THE UNIVERSITY OF THE WEST INDIES

Semester I ☐ Semester II ☐ Supplemental/Summer School ■

Examinations of December ☐ /April/May ☐ /July ■ 2011

Originating Campus:	Cave Hill	☐	Mona	■	St. Augustine	☐
Mode:	On Campus	■	By Distance	☐		

Course Code and Title: **COMP2101/CS20S Discrete Mathematics for Computer Scientists**

Date: Monday, July 25, 2011 Time: 9:00-11:00a.m.

Duration: 2 **Hours.** Paper No: **1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐ **1-100** ☐	

Auxiliary/Other material(s) – Please specify: None

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SUMMER 2010/2011

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) Translate or rewrite the following statement in words
If Barrette is the lecturer, then all students will not pass the examination.

i. Using its **contrapositive** and **inverse** **[2]**
ii. **Without using a conditional statement** **[1]**

(b) i. Prove by induction that
$\mathbf{1(1!) + 2(2!) + \ldots + n(n!) = (n+1)! - 1}$ for the positive integer n **[4]**

ii. Let the set X = {0,1,3,4} and Y = {0,3,4,5} and Z = {0,1,5,6}
Determine the set $(X \; \Delta \; Y) \; \Delta \; Z$,
where Δ represents the symmetric difference between two sets. **[1]**

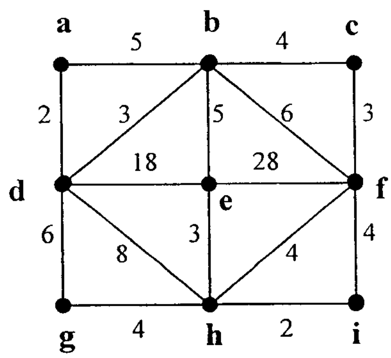
(c) Prove that in any set of n+1 integers from the set {1, 2, 3, 4, …, 2n},
two of the numbers must differ by 1. **[5]**

(d) Let S be the set of all cubes with length of one side, *l*.
Define $f : S \rightarrow [0,\infty)$ by $f(C) = \text{the volume of C, for all } C \,\varepsilon\, S$.
i. Define "injective" and show whether or not f is injective. **[2]**
ii. Define "surjective" and show whether or not f is surjective. **[2]**

(e) Represent the following expression as a binary tree
$((x+2)\,\hat{}\,3) * (y-(3+x)) - 5$ **[3]**

(f) Let $f_1(x)$ and $f_2(x)$ be functions defined $f_i: \mathbb{Z}^+ \rightarrow \mathbb{R}$
where $\mathbb{Z}^+$ is the set of Positive integers and
$\mathbb{R}$ is the set of Real numbers
Prove the following statement
$If \; f_1(x) = \Theta(g_1(x)) \; and \; f_2(x) = \Theta(g_2(x)), \text{then} \, (f_1 f_2)(x) = \Theta((g_1 g_2)(x))$ **[4]**

(g) Find a minimal spanning tree for the following graph. **[3]**



(h) Define R to be the relation defined on the set of ordered pairs of the set of positive integers S = {1, 2, 3, 4, 5, 6} such that

$(a,b)R(c,d)$ if and only if ad = bc

i. Show that the relation R is an equivalence relation. **[4]**

ii. Describe [(1,2)], that is, describe the equivalence class of (1,2). *Hint: ad = bc is equivalent to a/b = c/d* **[2]**

iii. List one member of each equivalence class **[3]**

(i) Find a formula for the sequence s defined by

$$s_n = \sum_{i=1}^{n} c_i$$

where c_i is a term in the sequence defined by $c_1 = 7$, $c_n = c_{n-1} + 5$, $n \geq 2$ **[4]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) Consider the geometric series:

$$\frac{5}{3} + \frac{50}{3} + \frac{500}{3} + \ldots$$

i. Find the common ratio. **[1]**

ii. What is the smallest value of n such that $S_n > 150$, where S_n is the sum of the first n terms of the series? **[3]**

(b) Consider the following recurrence relation

$s_0 = 1$

$s_1 = 1$

$s_n = -2s_{n-1} - s_{n-2}$ for $n \geq 2$

i. List the first five terms in the sequence generated **[1]**

ii. Find a closed form expression for s_n. **[5]**

QUESTION THREE (10 Marks)

(a) In a given city 3 percent of all licensed drivers will be involved in at least one car accident in any given year. Find the probability that among 120 licensed drivers randomly chosen in the city

i. only five will be involved in at least one accident in any given year **[1]**

ii. at most three will be involved in at least one accident in any given year **[2]**

(b) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters? Please note that only uppercase letters are being considered for the license plates. **[2]**

(c) A random experiment consists of rolling an unfair, six-sided die. The digit 3 is two times as likely to appear as the number 1. The number 1 is three times as likely to appear as the number 6. The number 6 is two times as likely to appear as each of the other numbers 2, 4 and 5

i. Assign probabilities to the six outcomes in the sample space **[3]**

ii. Suppose that the random variable X, is assigned the value of the digit that appears when the die is rolled. If the expected value is denoted by

$$E(X) = \sum_{i=0}^{n} p(x_i)X(x_i)$$ where $p(x_i)$ is the probability for the event x_i,

what is the expected value of X, $E(X)$? **[2]**

QUESTION FOUR (10 Marks)

(a) Assuming that any seven-digits could be used to form a telephone number

i. How many telephone numbers would not have any repeated digits?

ii. How many seven-digit telephone numbers would have at least one repeated digit? **[1]**

iii. What is the probability that a randomly chosen seven-digit telephone number would have at least one repeated digit? **[3]**

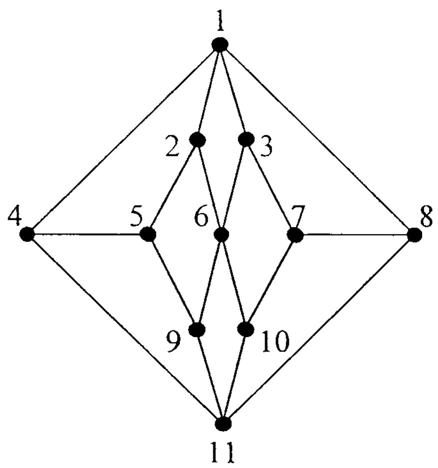
(b) The probability that a child exposed to a contagious disease will catch it is 0.25. Assuming that each child's exposure is independent of the others, what is the probability that the eleventh child exposed will be the fifth to catch it? **[2]**

(c) An automotive safety engineer claims that 1 in 10 automobile accidents is due to driver fatigue. What is the probability that at most 3 of 5 automobile accidents are due to driver fatigue? **[3]**

QUESTION FIVE (10 Marks)

(a) Consider the graph below

i. Prove that a Hamiltonian cycle does not exist in this graph **[3]**



ii. If a Hamiltonian path exists, describe one; otherwise, prove that a Hamiltonian path does not exist. **[1]**

(b) A small school has five teachers, Andy, Beth, Charl, Donnue and Eve. In the spring term, six courses, CS1, CS2, CS3, CS4, CS5 and CS6, are to be offered. Each teacher is qualified to teach one or more courses but each course is assigned at most one teacher. The school has the following information for each teacher.

Teacher	**Courses qualified for**
Andy	CS1, CS5, CS6
Beth	CS2, CS4
Charl	CS1, CS2, CS3
Donnue	CS3, CS4
Eve	CS2, CS6

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each teacher can be assigned to teach exactly one course or use Hall's Theorem to explain why no such way exists. **[3]**

END OF QUESTION PAPER