THE UNIVERSITY OF THE WEST INDIES

EXAMINATION of JULY 2007

Code and Name of Course: CS20S - Discrete Maths. for Comp. Science **Paper: B.Sc.**
Date and Time: Wednesday July 18, 2006 1:00-3:00pm Duration: **2 Hrs.**

INSTRUCTIONS TO CANDIDATES: This paper has 4 page(s) and 5 questions
Do Question ONE and any other TWO. Calculators are allowed in the examination.

SECTION ONE

Question 1 (30 Marks)

(a) i) Prove or disprove that $\neg(p \rightarrow q)$ is logically equivalent to $p \wedge \neg q$. [3]

ii) Hence, write the negation of the following statement as simply as possible:

"If she works, then she will earn money"

[1]

(b) Let the universal set $U = \{1,2,3,...,9,10\}$. Let the set $X = \{1,2,3,5\}$ and $Y = \{5,6,7,8,9\}$

i. What is the **symmetric difference** of sets X and Y? [3]

ii. List the elements of $(X \cap Y)^C$. [1]

(c) Prove that in any set of $n+1$ integers from the set $\{1,2,3,4,...,2n\}$, two of the numbers must differ by 1? [5]

(d) Determine whether the following statement is true or false. If the statement is false give a counterexample. Assume that the functions f, g and h take on only positive values.

$$\text{If } f(n) = O(g(n)) \text{ and } g(n) = O(f(n)), \text{ then } f(n) = \Theta(g(n))$$

[3]

(e) Prove by induction that $1(1!) + 2(2!) + ... + n(n!) = (n+1)! - 1$. [6]

(f) Is there a simple connected planar graph with 35 vertices and 100 edges? Justify your response.

[3]

(g) Prove that a simple, connected graph with $n > 1$ vertices and n-1 edges must contain a vertex of degree 1. [5]

SECTION TWO

Question 2 (15 Marks)

(a) How many ways are there to choose three different numbers each between, and inclusive of, 1 and 100 so that their sum is even? **[4]**

(b) Show that $2n + 3\log_2 n = \Theta(n)$. **[3]**

(a) Suppose that five University students, Mary, Tahir, Bob, Paul and Susan are members of four different committees, C_1, C_2, C_3, and C_4 as shown below.

Student	Committee
Mary	C_1, C_3 ,C_4
Tahir	C_4
Bob	C_1, C_2,C_4
Paul	C_1, C_3
Susan	C_2, C_3, C_4

Table 1

i. Let $G = (V, E)$ be a graph where $V = V_1 \cup V_2$. V_1 is the set of students V_2 is the set of committees. An edge joins a student to a committee C_j if and only if that student is a member of C_j. Draw a graph describing the information in Table 1 above. **[2]**

ii. Using an example, illustrate what is meant by a maximal matching. **[2]**

iii. Does there exist a complete matching in G? If there is a complete matching give an example, otherwise prove that none exist. **[4]**

Question 3 (15 Marks)

(a) Show, by giving a proof by contradiction, that if 100 balls are placed in nine boxes, there is at least one box contains 12 or more balls. **[3]**

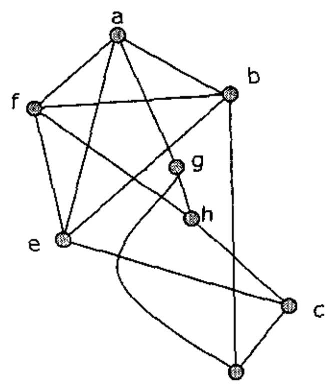
(b) Consider the random experiment of tossing eight fair coins simultaneously. What is the probability that the number of heads and the number of tails differ by at most 2 **[4]**

(c) Let $G = (V, E)$ be a graph.

i. What is meant by a planar representation of G? **[1]**

ii. State Kuratowski's Theorem. **[2]**

iii. Show that the graph G of Figure 1, below is not planar by using Kuratowski's Theorem. **[5]**



Question 4 (15 Marks)

(a) If the probability is 0.75 that a person will believe a rumour about the transgression of a certain politician, find the probabilities that the eighth person to hear the rumour will be the fifth to believe. **[4]**

(b) Given that the generating function for the sequence $\{1,1,1,1,...\}$ is $\frac{1}{1-x}$, derive the generating function for the sequence $\{4,2,4,2,4,2,...\}$. **[5]**

(c) Consider the following list of letters:

t c w q d k f a l e j h v

i. What is a binary search tree? **[2]**

ii. Use a binary search trees to sort the list of letters *(above)*, adding the nodes in the order specified. Provide adequate explanation for your work. **[4]**

Question 5 (15 Marks)

(d) In how many ways can one select a committee of four Republicans, three Democrats and two Independents from a group of 10 distinct Republicans, 12 distinct Democrats and four distinct Independents? [3]

(e) Suppose there are 90 small towns in a country. From each town there is a direct bus route to at least 50 towns. Is it possible to go from one town to any other town by bus possibly changing from one bus and then taking another bus to another town? [5]

(f) Consider the following recurrence relation

$$s_0 = 1,$$
$$s_n = 2s_{n-1} + 1 \text{ for } n \geq 1$$

i. List the first four terms in the sequence generated by the following recurrence relation. [2]

ii. Find a closed form expression for s_n. [5]

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