THE UNIVERSITY OF THE WEST INDIES

Semester I ■   Semester II □   Supplemental/Summer School □

Mid-Semester Examinations of: October ■   /February/March □   /June □ 2019/2020

Originating Campus: **Cave Hill** □   **Mona** ■   **St. Augustine** □

Mode: **On Campus** ■   **By Distance** □

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Science**

Date: **Thursday, October 17, 2019**   Time: **11:00 a.m.**

Duration: **1**   **Hour.**   Paper No: **1 (of 1)**

Materials required:

        Answer booklet: Normal ■   **Special** □   **Not required** □

                Calculator: Programmable □   **Non Programmable** ■   **Not required** □
        (where applicable)

  Multiple Choice answer sheets: numerical □   **alphabetical** □   **1-20** □   **1-100** □

Auxiliary/Other material(s) – Please specify:

Candidates are permitted to bring the following items to their desks: Pencil or pen, Ruler, ID card, Exam card

Instructions to Candidates: This paper has   2   pages &   6   questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

All questions are COMPULSORY.

Calculators are allowed.

1. (a) A shipment of 50 microprocessors of which three are defective. In
        how many ways can we select a set of five microprocessors
        containing at most one defective microprocessor?   **[1]**
    (b) By drawing a Tree Diagram, determine how many bit strings of
        length three have consecutive 1s?   **[3]**

2. Seven (7) plastic balls of different colours and varied weights are placed in
    a box. The Red ball is four times as likely to be pulled as the Orange,
    Yellow and Green balls. The Yellow ball is two times as likely to be pulled
    as the Blue and Indigo balls. The Blue ball is three times as likely to be
    pulled as the Violet ball. Assign probabilities to the seven outcomes in the
    sample space.   **[5]**

3. (a) Use Pascal's triangle to compute the values of
        $$\binom{6}{2} \quad and \quad \binom{8}{5}$$ **[2]**

    (b) Find the coefficient of p^6q in the expansion $(3p + 4q)^7$ **[2]**

    (c) In a given city, 1.5 percent of the citizens use social media as their
        main source of news. Find the probability that among 500 citizens
        chosen for a poll, at most two use social media as their main source
        of news.   **[2]**

4. Consider the recurrence function
    $$T(n) = 81T(n/3) + 81n^4$$
    Give an expression for the runtime $T(n)$ if the recurrence can be solved
    with the Master Theorem. Assume that $T(n) = 1$ for $n \leq 1$.   **[3]**

5. (a) By applying the principle of residue class replacement, determine the
        value of
        $$(57 \times 72 + 69) \bmod 7$$ **[2]**

    (b) List two positive and two negative values of each of the residue
        classes (mod 7).   **[2]**

    (c) Construct the Multiplication Table in Z_7.   **[3]**

6. Let $f_1(x), f_2(x), g_1(x)$ and $g_2(x)$ be functions defined $f_i, g_i : Z^+ \rightarrow R$
        where Z^+ is the set of Positive integers and
                    R is the set of Real numbers
    Prove the following statement
            If $f_1(x) = \Theta(g_1(x))$ and $f_2(x) = \Theta(g_2(x))$,

            then $f_1(x)f_2(x) = \Theta(g_1(x)g_2(x))$ **[5]**

END OF QUESTION PAPER