THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of December ■ /April/May ☐ /July ☐ 2013

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Scientists**

Date:	**Friday, December 13, 2013**	Time:	**1:00 p.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐	**1-100** ☐

Auxiliary/Other material(s) – Please specify: None

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) In an arithmetic series, the sum of the third term and the sixth term is 45. Three consecutive terms of the same series are $4x + 27$, $2x + 26$ and $3x + 34$. If the sum of the terms in the series is 105, find the number of terms in the series, n. **[5]**

(b) i. What is the row of Pascal's triangle containing the binomial coefficients $\binom{7}{k}$, $0 \le k \le 7$ **[1]**

ii. Expand $(-3p + 2q)^6$ using the Binomial Theorem. **[3]**

(c) If you studied intensely the probability of passing this COMP2201 final examination is 90%, if you studied lightly the probability of passing the examination is 50%, and if you studied none at all the probability of passing is 2%. The course tutor knows that 60% of the students studied intensely, 30% of them studied lightly and 10% did not find the time to study. What is the probability that you studied intensely, given that you pass this COMP2201 final examination? **[4]**

(d) Consider the recurrence function

$$T(n) = 4T(n/4) + n/3$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \le 1$. **[4]**

(e) Given G, a phrase-structure grammar.
Let $G = (V, T, S, P)$, where

V = {a, b, A, B, S}, T = {a, b}, S is the start symbol,
P = {S → ABb, A → BB, Bb → aA, B → ab, AB → b}.

What is the *language generated by G, L(G)*, that is the set of all strings of terminals that are derivable from the starting state *S*. **[3]**

(f) By constructing a Tree Diagram, determine how many 3-combinations are there of 4 objects (A, B, C, D)? **[2]**

(g) Find a closed form expression for the following recurrence relation

$s_0 = 0$
$s_1 = 1$
$s_n = s_{n-1} + 2s_{n-2}$ for $n \ge 2$ **[5]**

(h) Construct a finite-state machine that models a snack vending machine that accepts \$5, \$10 and \$20 coins. The machine has a cancel button (C) that may be pressed at any stage to cancel the purchase and have all collected money returned. When \$25 or more is deposited, the excess (over \$25) is returned, and the customer may press the green button (G) to receive the snack (S).

I. Represent the finite-state machine as a **state table**. **[4]**

II. Represent the finite-state machine as a **state diagram**. **[3]**

(i) Suppose that five politicians, Anderson, Brown, Curtis, Debue and Earle are members of executive committees, E_1, E_2, E_3, E_4, E_5 and E_6 based on their various levels of expertise.

Politician	**Executive Committees**
Anderson	E_1, E_2, E_4
Brown	E_1, E_4
Curtis	E_2, E_3, E_5, E_6
Debue	E_1, E_4
Earle	E_1, E_2

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each politician can be assigned to a committee such that each committee has no more than one of these five politicians, or use Hall's Theorem to explain why no such complete matching exists. **[3]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) Determine the order of growth of $\sum_{k=1}^{n+1} k^{m-1}$, if m is a positive integer? **[3]**

(b) If the probability is 0.55 that a student will attend a Computer Science conference if she is deemed to be an excellent student, find the probability that the seventh student to be deemed an excellent student will be the first to attend a Computer Science conference. **[1]**

(c) A computer access password consists of from three to five digits chosen from the 10 numeric digits with repetitions allowed. How many different passwords are possible? **[2]**

(d) Given seven courses C1, C2, C3, P1, P2, M1 and M2, and the following listing which shows courses for which examinations cannot be at the same time due to students' course selections:

Students who pursue Course C1 also pursue C2.
Those who pursue Course C3 also pursue P1, M1 and M2.
Students who pursue Course P1 also pursue C2 and C1.
Those who pursue Course P2 also pursue M2 and C2.
Those who pursue Course M1 also pursue M2 and C3.
Students who pursue Course M2 also pursue M1, C3 and P2.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and by determining the chromatic number χ, state the minimum number of time periods necessary for the examinations of the seven courses. **[2]**

QUESTION THREE (10 Marks)

(a) Consider the geometric series:

$$3 + 21/9 + 147/81 + 1029/729 + \dots$$

I. Determine a formula for S_n where S_n is the sum of the first *n* terms of the series? **[4]**

II. What is the limit of S_n **[1]**

(b) Let $f(n)$ be defined by

$$f(n) = 8f\left(\frac{n}{3}\right) + 2n^3$$

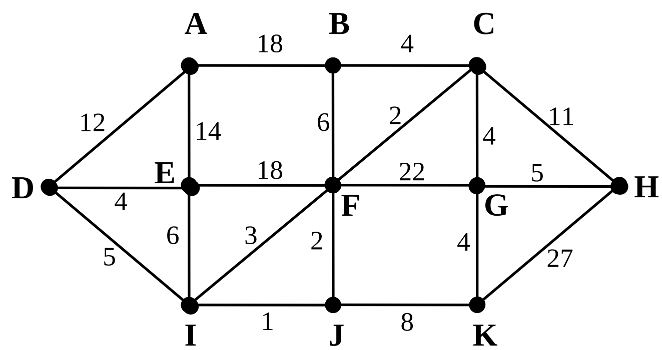
if $n > 1$ and $n = 3^m$, where m is a positive integer.

By using the principles of Recurrence Relation, find a general formula for $f(n)$. **[5]**

QUESTION FOUR (10 Marks)

(a) Find the generating function for the sequence
$$\{3,\ 5,\ 7,\ 9,\ 11,\ \ldots\}$$ **[5]**

(b) Find a minimal spanning tree for the following graph. **[3]**

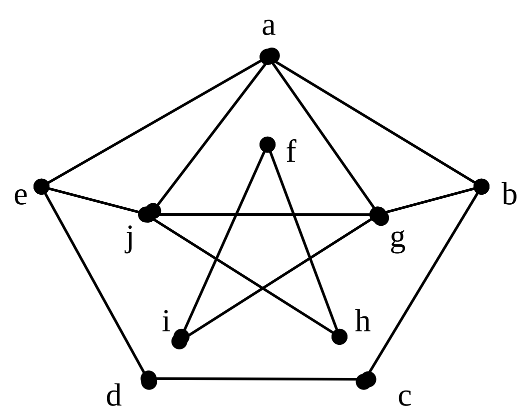


(c) By using the inclusion-exclusion principle, give a formula for the number of elements in the union of four sets A, B, C and D **[2]**

QUESTION FIVE (10 Marks)

(a) i. By using the Euler Theorem, prove whether the $K_{3,3}$ is planar. **[3]**

ii. Find an Euler Path, or state why one does not exist in the graph. **[2]**



(b) If there are 62 successful GSAT students that were placed in 9 secondary institutions, use the Pigeonhole Principle to show that there is an institution with at least seven of these GSAT students. **[2]**

(c) A random experiment consists of rolling an unfair, six-sided die. The number 6 is three times as likely to appear as the numbers 1, 2 and 4. The number 4 is three times as likely to appear as the number 3. The number 1 is two times as likely to appear as the number 5.
What are the probabilities to the six outcomes in the sample space. **[3]**

END OF QUESTION PAPER