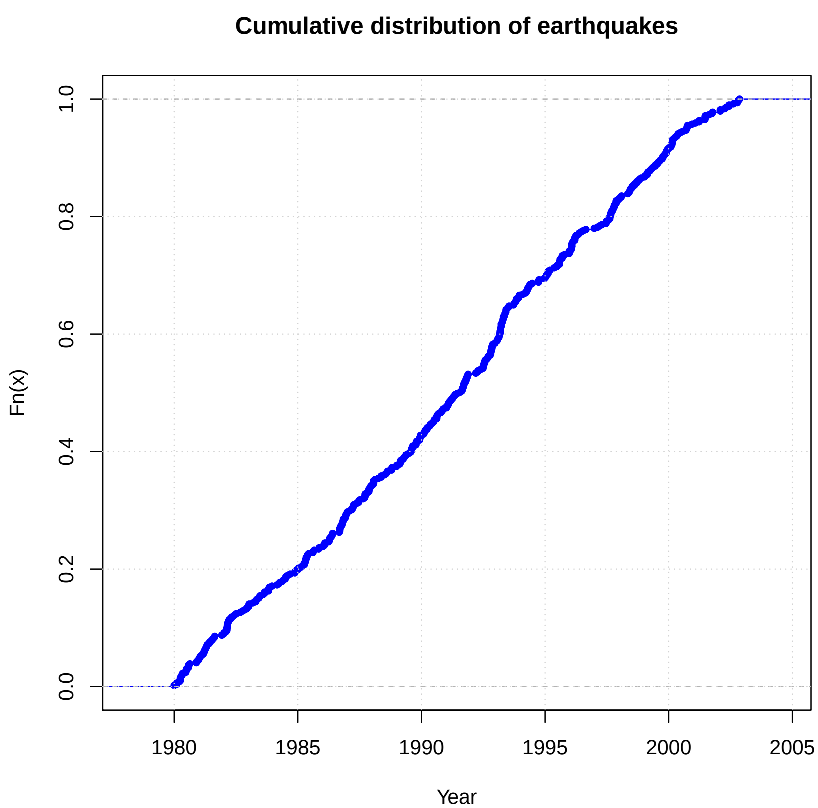
```
      pch = 20,
      col = "blue")
grid()
```



The statistical techniques applied in this study assume a stationary Poisson process. This graph shows that the events before 1979 are much sparser, and hence will interfere significantly with any statistical calculations. A careful look also reveals that another change takes place around 2002, when once again fewer events make up the database.

A stationary period appears to be present between May 1979 and November 2002 (for a total of 485 events): a plot of events against time yields the expected, and required, linear relation. Let us therefore restrict our data and exclude all events before 1979 and after 2002. While we do this, let us take the opportunity of adding a column with a time line, as the number of days elapsed since the first event in the (filtered) database. Show also the result of this restriction with a cumulative distribution plot.

```
events <- events %>%
    filter(year(UTC) > 1979, year(UTC) < 2003) %>%
    mutate(UTC_d = as.numeric(difftime(UTC,UTC[1], unit = "days")))

plot(ecdf(events$UTC_d/365.4 + year(events$UTC[1])),
     main = "Cumulative distribution of earthquakes",
     xlab = "Year",
     pch = 20,
     col = "blue")
grid()
```



We can verify to what extent this restricted data set is consistent with a Poisson Process and use the Kolmogorov-Smirnov test.

```
NOE <- length(events$UTC_d)
TL <- max(events$UTC_d)
```

```
Poisson_Data <- seq(1, TL, by=TL/NOE)
KS_Result <- ks.test(events$UTC_d,Poisson_Data)
KS_Result

Warning message:
In ks.test(events$UTC_d, Poisson_Data) :
  p-value will be approximate in the presence of ties

        Two-sample Kolmogorov-Smirnov test

data:  events$UTC_d and Poisson_Data
D = 0.057026, p-value = 0.4017
alternative hypothesis: two-sided
```

5.3 Sensitivity to data error

We may gain some idea of the sensitivity of the results to magnitude estimate errors. The simplest way of doing this, is by perturbing the actual magnitudes by e(0.0, 0.25) with these two lines of code and re-running the entire analysis.

```
#  My_perturb <- rnorm(length(events$Mag), mean=0.0, sd = 0.25)
#  events$Mag <- events$Mag + My_perturb
```

5.4 De-clustering

De-clustering of the catalogue might improve the Poissonian nature of the catalogue data, by eliminating any after-shocks.

We will do this by identifying the events that are deemed to be main shocks, and calculating both the time and space intervals imposed by the magnitude of each of the shocks.

We then check each event in the catalogue against all these limits: if an event falls within the limits, it gets flagged as an aftershock.

First, we set the main-shock magnitudes, with the calculated time and distance limits imposed by the actual magnitude (the values of parameters used to determine the size of the windows are taken from (Knopoff, Kagan, and Knopoff 1982))

```
Max_Mag <- 5.0
main_shocks <- filter(events, Mag > Max_Mag)
main_events <- mutate(main_shocks,
```

```
      delta_T = 10^(-0.31 + 0.46 * Mag),
      delta_D = 10^(-0.85 + 0.46 * Mag))
```

Step through all the events: check if the magnitude falls in the main shock series, then check the following events to see if these fall in the critical time-space window (remember that the distHaversine function returns distance between points (long, lat) in m, not km). To avoid clashes with main shocks, we shall take out the main shocks from the events set, and then add them again after processing.

```
events <- anti_join(events, main_shocks, by = "EVENT.ID")

Index <- 1
  while (Index < length(events$UTC)-1)
  {
      main_result <- main_events %>%
  mutate(test_dT = as.numeric(difftime(events$UTC[Index], UTC,
       unit = "days")),
 test_dD = distHaversine(cbind(Long, Lat),
 c(events$Long[Index], events$Lat[Index]))/1000
 ) %>%
  filter(test_dT > 0, test_dT < delta_T, test_dD < delta_D)
      if (nrow(main_result) > 0) {events$Aftershock[Index] = TRUE}

    Index <- Index + 1
    }

events <- full_join(events, main_shocks)
events <- arrange(events, UTC)

events <- filter(events, Aftershock == FALSE)

Joining, by = c("Mag", "UTC", "Lat", "Long", "Magnitude.Type", "Depth", "EVENT.ID", "A
```

5.5 EVD calculations

First, set the lower magnitude cut-off. We also have to define the time steps for the Generalised Extreme Value distribution (in days)

```
M_Min <- 2.5
delta_T <- 10
```

```
Time_Steps <- seq(20, 300, by = delta_T)
delta_T_Max <- last(events$UTC_d) - first(events$UTC_d)
```

To improve the acuracy of the results, a bootstrapping approach is highly effective. The idea behind the bootstap approach is that shuffling the magnitudes around amounts to a resampling of the population of the events, whilst maintaining the distribution in time. Set up the number of data shuffles to bootstrap the GEV parameter calculations.

The fitted parameters go into an 3-d array

We need to step through the entire events dataset in contiguous blocks of size Time_Steps[i], and determine the maximum magnitude in each of the intervals. A convenient way of doing this is by creating additional columns, containing the blocknumber (in effect the modulus of the day number of the event to the time step size). These numbers can then be used as groups, so that dplyr grouping can be brought into play.

Calculate the MLE of the GEV distribution and store the results The order is: T, loc, scale, shape, error_loc, error_scale, error_shape

```
Bootstrap_Total <- 100
shuffle_events <- events

GEV_Parameters <- array(0, c(length(Time_Steps),4,Bootstrap_Total))

for (Re_runs in 1:Bootstrap_Total){
shuffle_events$Mag <- sample(events$Mag)
for (i in 1:length(Time_Steps)){
    shuffle_events <- shuffle_events %>%
mutate(block = UTC_d %/% Time_Steps[i])
    Max_Mags <- shuffle_events %>%
group_by(block) %>%
summarize(value = max(Mag))
    GEV_Fit <- fgev(Max_Mags$value,std.err=F)
    GEV_Parameters[i, ,Re_runs] <-
c(Time_Steps[i],fitted.values(GEV_Fit))
}
}
```

Present the results by performing a statistical summary of the parameter estimates

```
GEV_Results <- array(0, c(length(Time_Steps),10))
```

```
for (i in 1:length(Time_Steps))
  {
  GEV_Results[i,1] <- Time_Steps[i]
  GEV_Results[i,2:4] <- quantile(GEV_Parameters[i,2,],
 probs=c(0.16,0.50,0.84))
  GEV_Results[i,5:7] <- quantile(GEV_Parameters[i,3,],
 probs=c(0.16,0.50,0.84))
  GEV_Results[i,8:10] <- quantile(GEV_Parameters[i,4,],
  probs=c(0.16,0.50,0.84))
  }

Shape <- GEV_Results[,9]
Scale <- GEV_Results[,6]
Location <- GEV_Results[,3]
```

Now we can calculate estimates of maximum magnitudes for arbitrary time in the future. Here we look at 1000, 2000, 3000, 4000 and 5000 years ahead

```
Tau <- c(365000*1:5)
Q <- 0.975

Quantiles <- array(0, c(length(Time_Steps),length(Tau)))
for (Tau_i in 1:length(Tau))
  {
      Quantiles[,Tau_i] <- Location +
  ((Tau[Tau_i]/(log(1/Q)*Time_Steps))^Shape - 1) * Scale / Shape
  }
```

5.6 Generating plots

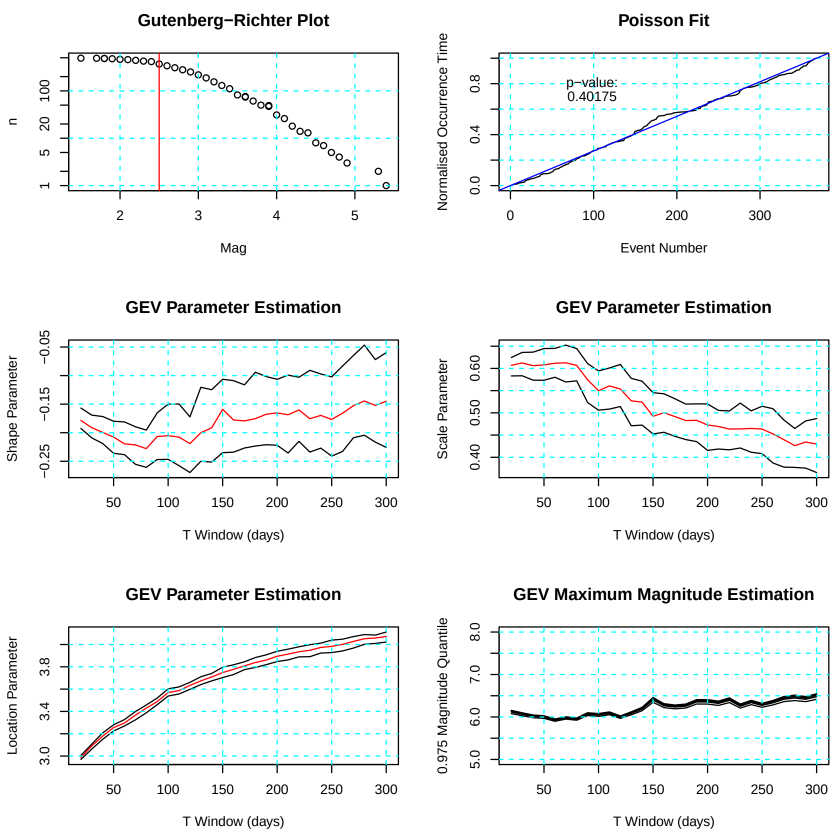
Calculate the Gutenberg-Richter relationship between magnitude and number of events smaller or equal to the given magnitude

```
GR <- events %>%
    count(Mag) %>%
    mutate(n = cumsum(n)) %>%
    mutate(n = max(n) - n + 1)
```

Make the actual plots

```
op <- par(mfrow = c(3,2))

plot(GR,log="y",main="Gutenberg-Richter Plot")
grid(lty=2,col=5)
abline(v=M_Min,col=2)
My_List <- subset(events, Mag > M_Min, select = c(UTC_d,Mag))

plot(My_List$UTC_d/My_List$UTC_d[length(My_List$UTC_d)],
     type="l",
     xlab="Event Number",
     ylab="Normalised Occurrence Time",
     main="Poisson Fit")
grid(lty=2,col=5)
abline(0,1/length(My_List$UTC_d),col=4)
text(NOE/5,0.8,"p-value:")
text(NOE/5,0.7,round(KS_Result$p.value,5))

matplot(GEV_Results[,1],GEV_Results[,8:10],
type="l",
lty=1,
col=c(1,2,1),
main="GEV Parameter Estimation",
xlab="T Window (days)",
ylab="Shape Parameter")
grid(lty=2,col=5)

matplot(GEV_Results[,1],GEV_Results[,5:7],
type="l",
lty=1,
col=c(1,2,1),
main="GEV Parameter Estimation",
xlab="T Window (days)",
ylab="Scale Parameter")
grid(lty=2,col=5)

matplot(GEV_Results[,1],GEV_Results[,2:4],
type="l",
lty=1,
col=c(1,2,1),
main="GEV Parameter Estimation",
```

```
xlab="T Window (days)",
ylab="Location Parameter")
grid(lty=2,col=5)

matplot(Time_Steps,Quantiles,ylim=c(5,8),
type="l",
lty=1,
col=1,
main="GEV Maximum Magnitude Estimation",
xlab="T Window (days)",
ylab="0.975 Magnitude Quantile")
grid(lty=2,col=5)

par(op)
```



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