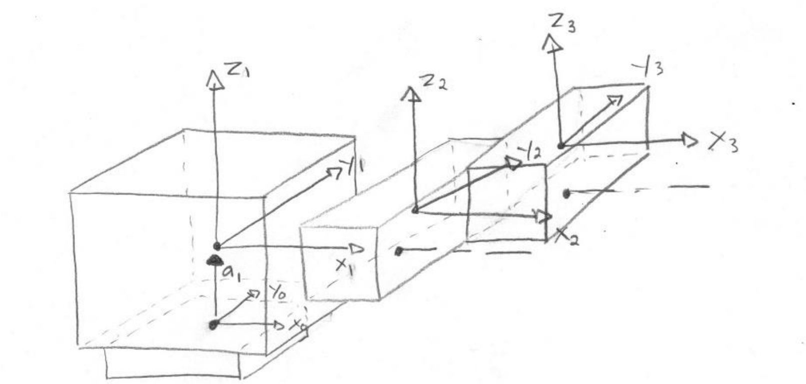
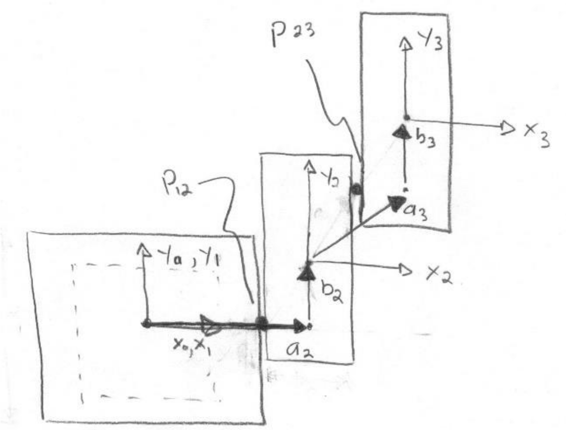
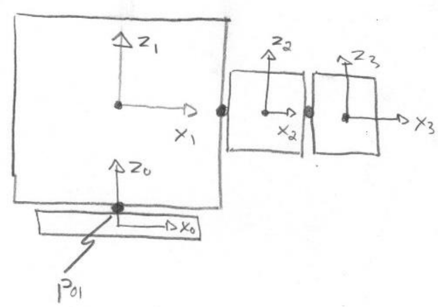
Homework 6 Solution

Key:	Black text	= Original problem statement
	Blue text	= Solution and remarks
	Final solutions appear in $\boxed{\text{boxes}}$	







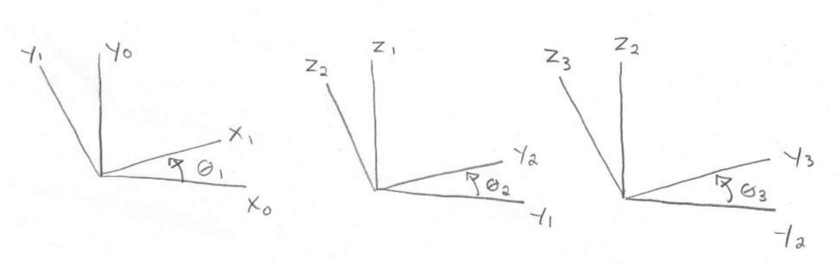
In the diagram above I've drawn in the a and b vectors, but you didn't have to.

1. (120 pts) You must derive expressions for the following things by hand, including any diagrams necessary to complete these derivations:

   - position and orientation of each link

$$o_1^0, R_1^0, o_2^0, R_2^0 \text{ (also requires } R_2^1), o_3^0, R_3^0 \text{ (also requires } R_3^2)$$

Let us start by drawing the rotations diagrams for each joint.



$$\boxed{R_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \qquad R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{bmatrix} \qquad R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{bmatrix}$$

$$\boxed{R_2^0 = R_1^0 R_2^1}$$

$$\boxed{R_3^0 = R_1^0 R_2^1 R_3^2 = R_2^0 R_3^2}$$

Positions of coordinate frames:

$$\boxed{o_1^0 = a_1 + R_1^0 b_1} \qquad \text{(given)}$$

$$o_2^0 = o_1^0 + R_1^0 o_2^1$$

$$= a_1 + R_1^0 b_1 + R_1^0 (a_2 + R_2^1 b_2)$$

$$\boxed{o_2^0 = a_1 + R_1^0 (b_1 + a_2) + R_2^0 b_2}$$

$$o_3^0 = o_2^0 + R_2^0 o_3^2$$

$$= a_1 + R_1^0 (b_1 + a_2) + R_2^0 b_2 + R_2^0 (a_3 + R_3^2 b_3)$$

$$\boxed{o_3^0 = a_1 + R_1^0 (b_1 + a_2) + R_2^0 (b_2 + a_3) + R_3^0 b_3}$$

- linear and angular velocity of each link

$$v_{0,1}^0,\ w_{0,1}^1,\ v_{0,2}^0,\ w_{0,2}^2 \text{ (also requires } w_{1,2}^2\text{)},\ v_{0,3}^0,\ w_{0,3}^3 \text{ (also requires } w_{2,3}^3\text{)}$$

$$v_{0,1}^0 = \dot{o}_1^0$$

$$= \frac{d}{dt}[a_1 + R_1^0 b_1] \qquad \text{Note } a_i,\ b_i \text{ are constant.}$$

$$\boxed{v_{0,1}^0 = R_1^0 \widehat{\omega_{0,1}^1} b_1}$$

$$v_{0,2}^0 = \dot{o}_2^0$$

$$= \frac{d}{dt}[a_1 + R_1^0 (b_1 + a_2) + R_2^0 b_2] \qquad \text{Note } a_i,\ b_i \text{ are constant.}$$

$$\boxed{v_{0,2}^0 = R_1^0 \widehat{\omega_{0,1}^1} (b_1 + a_2) + R_2^0 \widehat{\omega_{0,2}^2} b_2}$$

We will define $\omega_{0,2}^2$ shortly.

$$v_{0,3}^0 = \dot{o}_3^0$$

$$= \frac{d}{dt}\left[a_1 + R_1^0(b_1 + a_2) + R_2^0(b_2 + a_3) + R_3^0 b_3\right]$$

Note a_i, b_i are constant.

$$\boxed{v_{0,3}^0 = R_1^0 \widehat{\omega_{0,1}^1}(b_1 + a_2) + R_2^0 \widehat{\omega_{0,2}^2}(b_2 + a_3) + R_3^0 \widehat{\omega_{0,3}^3} b_3}$$

We will define $\omega_{0,3}^3$ shortly.

For the angular velocities, first define the angular velocities of each joint w.r.t. the one to which it is attached. We can get these by inspection from the rotation diagram in the previous section.

$$\boxed{\omega_{0,1}^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_1 = z_0^0 \dot{\theta}_1} \qquad \omega_{1,2}^2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \dot{\theta}_2 = x_1^1 \dot{\theta}_2 \qquad \omega_{2,3}^3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \dot{\theta}_3 = x_2^2 \dot{\theta}_3$$

We can get the remaining ω terms by noting that angular rates can simply be added when they are in the same frame.

$$\omega_{0,2}^2 = \omega_{0,1}^2 + \omega_{1,2}^2$$

$$\boxed{\omega_{0,2}^2 = {R_2^1}^T \omega_{0,1}^1 + \omega_{1,2}^2}$$

$$\omega_{0,3}^3 = \omega_{0,1}^3 + \omega_{1,2}^3 + \omega_{2,3}^3$$

$$\boxed{\omega_{0,3}^3 = {R_3^1}^T \omega_{0,1}^1 + {R_3^2}^T \omega_{1,2}^2 + \omega_{2,3}^3}$$

We note R_2^1 and R_3^2 , therefore we know R_3^1.

- linear and angular acceleration of each link (these will get lengthy—I strongly recommend that you group terms as much as possible, and that you immediately separate terms that involve the unknowns $\ddot{\theta}_1$, $\ddot{\theta}_2$, and $\ddot{\theta}_3$ from the other terms)

$$\dot{v}_{0,1}^0, \dot{w}_{0,1}^1, \dot{v}_{0,1}^0, \dot{w}_{0,2}^2, \dot{v}_{0,3}^0, \dot{w}_{0,3}^3$$

Let us first calculate the $\dot{\omega}$ terms, because they will show up in the expressions for $\dot{v}$.

$$\dot{\omega}_{0,1}^1 = \frac{d}{dt}\left[z_0^0 \dot{\theta}_1\right]$$

z_0^0 is constant

$$\boxed{\dot{\omega}_{0,1}^1 = z_0^0 \ddot{\theta}_1}$$

$$\dot{\omega}_{0,2}^2 = \frac{d}{dt}\left[R_1^2 \omega_{0,1}^1 + \omega_{1,2}^2\right]$$

Product rule

$$= R_1^2 \widehat{\omega_{2,1}^1} \omega_{0,1}^1 + R_1^2 \dot{\omega}_{0,1}^1 + \dot{\omega}_{1,2}^2$$

$\omega_{2,1}^1 = -\omega_{1,2}^1$

$$= -{R_2^1}^T \widehat{\omega_{1,2}^1} \omega_{0,1}^1 + {R_2^1}^T \dot{\omega}_{0,1}^1 + \dot{\omega}_{1,2}^2$$

$\dot{\omega}_{1,2}^2 = \frac{d}{dt}\left[\omega_{1,2}^2\right] = \frac{d}{dt}\left[x_1^1 \dot{\theta}_2\right] = x_1^1 \ddot{\theta}_2$

$$\boxed{\dot{\omega}_{0,2}^2 = -{R_2^1}^T \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 + {R_2^1}^T z_0^0 \ddot{\theta}_1 + x_1^1 \ddot{\theta}_2}$$

$\dot{\omega}_{0,3}^{3} = \frac{d}{dt}\left[R_1^3\omega_{0,1}^1 + R_2^3\omega_{1,2}^2 + \omega_{2,3}^3\right]$ Product rule

$= R_1^3\widehat{\omega_{3,1}^1}\omega_{0,1}^1 + R_1^3\dot{\omega}_{0,1}^1 + R_2^3\widehat{\omega_{3,2}^2}\omega_{1,2}^2 + R_2^3\dot{\omega}_{1,2}^2 + \dot{\omega}_{2,3}^3$ $\omega_{3,1}^1 = -\omega_{1,3}^1$

$= -R_1^3\widehat{\omega_{1,3}^1}\omega_{0,1}^1 + R_1^3\dot{\omega}_{0,1}^1 - R_2^3\widehat{\omega_{2,3}^2}\omega_{1,2}^2 + R_2^3\dot{\omega}_{1,2}^2 + \dot{\omega}_{2,3}^3$

$$\boxed{\dot{\omega}_{0,3}^{3} = -{R_3^1}^T\widehat{R_3^1\omega_{1,3}^3}\omega_{0,1}^1 - {R_3^2}^T\widehat{R_3^2\omega_{2,3}^3}\omega_{1,2}^2 + {R_3^1}^T z_0^0\ddot{\theta}_1 + {R_3^2}^T x_1^1\ddot{\theta}_2 + x_2^2\ddot{\theta}_3}$$

We effectively know $\omega_{1,3}^3$, so I won't substitute terms for it and make this expression any messier.

$\omega_{1,3}^3 = \omega_{1,2}^3 + \omega_{2,3}^3$

$= {R_3^2}^T\omega_{1,2}^2 + \omega_{2,3}^3$

Now let's find the translational velocities.

$\dot{v}_{0,1}^0 = \frac{d}{dt}\left[R_1^0\widehat{\omega_{0,1}^1}b_1\right]$ Product rule

$= R_1^0{\widehat{\omega_{0,1}^1}}^2 b_1 + R_1^0\widehat{\dot{\omega}_{0,1}^1}b_1$ Reverse order of cross product to isolate unknowns

$= R_1^0{\widehat{\omega_{0,1}^1}}^2 b_1 - R_1^0\widehat{b_1}\dot{\omega}_{0,1}^1$

$$\boxed{\dot{v}_{0,1}^0 = R_1^0{\widehat{\omega_{0,1}^1}}^2 b_1 - R_1^0\widehat{b_1}z_0^0\ddot{\theta}_1}$$

$\dot{v}_{0,2}^0 = \frac{d}{dt}\left[R_1^0\widehat{\omega_{0,1}^1}(b_1 + a_2) + R_2^0\widehat{\omega_{0,2}^2}b_2\right]$

$= R_1^0{\widehat{\omega_{0,1}^1}}^2(b_1 + a_2) + R_1^0\widehat{\dot{\omega}_{0,1}^1}(b_1 + a_2) + R_2^0{\widehat{\omega_{0,2}^2}}^2 b_2 + R_2^0\widehat{\dot{\omega}_{0,2}^2}b_2$ Isolate $\dot{\omega}_{0,1}^1$ and $\dot{\omega}_{0,2}^2$

$= R_1^0{\widehat{\omega_{0,1}^1}}^2(b_1 + a_2) + R_2^0{\widehat{\omega_{0,2}^2}}^2 b_2 - R_1^0(\widehat{b_1 + a_2})\dot{\omega}_{0,1}^1 - R_2^0\widehat{b_2}\dot{\omega}_{0,2}^2$ Substitute $\dot{\omega}_{0,2}^2$

$= R_1^0{\widehat{\omega_{0,1}^1}}^2(b_1 + a_2) + R_2^0{\widehat{\omega_{0,2}^2}}^2 b_2 - R_1^0(\widehat{b_1 + a_2})\dot{\omega}_{0,1}^1$
$- R_2^0\widehat{b_2}\left(-{R_2^1}^T\widehat{R_2^1\omega_{1,2}^2}\omega_{0,1}^1 + {R_2^1}^T z_0^0\ddot{\theta}_1 + x_1^1\ddot{\theta}_2\right)$

Expand

$= R_1^0{\widehat{\omega_{0,1}^1}}^2(b_1 + a_2) + R_2^0{\widehat{\omega_{0,2}^2}}^2 b_2 + R_2^0\widehat{b_2}{R_2^1}^T\widehat{R_2^1\omega_{1,2}^2}\omega_{0,1}^1 - R_1^0(\widehat{b_1 + a_2})z_0^0\ddot{\theta}_1 - R_2^0\widehat{b_2}{R_2^1}^T z_0^0\ddot{\theta}_1$
$- R_2^0\widehat{b_2}x_1^1\ddot{\theta}_2$

Put all unknowns together

$$\boxed{\dot{v}_{0,2}^0 = R_1^0{\widehat{\omega_{0,1}^1}}^2(b_1 + a_2) + R_2^0{\widehat{\omega_{0,2}^2}}^2 b_2 + R_2^0\widehat{b_2}{R_2^1}^T\widehat{R_2^1\omega_{1,2}^2}\omega_{0,1}^1 - \left(R_1^0(\widehat{b_1 + a_2}) + R_2^0\widehat{b_2}{R_2^1}^T\right)z_0^0\ddot{\theta}_1 - R_2^0\widehat{b_2}x_1^1\ddot{\theta}_2}$$

$$\dot{v}^0_{0,3} = \frac{d}{dt}\left[R^0_1\widehat{\omega^1_{0,1}}(b_1+a_2) + R^0_2\widehat{\omega^2_{0,2}}(b_2+a_3) + R^0_3\widehat{\omega^3_{0,3}}b_3\right]$$

$$= R^0_1\widehat{\omega^1_{0,1}}^{\,2}(b_1+a_2) + R^0_1\widehat{\dot{\omega}^1_{0,1}}(b_1+a_2) + R^0_2\widehat{\omega^2_{0,2}}^{\,2}(b_2+a_3) + R^0_2\widehat{\dot{\omega}^2_{0,2}}(b_2+a_3) + R^0_3\widehat{\omega^3_{0,3}}^{\,2}b_3 + R^0_3\widehat{\dot{\omega}^3_{0,3}}b_3$$

Isolate unknowns

$$= R^0_1\widehat{\omega^1_{0,1}}^{\,2}(b_1+a_2) + R^0_2\widehat{\omega^2_{0,2}}^{\,2}(b_2+a_3) + R^0_3\widehat{\omega^3_{0,3}}^{\,2}b_3 - R^0_1\widehat{(b_1+a_2)}\dot{\omega}^1_{0,1} - R^0_2\widehat{(b_2+a_3)}\dot{\omega}^2_{0,2} - R^0_3\widehat{b_3}\dot{\omega}^3_{0,3}$$

Substitue $\dot{\omega}^1_{0,1}$, $\dot{\omega}^2_{0,2}$, and $\dot{\omega}^3_{0,3}$

$$\begin{aligned} &= R^0_1\widehat{\omega^1_{0,1}}^{\,2}(b_1+a_2) + R^0_2\widehat{\omega^2_{0,2}}^{\,2}(b_2+a_3) + R^0_3\widehat{\omega^3_{0,3}}^{\,2}b_3 - R^0_1\widehat{(b_1+a_2)}z^0_0\ddot{\theta}_1 \\ &\quad - R^0_2\widehat{(b_2+a_3)}\left(-R^{1^T}_2\widehat{R^1_2\omega^2_{1,2}}\omega^1_{0,1} + R^{1^T}_2 z^0_0\ddot{\theta}_1 + x^1_1\ddot{\theta}_2\right) \\ &\quad - R^0_3\widehat{b_3}\left(-R^{1^T}_3\widehat{R^1_3\omega^3_{1,3}}\omega^1_{0,1} - R^{2^T}_3\widehat{R^2_3\omega^3_{2,3}}\omega^2_{1,2} + R^{1^T}_3 z^0_0\ddot{\theta}_1 + R^{2^T}_3 x^1_1\ddot{\theta}_2 + x^2_2\ddot{\theta}_3\right) \end{aligned}$$

Expand

$$\begin{aligned} &= R^0_1\widehat{\omega^1_{0,1}}^{\,2}(b_1+a_2) + R^0_2\widehat{\omega^2_{0,2}}^{\,2}(b_2+a_3) + R^0_3\widehat{\omega^3_{0,3}}^{\,2}b_3 - R^0_1\widehat{(b_1+a_2)}z^0_0\ddot{\theta}_1 \\ &\quad + R^0_2\widehat{(b_2+a_3)}R^{1^T}_2\widehat{R^1_2\omega^2_{1,2}}\omega^1_{0,1} - R^0_2\widehat{(b_2+a_3)}R^{1^T}_2 z^0_0\ddot{\theta}_1 - R^0_2\widehat{(b_2+a_3)}x^1_1\ddot{\theta}_2 \\ &\quad + R^0_3\widehat{b_3}R^{1^T}_3\widehat{R^1_3\omega^3_{1,3}}\omega^1_{0,1} + R^0_3\widehat{b_3}R^{2^T}_3\widehat{R^2_3\omega^3_{2,3}}\omega^2_{1,2} - R^0_3\widehat{b_3}R^{1^T}_3 z^0_0\ddot{\theta}_1 - R^0_3\widehat{b_3}R^{2^T}_3 x^1_1\ddot{\theta}_2 \\ &\quad - R^0_3\widehat{b_3}x^2_2\ddot{\theta}_3 \end{aligned}$$

Group

$$\boxed{\begin{aligned} \dot{v}^0_{0,3} &= R^0_1\widehat{\omega^1_{0,1}}^{\,2}(b_1+a_2) + R^0_2\widehat{\omega^2_{0,2}}^{\,2}(b_2+a_3) + R^0_3\widehat{\omega^3_{0,3}}^{\,2}b_3 + R^0_2\widehat{(b_2+a_3)}R^{1^T}_2\widehat{R^1_2\omega^2_{1,2}}\omega^1_{0,1} \\ &\quad + R^0_3\widehat{b_3}R^{1^T}_3\widehat{R^1_3\omega^3_{1,3}}\omega^1_{0,1} + R^0_3\widehat{b_3}R^{2^T}_3\widehat{R^2_3\omega^3_{2,3}}\omega^2_{1,2} \\ &\quad - \left(R^0_1\widehat{(b_1+a_2)}z^0_0 + R^0_2\widehat{(b_2+a_3)}R^{1^T}_2 z^0_0 + R^0_3\widehat{b_3}R^{1^T}_3 z^0_0\right)\ddot{\theta}_1 \\ &\quad - \left(R^0_2\widehat{(b_2+a_3)}x^1_1 + R^0_3\widehat{b_3}R^{2^T}_3 x^1_1\right)\ddot{\theta}_2 - R^0_3\widehat{b_3}x^2_2\ddot{\theta}_3 \end{aligned}}$$

Since we can find all the ω in the above equation, the only unknowns are the $\ddot{\theta}$. When I put this equation into Matlab, I will first have to define what each of the ω are. This will add extra lines of code but will make everything easier to read and debug.

- mass and moment of inertia of each link

$$m_1, J^1_1, m_2, J^2_2, m_3, J^3_3,$$

All the links are rectangular prisms, so the masses are easy to find.

$$\boxed{m_i = \rho_i dx_i dy_i dz_i \qquad \text{for } i = 1,2,3}$$

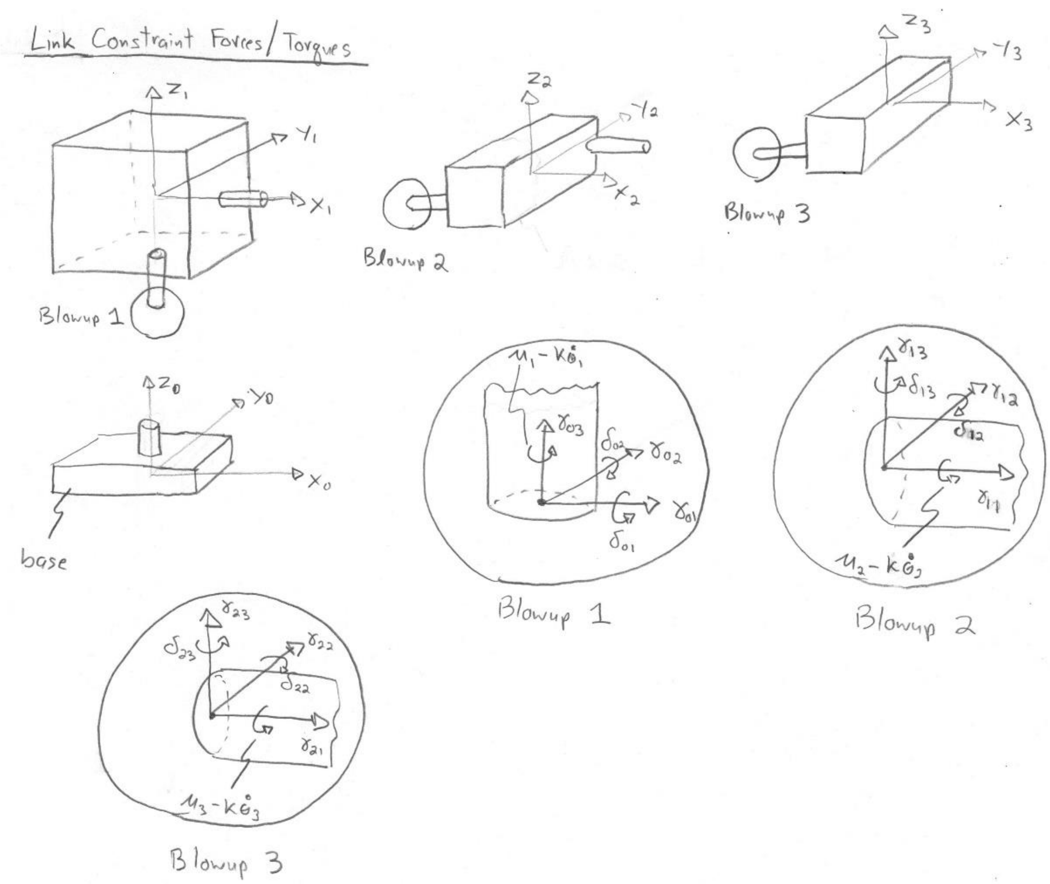
Since all the links are rectangular prisms, we can find the moment of inertia matrices using the equation we derived in homework 4.

$$\boxed{J_i^i = \begin{bmatrix} m_i\dfrac{dy_i^2 + dz_i^2}{12} & 0 & 0 \\ 0 & m_i\dfrac{dx_i^2 + dz_i^2}{12} & 0 \\ 0 & 0 & m_i\dfrac{dx_i^2 + dy_i^2}{12} \end{bmatrix} \quad \text{for } i = 1,2,3}$$

- constraint forces and torques

$$f_{01}^0, \tau_{01}^0, f_{12}^1, \tau_{12}^1, f_{23}^2, \tau_{23}^2$$

First sketch the revolute joints that connect each link.



From blowup 1 (joint 01) we have

$$\boxed{f_{01}^0 = \begin{bmatrix} \gamma_{01} \\ \gamma_{02} \\ \gamma_{03} \end{bmatrix}} \qquad \tau_{01}^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{01} \\ \delta_{02} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (u_1 - k\dot{\theta}_1) \Rightarrow \boxed{\tau_{01}^0 = S_{01} r_{01} + t_{01}(u_1 - k\dot{\theta}_1)}$$

From blowup 2 (joint 12) we have

$$\boxed{f_{12}^1 = \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \end{bmatrix}} \qquad \tau_{12}^1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{12} \\ \delta_{13} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} (u_2 - k\dot{\theta}_2) \Longrightarrow \boxed{\tau_{12}^1 = S_{12} r_{12} + t_{12}(u_2 - k\dot{\theta}_2)}$$

From blowup 3 (joint 23) we have

$$\boxed{f_{23}^2 = \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \\ \gamma_{23} \end{bmatrix}} \qquad \tau_{23}^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{22} \\ \delta_{23} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} (u_3 - k\dot{\theta}_3) \Longrightarrow \boxed{\tau_{23}^2 = S_{23} r_{23} + t_{23}(u_3 - k\dot{\theta}_3)}$$

Note that it wasn't absolutely necessary for us to write out f_{01}^0, f_{12}^1, and f_{23}^2 in terms of the γ_{ij} to finish solving the problem. In Newton's equations we will only be referencing the forces, not their components individually. However, breaking the forces into components is useful if we are interested in the forces acting between at the joints (if for instance we wanted to model structural failure at some maximum load).

Also note that $f_{12}^1 = -f_{21}^1$ (reaction forces).

- equations of motion
  - (a) write both Newton's Equation and Euler's Equation for each link separately
  - (b) plug in for the linear and angular accelerations $(\dot{v}_{0,1}^0, \dot{w}_{0,1}^1, \dot{v}_{0,1}^0, \dot{w}_{0,2}^2, \dot{v}_{0,3}^0, \dot{w}_{0,3}^3)$ and for the torques $(\tau_{01}^0, \tau_{12}^1, \tau_{23}^2)$
  - (c) put unknowns on left-hand-side and knowns on right-hand-side

Note that all the Newton's equations will be written in frame 0. This is useful because it is easy to think about translations w.r.t. some fixed frame (e.g. the Earth). On the other hand, all the Euler's equations will be written in the local frame. This done because the term J_i^i is constant in this frame (no need to use parallel axis theorem).

For the pin locations, we know p_{01}^0, p_{12}^1, and p_{23}^2 (these are defined in Matlab code). In the EE equations that follow I will not rewrite the p that we need in terms of these p values. I can do this because each of the p don't depend on any of the unknowns. I will instead have to add 3 extra lines to the GetRates function that calculates p in the frame we need using the point transformation equation.

Link 1 NE

$$m_1 \dot{v}_{0,1}^0 = f_{01}^0 - R_1^0 f_{12}^1 - m_1 z_0^0 g$$ where $g = 9.81\ m/s^2$

$$\dot{v}_{0,1}^0 = \frac{1}{m_1} f_{01}^0 - \frac{1}{m_1} R_1^0 f_{12}^1 - z_0^0 g$$ Substitute $\dot{v}_{0,1}^0$

$$R_1^0 \widehat{\omega_{0,1}^1}^2 b_1 - R_1^0 \widehat{b_1} z_0^0 \ddot{\theta}_1 = \frac{1}{m_1} f_{01}^0 - \frac{1}{m_1} R_1^0 f_{12}^1 - z_0^0 g$$ Unknowns to LHS, knowns to RHS

$$\boxed{-R_1^0 \widehat{b_1} z_0^0 \ddot{\theta}_1 - \frac{1}{m_1} f_{01}^0 + \frac{1}{m_1} R_1^0 f_{12}^1 = -R_1^0 \widehat{\omega_{0,1}^1}^2 b_1 - z_0^0 g}$$

Link 1 EE

$$J_1^1\dot{\omega}_{0,1}^1 + \widehat{\omega_{0,1}^1}J_1^1\omega_{0,1}^1 = R_1^{0^T}\tau_{01}^0 - \tau_{12}^1 + \widehat{p_{01}^1}R_1^{0^T}f_{01}^0 - \widehat{p_{12}^1}f_{12}^1$$

$$J_1^1 z_0^0\ddot{\theta}_1 + \widehat{\omega_{0,1}^1}J_1^1\omega_{0,1}^1 = R_1^{0^T}\left(S_{01}r_{01} + t_{01}\left(u_1 - k\dot{\theta}_1\right)\right) - \left(S_{12}r_{12} + t_{12}\left(u_2 - k\dot{\theta}_2\right)\right) + \widehat{p_{01}^1}R_1^{0^T}f_{01}^0 - \widehat{p_{12}^1}f_{12}^1$$

$$J_1^1 z_0^0\ddot{\theta}_1 + \widehat{\omega_{0,1}^1}J_1^1\omega_{0,1}^1 = R_1^{0^T}S_{01}r_{01} + R_1^{0^T}t_{01}\left(u_1 - k\dot{\theta}_1\right) - S_{12}r_{12} - t_{12}\left(u_2 - k\dot{\theta}_2\right) + \widehat{p_{01}^1}R_1^{0^T}f_{01}^0 - \widehat{p_{12}^1}f_{12}^1$$

Get unknowns on LHS, knowns on RHS

$$\boxed{J_1^1 z_0^0\ddot{\theta}_1 - R_1^{0^T}S_{01}r_{01} + S_{12}r_{12} - \widehat{p_{01}^1}R_1^{0^T}f_{01}^0 + \widehat{p_{12}^1}f_{12}^1 = R_1^{0^T}t_{01}\left(u_1 - k\dot{\theta}_1\right) - t_{12}\left(u_2 - k\dot{\theta}_2\right) - \widehat{\omega_{0,1}^1}J_1^1\omega_{0,1}^1}$$

Link 2 NE

$$m_2\dot{v}_{0,2}^0 = R_1^0 f_{12}^1 - R_2^0 f_{23}^2 - m_2 z_0^0 g$$

$$R_1^0\widehat{\omega_{0,1}^1}^2(b_1 + a_2) + R_2^0\widehat{\omega_{0,2}^2}^2 b_2 + R_2^0\widehat{b_2}R_2^{1^T}\widehat{R_2^1\omega_{1,2}^2}\omega_{0,1}^1 - \left(R_1^0\widehat{(b_1 + a_2)} + R_2^0\widehat{b_2}R_2^{1^T}\right)z_0^0\ddot{\theta}_1 - R_2^0\widehat{b_2}x_1^1\ddot{\theta}_2 = \frac{1}{m_2}R_1^0 f_{12}^1 - \frac{1}{m_2}R_2^0 f_{23}^2 - z_0^0 g$$

Get unknowns on LHS, knowns on RHS

$$\boxed{\begin{aligned}&-\left(R_1^0\widehat{(b_1 + a_2)} + R_2^0\widehat{b_2}R_2^{1^T}\right)z_0^0\ddot{\theta}_1 - R_2^0\widehat{b_2}x_1^1\ddot{\theta}_2 + \frac{1}{m_2}R_2^0 f_{23}^2 - \frac{1}{m_2}R_1^0 f_{12}^1 \\ &\qquad = -R_1^0\widehat{\omega_{0,1}^1}^2(b_1 + a_2) - R_2^0\widehat{\omega_{0,2}^2}^2 b_2 - R_2^0\widehat{b_2}R_2^{1^T}\widehat{R_2^1\omega_{1,2}^2}\omega_{0,1}^1 - z_0^0 g\end{aligned}}$$

Link 2 EE

$$J_2^2\dot{\omega}_{0,2}^2 + \widehat{\omega_{0,2}^2}J_2^2\omega_{0,2}^2 = R_2^{1^T}\tau_{12}^1 - \tau_{23}^2 + \widehat{p_{12}^2}R_2^{1^T}f_{12}^1 - \widehat{p_{23}^2}f_{23}^2$$

$$J_2^2\dot{\omega}_{0,2}^2 + \widehat{\omega_{0,2}^2}J_2^2\omega_{0,2}^2 = R_2^{1^T}\left(S_{12}r_{12} + t_{12}\left(u_2 - k\dot{\theta}_2\right)\right) - \left(S_{23}r_{23} + t_{23}\left(u_3 - k\dot{\theta}_3\right)\right) + \widehat{p_{12}^2}R_2^{1^T}f_{12}^1 - \widehat{p_{23}^2}f_{23}^2$$

$$J_2^2\dot{\omega}_{0,2}^2 = -\widehat{\omega_{0,2}^2}J_2^2\omega_{0,2}^2 + R_2^{1^T}S_{12}r_{12} + R_2^{1^T}t_{12}\left(u_2 - k\dot{\theta}_2\right) - S_{23}r_{23} - t_{23}\left(u_3 - k\dot{\theta}_3\right) + \widehat{p_{12}^2}R_2^{1^T}f_{12}^1 - \widehat{p_{23}^2}f_{23}^2$$

$$J_2^2\left(-R_2^{1^T}\widehat{R_2^1\omega_{1,2}^2}\omega_{0,1}^1 + R_2^{1^T}z_0^0\ddot{\theta}_1 + x_1^1\ddot{\theta}_2\right) = -\widehat{\omega_{0,2}^2}J_2^2\omega_{0,2}^2 + R_2^{1^T}S_{12}r_{12} + R_2^{1^T}t_{12}\left(u_2 - k\dot{\theta}_2\right) - S_{23}r_{23} - t_{23}\left(u_3 - k\dot{\theta}_3\right) + \widehat{p_{12}^2}R_2^{1^T}f_{12}^1 - \widehat{p_{23}^2}f_{23}^2$$

$$
\begin{aligned}
-J_2^2 R_2^{1^T} \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 + J_2^2 R_2^{1^T} z_0^0 \ddot{\theta}_1 + x_1^1 \ddot{\theta}_2 \\
&= -\widehat{\omega_{0,2}^2} J_2^2 \omega_{0,2}^2 + R_2^{1^T} S_{12} r_{12} + R_2^{1^T} t_{12}\left(u_2 - k\dot{\theta}_2\right) - S_{23} r_{23} - t_{23}\left(u_3 - k\dot{\theta}_3\right) \\
&+ \widehat{p_{12}^2} R_2^{1^T} f_{12}^1 - \widehat{p_{23}^2} f_{23}^2
\end{aligned}
$$

Get unknowns on LHS, knowns on RHS

$$
\boxed{
\begin{aligned}
J_2^2 R_2^{1^T} z_0^0 \ddot{\theta}_1 + x_1^1 \ddot{\theta}_2 - R_2^{1^T} S_{12} r_{12} + S_{23} r_{23} + \widehat{p_{23}^2} f_{23}^2 - \widehat{p_{12}^2} R_2^{1^T} f_{12}^1 \\
&= J_2^2 R_2^{1^T} \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 - \widehat{\omega_{0,2}^2} J_2^2 \omega_{0,2}^2 + R_2^{1^T} t_{12}\left(u_2 - k\dot{\theta}_2\right) - t_{23}\left(u_3 - k\dot{\theta}_3\right)
\end{aligned}
}
$$

Link 3 NE

$$
m_3 \dot{v}_{0,3}^0 = R_2^0 f_{23}^2 - m_3 z_0^0 g
$$

$$
\dot{v}_{0,3}^0 = \frac{1}{m_3} R_2^0 f_{23}^2 - z_0^0 g
$$

$$
\begin{aligned}
R_1^0 \widehat{\omega_{0,1}^1}^2 (b_1 + a_2) + R_2^0 \widehat{\omega_{0,2}^2}^2 (b_2 + a_3) + R_3^0 \widehat{\omega_{0,3}^3}^2 b_3 + R_2^0 (\widehat{b_2 + a_3}) R_2^{1^T} \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 \\
&+ R_3^0 \widehat{b_3} R_3^{1^T} \widehat{R_3^1 \omega_{1,3}^3} \omega_{0,1}^1 + R_3^0 \widehat{b_3} R_3^{2^T} \widehat{R_3^2 \omega_{2,3}^3} \omega_{1,2}^2 \\
&- \left(R_1^0 (\widehat{b_1 + a_2}) z_0^0 + R_2^0 (\widehat{b_2 + a_3}) R_2^{1^T} z_0^0 + R_3^0 \widehat{b_3} R_3^{1^T} z_0^0\right) \ddot{\theta}_1 \\
&- \left(R_2^0 (\widehat{b_2 + a_3}) x_1^1 + R_3^0 \widehat{b_3} R_3^{2^T} x_1^1\right) \ddot{\theta}_2 - R_3^0 \widehat{b_3} x_2^2 \ddot{\theta}_3 = \frac{1}{m_3} R_2^0 f_{23}^2 - z_0^0 g
\end{aligned}
$$

Get unknowns on LHS, knowns on RHS

$$
\boxed{
\begin{aligned}
-\left(R_1^0 (\widehat{b_1 + a_2}) z_0^0 + R_2^0 (\widehat{b_2 + a_3}) R_2^{1^T} z_0^0 + R_3^0 \widehat{b_3} R_3^{1^T} z_0^0\right) \ddot{\theta}_1 - \left(R_2^0 (\widehat{b_2 + a_3}) x_1^1 + R_3^0 \widehat{b_3} R_3^{2^T} x_1^1\right) \ddot{\theta}_2 \\
&- R_3^0 \widehat{b_3} x_2^2 \ddot{\theta}_3 - \frac{1}{m_3} R_2^0 f_{23}^2 \\
&= -z_0^0 g - R_1^0 \widehat{\omega_{0,1}^1}^2 (b_1 + a_2) - R_2^0 \widehat{\omega_{0,2}^2}^2 (b_2 + a_3) - R_3^0 \widehat{\omega_{0,3}^3}^2 b_3 \\
&- R_2^0 (\widehat{b_2 + a_3}) R_2^{1^T} \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 - R_3^0 \widehat{b_3} R_3^{1^T} \widehat{R_3^1 \omega_{1,3}^3} \omega_{0,1}^1 - R_3^0 \widehat{b_3} R_3^{2^T} \widehat{R_3^2 \omega_{2,3}^3} \omega_{1,2}^2
\end{aligned}
}
$$

Link 3 EE

$$
J_3^3 \dot{\omega}_{0,3}^3 + \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3 = R_3^{2^T} \tau_{23}^2 + \widehat{p_{23}^3} R_3^{2^T} f_{23}^2
$$

$$
J_3^3 \dot{\omega}_{0,3}^3 + \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3 = R_3^{2^T} \left(S_{23} r_{23} + t_{23}\left(u_3 - k\dot{\theta}_3\right)\right) + \widehat{p_{23}^3} R_3^{2^T} f_{23}^2
$$

$$
J_3^3 \dot{\omega}_{0,3}^3 = R_3^{2^T} S_{23} r_{23} + R_3^{2^T} t_{23}\left(u_3 - k\dot{\theta}_3\right) + \widehat{p_{23}^3} R_3^{2^T} f_{23}^2 - \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3
$$

$$
\begin{aligned}
J_3^3 \left(-R_3^{1^T} \widehat{R_3^1 \omega_{1,3}^3} \omega_{0,1}^1 - R_3^{2^T} \widehat{R_3^2 \omega_{2,3}^3} \omega_{1,2}^2 + R_3^{1^T} z_0^0 \ddot{\theta}_1 + R_3^{2^T} x_1^1 \ddot{\theta}_2 + x_2^2 \ddot{\theta}_3\right) \\
&= R_3^{2^T} S_{23} r_{23} + R_3^{2^T} t_{23}\left(u_3 - k\dot{\theta}_3\right) + \widehat{p_{23}^3} R_3^{2^T} f_{23}^2 - \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3
\end{aligned}
$$

$$
\begin{aligned}
-J_3^3 R_3^{1^T} R_3^1 \widehat{\omega_{1,3}^3} \omega_{0,1}^1 - J_3^3 R_3^{2^T} R_3^2 \widehat{\omega_{2,3}^3} \omega_{1,2}^2 + J_3^3 R_3^{1^T} z_0^0 \ddot{\theta}_1 + J_3^3 R_3^{2^T} x_1^1 \ddot{\theta}_2 + J_3^3 x_2^2 \ddot{\theta}_3 \\
= R_3^{2^T} S_{23} r_{23} + R_3^{2^T} t_{23}(u_3 - k\dot{\theta}_3) + \widehat{p_{23}^3} R_3^{2^T} f_{23}^2 - \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3
\end{aligned}
$$

Get unknowns on LHS, knowns on RHS

$$
\boxed{
\begin{aligned}
+J_3^3 R_3^{1^T} z_0^0 \ddot{\theta}_1 + J_3^3 R_3^{2^T} x_1^1 \ddot{\theta}_2 + J_3^3 x_2^2 \ddot{\theta}_3 - R_3^{2^T} S_{23} r_{23} - \widehat{p_{23}^3} R_3^{2^T} f_{23}^2 \\
= R_3^{2^T} t_{23}(u_3 - k\dot{\theta}_3) - \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3 + J_3^3 R_3^{1^T} R_3^1 \widehat{\omega_{1,3}^3} \omega_{0,1}^1 + J_3^3 R_3^{2^T} R_3^2 \widehat{\omega_{2,3}^3} \omega_{1,2}^2
\end{aligned}
}
$$

(d) write in matrix form as $F\gamma = h$ where γ is a column matrix of unknowns, so you can solve easily in MATLAB as $\gamma = F^{-1}h$ (note that I switched from "g" to "γ" this week because "g" is now being used to denote the acceleration of gravity)

$$
\begin{array}{c}
\begin{array}{ccccccccc}
\ddot{\theta}_1 & f_{01}^1 & r_{01} & \ddot{\theta}_2 & f_{12}^1 & r_{12} & \ddot{\theta}_3 & f_{23}^2 & r_{23}
\end{array} \\
\begin{bmatrix}
-R_1^0 \hat{b}_1 z_0^0 & -\frac{1}{m_1} I_{3\times3} & O_{3\times2} & O_{3\times1} & \frac{1}{m_1} R_1^0 & O_{3\times2} & O_{3\times1} & O_{3\times3} & O_{3\times2} \\
J_1^1 z_0^0 & -\widehat{p_{c1}^1} R_1^{0^T} & -R_1^{0^T} S_{01} & O_{3\times1} & \widehat{p_{12}^1} & S_{12} & O_{3\times1} & O_{3\times3} & O_{3\times2} \\
\alpha & O_{3\times3} & O_{3\times2} & -R_2^0 \hat{b}_2 x_1^1 & -\frac{1}{m_2} R_1^0 & O_{3\times2} & O_{3\times1} & \frac{1}{m_2} R_2^0 & O_{3\times2} \\
J_2^2 R_2^{1^T} z_0^0 & O_{3\times3} & O_{3\times2} & x_1^1 & -\widehat{p_{12}^2} R_2^{1^T} & -R_2^{1^T} S_{12} & O_{3\times1} & \widehat{p_{23}^2} & S_{23} \\
\beta & O_{3\times3} & O_{3\times2} & \Lambda & O_{3\times3} & O_{3\times2} & -R_3^0 \hat{b}_3 x_2^2 & -\frac{1}{m_3} R_2^0 & O_{3\times2} \\
J_3^3 R_3^{1^T} z_0^0 & O_{3\times3} & O_{3\times2} & J_3^3 R_3^{2^T} x_1^1 & O_{3\times3} & O_{3\times2} & J_3^3 x_2^2 & -\widehat{p_{23}^3} R_3^{2^T} & -R_3^{2^T} S_{23}
\end{bmatrix}
\end{array}
\begin{bmatrix}
\ddot{\theta}_1 \\ f_{01}^0 \\ r_{01} \\ \ddot{\theta}_2 \\ f_{12}^1 \\ r_{12} \\ \ddot{\theta}_3 \\ f_{23}^2 \\ r_{23}
\end{bmatrix}
\updownarrow 18
$$

$$\alpha = -\left(R_1^0 \left(\widehat{b_1 + a_2}\right) + R_2^0 \hat{b}_2 R_2^{1^T}\right) z_0^0$$

$$\beta = -\left(R_1^0 \left(\widehat{b_1 + a_2}\right) + R_2^0 \left(\widehat{b_2 + a_3}\right) R_2^{1^T} + R_3^0 \hat{b}_3 R_3^{1^T} \right) z_0^0$$

$$\Lambda = -\left(R_2^0 \left(\widehat{b_2 + a_3}\right) + R_3^0 \hat{b}_3 R_3^{2^T} \right) x_1^1$$

$$
h = \begin{bmatrix} -R_1^0 \widehat{\omega_{0,1}^1}^2 b_1 - z_0^0 g \\ {R_1^0}^T t_{01}\left(u_1 - k\dot{\theta}_1\right) - t_{12}\left(u_2 - k\dot{\theta}_2\right) - \widehat{\omega_{0,1}^1} J_1^1 \omega_{0,1}^1 \\ -R_1^0 \widehat{\omega_{0,1}^1}^2 (b_1 + a_2) - R_2^0 \widehat{\omega_{0,2}^2}^2 b_2 - R_2^0 \widehat{b_2} {R_2^1}^T \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 - z_0^0 g \\ J_2^2 {R_2^1}^T \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 - \widehat{\omega_{0,2}^2} J_2^2 \omega_{0,2}^2 + {R_2^1}^T t_{12}\left(u_2 - k\dot{\theta}_2\right) - t_{23}\left(u_3 - k\dot{\theta}_3\right) \\ \sigma \\ {R_3^2}^T t_{23}\left(u_3 - k\dot{\theta}_3\right) - \widehat{\omega_{0,3}^3} J_3^3 \omega_{0,3}^3 + J_3^3 {R_3^1}^T \widehat{R_3^1 \omega_{1,3}^3} \omega_{0,1}^1 + J_3^3 {R_3^2}^T \widehat{R_3^2 \omega_{2,3}^3} \omega_{1,2}^2 \end{bmatrix}
$$

Where

$$
\begin{aligned}
\sigma = -z_0^0 g - R_1^0 \widehat{\omega_{0,1}^1}^2 (b_1 + a_2) - R_2^0 \widehat{\omega_{0,2}^2}^2 (b_2 + a_3) - R_3^0 \widehat{\omega_{0,3}^3}^2 b_3 - R_2^0 (\widehat{b_2 + a_3}) {R_2^1}^T \widehat{R_2^1 \omega_{1,2}^2} \omega_{0,1}^1 \\
- R_3^0 \widehat{b_3} {R_3^1}^T \widehat{R_3^1 \omega_{1,3}^3} \omega_{0,1}^1 - R_3^0 \widehat{b_3} {R_3^2}^T \widehat{R_3^2 \omega_{2,3}^3} \omega_{1,2}^2
\end{aligned}
$$

2. (60 pts) You must implement everything marked "must change" in **hw6.m**. (You may, of course, also play around with anything marked "can change.") Submit a print-out *only* of the lines of code that are changed, in the order in which these lines appear in **hw6.m**.

```
function [R_1in0, R_2in1, R_3in2] = GetR(theta)

% Get R_1in0
s1 = sin(theta(1));     c1 = cos(theta(1));
s2 = sin(theta(2));     c2 = cos(theta(2));
s3 = sin(theta(3));     c3 = cos(theta(3));

R_1in0 = [c1 -s1  0;
          s1  c1  0;
          0   0   1];

R_2in1 = [1  0   0;
          0  c2 -s2;
          0  s2  c2];

R_3in2 = [1  0   0;
          0  c3 -s3;
          0  s3  c3];

function a_hat = wedge(a)

a_hat = [0 -a(3) a(2);
         a(3) 0 -a(1);
         -a(2) a(1) 0];

function [m, J] = GetmJ( link )
rho = link.rho;
dx  = link.dx;
dy  = link.dy;
dz  = link.dz;
m = rho*dx*dy*dz;
J = m/12*diag([dy^2+dz^2; dx^2+dz^2; dx^2 + dy^2]);


% - Mass and moment of inertia of link #1
[robot.link1.m, robot.link1.J_in1] = GetmJ ( robot.link1 );

% - Mass and moment of inertia of link #2
[robot.link2.m, robot.link2.J_in2] = GetmJ ( robot.link2 );

% - Mass and moment of inertia of link #3
[robot.link3.m, robot.link3.J_in3] = GetmJ ( robot.link3 );


    % Compute: orientation and position of link #1
    %   (R_1in0 and o_1in0)
    [R_1in0, R_2in1, R_3in2] = GetR(theta);
    o_1in0 = robot.a1 + R_1in0*robot.b1;

    % Compute: orientation and position of link #2
```

```
    %   (R_2in0 and o_2in0 ... hint: first compute R_2in1 and o_2in1)
    R_2in0 = R_1in0 * R_2in1;
    o_2in0 = robot.a1 + R_1in0*(robot.b1 + robot.a2) + R_2in0*robot.b2;

    % Compute: orientation and position of link #3
    %   (R_3in0 and o_3in0 ... hint: first compute R_3in2 and o_3in2)
    R_3in0 = R_2in0 * R_3in2;
    o_3in0 = robot.a1 +
R_1in0*(robot.b1+robot.a2)+R_2in0*(robot.b2+robot.a3)+R_3in0*robot.b3;

    % Compute: robot.link1.p_in0, robot.link2.p_in0, robot.link3.p_in0
    for i = 1:size(robot.link1.p_in1, 2)
        robot.link1.p_in0(:,i) = o_1in0 + R_1in0 * robot.link1.p_in1(:,i);
    end
    for i = 1:size(robot.link2.p_in2, 2)
        robot.link2.p_in0(:,i) = o_2in0 + R_2in0 * robot.link2.p_in2(:,i);
    end
    for i = 1:size(robot.link3.p_in3 , 2)
        robot.link3.p_in0(:,i) = o_3in0 + R_3in0 * robot.link3.p_in3(:,i);
    end


function [thetadot,thetadotdot] = GetRates(theta,thetadot,u1,u2,u3,robot)
%
% inputs:
%
%   theta       3x1 matrix of joint angles
%   thetadot    3x1 matrix of joint velocities
%   u1          torque applied by motor to link 1 through joint 1
%   u2          torque applied by motor to link 2 through joint 2
%   u3          torque applied by motor to link 3 through joint 3
%   robot       a whole bunch of parameters (see GetGeometryOfRobot)
%
% outputs:
%
%   thetadot    3x1 matrix of joint velocities
%   thetadotdot 3x1 matrix of joint accelerations

% Get all required R matrices
[R_1in0, R_2in1, R_3in2] = GetR(theta);
R_2in0 = R_1in0 * R_2in1;
R_3in0 = R_2in0 * R_3in2;
R_3in1 = R_2in1 * R_3in2;

a1 = robot.a1;
b1 = robot.b1;
a2 = robot.a2;
b2 = robot.b2;
a3 = robot.a3;
b3 = robot.b3;

% Get all coordinate frame position vectors
o_1in0 = a1 + R_1in0 * b1;
o_2in1 = a2 + R_2in1 * b2;
o_3in2 = a3 + R_3in2 * b3;
```

```
p_01in0 = robot.p_01in0;
p_12in1 = robot.p_12in1;
p_23in2 = robot.p_23in2;
p_01in1 = R_1in0'*(p_01in0 - o_1in0);
p_12in2 = R_2in1'*(p_12in1 - o_2in1);
p_23in3 = R_3in2'*(p_23in2 - o_3in2);

k = robot.kfriction;
g = 9.81;       % m/s/s

% Define the constant Matrices
z_0in0 = [0; 0; 1];
x_1in1 = [1; 0; 0];
x_2in2 = [1; 0; 0];
S01 = [1 0; 0 1; 0 0];
S12 = [0 0; 1 0; 0 1];
S23 = [0 0; 1 0; 0 1];
t01 = [0; 0; 1];
t12 = [1; 0; 0];
t23 = [1; 0; 0];

% Calculate angular rates
w_01in1 = z_0in0 * thetadot(1);
w_12in2 = x_1in1 * thetadot(2);
w_23in3 = x_2in2 * thetadot(3);
w_02in2 = R_2in1' * w_01in1 + w_12in2;
w_03in3 = R_3in1' * w_01in1 + R_3in2' * w_12in2 + w_23in3;
w_12in1 = R_2in1 * w_12in2;
w_13in3 = R_3in2' * w_12in2 + w_23in3;

% Redifine masses with shorter names to make code more readable
m1 = robot.link1.m;
m2 = robot.link2.m;
m3 = robot.link3.m;

% Redifine moments of inertia with shorter names to make code more readable
J_1in1 = robot.link1.J_in1;
J_2in2 = robot.link2.J_in2;
J_3in3 = robot.link3.J_in3;

z33 = zeros(3,3);
z31 = zeros(3,1);
z32 = zeros(3,2);

alpha  = -(R_1in0*wedge(b1+a2) + R_2in0*wedge(b2)*R_2in1')*z_0in0;
beta   = -(R_1in0*wedge(b1+a2) + R_2in0*wedge(b2+a3)*R_2in1' +
R_3in0*wedge(b3)*R_3in1')*z_0in0;
lambda = -(R_2in0*wedge(b2+a3) + R_3in0*wedge(b3)*R_3in2')*x_1in1;
```

```
F = [-R_1in0*wedge(b1)*z_0in0, -1/m1*eye(3,3), z32, z31, 1/m1*R_1in0, z32,
z31, z33, z32;
     J_1in1*z_0in0, -wedge(p_01in0)*R_1in0', -R_1in0'*S01, z31,
wedge(p_12in1), S12, z31, z33, z32;

     alpha, z33, z32, -R_2in0*wedge(b2)*x_1in1, -1/m2*R_1in0, z32, z31,
1/m2*R_2in0, z32;
     J_2in2*R_2in1'*z_0in0, z33, z32, x_1in1, -wedge(p_12in2)*R_2in1', -
R_2in1'*S12, z31, wedge(p_23in2), S23;

     beta, z33, z32, lambda, z33, z32, -R_3in0*wedge(b3)*x_2in2, -1/m3*R_2in0
z32;
     J_3in3*R_3in1'*z_0in0, z33, z32, J_3in3*R_3in2'*x_1in1, z33, z32,
J_3in3*x_2in2, -wedge(p_23in3)*R_3in2', -R_3in2'*S23];


h = [ -R_1in0*wedge(w_01in1)^2*b1 - z_0in0*g;
       R_1in0'*t01*(u1 - k*thetadot(1)) - t12*(u2-k*thetadot(2)) -
wedge(w_01in1)*J_1in1*w_01in1;

      -R_1in0*wedge(w_01in1)^2*(b1+a2) - R_2in0*wedge(w_02in2)^2*b2 ...
            - R_2in0*wedge(b2)*R_2in1'*wedge(R_2in1*w_12in2)*w_01in1 -
z_0in0*g;
       J_2in2*R_2in1'*wedge(R_2in1*w_12in2)*w_01in1 -
wedge(w_02in2)*J_2in2*w_02in2 ...
            + R_2in1'*t12*(u2-k*thetadot(2)) - t23*(u3 - k*thetadot(3));

      -z_0in0*g - R_1in0'*wedge(w_01in1)^2*(b1+a2) -
R_2in0*wedge(w_02in2)^2*(b2+a3) ...
            - R_3in0*wedge(w_03in3)^2*b3 ...
            - R_2in0*wedge(b2+a3)*R_2in1'*wedge(R_2in1*w_12in2)*w_01in1 ...
            - R_3in0*wedge(b3)*R_3in1'*wedge(R_3in1*w_13in3)*w_01in1 ...
            - R_3in0*wedge(b3)*R_3in2'*wedge(R_3in2*w_23in3)*w_12in2;
       R_3in2'*t23*(u3-k*thetadot(3)) - wedge(w_03in3)*J_3in3*w_03in3 ...
            + J_3in3*R_3in1'*wedge(R_3in1*w_13in3)*w_01in1 ...
            + J_3in3*R_3in2'*wedge(R_3in2*w_23in3)*w_12in2];

gamma = inv(F)*h;
thetadot = thetadot;
thetadotdot = [gamma(1); gamma(7); gamma(13)];
```