THE UNIVERSITY OF THE WEST INDIES

EXAMINATION OF DECEMBER 2007

Code and Name of Course: **CS20S – Discrete Mathematics for Computer Science** Paper: 1

Date and Time: **December 05, 2007 at 9am** Duration: **2 hours**

INSTRUCTIONS TO CANDIDATES: This paper has **2** page(s) and **4** questions

Answer question 1 in Section 1 and any other two questions from Section 2. Calculators are allowed.

Section 1

Question 1 *Compulsory* [30]

a. Let p be Erika reads The Gleaner, let q be Erika reads The Observer, and let r be Erika reads The Herald. Write each of the following in symbolic form:

(i) Erika reads The Gleaner or The Observer, but not the The Herald

(ii) Erika reads The Gleaner and The Observer, or she does not read The Gleaner and The Herald

(iii) It is not true that Erika reads The Gleaner but not The Herald

(iv) It is not true that Erika reads The Herald or The Observer but not The Gleaner

[4]

b. Consider the conditional proposition q: $(\exists x \in A)(x > c) \rightarrow (\forall y \in B)(y \geq 0)$. Write down the converse, inverse and contrapositive of the conditional proposition q. [6]
Which of these propositions are logically equivalent to q? Explain. [2]

c. Solve the following recurrence relation, using generating functions:
$S_0 = 0, S_1 = 1$ and $S_n = 2S_{n-1} - S_{n-2}$ for $n \geq 2$ [9]

d. Consider the random experiment of tossing nine fair coins. What is the probability that the number of heads and the number of tails differ by at most 3? [6]

e. Show that $7x^2$ is $O(x^3)$ [3]

OVER. . .

Section 2

Question 2 *Proofs, Counting and Equivalence Relations* **[15]**

a. Use mathematical induction to prove this formula for the sum of a finite number of terms of a geometric progression:
$\sum_{j=0}^{n} ar^j = a + ar + ar^2 + ... + ar^n = \frac{ar^{n+1} - a}{r - 1}$ when $r \neq 1$,
where n is a non-negative integer. **[6]**

b. Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there? **[4]**

c. What is an equivalence relation? Equivalence relation R partitions set S = {1,2,3,4,5,6} into the partition $A_1 = \{1, 2, 3\}, A_2 = \{4, 5\}, A_3 = \{6\}$. List the ordered pairs of the equivalence relation R. **[5]**

Question 3 *Graphs* **[15]**

a. What makes a graph planar? Draw graphs $K_{3,3}$ and K_5. **[4]**

b. Show that while K_4 is planar both $K_{3,3}$ and K_5 are not. **[6]**

c. Define the following terms in a graph G: Euler path, Hamiltonian circuit, connected graph. Draw an example of each. **[5]**

Question 4 *Trees* **[15]**

a. The monthly lease costs for lines in a computer network connecting various towns is given below:
A to B : $2000; B to C : $800; C to D : $700; A to D : $1200; A to E : $900; A to C : $2200; E to C : $1400; E to B : $1600; E to D : $1300

(i) Draw a weighted graph of the towns A, B, C, D and E showing these costs. **[2]**

(ii) Find a minimum-cost communication network connecting all the computers represented by the weighed graph in (i) above. **[6]**

b. A binary tree is used to represent an expression. Internal nodes of the tree represent arithmetic operators and leaf nodes represent variables (x, y and z in our example). A pre-order traversal of the tree yields the following: *+x/yxz.

(i) Draw the binary tree thus represented. **[2]**

(ii) Give the output of in-order and post-order traversals of the tree. **[4]**

(iii) If x = 2, y = 6 and z = 3 what is the value of the expression represented? **[1]**

***** End Of Question Paper *****