THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of December ■ /April/May ☐ /July ☐ 2014

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Scientists**

Date:	**Monday, December 15, 2014**	Time:	**1:00 p.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐ **1-100** ☐	

Auxiliary/Other material(s) – Please specify: None

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 6 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) i. By constructing a Tree Diagram, determine how many 2-pemutations are there of 4 objects (A, B, C, D)? **[3]**

ii. Within a certain university, students are assigned separately uniquely built male and female lockers according to their gender. The ratio between male and female students would have been 1:1 if there were not an odd number of students. If there are 109 students registered to the university and 54 lockers of each type (male and female), show that if each student uses a locker, then at least one locker is shared by two students. **[2]**

(b) Consider the recurrence function

$$T(n) = 32T(n/2) + 5n^3 \log n$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \leq 1$. **[4]**

(c) In a given city 3 percent of all licensed drivers will be involved in at least one car accident in any given year. Find the probability that the two hundredth licensed driver randomly chosen in the city will be the first to be involved in at least one car accident in a given year. **[2]**

(d) Prove whether the following statement is true or false. Assume that the functions f, *g* and *h* take on only positive values.

$$f(n) = \Theta(h(n)) \textit{ and } g(n) = \Theta(h(n)), \text{ then } f(n)g(n)h(n) = \Theta(h(n))$$ **[4]**

(e) i. Derive a formula for S_n, the sum of the first n terms of a Geometric Progression, where the first term is a and the common ratio, r is 1. **[2]**

ii. Consider the geometric series:

$$21 + 15 + 75/7 + 375/49 + \ldots$$

I. Determine a formula for S_n where S_n is the sum of the first n terms of the series? **[4]**

II. What is the limit of S_n **[1]**

(f) Given G, a phrase-structure grammar.
Let $G = (V, T, S, P)$, where

V = {a, b, A, B, S}, T = {a, b}, S is the start symbol,

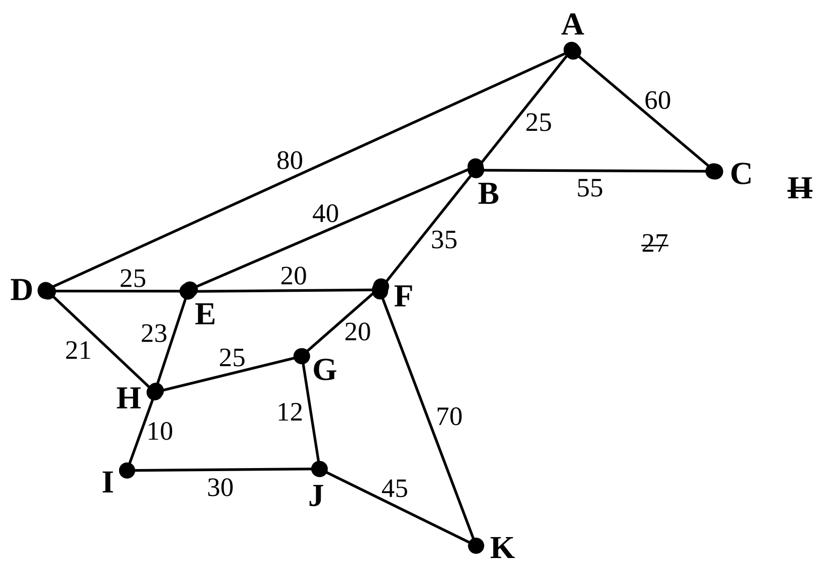
P = {S → AbB, A → bBB, Bb → aA, B → ba, AB → aA}.

What is the *language generated by G, L(G),* that is the set of all strings of terminals that are derivable from the starting state *S.* **[3]**

(g) Find the generating function for the sequence

{3, 6, 9, 12, 15, …} **[4]**

(h) Find a minimal spanning tree for the following graph. **[4]**



(h) Construct a finite-state machine that models a newspaper vending machine that accepts \$5, \$10 and \$20 coins. The machine has an automatic locking door (D) that is unlocked only after \$30 or more have been inserted. Once the door is unlocked the customer opens it and takes a paper, and closes the door. No change is ever returned no matter how much extra money has been inserted. The next customer starts with no credit. While the door has not been opened, the customer may press a red cancel button (C) to cancel the purchase and have all collected money returned.

I. Represent the finite-state machine as a **state table**. **[4]**

II. Represent the finite-state machine as a **state diagram**. **[3]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) Find a closed form expression for the following recurrence relation

$s_0 = 2$

$s_n = -3s_{n-1} + 2$ for $n \geq 1$ **[6]**

(b) Given nine courses CS1, CS2, CS3, P1,P2, P3, M1, M2 and M3, and the following listing which shows courses for which exams cannot be at the same time due to students' course selections:

Students who pursue Course CS1 also pursue CS3 and P1.
Those who pursue Course CS2 also pursue CS1, P2, M2 and M3.
Students who pursue Course P3 also pursue CS2, CS3 and P1.
Those who pursue Course M1 also pursue M2, P1 and CS3.
Students who pursue Course M2 also pursue M1, M3, P2 and CS2.
Those who pursue Course M3 also pursue M2, CS2 and P1.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and thereby determine the minimum number of time periods necessary for the nine courses. **[2]**

QUESTION THREE (10 Marks)

(a) In an arithmetic series, the sum of the third term and the sixth term is 12.5 . Three consecutive terms of the same series are $3x - 2$, $2x + 9$ and $4x + 3.5$. If the sum of the terms in the series is 50, find the number of terms in the series, n. **[4]**

(b) Let a,b,c be integers such that $a \geq 1$, $b > 1$ and $c > 0$.
Let $f: N \rightarrow R$ be functions
where N is the set of Natural numbers and R is the set of Real numbers
Given that

$f(1) = c$

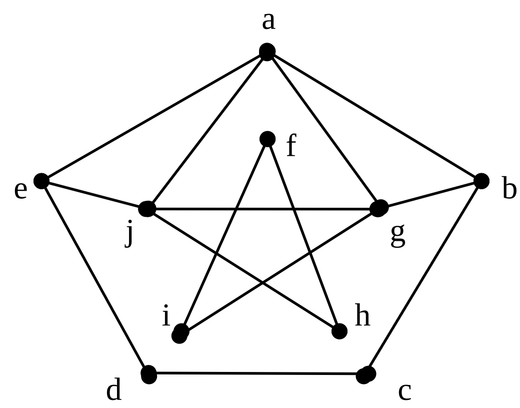
$n = b^k$, where k is a positive integer greater than 1.

$$f(n) = a^k f(n/b^k) + c\sum_{i=0}^{k-1} a^i$$

Show that $$f(n) = \Theta(a^{\log_b n})$$ **[6]**

QUESTION FOUR (10 Marks)

(a) Find a Hamiltonian Cycle, or state why one does not exist in the graph. **[3]**



(b) i. What is the row of Pascal's triangle containing the binomial coefficients $\binom{6}{k},\ 0 \le k \le 6$ **[1]**

ii. Expand $(x - 3y)^5$ using the Binomial Theorem. **[2]**

(c) A random experiment consists of rolling an unfair, six-sided die. The number 1 is two times as likely to appear as the numbers 2, 3 and 5. The number 3 is three times as likely to appear as the number 4. The number 1 is three times as likely to appear as the number 6.

i. Assign probabilities to the six outcomes in the sample space **[3]**

ii. Suppose that the random variable X, is assigned the value of the digit that appears when the die is rolled. If the expected value is denoted by

$E(X) = \sum_{i=0}^{n} p(x_i)X(x_i)$ where $p(x_i)$ is the probability for the event x_i,

what is the expected value of X? **[1]**

QUESTION FIVE (10 Marks)

(a) If a student does not study at all for this COMP2201 examination, the probability of passing the examination is 3%. If one studies at an average level, the probability of passing the examination is 51% whereas if study is done intensely, the probability of passing the COMP2201 examination is 92%. The course lecturer is sure that 8% of students do not study at all, 67% of them study at an average level and 25% of them study intensely. Given that you pass this COMP2201 final examination, what is the probability that you did not study at all? **[4]**

(b) There is a certain parish that has six towns. Suppose that six Travelling salespersons, Andrew, Bob, Carol, Donovan, Erik and Franz sell their products within the six towns, T_1, T_2, T_3, T_4, T_5 and T_6 as stated below.

Travelling Salespersons	**Towns**
Andrew	T_1, T_6
Bob	T_2, T_4
Carol	T_1, T_2, T_4
Donovan	T_4, T_6
Erik	T_1, T_2, T_6
Franz	T_1, T_3, T_5, T_6

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each travelling salesperson can be assigned to a single town such that each town is visited by a salesperson and no more than one of the six salespersons visit a town, or use Hall's Theorem to explain why no such complete matching exists. **[3]**

END OF QUESTION PAPER