COMP2201 – Discrete Mathematics
Matching

1. A small school has five teachers, Andy, Beth, Charl, Donnue and Eve. In the spring term, six courses, CS1, CS2, CS3, CS4, CS5 and CS6, are to be offered. Each teacher is qualified to teach one or more courses. The school has the following information for each teacher.

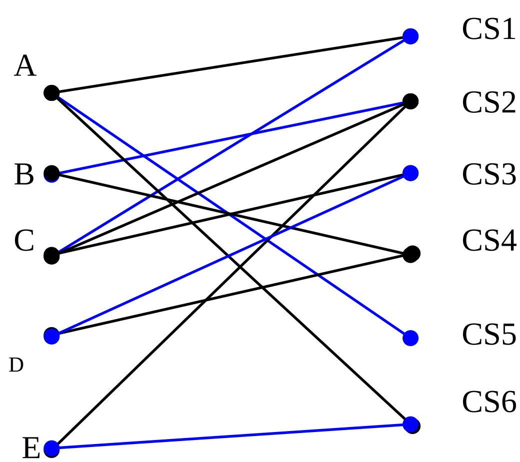
Teacher	**Courses qualified for**
Andy	CS1, CS5, CS6
Beth	CS2, CS4
Charl	CS1, CS2, CS3
Donnue	CS3, CS4
Eve	CS2, CS6

i. Model the above situation as a matching network
ii. Find a maximal matching
iii. Find a way in which each teacher can be assigned to teach a course or use Hall's Theorem to explain why no such way exists.

Answer 1.

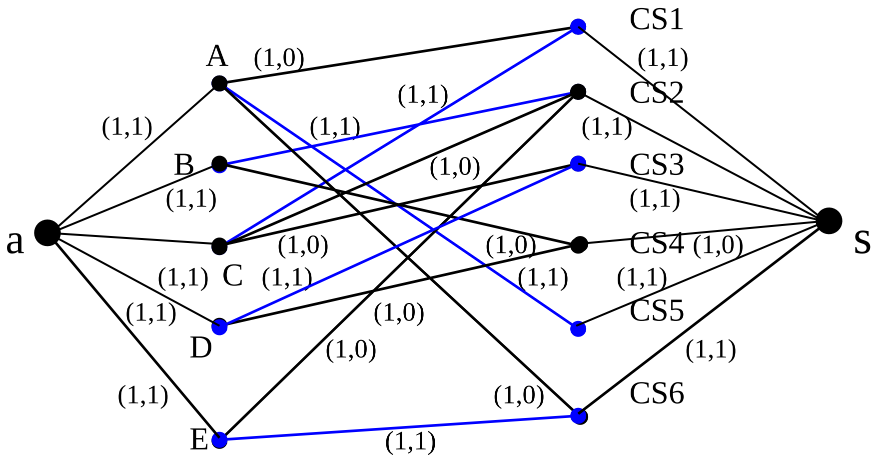
i. Let A be Andy, B be Beth, C be Charl, D be Donnue and E be Eve

[SIMPLE MATCHING NETWORK]



OR

[COMPLEX MATCHING NETWORK – For Knowledge Purposes Only]



ii. A-CS5, B-CS2, C-CS1, D-CS3, E-CS6

or

A-CS1, B-CS2, C-CS3, D-CS4, E-CS6

or

…

iii. Assuming that each teacher is allowed to teach exactly one course

[Same as the maximal matching]

A-CS5, B-CS2, C-CS1, D-CS3, E-CS6

or

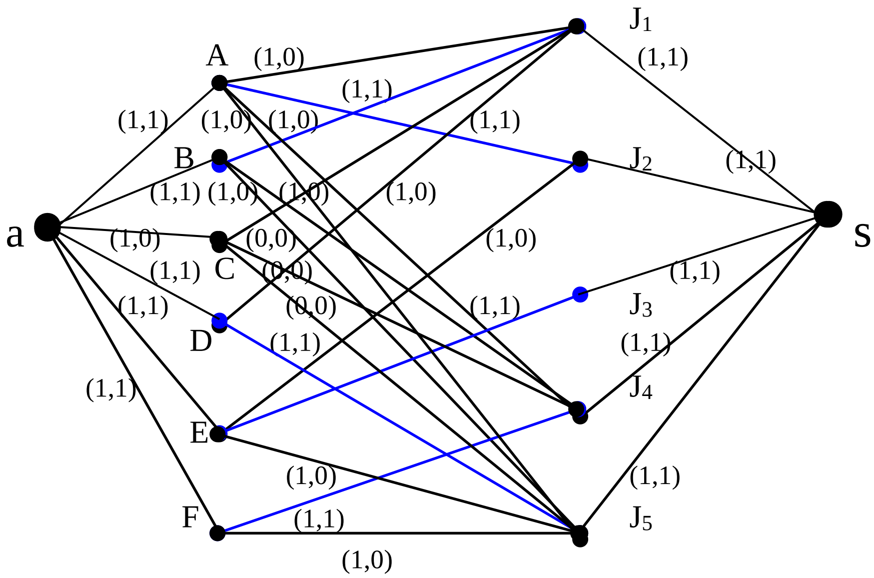
A-CS1, B-CS2, C-CS3, D-CS4, E-CS6

2. Applicant A is qualified for jobs J_1, J_2, J_4 and J_5; B is qualified for jobs J_1, J_4 and J_5; C is qualified for jobs J_1, J_4 and J_5; D is qualified for jobs J_1 and J_5. E is qualified for jobs J_2, J_3 and J_5; F is qualified for jobs J_4 and J_5.

Model the above situation as a matching network
Find a maximal matching
Show a complete matching or Use Hall's Theorem to show that one does not exist.

Answer 2.

i.



ii. A maximal Matching is found and is highlighted in the Matching Network.

A-J_2, B-J_1, D-J_5, E-J_3, F-J_4

or

A-J_2, C-J_1, D-J_5, E-J_3, F-J_4

or

…

iii. A Complete Matching does not exist.

By Hall's Theorem

If G is a directed, bipartite graph with disjoint sets of vertices V and W and with directed edges from V to W.

Let $S \subseteq V$.
Let $R(S) = \{w \in W \mid v \in S \text{ and } (v, w) \text{ is an edge in } G\}$
Then: there exists a *complete matching* in G if and only if
$|S| \leq |R(S)|$ for all $S \subseteq V$.

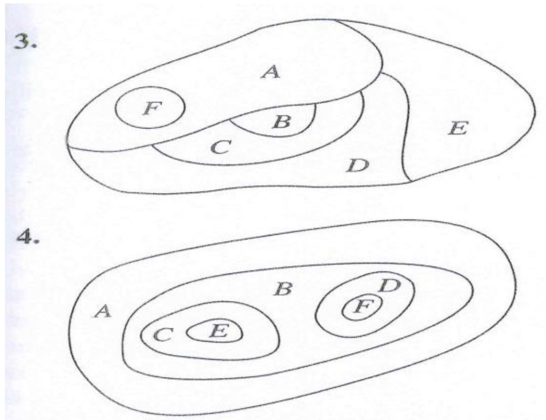
As $V = \{A, B, C, D, E, F\}$
Let $S = \{B, C, D, F\}$ $\rightarrow$ $|S| = 4$
$R(S) = \{1, 4, 5\}$ $\rightarrow$ $|R(S)| = 3$

As $|S| = 4$ NOT $\leq$ $3 = |R(S)|$

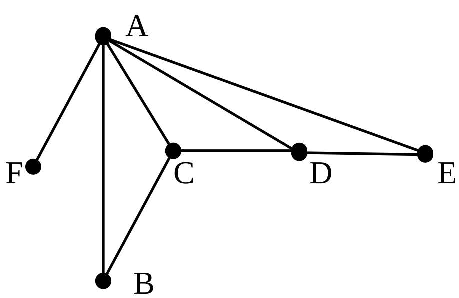
By Hall's Theorem
A Complete Matching Does Not Exist

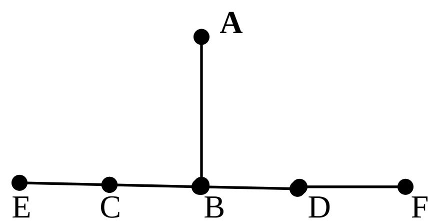
Colouring

3. Construct the Dual Graph, G and determine the chromatic number of G, $\chi(G)$ in exercises 3.3 to 3.4



Answer 3.3 and 3.4





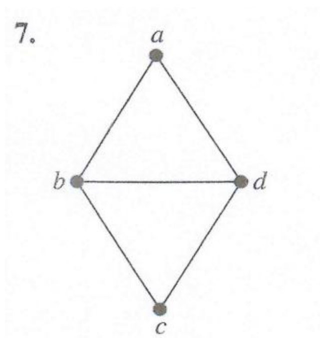
Chromatic Number is 3

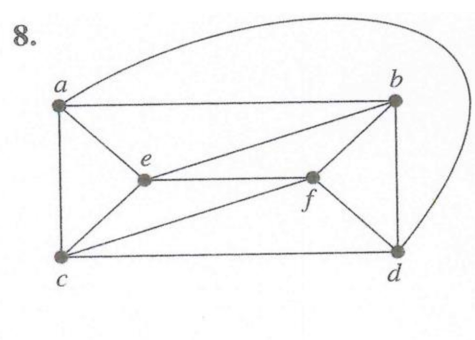
- A – Colour 1
- B – Colour 2
- C – Colour 3
- D – Colour 2
- E – Colour 3
- F – Colour 2

Chromatic Number is 2

- A – Colour 1
- B – Colour 2
- C – Colour 1
- D – Colour 1
- E – Colour 2
- F – Colour 2

4. Find the chromatic number of the given graphs in exercises 4.7 to 4.8





Answer 4.7 to 4.8

4.7. Chromatic Number is 3
a – Colour 1
b – Colour 2
c – Colour 3
d – Colour 1

4.8. Chromatic Number is 3
a – Colour 1
b – Colour 2
c – Colour 1
d – Colour 2
e – Colour 1
f – Colour 2
g – Colour 3

5. Given courses 1 to 7 and the listing which shows courses for which exams cannot be at the same time due to student schedules, construct the graph representing the scheduling of final exams and by using coloring, schedule the final exams and thereby determine the minimum number of time periods necessary for the seven courses.

Students who pursue Course 1 also pursue Courses 2, 3, 4 and 7.
Those who pursue Course 2 also pursue Courses 3, 4, 5 and 7.
Students who pursue Course 3 also pursue 1, 4, 6 and 7.
Those who pursue Course 4 also pursue Courses 2, 3, 5 and 6.
Students who pursue Course 5 also pursue Courses 6 and 7.
Those who pursue Course 7 also pursue Courses 1, 2 and 6.

Answer 5.

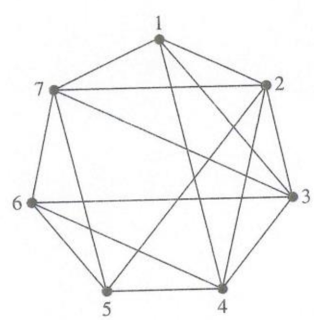


FIGURE 8 The Graph Representing the Scheduling of Final Exams.

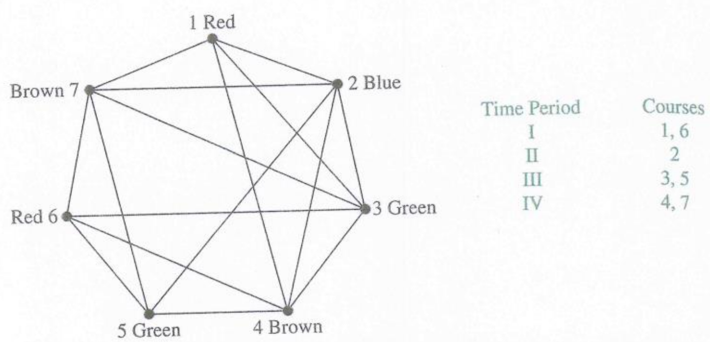


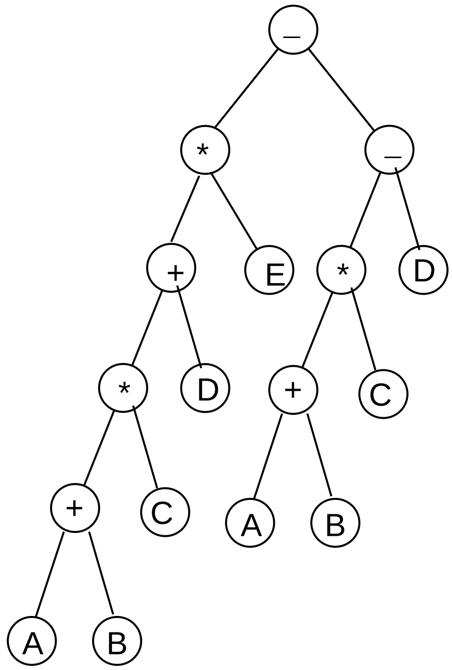
FIGURE 9 Using a Coloring to Schedule Final Exams.

Trees

6. Represent the following expression as a binary tree and write the prefix and postfix forms of the expression.

$$(((A+B)*C+D)*E)-((A+B)*C-D)$$

Answer 6.



Prefix

– * + * + A B C D E – * + A B C D

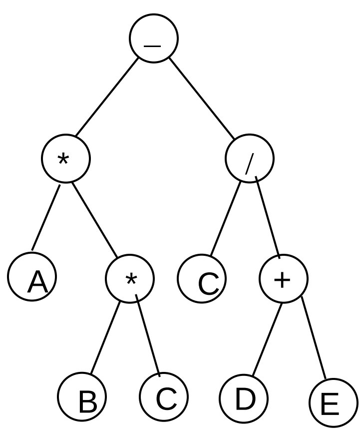
Postfix

A B + C * D + E * A B + C * D – –

7. Represent the postfix expression as a binary tree and write the prefix form, the usual infix form, and the fully parenthesized infix form of the expression.

A B C * * C D E + / -

Answer 7.



Prefix

– * A * B C / C + D E

Infix

A * B * C – C / D + E

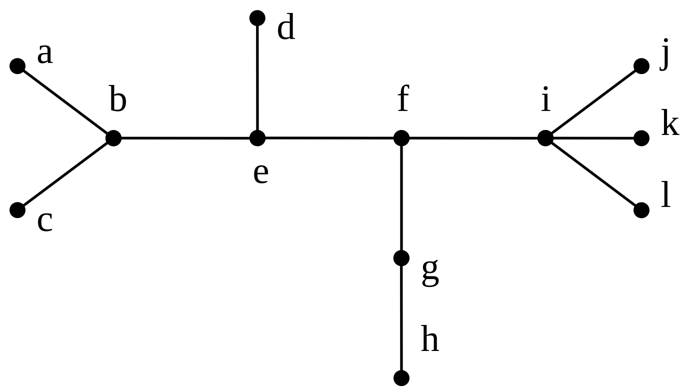
Parenthesized Infix

A * (B * C) – C / (D + E)

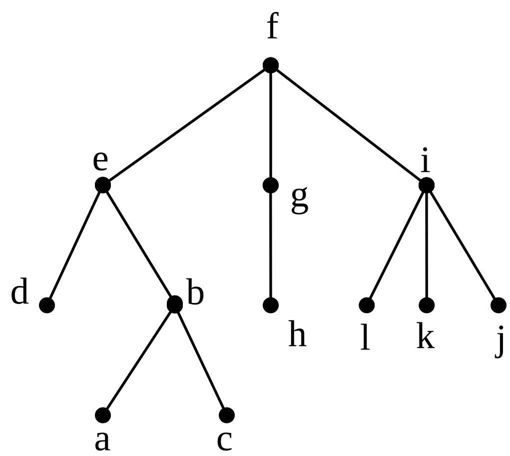
Fully Parenthesized Infix

((A * (B * C)) – (C / (D + E)))

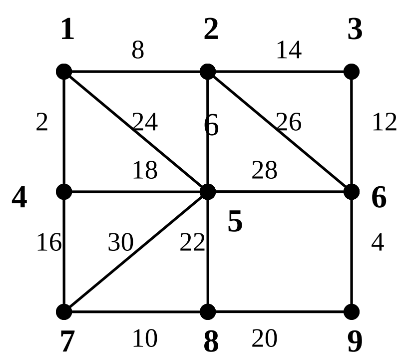
8. Draw the free tree of the graph below as a rooted tree with root f.



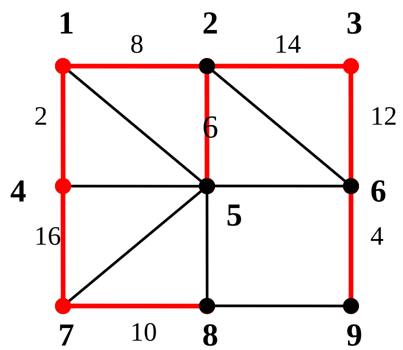
Answer 8.



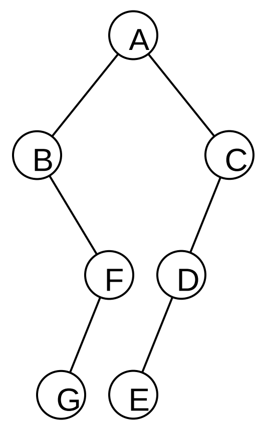
9. Find a minimal spanning tree for the following graph.



Answer 9.



10. For the following binary tree, list the order in which the vertices are processed using preorder, inorder and postorder traversal.



Answer 10.

Preorder

A B F G C D E

Inorder

B G F A E D C

Postorder

G F B E D C A