THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of: December ■ /April/May ☐ /July ☐ 2018

Originating Campus:	**Cave Hill**	■	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Science**

Date:	**Friday, December 21, 2018**	Time:	**1:00 p.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐			
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐			
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20**	☐	**1-100**	☐	

Auxiliary/Other material(s) – Please specify:

Candidates are permitted to bring the following items to their desks: Pencil or pen, Ruler, ID card, Exam card

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Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) i. What is the row of Pascal's triangle containing the binomial coefficients

$$\binom{9}{k}, 0 \leq k \leq 9$$ **[1]**

ii. Write a formula to determine the number of strings that can be formed by ordering the letters APPARATUS. **[1]**

(b) Suppose that six politicians with surnames Usher, Valentine, Wallace, Xanders, Yap and Zephers are members of executive committees, E_1, E_2, E_3, E_4, E_5 and E_6 based on their various levels of expertise.

Politician	**Executive Committees**
Usher	E_2, E_4
Valentine	E_1, E_3, E_4, E_6
Wallace	E_2, E_4, E_5, E_6
Xanders	E_4, E_5
Yap	E_2, E_5, E_6
Zephers	E_2, E_6

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each politician can be assigned to one committee such that each committee has no more than one of these six politicians, or use Hall's Theorem to explain why no complete matching exists. **[3]**

(c) A random experiment consists of rolling an unfair, six-sided die. The number 4 is two times as likely to appear as the numbers 1 and 6. The number 6 is two times as likely to appear as the numbers 2 and 5. The number 1 is four times as likely to appear as the number 3.

i. Assign probabilities to the six outcomes in the sample space **[3]**

ii. Suppose that the random variable X, is assigned the value of the digit that appears when the die is rolled. If the expected value is denoted by

$E(X) = \sum_{i=0}^{n} p(x_i)X(x_i)$ where $p(x_i)$ is the probability for the event x_i,

what is the expected value of X? **[1]**

(d) Find a closed form expression for the following recurrence relation

$s_0 = 1$

$s_1 = 1$

$s_n = -2s_{n-1} - s_{n-2}$ for $n \geq 2$ **[6]**

(e) Consider the arithmetic series:

$$3/2 + 9/2 + 15/2 + 21/2 + \ldots$$

What is the smallest value of *n* such that $S_n > 400$, where
S_n is the sum of the first *n* terms of the series? **[4]**

(f) Given *G*, a phrase-structure grammar.
Let *G = (V, T, S, P)*, where

V = {a, b, A, B, S}, *T* = {a, b}, S is the start symbol,

P = {S → aAbB, A → BB, bB → aA, B → ab, Ab → Bb}.

What is the *language generated by G*, *L(G)*, that is the set of all strings of terminals that are derivable from the starting state *S*. **[3]**

(g) Consider the recurrence function

$$T(n) = 81\,T(n/3) + 9n^3$$

Give an expression for the runtime *T(n)* if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \leq 1$. **[4]**

(h) Find the generating function for the sequence

$$\{\cancel{0}\ 1, 4, 7, 10, 13, \ldots\}$$ **[4]**

(i) Construct a finite-state machine that models a vending machine for tin juices that accepts \$50 and \$100 notes. The machine has a red button (R) that may be pressed at any stage to cancel the purchase and have all collected money returned. When \$250 or more is deposited, the excess (over \$250) is automatically returned, and the customer may press the green button (G) to receive a tin juice (TJ). The machine also has a yellow button (Y) that may be pressed at any stage to donate \$50 to the vending company for their venture to minimize the use of plastic. When a donation takes place, the sum collected over \$50 is returned.

I. Represent the finite-state machine as a **state table**. **[4]**

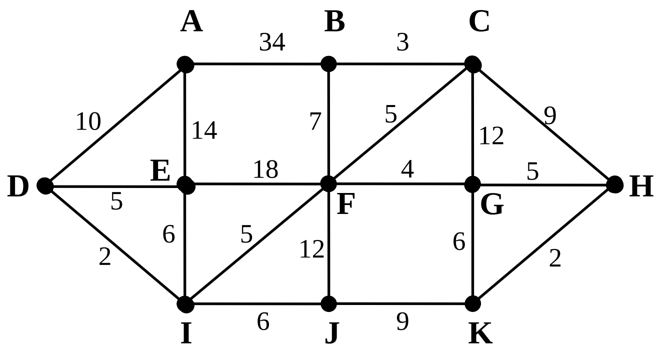
II. Represent the finite-state machine as a **state diagram**. **[3]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) Expand $(-4p + q)^6$ using the Binomial Theorem. **[2]**

(b) Find a minimal spanning tree for the following graph. **[4]**



(c) If the probability is 0.75 that a person will believe a rumor about the transgression of a certain teacher, find the probability that the sixteenth person to hear the rumor will be the ninth to believe it. **[2]**

(d) If there are 283 JUTC buses in operation at a point in time. At that time, the buses are confirmed to be transporting 9,342 commuters. Use the Pigeonhole Principle to show that at the same point in time, there is at least one of the JUTC buses in operation that is transporting thirty-four commuters. **[2]**

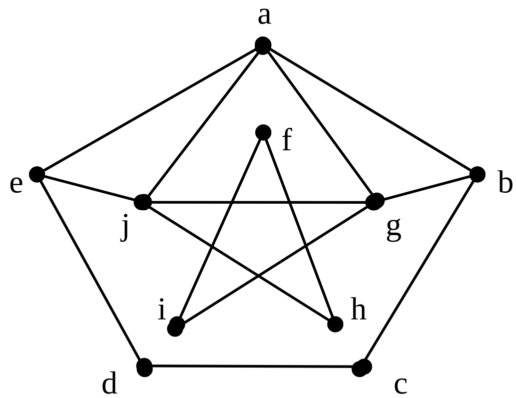
QUESTION THREE (10 Marks)

(a) Consider the geometric series:

$$1 + 5/7 + 25/49 + 125/343 + \ldots$$

Determine a formula for S_n, the sum of the first n terms of the series? **[4]**

(b) Find a Hamiltonian Cycle, or state why one does not exist in the graph.



[2]

(c) Given the following postfix expression:

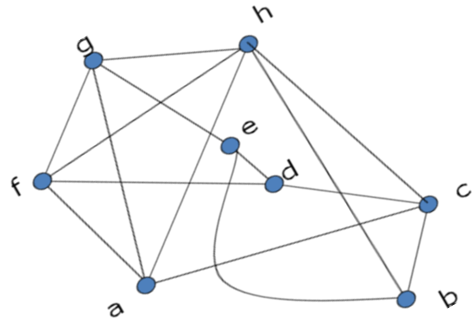
A B / C + D E * F * –

i. Represent the expression as a binary tree **[2]**

ii. Write the prefix and infix expression of the binary tree represented. **[2]**

QUESTION FOUR (10 Marks)

(a) Using Kuratowski's Theorem or otherwise, show that the graph below is not planar



[4]

(b) Let a,b,c be integers such that $a \geq 1$, $b > 1$ and $c > 0$.

Let $f: N \rightarrow R$ be functions

where N is the set of Natural numbers and R is the set of Real numbers

Given that

$f(1) = c$

$n = b^k$, where k is a positive integer greater than 1.

$$f(n) = a^k f(n/b^k) + c\sum_{i=0}^{k-1} a^i$$

Show that $f(n) = \Theta(a^{\log_b n})$ **[6]**

QUESTION FIVE (10 Marks)

(a) Given courses CS1, CS2, CS3, P1,P2, M1, M2 and M3, and the following listing which shows courses for which exams cannot be at the same time:

Students who pursue Course CS1 also pursue CS3 and P1.

Those who pursue Course CS2 also pursue CS1, P2, M2 and M3.

Students who pursue Course CS3 also pursue CS2 and P1.

Those who pursue Course M1 also pursue M2 and CS3.

Students who pursue Course M2 also pursue M1, M3, P1, P2 and CS2.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and by determining the chromatic number χ, state the minimum number of time periods necessary for the examinations of the eight courses. **[2]**

(b) Let $f(n)$, $g(n)$ and $h(n)$ be functions defined $f: Z^+ \rightarrow Z^+$

where Z^+ is the set of Positive integers

Prove whether the following statement is true or false.

$$If\ f(n) = \Theta(h(n)),\ \text{and}\quad g(n) = \Theta(h(n))$$
$$\text{then}\quad (fgh)(n) = \Theta(h(n))$$ **[4]**

(c) By constructing a Tree Diagram, determine how many 3-permutations that do not end with D, are there of 4 objects (A, B, C, D)? **[2]**

END OF QUESTION PAPER