THE UNIVERSITY OF THE WEST INDIES

ASSIGNMENT 2

SEMESTER I, 2020/2021

Code and Name of Course: **COMP2201 – Discrete Mathematics**

Assignment 2 - Group Assignment	
AREA	**DESCRIPTION**
Objectives	To have students apply the knowledge garnered during weeks 4 to 9 To have students work within a group setting with the allocation of work but still ensuring that the knowledge is spread throughout the group
Title	Assignment 2 – Group Assignment
Deliverable	The answers for the questions which follow the given instructions. A summary of the contribution made by each student in the group.
Instructions	1. Review Lectures of weeks 4 to 9 of the course, the COMP2201 Text and any other related Discrete Mathematics material 2. Read the Assignment 2 Sheet thoroughly 3. Submit the gradable solution by using the COMP2201 Assignment 2 - Group located within the **ASSIGNMENTS** Section of the OURVLE COMP2201 Course Environment. This may also be accessed by choosing the Assignments section below “Activities”
Format	The solution for this assignment must be submitted as a Microsoft Word document. Your ID number should form part of the Microsoft Word file name. The file name should take the format “COMP2201 Assignment 2 Semester 1 2020-2021 XXXXXXXX YYYYYYYY ZZZZZZZZ” where XXXXXXXX, etc. represents the student ID numbers.

Upload Constraint	The assignment should be uploaded in the relevant space provided in OURVLE (See "Instructions" section above). A message indicating "File uploaded successfully" will acknowledge that the file has been sent successfully. Do **NOT** assume your project has been received if you do not get this acknowledgement.
Group Restrictions	This is a group assignment. Each group must have a minimum of 2 members and a maximum of three (3) members.
Late Assignments	Late assignments are accepted. These are however graded then 25% deducted for each day of late submission.
Expectation	It is expected that students will discuss means to a solution within their group. The actual work written is expected to reflect the group's decision. Where replication of work is identified between groups, each paper will be graded. The allocated grade to each group's piece of work for where this anomaly is identified will be the grade divided by the number of replications discovered.
Scoring Rubric	Your electronic submission will be evaluated on: 1. The response submitted for each question (See detail individual marks below) – 95 marks 2. Group Dynamics i.e. for accomplishing the task as a group – 5 marks. Marks allocated based on correct inclusion of each student id number and name and a brief description as to the contribution made within the group. The actual grade of 100 marks will be displayed. The actual grade allocated is the percentage of the maximum marks (8 points)
Due Date	**Sunday, November 15, 2020**

Question 1 [5 mks]

In a geometric series, the sum of the third term and the fifth term is $68\frac{86}{125}$.

Three consecutive terms of the same series are $3\frac{6}{25}x$, $5\frac{104}{125}x$ and $10\frac{311}{625}x$. If x is equal to the sixth term in the series, and the sum of the terms in the series is $2\frac{10856}{59049}x$, find the number of terms in the series. **[5]**

Question 2 [5 mks]

Let $f_j(n)$, $g_j(n)$ and $h_j(n)$ be functions defined f_j , g_j , h_j: $R^+ \rightarrow R^+$

where Z^+ is the set of Positive integers and

R is the set of Real numbers

Determine the truth of the following statement

$$If\ f_1(n) = \Theta\big(f_2(n)\big)\ ,\ g_1(n) = \Theta\big(g_2(n)\big), \text{ and } h_1(n) = \Theta\big(h_2(n)\big)$$

$$\text{then} \qquad (f_1)(n) + (f_1g_1)(n) + (f_1g_1h_1)(n) = \Theta((f_2g_2h_2)(n))$$ **[5]**

It is known that $(z_1\, z_2\, z_3)(n) \equiv z_1\,(z_2\,(z_3(n)))$.

Question 3 [5 mks]

By applying the principles of Modular Arithmetic

(a) Examine the Addition and Multiplication Tables of Z_7, Z_8 and Z_9, and describe the patterns of the Addition Table and Multiplication Table of Z_n. **[2]**

(b) Use the pattern described in part (a) to determine the values of the Addition and Multiplication Tables of Z_{10}. **[1]**

(c) Construct the Addition and Multiplication Tables of Z_{10}, and determine the approximate percentage of the Addition and Multiplication Tables that may be determined from pattern matching. **[2]**

Question 4 [6 mks]

Consider the recurrence function

$$T(n) = 128T(n/4) + 2^9n^3 log\ n$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \leq 1$. **[6]**

Question 5 [6 mks]
Find the generating function for the sequence

$\{140, \; 640, \; 1860, \; 4160, \; 7900, \; \ldots\}$ **[6]**

Question 6 [7 mks]
Find a closed form expression for the following recurrence relation

$s_0 = 0$
$s_1 = 1$
$s_n = s_{n-1} + 12s_{n-2}$ for $n \geq 2$ **[7]**

Question 7 [6 mks]
Each integer in Z_n represents a residue class or congruence class.

(a) Define the set of congruence classes of (mod 12). **[1]**

(b) List the congruence classes (mod 12). **[1]**

(c) A certain country C has created 223 Zoom Meetings to provide for free primary education classes. For ease of reference, the Zoom Meeting Ids are uniquely assigned and referred as Education Zoom Ids

$$0, \; 1, \; 2, \; 3, \; \ldots, \; 222.$$

Parents or guardians are required to access a certain Primary Education website and register for the classes. In the registration process, each parent or guardian is assigned a unique 10-digit Registration number that begins with one pf the digits 1 to 9. The parent or guardian is assigned one of the Education Zoom Ids based on the unique 10-digit number and the result of the hashing function h. The Registration number is defined as R and the hashing function h is defined as

$$h(R) = R \bmod 223.$$

If Patrice McFarlane is a parent who accessed the Primary Education website of country C and assigned Registration number of 8487610023, by simplifying the Registration number to a set of addition and multiplication of positive integers, apply the principles of modular arithmetic and illustrate to which Education Zoom Id, Patrice McFarlane would be assigned. **[4]**

Question 8 [6 mks]
Find a formula for the series c defined by

$$c_n = \sum_{i=1}^{n} b_i$$ Where b is the sequence $\{1, 1, 2, 3, 5, 2, 6, 8, 9, 4, 18, 13, \ldots\}$ **[6]**

Question 9 [4 mks]

Determine the order of growth of $\sum_{k=1}^{n}(k-1)n(n+1)$. **[4]**

Question 10 [9 mks]

A coding system encodes messages using strings of base 9 digits. A codeword is considered valid if and only if it contains an odd number of zeroes.

i. Find a recurrence relation for the number of valid codewords of length n. State initial conditions. **[4]**

ii. Solve this recurrence relation using generating functions. **[5]**

Question 11 [4 mks]

A set of students was surveyed and provided a response on the Likert scale concerning whether online classes should continue the following school year. If the majority response is Strongly Agree, the probability of online classes continuing the following year is 78%. If the majority response is Agree, the probability of online classes continuing the following year is 64%. If the majority response is Neutral, the probability of the is 53%; however, if the majority response is Disagree, the probability of online classes is 37%. If the majority response is Strongly Disagree, the likelihood of online classes continuing the following year is only 16%.

Concerning whether online classes should continue the following school year, the survey revealed the results were 13% of respondents Strongly Agree, 27% Agree, 20% Neutral, 28% Disagree, and 12% Strongly Disagree.

i. Given that online classes actually continue the following school year, what is the probability that the majority response of the survey was Neutral? **[2]**

ii. Draw the Probability Tree that represents the given scenario. **[2]**

Question 12 [7 mks]

The Pigeonhole Principle suggests that where there are 367 people, at least two people must share the same birthday. Research the principles of the Birthday Problem, and use these principles to:

i. Show that the number of people randomly chosen is significantly less than that suggested by the Pigeonhole Principle. **[3]**

ii. Find the smallest number of people you need to choose at random so that the probability that at least two of them share the same birthday, is in excess of 90% **[3]**

iii. State any assumptions made for parts i. and ii. **[1]**

Question 13 [7 mks]

Let $f(n)$ be defined by

$$f(1) = 7$$
$$f(n) = 64f\left(\frac{n}{4}\right) + 9n^5 \log n$$

if $n > 1$ and $n = 4^k$, where k is a positive integer.

i. By using the principles of Recurrence Relation, find a general formula for $f(n)$ **[4]**

ii. Hence show that $f(n) = \Theta(n^5 \log n)$. **[3]**

Question 14 [8 mks]

Find the generating function for the sequence

$\{1, 225, 2227, 9283, 26349, 60061, \ldots\}$ **[8]**

Question 15 [10 mks]

Given the following concerning an arithmetic series and a geometric series:

- The second term of the geometric series is the same as the sixth term of the arithmetic series. Additionally, the negation of the seventh term of the arithmetic series is the same as the sum of the second and third terms of the geometric series.

- The first term of the geometric series exceeds the first term of the arithmetic series by $17\frac{5}{16}$.

- The sum of the first ten terms of the arithmetic series, S_{AP-10} and the sum of the first two terms of the geometric series, S_{GP-2} are related by the formula

$$S_{AP-10} + 16S_{GP-2} + 9 = 0$$

What is the total of the third term of the geometric series and the sum of the first twelve terms of the arithmetic series? **[10]**