THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of December ■ /April/May ☐ /July ☐ 2010

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2101/CS20S Discrete Mathematics for Computer Scientists**

Date:	**Thursday, December 16, 2010**	Time:	**9:00 a.m.**
Duration:	**2 Hours.**	Paper No:	

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐ **1-100** ☐	

Auxiliary/Other material(s) – Please specify:

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SEMESTER 1 2010/2011

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) i. Let p be John reads The Herald, let q be John reads The Gleaner, and let r be John reads The Observer. Write the following in symbolic form:

It is not true that John reads The Observer or The Gleaner but not The Herald. [1]

ii. Write the negation of the following statement as simply as possible:

If Romelda Aiken is the goal shooter, then some matches will be won. [2]

iii. Prove by induction that

$\mathbf{1(1!) + 2(2!) + \ldots + n(n!) = (n+1)! - 1}$ for the positive integer n [4]

iv. Let the set X = {0,1,3,4} and Y = {0,3,4,5} and Z = {0,1,5,6}

Determine the set $(X \,\Delta\, Y) \,\Delta\, Z$,

where Δ represents the symmetric difference between two sets. [1]

(b) Assuming that any seven digits could be used to form a telephone number

i. How many telephone numbers would not have any repeated digits? [1]

ii. How many seven-digit telephone numbers would have at least one repeated digit? [1]

iii. What is the probability that a randomly chosen seven-digit telephone number would have at least one repeated digit? [2]

iv. By using the inclusion-exclusion principle, give a formula for the number of elements in the union of four sets A_1, A_2, A_3, and A_4 [1]

(c) Let S be the set of all circles in a plane. Define $f : S \rightarrow [0,\infty)$ by

$f(C) =$ the area of C, for all $C \in S$.

i. Show whether or not f is injective? [3]

ii. Show whether or not f is surjective? [3]

(d) Find a closed form expression for the following recurrence relation

$s_0 = 0$

$s_1 = 1$

$s_n = 2s_{n-1} - s_{n-2}$ for $n \geq 2$ [5]

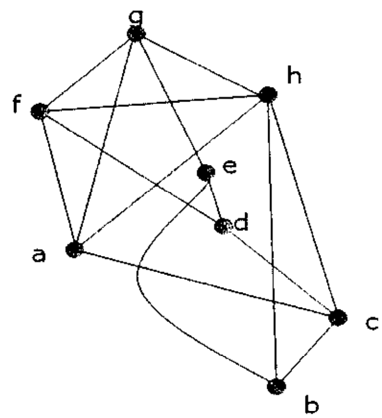
(e) Represent the following expression as a binary tree

$(a \wedge 3 - 5)/(b * (3 + c)) - 2$ [2]

(f) Prove whether the following statement is true or false. If the statement is false, give a counterexample. Assume that the functions g and h take on only positive values.

$$\text{If } g(x) = O(h(x)) \text{ and } h(x) = O(g(x)), \text{ then } g(x) = \Theta(h(x))$$ **[3]**

(g) Using Kuratowski's Theorem or otherwise, show that the graph below is not planar **[3]**



(h) Define R to be the relation defined on the set of ordered pairs of the set of positive integers $\mathbb{Z}$, such that

$(a_1,a_2)R(b_1,b_2)$ if and only if $a_1 \leq b_1$ and $a_1 + a_2 \leq b_1 + b_2$

Prove that the relation R is a partial order. **[4]**

(i) Find a formula for the sequence c defined by

$$c_n = \sum_{i=1}^{n} b_i$$

where b is the sequence defined by $b_1 = 2,\ b_n = 3 + b_{n-1},\ n \geq 2$ **[4]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) Given eight courses CS1, CS2, CS3, CS4, M1, M2, M3, and M4, and the following listing which shows courses for which exams cannot be at the same time due to students' course selections:

Students who pursue Course CS1 also pursue CS2 and M1.
Those who pursue Course CS3 also pursue CS1, CS2, M2 and M4.
Students who pursue Course CS4 also pursue CS1, CS2, CS3 and M4.
Those who pursue Course M1 also pursue M4, CS1, CS2 and CS3.
Students who pursue Course M2 also pursue M3, M4, CS1 and CS2.
Those who pursue Course M3 also pursue CS1, CS2, CS3 and CS4.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and thereby determine the minimum number of time periods necessary for the eight courses. **[4]**

(b) Consider the geometric series:

$$\frac{5}{3} + \frac{50}{3} + \frac{500}{3} + \ldots$$

i. Find the common ratio. **[1]**

ii. What is the smallest value of n such that $S_n > 150$, where S_n is the sum of the first n terms of the series? **[3]**

QUESTION THREE (10 Marks)

(a) Given that the equivalence relation, R partitions set S = {1,2,3,4,5,6} into the partition $A_1 = \{1,2,3\}$, $A_2 = \{4,5\}$, $A_3 = \{6\}$. List the ordered pairs of the equivalence relation R. **[3]**

(b) Consider the random experiment of tossing nine fair coins. What is the probability that the number of heads and the number of tails differ by at most 3? **[2]**

(c) A random experiment consists of rolling an unfair, six-sided die. The number 6 is three times as likely to appear as the numbers 2 and 4. The number 2 is two times as likely to appear as the number 1. The number 4 is three times as likely to appear as the numbers 3 and 5

i. Assign probabilities to the six outcomes in the sample space **[3]**

ii. Suppose that the random variable X, is assigned the value of the digit that appears when the die is rolled. If the expected value is denoted by

$$E(X) = \sum_{i=0}^{n} p(x_i)X(x_i) \text{ where } p(x_i) \text{ is the probability for the event } x_i,$$

what is the expected value of X? **[2]**

QUESTION FOUR (10 Marks)

(a) The probability that a child exposed to a contagious disease will catch it is 0.25. Assuming that each child's exposure is independent of the others, what is the probability that the eleventh child exposed will be the fifth to catch it? **[2]**

(b) In a given city 3 percent of all licensed drivers will be involved in at least one car accident in any given year. Find the probability that among 200 licensed drivers randomly chosen in the city at most three will be involved in at least one accident in any given year. **[3]**

(c) Let f and g be functions defined on $\mathbb{N} \rightarrow R$

where $\mathbb{N}$ is the set of Natural numbers and

R is the set of Real numbers

i. Explain what is meant by f is "Big Oh" of g. **[2]**

ii. Show that $\quad$ *If* $f = O(g)$, then $f + g = O(g)$ **[3]**

QUESTION FIVE (10 Marks)

(a) i. Give a formula for the number of ways that 8 balls may be selected in any order from a box of 50 distinct balls. **[1]**

ii. Show, by giving a proof by contradiction, that if 50 balls are placed in 7 boxes, there is at least one box that contains 8 or more balls. **[2]**

(b) Suppose that five University students, Amelia, Betty, Carol, Debby and Elle are members of the committees, C_1, C_2, C_3, C_4, C_5 and C_6.

Student	**Committees**
Amelia	C_1, C_3, C_5, C_6
Betty	C_2, C_4
Carol	C_1, C_2, C_4
Debby	C_1, C_4
Elle	C_1, C_2

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each student can be assigned to a committee such that each committee has no more than one of these five students, or use Hall's Theorem to explain why no such complete matching exists. **[4]**

END OF QUESTION PAPER