COMP2201 – Discrete Mathematics
Probability Tree and Probability Distributions

1. Suppose that if you study none at all, the probability of passing the course COMP2201 is 10%, if you studied lightly, the probability of passing the course is 50%, and if you studied intensely, the probability of passing the course is 85%. Research shows that only 5% of the students do no study at all, 25% of them study lightly and 70% actually study intensely.
   i. Given that you pass the course, what is the probability that you studied none at all?
   ii. Draw the Probability Tree to represent the given scenario.

Solution

i.

Let N - Study None at all
L - Study Lightly
I - Studied Intensely

Y - Passing the course COMP2201

Given

P(Y|N) = 0.10
P(Y|L) = 0.50
P(Y|I) = 0.85

P(N) = 0.05
P(L) = 0.25
P(I) = 0.70

Required

P(N|Y)

Recall that

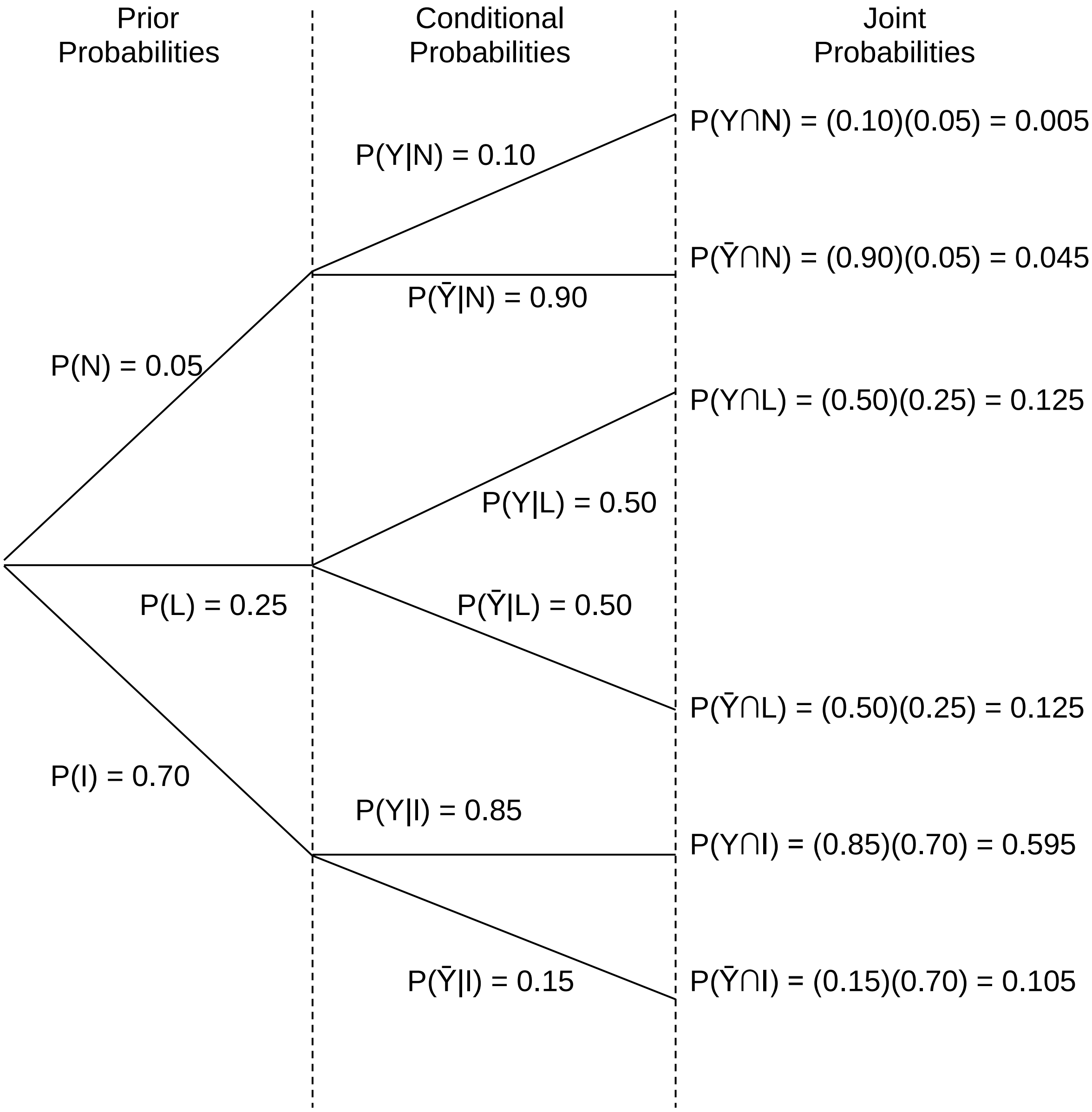
$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) + \ldots}$$

P(N|Y) = [P(Y|N)·P(N)] / [P(Y|N)·P(N) + P(Y|L)·P(L) + P(Y|I)·P(I)]

P(N|Y) = [0.10 × 0.05] / [0.10 × 0.05 + 0.50 × 0.25 + 0.85 × 0.70]

= 0.006897

ii.



2. An automotive safety engineer claims that 1 in 10 automobile accidents is due to driver fatigue. What is the probability that at least 3 of 5 automobile accidents are due to driver fatigue?

Solution

Binomial

$$b(x; n, \theta) = C_x^n \, \theta^x (1-\theta)^{n-x} \text{ for } x = 0,1,2.., n$$

$$b(\geq 3;5,0.1) = b(3;5,0.1) + b(4;5,0.1) + b(5;5,0.1)$$

$$= {}_5C_3 \times 0.1^3 \times 0.9^{5-3} + {}_5C_4 \times 0.1^4 \times 0.9^{5-4} + {}_5C_5 \times 0.1^5 \times 0.9^{5-5}$$

$$= 0.00856$$

3. In a certain city, incompatibility is given as the legal reason in 70 percent of all divorce cases. Find the probability that five of the next six divorce cases filed in this city will claim incompatibility as the reason.

Solution

Binomial

$$b(x; n, \theta) = C_x^n \, \theta^x (1-\theta)^{n-x} \text{ for } x = 0,1,2.., n$$

b(5;6,0.7) = $_6C_5$ x 0.7^5 x 0.3^{6-5}

= 0.3025

4. If the probability is 0.75 that a person will believe a rumour about the transgression of a certain politician, find the probabilities that the fifteenth person to hear the rumour will be the tenth to believe it.

Solution

Negative Binomial

$$b^*(x; k, \theta) = C_{k-1}^{x-1} \, \theta^k (1-\theta)^{x-k} \text{ for } x = k, k+1, k+2$$

b*(15;10,0.75) = $_{15-1}C_{10-1}$ x 0.75^{10} x 0.25^{15-10}
= *0.1101*

5. An expert sharpshooter misses a target 5 percent of the time. Find the probability that she will miss the target for the first time on the fifteenth shot.

Solution

Geometric

$$g(x;\theta) = \theta(1-\theta)^{x-1} \text{ for } x = 1,2,3,\ldots$$

g(15;0.05) = 0.05 (0.95^{15-1})

= *0.02438*

Negative Binomial

$$b^*(x; k, \theta) = C_{k-1}^{x-1} \, \theta^k (1-\theta)^{x-k} \text{ for } x = k, k+1, k+2$$

b*(15;1,0.05) = $_{15-1}C_{1-1}$ x 0.05^1 x 0.95^{15-1}

= *0.02438*

6. In a given city 4 percent of all licensed drivers will be involved in at least one car accident in any given year. Find the probability that among 150 licensed drivers randomly chosen in the city

a) only five will be involved in at least one accident in any given year

b) at most three will be involved in at least one accident in any given year

Solution

Poisson or Binomial

(a) ***Poisson***

$$p(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$
$$x = 0,1,2,...$$

As $\lambda = n\theta = 150 \times 0.04 = 6$

$p(5;6) = (6^5 \times e^{-6}) / 5!$

$= 0.161$

Binomial

$$b(x; n, \theta) = C_x^n \theta^x (1-\theta)^{n-x} \text{ for } x = 0,1,2.., n$$

$b(5;150,0.04) = {}_{150}C_5 \times 0.04^5 \times 0.96^{145}$

$= 0.163$

(b) ***Only Poisson shown (either may be used but Poisson is more easily used)***

$$p(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$
$$x = 0,1,2,...$$

$p(\leq 3;6) = p(0;6) + p(1;6) + p(2;6) + p(3;6)$

$= (6^0 \times e^{-6}) / 0! + (6^1 \times e^{-6}) / 1! + (6^2 \times e^{-6}) / 2! + (6^3 \times e^{-6}) / 3!$

$= 0.151$