COMP2201 – Discrete Mathematics
Counting

Question 1

There are four bus lines between *A* and *B*; and three bus lines between *B* and *C*. In how many ways can a man travel :

a) by bus from *A* to *C* by way of *B*?

= 4 x 3 = 12

OR

= $_4P_1$ x $_3P_1$ = 4 x 3 = 12

b) round - trip bu bus from *A* to *C* by way of *B*?

= 12 x 12 = 144

c) round - trip by bus from *A* to *C* by way of *B*, if he does not want to use a bus line more than once?

= 12 x (3-1) x (4-1) = 72

Question 2

Suppose repititions are not permitted

a) How many three - digit numbers can be formed from the six digits 2,3,5,6,7 and 9?

= 6 x 5 x 4 or $_6P_3$ = 120

b) How many of these numbers are less than 400?

= 2 x 5 x 4 = 40

c) How many are even?

= 5 x 4 x 2 = 40

Question 3

Find the number of ways that a party of seven persons can arrange themselves :

a) In a row of seven chairs

= 7! = 5040

b) around a circular table

= (7 - 1)! = 720

Question 4

Find the number of distinct permutaions that can be formed from all the letters of each word :

a) RADAR

= $n! / (n_1! \times n_2! \times \ldots \times n_t!)$ = 5! / (2! x 2! x 1!) = 30

b) UNUSUAL

= $n! / (n_1! \times n_2! \times \ldots \times n_t!)$ = 7! / (3! x 1! x 1! x 1! x 1!) = 840

Question 5

In how many ways can four mathematics books, three hisory books, three chemistry books, and two sociology books be arranged on a shelf so that all books of the same subject are together?

$= \quad 4 \times 3 \times 2 \times 1 \quad = \quad 24$

Question 6

A bag contains six white marbles and five red marbles. Find the number of ways four marbles can be drawn from the bag if

a) they can be any colour

Assuming Distinct

$= \quad {}_{11}C_4 \quad = \quad 11! / [4! \times (11 - 4)!] \quad = \quad 330$

or (but much longer)

$= \quad {}_6C_4 + {}_6C_3 \times {}_5C_1 + {}_6C_2 \times {}_5C_2 + {}_6C_1 \times {}_5C_3 + {}_5C_4$

$= \quad 15 + 100 + 150 + 60 + 5 \quad = \quad 330$

b) two must be white and two red

$= \quad {}_6C_2 \times {}_5C_2 \quad = \quad 15 \times 10 \quad = \quad 150$

c) they must all be of the same colour

$= \quad {}_6C_4 + {}_5C_4 \quad = \quad 15 + 5 \quad = \quad 20$

Question 7

How many committees of five with a given chairperson can be selected from 12 persons?

Assuming that Chairperson is INCLUDED in the 12 persons

$= \quad {}_{11}C_4 \quad = \quad 11! / [4! \times (11 - 4)!] \quad = \quad 330$

Assuming that Chairperson is NOT INCLUDED in the 12 persons

$= \quad {}_{12}C_4 \quad = \quad 12! / [4! \times (12 - 4)!] \quad = \quad 495$

Question 8

In how many ways can nine students be partitioned into three teams containing four, three and two students, respectively?

$= \quad {}_9C_4 \times {}_5C_3 \times {}_2C_2 \quad = \quad 126 \times 10 \quad 1 \quad = \quad 1260$

OR

$= \quad n! / (n_1! \times n_2! \times \ldots \times n_t!) \quad = \quad 9! / (4! \times 3! \times 2!) \quad = \quad 1260$

Question 9

There are 12 students in a class. In how many way can the 12 students take four different tests if three students are to take each test?

$= {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3 = 220 \times 84 \times 20 \times 1 = 369600$

OR

$= n! / (n_1! \times n_2! \times \ldots \times n_t!) = 12! / (3! \times 3! \times 3! \times 3!) = 369600$

Question 10

In how many way can 12 students be partitioned into four teams so that each team contains three students?

$= n! / (n_1! \times n_2! \times \ldots \times n_t!) = 12! / 3!^4 = 369600$