THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of December ■ /April/May ☐ /July ☐ 2015

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Scientists**

Date:	**Thursday, December 3, 2015**	Time:	**4:00 p.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐	**1-100** ☐

Auxiliary/Other material(s) – Please specify: None

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) Suppose that five teachers, Paul, Quilston, Ray, Sandra and Thomas are members of executive committees, E_1, E_2, E_3, E_4, E_5 and E_6 based on their various levels of expertise.

Politician	**Executive Committees**
Paul	E_1, E_2, E_4
Quilston	E_1, E_4
Ray	E_2, E_3, E_5, E_6
Sandra	E_1, E_4
Thomas	E_1, E_2

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each teacher can be assigned to a committee such that each committee has no more than one of these five teachers, or use Hall's Theorem to explain why no such complete matching exists. **[3]**

(b) A random experiment consists of rolling an unfair, six-sided die. The number 6 is two times as likely to appear as the numbers 4 and 1. The number 4 is three times as likely to appear as the numbers 2 and 5. The number 1 is two times as likely to appear as the number 3.

i. Assign probabilities to the six outcomes in the sample space **[3]**

ii. Suppose that the random variable X, is assigned the value of the digit that appears when the die is rolled. If the expected value is denoted by

$E(X) = \sum_{i=0}^{n} p(x_i)X(x_i)$ where $p(x_i)$ is the probability for the event x_i,

what is the expected value of X? **[1]**

(c) Find a closed form expression for the following recurrence relation

$s_0 = 1$

$s_1 = 1$

$s_n = 2s_{n-1} - s_{n-2}$ for $n \geq 2$ **[6]**

(d) Consider the geometric series:

$$3 + 15/7 + 75/49 + 375/343 + \ldots$$

i. Determine the common ratio, r. **[1]**

ii. What is the fifth term of the series? **[1]**

(e) Consider the arithmetic series:

$$7/2 \ + \ 15/2 + 23/2 \ + \ 31/2 \ + \ \ldots$$

What is the smallest value of n such that $S_n > 600$, where S_n is the sum of the first n terms of the series? **[4]**

(f) Given G, a phrase-structure grammar.
Let $G = (V, T, S, P)$, where

$V = \{a, b, A, B, S\}$, $T = \{a, b\}$, S is the start symbol,
$P = \{S \rightarrow ABb, A \rightarrow BB, Bb \rightarrow aA, B \rightarrow ab, AB \rightarrow b\}$.

What is the *language generated by* G, $L(G)$, that is the set of all strings of terminals that are derivable from the starting state S. **[3]**

(g) Consider the recurrence function

$$T(n) = 32T(n/2) \ + \ 5n^2 \log n$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \leq 1$. **[4]**

(h) Find the generating function for the sequence

$$\{0, \ 3, \ 6, \ 9, \ 12, \ \ldots\}$$ **[4]**

(i) Construct a finite-state machine that models a vending machine for flavoured water that accepts \$10 and \$20 coins. The machine has a yellow button (Y) that may be pressed at any stage to cancel the purchase and have all collected money returned. When \$50 or more is deposited, the excess (over \$50) is returned, and the customer may press the blue button (B) to receive a flavoured water (W).

I. Represent the finite-state machine as a **state table**. **[4]**

II. Represent the finite-state machine as a **state diagram**. **[3]**

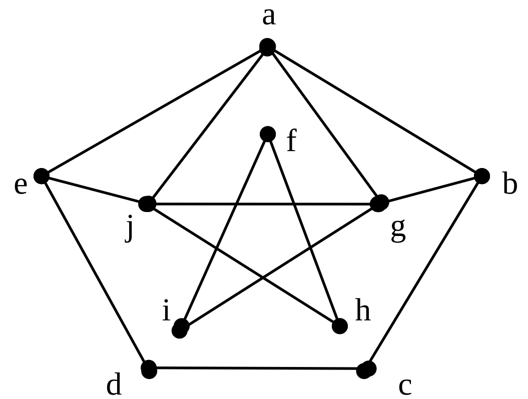
SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) i. What is the row of Pascal's triangle containing the binomial coefficients $\binom{6}{k}$, $0 \le k \le 6$ **[1]**

ii. Expand $(-3p + 2q)^4$ using the Binomial Theorem. **[2]**

(b) Find a Hamiltonian Cycle, or state why one does not exist in the graph. **[3]**



(c) Given the following postfix expression:

A B / C D * – E F + *

i. Represent the expression as a binary tree **[2]**

ii. Write the prefix and infix expression of the binary tree represented. **[2]**

QUESTION THREE (10 Marks)

(a) Given nine courses CS1, CS2, CS3, CS4, P1,P2, M1, M2 and M3, and the following listing which shows courses for which exams cannot be at the same time due to students' course selections:

Students who pursue Course CS1 also pursue CS3 and P1.
Those who pursue Course CS2 also pursue CS1, P2, M2 and M3.
Students who pursue Course CS4 also pursue CS2, CS3 and P1.
Those who pursue Course M1 also pursue M2 and CS3.
Students who pursue Course M2 also pursue M1, M3, P1, P2 and CS2.
Those who pursue Course M3 also pursue M2, CS2 and P1.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and thereby determine the minimum number of time periods necessary for the nine courses. **[2]**

(b) Prove whether the following statement is true or false. Assume that the functions f, g and h take on only positive values.

$$f(n) = \Theta(h(n))\ and\ g(n) = \Theta(h(n)),\quad \text{then}\quad f(n)g(n)h(n) = \Theta(h(n))$$ **[4]**

(c) If there are 54 students who have completed a Computer Science course and 11 possible grades that could have been attained, use the Pigeonhole Principle to show that there is a grade that at least five students attained. **[2]**

QUESTION FOUR (10 Marks)

(a) If you studied intensely the probability of passing this COMP2201 final examination is 87%, if you studied lightly the probability of passing the examination is 45%, and if you studied none at all the probability of passing is 2%. The course tutor knows that 65% of the students studied intensely, 30% of them studied lightly and 5% did not find the time to study. What is the probability of passing this COMP2201 final examination? **[4]**

(b) If the probability is 0.85 that a person will believe a rumor about the transgression of a certain politician, find the probabilities that the fifteenth person to hear the rumor will be the tenth to believe it. **[2]**

(c) By constructing a Tree Diagram, determine how many 3-pemutations are there of 5 objects (A, B, C, D, E)? [4]

QUESTION FIVE (10 Marks)

(a) Consider the geometric series:

$$\frac{5}{3} + \frac{50}{3} + \frac{500}{3} + \ldots$$

What is the smallest value of n such that $S_n > 150$, where

S_n is the sum of the first n terms of the series? [**4]**

(b) Let a,b,c be integers such that a ≥ 1, b > 1 and c > 0.

Let $f\!: N \rightarrow R$ be functions

where N is the set of Natural numbers and R is the set of Real numbers

Given that

$f(1) = c$

$n = b^k$, where k is a positive integer greater than 1.

$$f(n) = a^k f(n/b^k) + c\sum_{i=0}^{k-1} a^i$$

Show that $$f(n) = \Theta(a^{\log_b n})$$ **[6]**

END OF QUESTION PAPER