THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of December ■ /April/May ☐ /July ☐ 2009

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2101/CS20S Discrete Mathematics for Computer Scientists**

Date: Wednesday, December 16, 2009   Time: 1:00 - 3:00 p.m.

Duration: **2 Hours.**   **Paper No:**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20 ☐ 1-100**	☐

Auxiliary/Other material(s) – Please specify:

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examinations consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) Prove the following

i. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$ **[2]**

ii. $1^3 + 2^3 + \ldots + n^3 = (n\,(n+1)/2)^2$ for the positive integer n **[4]**

(b) By using the inclusion-exclusion principle

i. Calculate the number of bit strings of length 8 that begin with two 0s, have seven consecutive 0s, or end with a 1 bit **[3]**

ii. If a random experiment is selecting bit strings of length 8, what is the probability of selecting bit strings that begin with two 0s, have seven consecutive 0s, or end with a 1 bit **[1]**

iii. Give a formula for the number of elements in the union of four sets A_1, A_2, A_3, and A_4 **[1]**

(c) Let the set A = {0,2,4,6,8,10} and B = {0,1,3,4,5,7} and C = {4,5,6,7,8,9,10}

i. List the elements of $A - ((A \cap B) \cup C)$ **[1]**

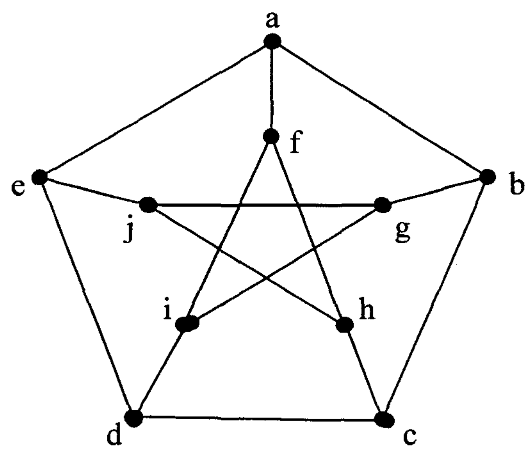
ii. Where Δ represents the symmetric difference between two sets, determine the set $A \;\Delta\; (B \;\Delta\; C)$ **[2]**

(d) Use generating functions to solve the following recurrence relation

$a_0 = 1$

$a_n = 3a_{n-1} + 2$ for $n \geq 1$ **[5]**

(e) Prove that a Hamiltonian cycle does not exist in the graph below **[2]**



(f) In an arithmetic series, the sum of the third term and the sixth term is 35. The fifth term is greater than the second term by 12.

i. Find the first term and the common difference. **[1]**

ii. What is the smallest value of n such that $S_n > 600$, where S_n is the sum of the first n terms of the series? **[3]**

(g) If $f: R \rightarrow R$ is a function and c is a nonzero real number, the function $(c \cdot f): R \rightarrow R$ is defined by the formula $(c \cdot f)(x) = c \cdot f(x)$
where R is the set of Real numbers
Show that if f is bijective, $c \cdot f$ is also bijective **[3]**

(h) Let S be the set of 9-character strings of uppercase and lowercase English letters.
Let p = ComPUtiNg
Define $x \, R \, y$ to mean that for every positive integer n, the nth character in x and y are the same letter, either uppercase or lowercase.

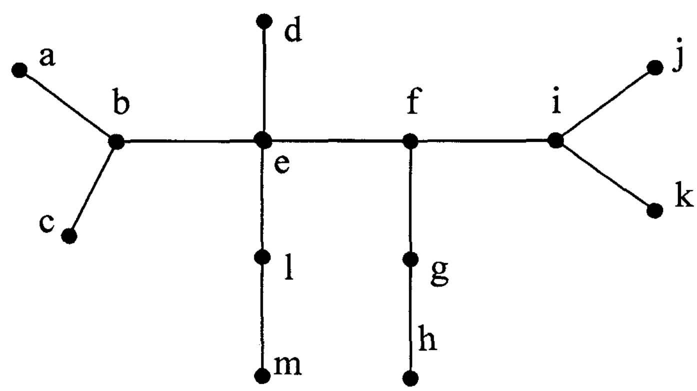
i. Show that the given relation R is an equivalence relation on the set S. **[3]**

ii. Describe the equivalence class containing the given element p in S. **[2]**

iii. Show whether the following collection of subsets is a partition on the set S:
Strings that begin with a uppercase vowel, Strings that begin with a lowercase vowel, Strings that begin with an uppercase consonant, and Strings that begin with a lowercase consonant. **[2]**

(i) i. Represent the following expression as a binary tree
$$((x+2)\wedge 3)*(y-(3+x))-5$$ **[3]**

ii. Draw the free tree of the graph below as a rooted tree with root f.

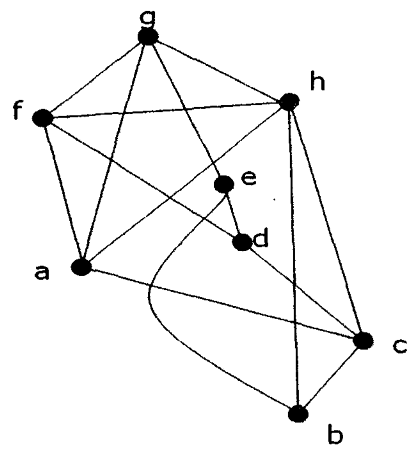


[2]

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

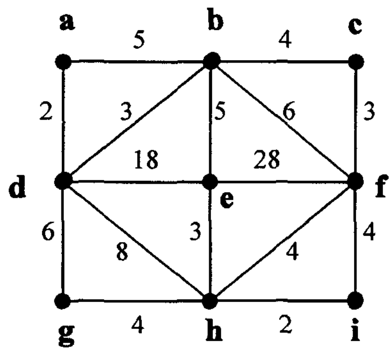
(a) Using Kuratowski's Theorem or otherwise, show that the graph below is not planar **[3]**



(b) Consider the K_5 graph.
If an Eulerian cycle exists, describe one; otherwise prove that an Eulerian cycle does not exist in the K_5 graph **[2]**

(c) Prove that a tree with n vertices has n-1 edges. **[2]**

(d) Find a minimal spanning tree for the following graph. **[3]**



QUESTION THREE (10 Marks)

Let a,b,c be integers such that $a \geq 1$, $b > 1$ and $c > 0$. Let $f: N \rightarrow R$ be functions where N is the set of Natural numbers and R is the set of Real numbers such that

$$f(1) = c \quad \text{and} \quad f(n) = af(n/b) + c$$

for $n = b^k$, where k is a positive integer greater than 1.

(a) By using the principles of Recurrence Relation, find a general formula for $f(n)$ **[5]**

(b) Hence show that if $a \neq 1$, then $f(n) = \dfrac{c(an^{\log_b a} - 1)}{a-1} = \Theta(n^{\log_b a})$ **[5]**

QUESTION FOUR (10 Marks)

(a) Let $f_1(x)$ and $f_2(x)$ be functions defined $f_i : Z^+ \to R$
where Z^+ is the set of Positive integers and
R is the set of Real numbers

Prove the following statement

$$\text{If } f_1(x) = \Theta(g_1(x)) \text{ and } f_2(x) = \Theta(g_2(x)), \text{ then } (f_1 f_2)(x) = \Theta((g_1 g_2)(x))$$ **[4]**

(b) What is the limit of $\sum_{k=0}^{n} x^k$ as $n \to \infty$? State all necessary conditions for this limit to exist. **[3]**

(c) List the first five terms in the sequence generated by the following recurrence relation

$s_0 = 6$

$s_1 = 30$

$s_n = 5s_{n-1} - 6s_{n-2}$ for $n \geq 2$ **[1]**

(d) Using the recurrence relation in part (c), what is the order of growth of the number of steps to calculate the n^{th} term in the sequence? **[2]**

QUESTION FIVE (10 Marks)

(a) In a given city 3 percent of all licensed drivers will be involved in at least one car accident in any given year. Find the probability that among 120 licensed drivers randomly chosen in the city

i. only five will be involved in at least one accident in any given year **[1]**

ii. at most three will be involved in at least one accident in any given year **[2]**

(b) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters? **[2]**

(c) A random experiment consists of rolling an unfair, six-sided die. The digit 3 is two times as likely to appear as the number 1. The number 1 is three times as likely to appear as the number 6. The number 6 is two times as likely to appear as each of the other numbers 2, 4 and 5

i. Assign probabilities to the six outcomes in the sample space **[3]**

ii. Suppose that the random variable X, is assigned the value of the digit that appears when the die is rolled. If the expected value is denoted by

$E(X) = \sum_{i=0}^{n} p(x_i) X(x_i)$ where $p(x_i)$ is the probability for the event x_i,

what is the expected value of X? **[2]**

END OF QUESTION PAPER