# THE UNIVERSITY OF THE WEST INDIES

## ASSESSED TUTORIAL: SEMESTER 1 SAMPLE

Code and Name of Course: **COMP2201 Discrete Mathematics for Computer Scientists**

Date and Time: **Sample** Duration: **One (1) Hour**

**INSTRUCTIONS TO CANDIDATES:**
Do ALL Questions. Calculators are allowed in the examination.

**YOU ARE REQUIRED TO SHOW ONLY THE FORMULA WITH THE NECESSARY VALUES INSERTED. E.g. 2 x 4 or $3 \cdot 2^6$ or 4! or $^{13}C_5$ or $^{10}C_4\,^{12}C_3\,^{4}C_2$ or $^{6+6-1}C_{6-1}$ or $(^{20}C_5)^2$ or 6·9·7 + 6·9·4 + 6·7·4 + 9·7·4 or 8!/(3!2!), etc.**
**You do not earn any marks for showing the complete solution**

1. Suppose we have the digits 4, 3, 5, 7, 8 and 9, and repetitions are not permitted.
   (a) How many three-digit numbers can be formed?
   **Ans.** **6·5·4 or $^{6}P_3$**
   (b) How many are even?
   **Ans.** **5·4·2 or $^{5}P_2$ x 2**
   (c) How many of these three-digit numbers are less than 500?
   **Ans.** **2·5·4 or 2 x $^{5}P_2$**

2. (a) Determine the number of strings that can be formed by ordering the letters FIRST.
   **Ans.** **5!**
   (b) Find the number of permutations that can be formed from the letters of the word ELEVEN beginning with the letter E and ending with the letter N?
   **Ans.** **1 x 4!/2! x 1**

3. There is a set of five distinct art books, three distinct mathematics books, and two distinct computer science books.
   (a) In how many ways can these books be arranged on a shelf?
   **Ans.** **10!**
   (b) In how many ways can these books be arranged on a shelf if all books of the same discipline are grouped together?
   **Ans.** **3!5!2!3!**
   From a set of six distinct history books, nine distinct classics books, seven distinct law books, and four education books
   (c) How many ways can we select three books each from a different subject?
   **Ans.** **6·9·7 + 6·9·4 + 6·7·4 + 9·7·4**

4. (a) In how many ways can three teams containing four, two and two persons be selected from a group of eight persons?

**Ans.** $8!/(4!2!2!)$ **or** ${}^{8}C_{4}{}^{4}C_{2}{}^{2}C_{2}$

In view of a bag containing 20 distinct balls – six red, six green, and eight purple

(b) In how many ways can we draw five balls, then replace the balls, and then draw five more balls.

**Ans.** $({}^{20}C_{5})^{2}$

(c) In how many ways can we draw five balls without replacing them, then draw five more balls.

**Ans.** ${}^{20}C_{5}{}^{15}C_{5}$ **[or though incorrect** ${}^{20}C_{10}$**]**

5. (a) Suppose there are 10 roads from Oyster to Manchester and five roads from Manchester to Newfoundland. How many routes are there from Oyster to Newfoundland passing through Manchester?

**Ans.** $10 \cdot 5$

(b) How many eight-bit strings have either the second or the fourth bit 1 (or both)?

**Ans.** $2^{7} + 2^{7} - 2^{6}$ **or** $3 \times 2^{6}$

(c) A man has eight pairs of shoes, four pairs of pants, and five shirts. How many different outfits are possible?

**Ans.** $8 \cdot 4 \cdot 5$

6. (a) How many permutations are there of A, B, C, D?

**Ans.** $4!$

(b) How many 5-permutations are there of 11 distinct objects?

**Ans.** ${}^{11}P_{5}$

(c) The letters ABCDE are to be used to form strings of length 3. How many strings can be formed if we allow repetitions?

**Ans.** $5^{3}$

(d) The letters ABCDE are to be used to form strings of length 3. How many strings do not contain the letter A if repetitions are not allowed?

**Ans.** ${}^{4}P_{3}$

7. (a) A shipment of 100 compact disks contains five defective disks. In how many ways can we select a set of four compact disks that contains more defective than non-defective disks?

**Ans.** ${}^{5}C_{3}{}^{95}C_{1} + {}^{5}C_{4}$

(b) How many six-card hands chosen from an ordinary 52-card deck contain three cards of one suit and three cards of another suit?

**Ans.** ${}^{4}C_{2}({}^{13}C_{3})^{2}$

(c) Find the number of integer solutions of

$$x_1 + x_2 + x_3 = 15$$

subject to $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

**Ans.** ${}^{15+3-1}C_{15}$ **or** ${}^{15+3-1}C_{3-1}$

8. (a) In view of piles of identical green, blue and red balls where each pile contains 10 balls, in how many ways can 10 balls be selected if exactly one green ball must be selected?

**Ans.** ${}^{9+2-1}C_{9}$ **or** ${}^{9+2-1}C_{2-1}$

(b) A coin is flipped 10 times. How many outcomes have a head on the fifth toss?

**Ans.** $2^9$

**(if order is important)**

**or** ${}^{9+2-1}C_{2-1}$ **or 10**

**(if order is not important – but question suggest order)**

(c) Consider selections among Adventure books, Mystery books, Historical books, Comics, Romance novels and Educational books. How many ways are there to select any six of these?

**Ans.** ${}^{6+6-1}C_{6-1}$

9. A six-person committee composed of Alan, Beth, Carr, David, Essy, and Fran is to select a chairperson, secretary, and treasurer.

(a) How many selections exclude Carr?

**Ans.** $5 \cdot 4 \cdot 3$

(b) How many selections are there in which neither Beth nor Fran is an officer?

**Ans.** $4 \cdot 3 \cdot 2$

10. (a) In how many ways can we select a committee of four from a group of 12 persons?

**Ans.** ${}^{12}C_4$

(b) A club consists of six distinct men and seven distinct women. In how many ways can we select a committee of five persons?

**Ans.** ${}^{13}C_5$

(c) A club consists of six distinct men and seven distinct women. In how many ways can we select a committee of four persons that has at most one man?

**Ans.** ${}^{6}C_1 {}^{7}C_3 + {}^{7}C_4$

(d) In how many ways can we select a committee of four Democrats, three Republicans, and two Independents from a group of 10 distinct Democrats, 12 distinct Republicans, and four distinct Independents?

**Ans.** ${}^{10}C_4 {}^{12}C_3 {}^{4}C_2$

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