Hill Climbing vs Simulated Annealing

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Abstract

Documentul prezinta o analiza ampla asupra algoritmilor de Hill Climbing si Simulated Annealing, punand in prim-plan performantele acestora, precum si particularitatile lor. Vom utiliza variatii ale algoritmului Hill Climbing si impactul functiilor neighbour pentru rezultatele obtinute.

1 Introducere

Scopul final al temei este atingerea minimului unei functii complexe utilizand variatii ale algoritmului de Hill Climbing si Simulated Annealing. Hill Climbing este o tehnica de optimizare matematica din familia cautarilor locale care gaseste solutii optime pentru problem convexe, iar pentru celelalte functii se opreste doar la optime locale ce nu sunt neaparat cele mai bune din spatiul de cautare. Pentru a observa particularitatile, vom executa o serie de experimente in care vom utiliza functiile DeJong, Schwefel's, Rastrigin's, Michaelwicz's , analizand performantele pentru 5,10,30 de dimensiuni in cazul fiecarei functii. Atat algoritmii de Hill Climbing, cat si cel de Simulated Annealing cauta in maniera euristica solutia optima. In continuare, vom urmari modul de functionare si rezultatele obtinute, precum si diferentele dintre cele 4 variante propuse: Best Improvement Hill Climbing, First Improvement Hill Climbing, Worst Improvement Hill Climbing si Simulated Annealing.

1.1 Functii

De Jong's function 1[0]:

$$f(x) = \sum_{i=1}^{n} x_i^2, x_i \in [-5.12, 5.15]$$

Schwefel's function 7[0]:

$$f(x) = \sum_{i=1}^{n} -x_i \cdot sin(\sqrt{|x_i|}), x_i \in [-500, 500]$$

Rastrigin's function 6[0]:

$$f(x) = A \cdot n + \sum_{i=1}^{n} \left[x_i^2 - A \cdot cos(2\pi x_i)\right], A = 10, x_i \in [-5.12, 5.15]$$

Michalewicz's function 12[0]:

$$f(x) = -\sum_{i=1}^{n} sin(x_i) \cdot (sin(\frac{i \cdot x_i^2}{\pi}))^{2 \cdot m}, i = 1 : n, m = 10, 0 \leq x_i \leq \pi$$

2 Metode

2.1 Hill Climbing

Hill Climbing este o tehnica care porneste de la o solutie aleatoare si va incerca sa gaseasca optimul prin imbunatatiri successive. In cazul in care functia nu este convexa, exista riscul blocarii in minime locale, moment in care se poate reporni algortmul dintr-un nou punct aleator. O imbunatatire pentru aceasta problema este agloritmul de Simulated Annealing.

Pasi de executie:

Fie S $\in \omega$ [1], o solutie generata aleator

Cat timp nu am gasit rezultatul si k [2] $< k_{max}$

Alegem n $\in$ neighbours[3](S)

Daca $f(n) < f(S) => S = n$

O particularitate a implementarii utilizate este reprezentarea solutiei sub forma binara, mai exact un vector de lungime $d * \lceil log_2((b - a) * 10^p)\rceil$, unde d este dimensiunea functiei, a si b sunt capetele de interval, iar p este precizia utilizata in reprezentarea numerica.

In cadrul procedurii de initializare vom construi aleator o solutie de pornire S $\in \omega$, spatiul de cautare.

Functia neighbors(s) difera in functie de variatia algoritmului Hill Climbing. La baza, toate folosesc o notiune de vecinatate comuna. In acest context, consideram vecin orice reprezentare binara ce difera de S printr-un singur bit.

Cautarea se opreste atunci cand k, contorul iteratiei curente, ajunge la valoarea k_{max} sau cand evaluarea solutiei $f(S) < \epsilon$. Valoarea acestui ϵ este determinat de minimul functiei pentru care vom executa experimentul.

[1] ω = spatiul de cautare

[2] k = pasul curent

[3] neighbours(S) = multimea vecinilor lui S

2.1.1 First Improvement

Vom alege primul vecin cu o solutie imbunatatita fata de cea originala.

2.1.2 Best Improvement

Dintre vecinii generati il vom alege pe cel cu cel mai bun improvement.

2.1.3 Worst Improvement

Vom alege vecinul cu imbunatatirea cea mai slaba.

2.2 Simulated Annealing

Simulated Annealing este un algoritm capabil sa scape din optimele locale. Este o tehnica populara datorita usurintei de implementare, proprietatilor de convergenta si utilizarea Hill Climbing pentru evadarea din minime locale. Numele provine de la analogia cu procesul de incalzire, urmat de racire lenta pentru a favoriza solidificarea uniforma, fara defecte. Daca procesul de racire este suficient de lent, atunci produsul va avea o integritate structurala superioara. Algoritmul simuleaza acest comportament termodinamic in cautarea minumului global. Sa adopta o miscare iterativa in functie de variatia temperaturii pentru a imita acest proces. Pornim de la o temperatura de start arbitrar aleasa. Numarul de vecini verificati la fiecare pas este proportional cu temperatura. Dintre acestia, alegem solutia mai buna, insa exista probabilitatea alegerii unei solutii mai slabe (bazata pe formula T), in speranta ca vecinii acestei noi solutii ar putea duce totusi la un rezultat mai bun. Totodata generarea vecinilor este influentata de temperatura, aceasta nefiind limitata la editarea unui singur bit, spre deosebire de implementarea folosita in cadrul Hill Climbingului.

Fie S $\in \omega$, o solutie generata aleator
$T = T_{initial}$
Cat timp $f(S) \geq \epsilon$ sau $k < k_{max}$:
    $M_k = MT_k(T)$[4]
    Pentru $i = 0, i < M_k$:
        Fie $S' \in neighbours(S, T)$[5]
        Daca $f(S') < f(S)$[6] $=> S = S'$
        Altfel, S = S', cu probabilitatea $exp(\frac{-|f(S') - f(S)|}{T})$
    T = q(T)[7]

Reprezentarea solutiei se face in acelasi mod sugerat la algoritmul de Hill Climbing, insa pentru generarea vecinilor propunem o varianta influentata de temperatura, ce ofera rezultate mai bune comparative cu schimbarea aleatorie a unui

[4] $MT_k(T)$ = functia ce returneaza numarul de vecini verificati la temperatura T;
[5] neighbours(S,T) = vecinii lui S, in functie de temperatura T;
[6] f(S) = functia de optimizat
[7] q(T) = functia de racire

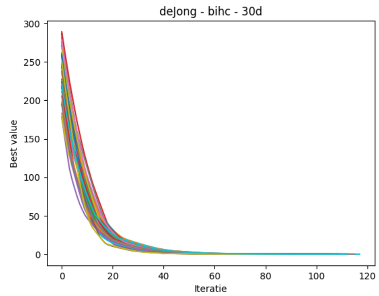
singur bit (in special in testele pe probleme cu dimensionalitate crescuta). Astfel, numarul de biti ce pot fi modificati in mod aleatoriu sunt proportionali cu numarul de dimensiuni ale problemei si temperatura curenta. In executie putem observa ca la inceput (T este mare), algoritmul exploreaza un numar mai mare de vecini (conform valorii lui $MT_k(T)$), si mai variati ($neighbours(S, T)$), urmand ca, impreuna cu scaderea T, numarul de vecini verificati sa scada, impreuna cu variatia acestora.

3 Descrierea experimentelor

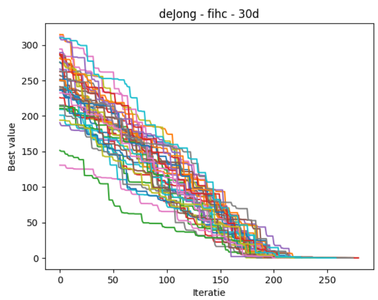
Am rulat algoritmii pe cele 4 functii prezentate mai sus, DeJong, Schwefel's, Rastrigin's, Michaelwicz's, cu cele 3 variante de dimensionalitate, $d \in \{5, 10, 30\}$. fiecare experiment a fost rulat de 40 de ori, pentru a obtine o viziune de ansamblu cat mai corecta asupra performantei fiecarui algoritm. In experimente, am utilizat $k_{max} = 5000$ si am observat ca aceasta valoare este suficent de mare pentru a produce rezultate satisfacatoare in cele mai multe cazuri. In ceea ce priveste precizia, aceasta a fost de 5 zecimale. experimentele au fost rulate cu o implementare in C++17 a algoritmilor descrisi, pe un sistem Ubuntu 20.04.5. Pentru a reduce considerabil timpul de executie am implementat o functionalitate de "kill on plateau", care opreste fortat un experiment Hill Climbing daca evaluarea nu se modifica timp de 150 de executii succesive. In cadrul Simulated Annealing am folosit $T_{initial} = 100$, si $q(T) = T \cdot 0.99$

4 Rezultate experimentale

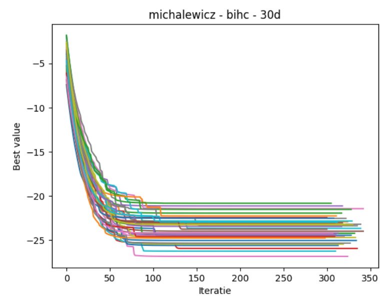
In urma rularii experimentelor am colectat urmatoarele rezultate. In sectiunea de mai jos se poate observa progresul algoritmilor de cautare a minimului functiilor, in raport cu iteratia curenta, precum si rezultatele fiecarei rulari.
Au fost executate 40 de rulari pentru fiecare permutare (functie - algoritm - dimensionalitate), exceptand Rastring - Simulated Annealing - 30 de dimensiuni, unde au fost executate doar 30 de iteratii.
Numarul redus de iteratii este dat de conditia de early stopping bazata pe ϵ, dar si de oprirea algoritmului in cazul in care acesta nu gaseste o solutie diferita (valabil doar pentru Hill Climbing).



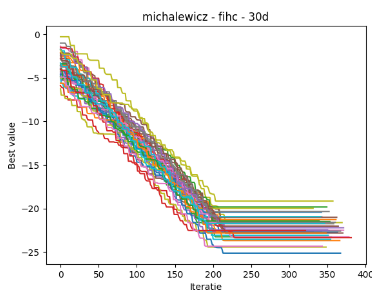
(a) deJong - Best Improvement



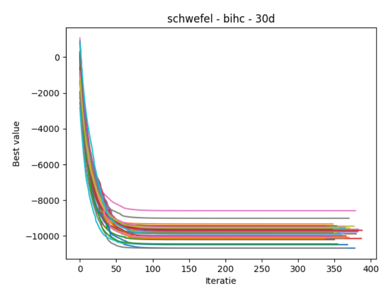
(b) deJong - First Improvement



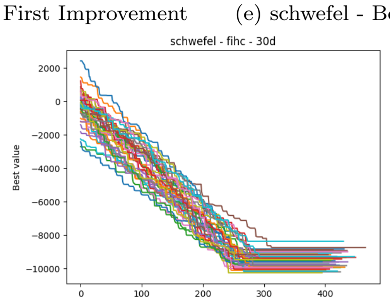
(c) michalewicz - Best Improvement



(d) michalewicz - First Improvement



(e) schwefel - Best Improvement



(f) schwefel - First Improvement

Figure 1: Evolutia minimului in functie de numarul de iteratii

4.1 Best Improvement Hill Climbing

Functie	D	Min	Max	Avg	Std	Mean	Q1	Q3
deJong	5	0.0031	0.00967	0.00754	0.0015	0.00754	0.00675	0.00871
deJong	10	0.0071	0.00998	0.00881	0.00081	0.00881	0.00825	0.00952
deJong	30	0.00905	0.00997	0.0096	0.00027	0.0096	0.00941	0.00983
schwefel	5	-1976.16	-1142.96	-1636.1385	213.49224	-1636.1385	-1824.62	-1503.295
schwefel	10	-3824.77	-2497.02	-3290.29575	288.24926	-3290.29575	-3473.8375	-3079.9875
schwefel	30	-10686.8	-8589.48	-9823.833	417.32899	-9823.833	-10095.375	-9553.8375
michalewicz	5	-4.6094	-2.92622	-3.83783	0.49337	-3.83783	-4.21466	-3.50423
michalewicz	10	-9.0125	-5.95644	-7.65102	0.763	-7.65102	-8.21891	-7.23091
michalewicz	30	-26.817	-20.8159	-23.66052	1.38422	-23.66052	-24.5724	-22.88192
rastringin	5	2.23078	18.4765	9.73974	3.57531	9.73974	7.56813	12.13152
rastringin	10	8.15926	36.0616	20.49974	7.58287	20.49974	14.78742	25.63278
rastringin	30	41.2225	85.7363	63.63507	10.09825	63.63507	56.53042	70.0331

Tabel 1: Rezultate Best Improvement Hill Climbing

D = numarul de dimensiuni; Min = valoarea minima; Max = valoarea maxima; Avg = media valorilor; Std = deviatia standard; Mean = mediana;

4.1.1 DeJong

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
9	0.00741055	37	0.00945665	98	0.0097818
13	0.00873875	36	0.00995516	105	0.00955233
10	0.00740351	25	0.00824943	110	0.00954661
14	0.00672896	30	0.00963109	113	0.00991742
10	0.00945906	26	0.0097432	99	0.00904748
14	0.0081993	42	0.00952353	95	0.00983263
12	0.00747393	32	0.00853148	116	0.00924658
20	0.00941347	33	0.00855225	113	0.0093391
15	0.00507305	37	0.00813293	101	0.0092121
14	0.00310352	23	0.00883615	109	0.0096454
20	0.00715597	28	0.00824294	114	0.00919527
12	0.00870511	32	0.00899718	109	0.00917949
14	0.00849124	28	0.00966091	98	0.00980379
13	0.00807148	29	0.00877622	113	0.00970863
13	0.00676342	31	0.00787445	103	0.00987692
12	0.00810789	25	0.00941074	105	0.00994366
10	0.00949253	34	0.00727477	104	0.00984468
16	0.00723725	29	0.00891725	93	0.00941515
12	0.00502516	32	0.00997914	97	0.00972564
17	0.00967497	24	0.00888079	118	0.00974108
18	0.00661454	29	0.00710258	104	0.00916784
12	0.00630787	31	0.00961702	106	0.00955483
18	0.00759328	32	0.00817876	97	0.00987057
18	0.00806884	30	0.00773066	88	0.00967243
11	0.00883378	25	0.00931274	93	0.00996626
16	0.00861964	35	0.00828347	107	0.00967472
21	0.00819812	28	0.00996993	107	0.00983989
15	0.003684	29	0.00886312	116	0.00925836
15	0.00788467	28	0.00952086	103	0.00962401
14	0.00628274	39	0.00726713	104	0.00951178
15	0.00631891	33	0.0094051	102	0.00939227
13	0.00928155	36	0.0072487	98	0.00990229
14	0.00758431	36	0.00931725	107	0.00973772
18	0.00882269	31	0.00959758	110	0.00960172
14	0.00675669	27	0.00851542	100	0.00904605
12	0.0072586	30	0.00811873	96	0.0098776
13	0.00634586	28	0.00845411	107	0.00972807
14	0.00723907	26	0.00913955	110	0.00942123
11	0.00915484	23	0.00971933	87	0.00968381
15	0.00921411	28	0.00847909	113	0.00994111

4.1.2 Schwefel's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
191	-1622.15	225	-2979.85	343	-9873.38
191	-1942.03	227	-2889.18	349	-9335.01
186	-1508.04	235	-3076.38	359	-9528.32
194	-1508.04	229	-3559.95	367	-10002.0
182	-1857.83	242	-3824.77	358	-9856.27
187	-1372.79	233	-3618.48	351	-10206.3
201	-1142.96	227	-3081.19	384	-9626.9
198	-1503.71	236	-3384.21	364	-9555.15
187	-1626.48	230	-3288.77	378	-9456.66
199	-1821.97	241	-2952.46	357	-9863.81
188	-1707.69	227	-3318.08	379	-10683.1
204	-1361.24	236	-2497.02	346	-9502.88
197	-1499.79	231	-3259.03	354	-9436.32
184	-1623.66	227	-3115.33	388	-10144.4
204	-1695.29	222	-3764.11	351	-9792.56
191	-1699.72	223	-3481.0	381	-9879.87
183	-1503.32	235	-3118.64	380	-8589.48
196	-1368.77	223	-3471.45	364	-10133.1
188	-1684.74	228	-3433.57	372	-9568.14
180	-1929.56	223	-3537.81	366	-9597.41
191	-1860.44	229	-2958.4	383	-9753.93
186	-1725.5	235	-3267.73	371	-10082.8
197	-1832.57	211	-3464.98	335	-10176.8
199	-1577.05	231	-3549.3	389	-9687.77
188	-1503.22	224	-3187.76	382	-9766.35
186	-1389.59	229	-3072.03	339	-9440.85
195	-1346.63	216	-3312.41	366	-9512.81
178	-1976.16	231	-3200.23	368	-10686.8
203	-1553.78	237	-2702.48	354	-9810.38
180	-1963.79	233	-2994.45	345	-10462.6
190	-1788.97	217	-3211.78	369	-10488.2
194	-1895.32	219	-3057.94	338	-10138.9
180	-1620.08	220	-3699.36	355	-10447.2
200	-1792.17	220	-3276.77	348	-9632.15
180	-1857.44	230	-3350.02	361	-9905.4
189	-1673.25	232	-3452.41	349	-9836.11
194	-1583.41	222	-3713.27	348	-10057.0
193	-1151.31	219	-3450.07	371	-9012.85
185	-1467.18	223	-3671.17	357	-9549.9
188	-1907.9	236	-3367.99	364	-9873.46

4.1.3 Rastrigin's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
184	17.001	221	8.15926	335	62.1094
183	9.62585	216	8.46159	333	51.3102
182	7.21063	216	12.2208	331	68.6952
182	7.1542	214	19.2369	346	57.0416
193	8.63089	224	20.647	337	68.9271
185	12.1542	227	30.0668	335	69.9869
183	12.3699	216	14.3648	338	45.9088
185	8.15421	207	31.451	324	49.1658
184	4.46662	220	14.8668	328	62.8648
177	6.96471	229	12.1492	356	63.3187
187	2.23078	220	27.6121	345	54.9969
185	13.9033	208	31.8227	344	54.2475
188	8.90325	205	19.0987	357	67.9045
181	18.4765	220	16.3497	343	62.6989
180	10.2106	218	32.376	345	83.4286
181	2.23078	221	26.2681	332	71.8696
184	12.3185	208	8.68226	332	69.2497
190	12.1391	222	20.5704	327	68.8285
183	6.96976	225	19.7649	330	51.0176
184	8.21064	215	20.0373	324	65.7368
192	9.38498	212	25.421	342	71.6334
186	12.1593	211	20.519	336	65.7781
181	16.4709	223	14.5493	357	64.1006
179	7.6873	213	10.6208	350	58.8383
180	11.1593	215	36.0616	358	49.4382
184	3.23585	218	35.9907	341	82.0802
179	11.8368	227	22.1089	325	52.511
181	12.129	210	15.4061	321	71.64
182	11.8368	219	16.4011	335	70.1717
190	11.1593	206	13.3749	361	70.9043
193	6.15924	207	30.4374	350	60.6753
183	9.62585	213	18.7602	328	58.3367
179	12.1542	218	21.6256	328	41.2225
185	7.97986	214	18.775	326	66.3065
184	9.97483	215	13.6621	340	69.5137
183	8.90831	213	33.5137	371	85.7363
182	11.8368	219	20.77	336	48.5712
181	9.9184	220	18.8617	340	62.8496
180	11.1905	217	17.6108	344	70.227
178	5.45653	222	21.3135	358	75.5613

4.1.4 Michaelwicz's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
181	-3.91901	211	-7.52875	329	-24.4255
178	-4.49514	204	-7.7156	310	-23.0269
178	-2.92622	220	-7.28144	318	-21.9279
178	-4.16151	213	-8.40654	336	-25.9214
181	-4.48895	220	-7.15179	319	-21.1188
195	-2.98061	216	-8.21477	312	-24.2235
182	-3.71147	209	-6.69316	343	-21.426
181	-3.01791	205	-7.60174	340	-23.1854
177	-3.23221	215	-7.99394	320	-25.4274
182	-3.96093	216	-8.38886	311	-26.2258
189	-3.54911	210	-8.2677	335	-25.0212
175	-4.49528	237	-6.34907	325	-22.9107
185	-4.0395	218	-5.95644	329	-21.5098
187	-3.74104	213	-7.78239	301	-22.4348
186	-4.22092	206	-7.07897	311	-24.391
181	-2.95064	217	-7.9956	343	-23.9715
190	-3.16161	212	-6.4108	325	-26.817
180	-4.43745	214	-7.93226	336	-23.359
181	-3.90044	214	-8.51057	322	-23.1849
181	-4.4524	227	-6.26587	340	-23.6549
187	-3.76895	221	-8.09958	314	-25.5861
185	-3.26428	211	-5.97597	312	-22.2482
189	-4.36977	220	-8.23133	333	-24.2091
181	-3.53663	199	-8.89748	299	-22.9012
183	-3.96863	211	-7.74507	318	-23.4319
185	-3.77944	206	-7.74161	319	-23.0107
180	-3.56101	210	-7.38238	321	-24.5919
10	-4.6094	211	-7.1911	323	-22.5224
190	-3.26695	207	-7.24418	334	-24.6818
181	-3.74631	211	-7.79846	313	-22.9514
183	-4.01962	211	-8.59074	309	-22.497
182	-4.16986	211	-7.58716	299	-24.7433
187	-3.27752	221	-9.0125	306	-20.8159
185	-4.21258	217	-7.47224	310	-24.5659
181	-4.19772	226	-8.17173	314	-24.1957
185	-3.40702	208	-8.56855	301	-23.7356
179	-4.45023	217	-7.48755	308	-24.183
189	-4.47331	218	-6.8604	328	-25.3191
182	-3.74102	212	-8.4902	331	-23.243
185	-3.85077	205	-7.9664	330	-22.8241

4.2 First Improvement Hill Climbing

Functie	D	Min	Max	Avg	Std	Mean	Q1	Q3
deJong	5	7e-05	0.00933	0.00379	0.00263	0.00379	0.00128	0.0054
deJong	10	0.0	0.00921	0.00367	0.00281	0.00367	0.00112	0.00518
deJong	30	0.00011	0.00902	0.00284	0.0025	0.00284	0.001	0.00382
schwefel	5	-1941.92	-1227.17	-1537.06	172.0996	-1537.06	-1617.4275	-1454.26
schwefel	10	-3871.0	-2504.57	-3123.5345	271.11565	-3123.5345	-3325.1775	-2986.6025
schwefel	30	-10266.0	-8379.34	-9522.6035	443.35171	-9522.6035	-9886.5825	-9186.51
michalewicz	5	-4.65392	-1.522	-3.47995	0.65029	-3.47995	-3.83833	-3.14386
michalewicz	10	-8.5915	-5.30489	-7.15793	0.87425	-7.15793	-7.7947	-6.44126
michalewicz	30	-25.1299	-19.1537	-22.10534	1.27305	-22.10534	-22.81755	-21.2869
rastringin	5	4.2207	29.9895	12.22156	4.95122	12.22156	9.56563	13.79415
rastringin	10	10.2006	48.1547	24.65029	9.88588	24.65029	17.333	30.89888
rastringin	30	47.4789	103.256	74.04023	15.01948	74.04023	64.21623	86.21738

Tabel 2: Rezultate First Improvement Hill Climbing

D = numarul de dimensiuni; Min = valoarea minima; Max = valoarea maxima; Avg = media valorilor; Std = deviatia standard; Mean = mediana;

4.2.1 DeJong

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
39	0.00278041	77	0.00105153	213	0.00886919
34	0.00227812	73	0.00921444	207	0.00420109
39	7.02881e-05	75	6.91732e-05	231	0.00135511
41	0.00513169	87	0.00186273	259	0.000680126
36	0.00567272	78	0.00812026	273	0.00142574
43	0.000151286	71	0.00377374	256	0.00144646
33	0.000937598	78	0.00850846	268	0.00113282
28	0.00306561	55	0.00706916	259	0.000247669
41	0.00449647	79	0.00209281	249	0.00822444
36	0.00270573	69	0.00152933	267	0.00719104
36	0.00345674	65	0.000616024	257	0.00543694
41	0.000480442	69	0.00678568	280	0.000509116
42	0.00383034	86	1.08361e-06	220	0.00117069
32	0.00612875	83	0.00222436	260	0.000113624
45	0.00884135	84	0.00302021	242	0.000677239
23	0.00932605	76	0.00324534	276	0.0059376
42	0.00179755	85	8.44409e-06	211	0.00901713
38	0.00257825	80	0.00677328	245	0.00200186
40	0.00531235	89	0.00277218	251	0.000847192
39	0.00511631	75	0.00129752	251	0.00282281
40	0.00111057	93	0.00488294	249	0.00116525
30	0.000811197	83	0.000252923	264	0.00213592
33	0.000868352	66	0.00464312	249	0.00130382
37	0.000944787	98	0.00485024	246	0.00335078
34	0.00587455	71	0.00813082	258	0.00252596
45	0.001883	60	0.00853026	269	0.00369022
27	0.00858225	75	0.00429329	255	0.00285297
36	0.00823505	77	0.00503296	257	0.00518359
33	0.00529529	90	0.000483015	275	0.00570218
45	0.000842079	89	0.000209036	227	0.00105412
42	0.00314487	72	0.00463371	272	0.0063727
39	0.00075223	70	0.000533675	235	0.00267846
34	0.00677355	69	0.00772393	205	0.00084528
32	0.00482311	77	0.00139493	280	0.000477451
33	0.00756342	75	0.004582	234	0.000124741
45	0.0036135	87	0.00114005	247	0.00140957
36	0.00133432	77	0.00405972	243	0.00369735
40	0.00336208	79	0.0010477	275	0.00367955
39	0.00439619	82	0.00563014	257	0.00178847
33	0.00738784	82	0.00483121	259	0.000400611

4.2.2 Schwefel's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
207	-1475.94	255	-3068.39	432	-8842.73
195	-1461.33	250	-2866.55	402	-10115.0
209	-1358.08	251	-2993.25	426	-9952.34
188	-1476.04	256	-3078.98	411	-9597.3
207	-1465.97	240	-3325.03	423	-9621.53
196	-1929.45	250	-2935.24	414	-8873.23
192	-1826.92	249	-3443.23	396	-9875.55
201	-1388.63	254	-3070.32	427	-9076.68
195	-1584.31	249	-3069.81	435	-9361.22
209	-1607.06	251	-2504.57	412	-9082.31
198	-1262.43	246	-3111.62	417	-9546.14
206	-1578.87	265	-3084.61	410	-9188.59
203	-1538.59	249	-3010.05	422	-9341.17
192	-1704.77	238	-3421.81	451	-9345.58
202	-1346.63	247	-2801.01	420	-9280.47
193	-1504.84	268	-2821.98	467	-8754.04
196	-1484.6	235	-3159.74	402	-10007.7
203	-1615.51	260	-3517.47	426	-10210.9
198	-1463.46	248	-3147.28	396	-10266.0
196	-1588.56	248	-3009.39	421	-10163.6
201	-1258.68	235	-3358.75	424	-9807.97
193	-1623.18	236	-3227.72	388	-9146.78
200	-1446.04	257	-2593.91	410	-9550.47
188	-1860.65	250	-3191.06	408	-10047.3
193	-1581.39	236	-3871.0	438	-9612.55
210	-1333.84	252	-3035.49	437	-8934.93
191	-1681.19	228	-3325.62	438	-9009.28
206	-1792.07	262	-2826.0	409	-9789.8
203	-1467.69	233	-3337.0	433	-9938.83
206	-1457.0	253	-3400.75	431	-8379.34
199	-1692.78	246	-3296.26	396	-9598.93
193	-1227.17	253	-2540.91	433	-9919.68
194	-1707.28	247	-3132.87	407	-9727.91
209	-1487.77	256	-3129.26	428	-9441.31
196	-1941.92	238	-3346.07	418	-9750.06
197	-1486.9	263	-3240.07	406	-9180.27
193	-1490.14	241	-3212.11	413	-9957.17
205	-1446.04	248	-3515.19	436	-9833.36
204	-1338.07	244	-2966.66	413	-9476.76
189	-1500.61	240	-2954.35	449	-9299.36

4.2.3 Rastrigin's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
190	8.95463	229	10.2006	376	67.6488
185	9.97483	232	22.1905	379	88.7215
187	11.1391	220	34.3989	371	54.0155
191	10.2157	220	10.3799	355	103.02
188	12.3901	248	13.8664	395	64.0438
190	11.1542	220	15.868	353	71.0881
189	18.159	223	28.3076	371	92.0662
192	10.4717	211	47.7345	366	47.4789
197	8.63089	228	12.3897	375	82.8027
178	9.94958	219	16.4213	358	85.6934
183	10.1904	223	28.0938	360	50.3817
190	9.62585	220	21.3295	365	81.6911
195	21.8065	235	18.8466	374	71.4415
186	18.4765	217	12.1489	366	77.2785
188	8.68731	218	13.8516	383	82.5821
179	12.9033	242	34.4312	345	59.4382
189	10.4717	234	31.0938	353	89.0134
189	10.2106	232	15.9333	368	69.3907
191	29.9895	226	24.3961	365	68.6845
189	14.1189	218	25.3245	347	90.9522
188	12.9748	228	19.3951	371	62.2919
190	5.995	237	30.5342	399	87.9785
186	14.2356	227	18.159	341	97.1563
195	9.38498	228	30.8339	377	68.7406
184	13.2157	221	48.1547	389	73.8606
185	12.3699	210	42.1375	374	47.5536
190	9.21568	223	20.4263	379	80.4089
188	4.70243	213	17.6369	349	103.256
181	21.1952	223	35.3344	361	61.8084
184	13.8267	226	27.0421	387	64.723
179	12.3185	215	22.3447	377	75.9988
188	7.21568	210	19.7649	380	87.7893
197	10.4515	239	15.3948	368	67.4675
181	11.134	220	33.5137	385	47.7758
189	7.45149	229	20.6622	372	65.0177
189	4.2207	221	39.7677	380	100.814
182	21.2205	217	21.3699	364	64.2737
182	16.235	226	24.2297	367	67.8707
184	10.1955	217	40.441	376	63.7738
194	13.7833	227	21.6622	363	75.6173

4.2.4 Michaelwicz's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
184	-3.91762	219	-8.56255	344	-20.9982
193	-3.25114	224	-6.40385	362	-21.2797
184	-3.06875	227	-8.54276	356	-23.2527
181	-3.64468	233	-7.23734	332	-22.6188
185	-3.82702	232	-7.86765	343	-22.7217
186	-3.18861	225	-8.22715	343	-20.9711
196	-4.24053	226	-6.42083	372	-22.5084
191	-3.52292	225	-6.68861	368	-22.6011
191	-2.56576	218	-7.37778	370	-21.6248
32	-4.65392	211	-7.15245	352	-23.0847
189	-3.83834	226	-7.32967	364	-22.7443
190	-3.68303	222	-6.81673	367	-23.684
187	-4.2402	229	-6.89512	350	-19.8347
188	-4.45538	234	-7.47722	359	-22.0191
27	-4.64018	222	-7.53093	353	-22.5475
184	-2.81895	221	-6.14023	363	-21.0182
191	-4.45168	219	-7.88927	379	-23.2564
188	-4.18166	228	-5.82743	359	-21.7562
185	-3.5712	215	-8.5915	332	-19.9835
185	-3.62876	224	-8.37497	342	-20.9377
184	-2.34052	208	-8.4453	343	-20.4271
188	-3.76038	214	-7.91413	361	-21.2893
189	-2.94917	220	-6.89767	354	-21.4995
193	-2.77888	228	-7.6452	382	-23.3527
182	-3.91551	217	-7.10354	351	-22.4588
194	-3.68062	217	-7.19112	371	-22.8173
187	-3.59928	215	-8.15393	350	-21.6043
191	-3.56254	216	-5.79029	353	-20.3487
195	-3.1699	229	-7.77039	349	-24.4378
182	-2.4157	220	-6.19135	355	-23.5506
195	-1.522	222	-5.83384	368	-25.1299
190	-2.87772	213	-5.59319	349	-22.8183
191	-2.98429	226	-7.37838	340	-21.4458
186	-3.78842	226	-6.44807	322	-22.5362
188	-3.16889	223	-6.15803	372	-22.2148
196	-3.5874	218	-5.30489	365	-22.0312
177	-3.31768	232	-7.59173	344	-24.3411
192	-3.35378	226	-7.3878	361	-21.5877
184	-3.19674	231	-6.79513	358	-19.1537
195	-3.83833	214	-7.36901	347	-21.7261

4.3 Worst Improvement Hill Climbing

Functie	D	Min	Max	Avg	Std	Mean	Q1	Q3
deJong	5	7.26619	69.4381	39.1168	17.28675	39.1168	22.0553	52.00228
deJong	10	25.0007	157.062	73.92855	28.19867	73.92855	55.95875	91.0774
deJong	30	171.681	350.822	257.64138	45.19511	257.64138	226.27625	295.6205
schwefel	5	-1065.82	789.703	-59.05979	390.72701	-59.05979	-295.99525	171.05825
schwefel	10	-1298.96	1534.75	-95.05012	557.36407	-95.05012	-530.42425	311.304
schwefel	30	-2593.51	2029.8	-183.27547	1040.74827	-183.27547	-867.4525	301.48125
michalewicz	5	-2.26767	-0.00014	-0.68082	0.59202	-0.68082	-0.92793	-0.18006
michalewicz	10	-2.56094	-0.00015	-0.92101	0.73768	-0.92101	-1.46101	-0.30833
michalewicz	30	-7.25287	-1.41125	-3.75662	1.14828	-3.75662	-4.559	-3.09998
rastringin	5	46.1365	135.752	92.17657	22.58269	92.17657	76.2215	109.5865
rastringin	10	135.028	234.246	184.74223	28.2098	184.74223	164.008	206.35825
rastringin	30	463.367	681.352	547.58785	52.20574	547.58785	502.13625	587.046

Tabel 3: Rezultate Worst Improvement Hill Climbing

D = numarul de dimensiuni; Min = valoarea minima; Max = valoarea maxima; Avg = media valorilor; Std = deviatia standard; Mean = mediana;

4.3.1 DeJong

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
151	22.1352	151	48.2108	151	297.738
151	31.6774	151	67.3849	151	229.235
151	51.8961	151	157.062	151	332.795
151	17.4403	151	58.0601	151	239.995
151	33.4879	151	62.3002	151	297.481
151	65.968	151	40.9672	151	281.012
151	16.4809	151	68.6742	151	196.471
151	47.0698	151	36.1041	151	171.681
151	20.0056	151	72.2262	151	240.254
151	66.1571	151	59.4827	151	298.636
151	54.7622	151	49.4704	151	317.216
151	67.8197	151	107.303	151	268.604
151	53.5251	151	98.3072	151	257.334
151	43.7441	151	49.9742	151	172.874
151	28.8216	151	76.6268	151	244.525
151	18.016	151	82.4484	151	267.571
151	54.8042	151	81.2453	151	210.841
151	38.4895	151	65.0278	151	350.822
151	42.5198	151	130.482	151	322.691
151	17.9307	151	57.0625	151	328.987
151	43.8818	151	63.4555	151	202.881
151	39.2313	151	89.6039	151	247.314
151	36.8763	151	112.343	151	223.112
151	69.4381	151	95.4979	151	288.52
151	40.6953	151	53.6228	151	297.209
151	21.8156	151	47.9004	151	254.905
151	7.26619	151	77.6389	151	255.394
151	16.1697	151	65.6659	151	258.346
151	15.2589	151	104.548	151	206.219
151	41.6667	151	75.9546	151	230.945
151	68.8805	151	60.6727	151	183.54
151	52.3208	151	99.5072	151	204.417
151	23.1778	151	34.4949	151	314.493
151	45.6558	151	57.4647	151	295.091
151	32.92	151	46.8285	151	293.148
151	13.5304	151	25.0007	151	226.351
151	55.5434	151	56.7374	151	272.794
151	51.0879	151	121.885	151	247.776
151	48.3485	151	112.766	151	226.052
151	48.1558	151	87.1339	151	250.385

4.3.2 Schwefel's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
151	234.023	151	-920.801	151	-1050.31
151	524.835	151	40.1628	151	-196.863
151	-206.352	151	309.72	151	-1307.52
151	495.896	151	-204.424	151	-894.724
151	-530.09	151	116.031	151	-3.1237
151	-41.7037	151	-505.264	151	112.807
151	-466.765	151	-650.199	151	-708.53
151	-801.623	151	145.596	151	-972.42
151	73.4541	151	579.211	151	-909.484
151	-272.986	151	-776.84	151	-2593.51
151	130.536	151	-284.235	151	84.1178
151	203.822	151	-114.703	151	292.153
151	77.4341	151	-124.601	151	2029.8
151	-318.509	151	-278.574	151	399.576
151	134.392	151	-712.208	151	-454.2
151	8.97907	151	-745.699	151	813.161
151	40.9497	151	42.8003	151	1240.81
151	-273.101	151	-1298.96	151	1162.22
151	-150.734	151	-1149.18	151	717.247
151	-290.596	151	353.143	151	1910.75
151	-162.045	151	399.349	151	-299.025
151	54.834	151	77.5641	151	-559.559
151	693.579	151	-54.9531	151	95.2775
151	-1065.82	151	130.89	151	-2120.36
151	656.822	151	-59.7129	151	-228.859
151	-151.232	151	596.81	151	1753.15
151	-182.414	151	1534.75	151	-858.362
151	112.569	151	-96.4461	151	-807.287
151	-356.969	151	-184.448	151	-8.82448
151	-380.783	151	477.765	151	10.9875
151	288.196	151	775.603	151	-1053.96
151	789.703	151	362.563	151	-401.314
151	-475.082	151	117.877	151	-717.47
151	-589.084	151	316.056	151	-600.63
151	253.451	151	404.405	151	1923.17
151	-312.193	151	-605.905	151	29.9464
151	161.222	151	-83.4708	151	-753.133
151	-246.399	151	-814.776	151	-1469.09
151	200.567	151	-706.214	151	329.466
151	-223.175	151	-210.687	151	-1267.1

4.3.3 Rastrigin's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
151	83.7316	151	140.633	151	543.864
151	123.213	151	155.809	151	494.856
151	63.6074	151	171.582	151	508.788
151	71.9326	151	205.634	151	497.59
151	135.752	151	194.913	151	583.32
151	95.2699	151	184.99	151	496.84
151	73.8366	151	150.561	151	573.97
151	46.1365	151	174.717	151	622.944
151	115.586	151	234.246	151	522.04
151	98.5884	151	227.665	151	496.649
151	128.812	151	224.71	151	543.179
151	84.5129	151	208.531	151	502.68
151	61.5905	151	232.486	151	532.798
151	88.3901	151	146.634	151	517.134
151	52.1684	151	219.113	151	485.529
151	104.215	151	158.305	151	502.833
151	82.4401	151	172.324	151	612.465
151	105.851	151	184.448	151	599.111
151	77.8519	151	169.041	151	517.97
151	110.582	151	166.7	151	514.903
151	93.2395	151	197.365	151	637.067
151	109.594	151	221.608	151	568.779
151	102.011	151	158.874	151	482.916
151	123.651	151	135.028	151	486.935
151	81.8224	151	195.549	151	570.15
151	109.584	151	215.704	151	572.881
151	69.2734	151	179.145	151	463.367
151	128.061	151	204.226	151	492.582
151	76.5104	151	205.297	151	578.435
151	98.3987	151	167.671	151	524.757
151	87.3717	151	167.804	151	531.513
151	127.274	151	156.068	151	622.443
151	83.1579	151	162.856	151	500.505
151	64.1495	151	177.01	151	522.795
151	85.1348	151	201.9	151	681.352
151	91.0739	151	210.227	151	609.492
151	127.452	151	230.274	151	598.224
151	69.0079	151	136.792	151	607.763
151	80.873	151	164.392	151	564.029
151	75.3548	151	178.857	151	618.066

4.3.4 Michaelwicz's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
151	-0.000919022	151	-1.31587	151	-3.08182
151	-1.93677	151	-1.22067	151	-2.41921
151	-0.110285	151	-2.56094	151	-4.34027
151	-0.795048	151	-1.7387	151	-3.91408
151	-0.697541	151	-0.554061	151	-3.32922
151	-0.738638	151	-0.430726	151	-6.22236
151	-0.922091	151	-0.558981	151	-4.6679
151	-1.48378	151	-0.0409321	151	-4.9772
151	-0.927515	151	-1.21118	151	-2.10446
151	-0.441819	151	-0.109014	151	-3.27573
151	-1.07311	151	-0.968891	151	-7.25287
151	-0.929182	151	-0.369523	151	-3.31543
151	-0.816177	151	-2.40594	151	-3.3552
151	-0.00230249	151	-1.52689	151	-2.72588
151	-0.0844607	151	-0.559163	151	-3.38287
151	-0.0433656	151	-1.29554	151	-2.91398
151	-0.920911	151	-1.54856	151	-3.22474
151	-0.407396	151	-0.863304	151	-3.20387
151	-0.215984	151	-0.311879	151	-3.10725
151	-0.65591	151	-0.293469	151	-3.24767
151	-0.374278	151	-0.0143385	151	-5.31117
151	-0.399171	151	-2.48395	151	-1.41125
151	-0.30153	151	-0.689245	151	-3.69546
151	-2.16136	151	-0.561425	151	-3.44885
151	-0.675746	151	-0.318332	151	-2.72313
151	-1.48088	151	-1.61743	151	-3.10604
151	-1.1881	151	-0.732709	151	-3.3836
151	-0.000136488	151	-0.1178	151	-2.83632
151	-1.40049	151	-1.44493	151	-3.57714
151	-0.160983	151	-0.078728	151	-5.22624
151	-0.186414	151	-0.0286891	151	-3.33797
151	-0.159137	151	-0.000154134	151	-4.54976
151	-0.711472	151	-1.98371	151	-4.57619
151	-0.287062	151	-1.50925	151	-4.90761
151	-0.0679761	151	-1.15055	151	-2.3939
151	-1.21933	151	-0.998724	151	-2.7352
151	-0.0961441	151	-0.085527	151	-5.0684
151	-2.26767	151	-0.297692	151	-5.11572
151	-0.59562	151	-2.16338	151	-4.24567
151	-0.296136	151	-0.679775	151	-4.55327

4.4 Simulated Annealing

Functie	D	Min	Max	Avg	Std	Mean	Q1	Q3
deJong	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
deJong	10	0.00012	0.00315	0.00084	0.00062	0.00084	0.00044	0.00106
deJong	30	0.00631	0.00997	0.00914	0.00078	0.00914	0.00884	0.00975
schwefel	5	-2094.91	-2094.29	-2094.69725	0.14073	-2094.69725	-2094.81	-2094.6
schwefel	10	-4189.51	-4180.31	-4188.54375	1.78991	-4188.54375	-4189.345	-4188.94
schwefel	30	-12568.1	-12533.5	-12559.1275	9.52701	-12559.1275	-12566.125	-12557.675
michalewicz	5	-4.68766	-4.53266	-4.63375	0.05388	-4.63375	-4.67053	-4.61239
michalewicz	10	-9.61505	-8.93424	-9.47753	0.14186	-9.47753	-9.589	-9.42567
michalewicz	30	-28.8369	-26.966	-28.13339	0.39802	-28.13339	-28.38828	-27.86085
rastringin	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
rastringin	10	0.0	1.23582	0.04278	0.22155	0.04278	0.0006	0.00223
rastringin	30	2.47163	12.3582	6.40533	2.39804	6.40533	4.9433	7.97337

Tabel 4: Rezultate Simulated Annealing

D = numarul de dimensiuni; Min = valoarea minima; Max = valoarea maxima; Avg = media valorilor; Std = deviatia standard; Mean = mediana;

4.4.1 DeJong

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
0	2.22445e-08	0	0.00083979	72	0.00925769
0	1.37067e-07	0	0.000457678	65	0.00995468
0	2.06495e-07	0	0.00315491	64	0.0098022
0	3.40701e-08	0	0.000404271	76	0.00846011
0	2.66958e-07	0	0.000452174	72	0.00950404
0	8.70945e-08	0	0.00057638	66	0.00981825
0	2.22445e-08	0	0.000499721	66	0.00970706
0	1.21999e-07	0	0.00129202	70	0.00997492
0	1.44244e-08	0	0.00279777	72	0.00989542
0	3.00646e-08	0	0.00105347	76	0.00893305
0	6.97376e-08	0	0.000986586	66	0.00825613
0	1.23263e-08	0	0.000296468	71	0.00838652
0	2.96832e-08	0	0.000969326	70	0.00631395
0	7.52689e-08	0	0.000734189	70	0.00855773
0	2.79666e-08	0	0.00157498	70	0.0077946
0	1.32799e-08	0	0.000392187	64	0.00988185
0	1.59502e-08	0	0.00137282	72	0.00976745
0	7.88929e-08	0	0.000184224	65	0.00773153
0	7.69855e-08	0	0.000128956	71	0.00946489
0	1.23906e-07	0	0.00116908	64	0.00911164
0	2.75851e-08	0	0.000354704	75	0.00925823
0	7.10727e-08	0	0.000439008	69	0.00905949
0	2.58685e-08	0	0.00117164	68	0.00958698
0	8.36612e-08	0	0.0005811	64	0.00944883
0	1.5347e-07	0	0.000773117	65	0.00911271
0	3.23535e-08	0	0.000659649	63	0.00961661
0	1.21618e-07	0	0.000435689	63	0.0099074
0	2.66314e-08	0	0.000653721	68	0.00839778
0	3.59774e-08	0	0.00147556	68	0.00981046
0	1.52898e-07	0	0.000870996	71	0.00974428
0	1.75023e-07	0	0.000929959	60	0.00893386
0	2.30074e-08	0	0.00024993	69	0.00955693
0	2.37704e-08	0	0.000771599	69	0.0081137
0	2.31982e-08	0	0.000629846	66	0.00914473
0	5.4288e-08	0	0.000123502	63	0.00796406
0	1.56141e-07	0	0.00106089	66	0.0092541
0	1.11819e-08	0	0.00129248	71	0.00981767
0	5.65053e-09	0	0.000234734	67	0.0096426
0	1.08838e-07	0	0.000913561	69	0.00969259
0	3.0396e-07	0	0.000483841	69	0.00906873

4.4.2 Schwefel's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
0	-2094.71	1	-4189.22	84	-12560.2
0	-2094.49	1	-4189.05	126	-12559.0
0	-2094.81	1	-4189.19	102	-12556.8
0	-2094.5	2	-4189.18	95	-12564.8
0	-2094.71	1	-4188.62	116	-12563.9
0	-2094.6	1	-4189.25	141	-12566.1
0	-2094.5	2	-4188.95	100	-12562.9
0	-2094.91	2	-4189.15	87	-12561.9
1	-2094.7	1	-4189.22	205	-12568.1
0	-2094.29	2	-4188.92	112	-12567.2
0	-2094.6	1	-4189.13	107	-12564.8
0	-2094.81	4	-4189.26	93	-12564.8
0	-2094.81	1	-4189.27	4999	-12533.5
0	-2094.84	3	-4189.12	71	-12545.4
0	-2094.81	3	-4189.39	81	-12540.8
1	-2094.91	0	-4184.41	79	-12561.7
0	-2094.91	0	-4185.38	103	-12540.8
0	-2094.5	2	-4189.37	70	-12548.9
0	-2094.71	0	-4187.21	117	-12567.4
0	-2094.65	1	-4189.12	126	-12560.6
1	-2094.81	1	-4189.46	120	-12566.3
0	-2094.71	1	-4189.29	4999	-12533.8
0	-2094.6	4	-4189.5	42	-12544.3
0	-2094.81	1	-4189.1	132	-12563.3
0	-2094.71	1	-4180.31	105	-12565.9
0	-2094.7	2	-4189.36	110	-12557.6
0	-2094.71	1	-4189.38	121	-12567.9
0	-2094.5	5	-4189.18	83	-12547.7
0	-2094.91	2	-4189.49	112	-12558.5
0	-2094.81	1	-4189.38	91	-12566.0
0	-2094.5	2	-4189.49	61	-12557.7
0	-2094.6	1	-4186.6	100	-12559.5
0	-2094.6	0	-4189.51	106	-12568.1
0	-2094.71	2	-4189.05	115	-12567.1
0	-2094.6	1	-4189.03	161	-12567.8
0	-2094.81	2	-4188.87	148	-12558.0
0	-2094.7	4	-4188.94	112	-12566.5
0	-2094.71	1	-4185.12	93	-12564.5
0	-2094.81	4	-4188.94	110	-12558.8
0	-2094.81	3	-4189.34	105	-12566.2

4.4.3 Rastrigin's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
6	4.73005e-09	62	3.93295e-06	4999	3.70747
0	4.73005e-09	7	0.000598023	4999	4.9433
4	4.73005e-09	5	0.00259823	999	6.17911
0	4.73005e-09	7	0.000962261	4999	8.15952
0	4.73005e-09	10	0.00152401	4999	4.94327
0	4.73005e-09	19	0.000457193	999	8.65076
2	4.73005e-09	24	0.000308609	4999	8.65074
0	4.73005e-09	3	0.00431676	4999	8.65077
4	4.73005e-09	30	0.000454544	4999	3.70747
0	4.73003e-09	1	0.000884142	167	6.17914
0	4.73005e-09	8	0.00106053	4999	4.9433
0	4.73005e-09	16	0.00995165	4999	4.9433
1	4.73005e-09	5	0.00240344	4999	7.17912
0	4.73005e-09	21	0.000109608	4999	8.65074
3	4.73005e-09	1	0.000777274	91	5.00679
0	4.73005e-09	10	0.00336022	4999	7.41493
6	4.73005e-09	4	0.00135193	4999	6.23421
14	4.73005e-09	0	0.00348071	4999	3.70747
1	4.73005e-09	2	0.000608203	4999	6.17911
14	1.36698e-06	17	0.000627421	999	8.65073
0	3.81943e-06	10	0.00491938	4999	7.41493
0	4.73005e-09	5	0.000928441	4999	2.47163
0	4.73005e-09	1	0.00172091	4999	6.17912
3	4.73005e-09	1	0.000742777	4999	2.47165
0	3.85727e-06	4	0.000334539	4999	6.17911
0	4.73005e-09	3	0.000887085	4999	3.70746
4	4.73005e-09	20	0.000314831	999	6.17909
0	4.73005e-09	9	0.000829859	4999	12.3384
2	4.73005e-09	4999	1.23582	4999	12.3582
0	4.73005e-09	16	0.00115596	999	6.17909

4.4.4 Michaelwicz's

ultima iteratie 5D	optim	ultima iteratie 10D	optim	ultima iteratie 30	optim
0	-4.61055	4999	-9.58948	4999	-28.112
5	-4.68766	4999	-9.57135	4999	-27.6084
4999	-4.53325	4999	-9.33733	4999	-28.4125
5	-4.68765	4999	-9.46097	4999	-28.1609
0	-4.63727	4999	-9.33167	4999	-28.3601
4999	-4.53325	4999	-9.46934	4999	-28.3142
0	-4.61223	4999	-9.46311	4999	-27.8019
0	-4.65859	4999	-9.48002	4999	-28.5127
4999	-4.53325	4999	-9.59586	4999	-26.966
1	-4.659	4999	-9.56507	4999	-28.1303
0	-4.63976	4999	-9.42583	4999	-28.4408
4	-4.68691	4999	-9.55616	4999	-27.828
4999	-4.53706	4999	-9.51947	4999	-27.5346
0	-4.61508	4999	-9.46585	4999	-27.5804
3	-4.66864	4999	-9.50571	4999	-27.8961
2	-4.68766	4999	-9.4252	4999	-27.8718
16	-4.68308	4999	-9.40121	4999	-28.15
4999	-4.53766	4999	-9.59205	4999	-28.2597
13	-4.64082	4999	-9.59802	4999	-28.3802
0	-4.67023	4999	-9.3757	4999	-27.3645
0	-4.66775	4999	-9.20041	4999	-28.3461
2	-4.64146	22	-9.60776	4999	-28.1808
4	-4.66863	4999	-9.50907	4999	-28.8303
0	-4.68762	4999	-9.56914	4999	-28.5579
2	-4.64148	4999	-9.57752	4999	-27.9998
5	-4.64513	4999	-9.46346	4999	-27.8941
0	-4.63916	85	-9.61505	4999	-28.2618
4	-4.6875	4999	-9.17662	4999	-28.7233
0	-4.64464	4999	-9.57624	4999	-27.7764
0	-4.61244	4999	-9.58884	4999	-28.6183
4999	-4.53766	4999	-9.59118	4999	-27.825
0	-4.68593	4999	-9.59718	4999	-28.5061
14	-4.67038	36	-9.60006	4999	-27.7319
13	-4.68313	4999	-8.93424	4999	-28.5484
4999	-4.53766	4999	-9.59097	4999	-28.1825
0	-4.67022	4999	-9.55334	4999	-28.1634
0	-4.6396	4999	-9.4885	4999	-28.1628
4999	-4.53266	4999	-9.24131	4999	-28.1382
21	-4.6664	4999	-9.40388	4999	-28.8369
2	-4.67099	4999	-9.48705	4999	-28.3663

5 Comparatii si discutie

Dupa cum se observa din experimente, toti algoritmii, mai putin cel de Worst Improvement, ajung la rezultate bune. Din punct de vedere al vitezei de executie, Best Improvement este cel mai rapid, fiind urmat la diferenta mica de First Improvement. Simulated Annealing este ceva mai lent, insa ajunge la un optim mai bun decat ceilalti algoritmi. Ca optimizari, oprim algoritmul de Hill Climbing cand nu observam variatii ale candidatului curent, reducand semnificativ numarul de pasi executati si timpul necesar rularii testelor. In cazul in care observam ca acest algoritm gaseste solutii diferite, numarul maxim de iterari este 5000.
In cazul Simulated Annealing, convergenta catre solutia optima este mai rapida decat in cazul algoritmilor din familia Hill Climbing, pentru problemele de dimensionalitate redusa fiind capabil sa gaseasca o solutie optima $< \epsilon$ chiar din prima rulare. Acest fapt este dat si de functia de vizitare a vecinilor, ce exploreaza un numar mai mare de vecini atunci cand temperatura este ridicata, dar si datorita functiei de generare a vecinilor, care adapteaza variatia acestora la temperat curenta. Similar, Simulated Annealing are un numar maxim de iterari egal cu 5000, insa pentru fiecare iterare exploreaza un numar variabil de vecini. Functia $neighbours(S, T)$, folosita in cadrul algoritmului de Simulated Annealing alege randomizat un numar de biti cel mult egal cu α pe care ii va modifica. $\alpha = min(d \cdot \frac{1}{\frac{100}{T}}, 1)$, unde d = dimensionalitatea problemei si T = temperatura.

6 Concluzii

In concluzie, se observa o diferenta intre modul de functionare al algoritmilor din familia Hill Climbing si cel de Simulated Annealing, in special la functiile mai complexe. Astfel, Hill Climbing este capabil sa gaseasca cu usurinta minimul global pentru functia DeJung, aceasta fiind convexa in natura. Performanta pe celelalte 3 functii este puternic influentata de punctul initial de pornire al algoritmului, forma lor facilitand blocarea in minime locale.

Un punct important este performanta redusa al algoritmului de Worst Improvement Hill climbing, care pare mai vulnerabil la blocarea intr-o minima locala / platou. (Posibil implementarea acestuia lasa de dorit in experimentele efectuate).

Totusi, First Improvement si Best Improvement au convergenta inspre punctual de minim global. Exista o diferenta intre modul in care acestea converg, Best Improvement Hill Climbing avand o convergenta mai rapida si mai lina, in vreme ce First Improvement Hill Climbing este mai lenta / abrupta. Convergenta initiala este rapida, algoritmul continuand rularea pana la conditia de kill-on-plateou, in unele cazuri, sau pana la finalizarea optimizarii.

In testele initiale executate cu Simulated Annealing, functia de neighbor modifica doar un singur bit aleator atunci cand este apelata. Acest lucru este suficient, iar algoritmul reuseste (in timp util) sa gaseasca solutia optima pe problemele de 5-10 dimensiuni. Problema apare, insa, la rularea acestuia pe functia Rastring (30 dimensiuni), unde convergenta sa este foarte lenta. Acest lucru a fost imbunatatit considerabil de modificarea adusa algoritmului. O directie de cercetare ulterioara este modificarea functiei de alegere a vecinului, pentru a imbunatati procesul ce leaga variatia acestora in functie de temperatura. In implementarea actuala, chiar daca convergenta este foarte rapida, stabilizarea necesita un timp ridicat pana cand temperatura va ajunge la zero.

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