THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of: December ■ /April/May ☐ /July ☐ 2019

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Science**

Date:	**Friday, December 20, 2019**	Time:	**1:00 p.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐		
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐		
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20**	☐	**1-100**	☐

Auxiliary/Other material(s) – Please specify:

Candidates are permitted to bring the following items to their desks: Pencil or pen, Ruler, ID card, Exam card

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Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) In an arithmetic series, the sum of the second term and the fifth term is 49. Three consecutive terms of the same series are $5x - 38$, $3x - 7$ and $2x + 11$. If the sum of the terms in the series is 345

i. Find x **[1]**

ii. Find the common difference, d and the first term, a **[1]**

iii. Find the number of terms in the series, n **[2]**

(b) Given the following prefix expression:

$$+ \; - \; * \; G \; * \; F \; A \; B \; / \; C \; * \; D \; E$$

Represent the expression as a binary tree **[2]**

(c) Find the generating function for the sequence

$$\{2,\ 5,\ 10,\ 17,\ 26,\ \ldots\}$$ **[4]**

(d) A random experiment consists of rolling an unfair, six-sided die. The number 5 is three times as likely to appear as the numbers 1 and 6. The number 1 is three times as likely to appear as the numbers 3 and 4. The number 4 is two times as likely to appear as the number 2. What are the probabilities to the six outcomes in the sample space. **[4]**

(e) Consider the congruence class (mod 9).

i. List one positive and one negative value of each of the residue classes (mod 9). **[2]**

ii. Construct the Addition Table in Z_9. **[2]**

iii. By applying the principle of residue class replacement, determine the value of

$$(82 + 66 \times 101 + 22) \bmod 9$$ **[2]**

(f) Find a closed form expression for the following recurrence relation

$s_0 = 1$

$s_1 = 1$

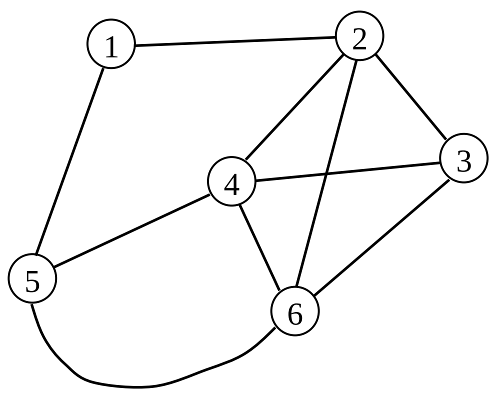
$s_n = 2s_{n-1} - s_{n-2}$ for $n \geq 2$ **[6]**

(g) Construct a finite-state machine that models a newspaper vending machine that accepts \$10 and \$20 coins. The machine has an automatic locking door (D) that is unlocked only after \$40 or more have been inserted. Once the door is unlocked the customer opens it and takes a paper, and closes the door. No change is ever returned no matter how much extra money has been inserted. The next customer starts with no credit. While the door has not been opened, the customer may press a red cancel button (C) to cancel the purchase and have all collected money returned.

I. Represent the finite-state machine as a **state table**. **[4]**

II. Represent the finite-state machine as a **state diagram**. **[3]**

(h) Given the following undirected graph G with vertices 1, 2, 3, 4, 5 and 6



i. List the clique(s) with 3 or more vertices that are within graph G **[1]**

ii. Identify the maximal clique(s) **[1]**

iii. Find the maximum clique(s) or state why there is no maximum cliques in the graph G **[1]**

iv. Determine the clique number $\omega(G)$ **[1]**

(i) Given *G*, a phrase-structure grammar.

Let $G = (V, T, S, P)$, where

$V = \{a, b, A, B, S\}$, $T = \{a, b\}$, S is the start symbol,

$P = \{S \rightarrow AbB, A \rightarrow bBB, Bb \rightarrow aA, B \rightarrow ba, AB \rightarrow aA\}$.

What is the *language generated by G*, $L(G)$, that is the set of all strings of terminals that are derivable from the starting state *S*. **[3]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) What is the row of Pascal's triangle containing the binomial coefficients

$$\binom{10}{k}, 0 \le k \le 10$$ **[1]**

(b) By using the Euler Theorem, prove whether the $K_{3,3}$ is planar. **[3]**

(c) There is a certain parish that has six towns. Suppose that six Travelling salespersons, Ann, Beth, Carla, Debbie, Emanuel and Frankson sell their products within the six towns, T_1, T_2, T_3, T_4, T_5 and T_6 as stated below.

Travelling Salespersons	**Towns**
Ann	T_1
Beth	T_2, T_4, T_6
Carla	T_1, T_3, T_5, T_6
Debbie	T_4, T_6
Emanuel	T_1, T_2, T_6
Frankson	T_1, T_2, T_4, T_6

v. Model the above situation as a matching network **[2]**

vi. Find a maximal matching **[1]**

vii. Find a way in which each travelling salesperson can be assigned to a single town such that each town is visited by a salesperson and no more than one of the six salespersons visit a town, or use Hall's Theorem to explain why no such complete matching exists. **[3]**

QUESTION THREE (10 Marks)

(a) Consider the recurrence function

$$T(n) = 81T(n/3) + 3n^3 \log n$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \le 1$. **[4]**

(b) Expand $(2q - 3p)^5$ using the Binomial Theorem. **[2]**

(c) What is the order of growth of $\sum_{k=1}^{n}(k+1)^m$ where m is a positive integer? **[4]**

QUESTION FOUR (10 Marks)

(a) Given courses CS1, CS2, P1, P2, M1 and M2, and the following listing which shows courses for which exams cannot be at the same time due to course selections:

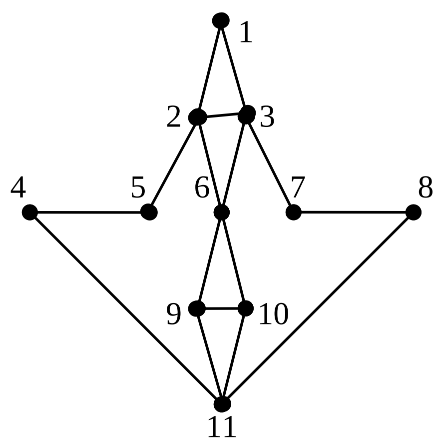
Students who pursue Course CS1 also pursue P1.
Those who pursue Course CS2 also pursue CS1, P2 and M2.
Those who pursue Course P2 also pursue M2, CS2 and P1.
Those who pursue Course M1 also pursue M2 and P1.
Students who pursue Course M2 also pursue M1, P2 and CS2.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and determine the minimum number of time periods necessary for the six courses. **[2]**

(b) If you studied intensely the probability of passing this COMP2201 final examination is 93%, if you studied lightly the probability of passing the examination is 52%, and if you studied none at all the probability of passing is 1%. The course tutor knows that 70% of the students studied intensely, 20% of them studied lightly and 10% did not find the time to study. What is the probability that you pass this COMP2201 final examination? **[4]**

(c) Find a Hamiltonian cycle, or state why one does not exist in the graph. **[2]**



QUESTION FIVE (10 Marks)

(a) Consider the geometric series:

$$1 + 5/7 + 25/49 + 125/343 + \dots$$

i. What is the sixth term of the series? **[1]**

ii. What is the smallest value of n such that $S_n > 3$, where S_n is the sum of the first n terms of the series? **[3]**

(b) Let $f(n)$ be defined by

$$f(n) = 64f\left(\frac{n}{4}\right) + 4n^2$$

if $n > 1$ and $n = 4^m$, where m is a positive integer.

By using the principles of Recurrence Relation, find a general formula for $f(n)$. **[6]**

END OF QUESTION PAPER