THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of December ■ /April/May ☐ /July ☐ 2012

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐
Mode:	**On Campus**	■	**By Distance**	☐		

Course Code and Title: **COMP2101/CS20S Discrete Mathematics for Computer Scientists**

Date:	**Thursday, December 6, 2012**	Time:	**9:00 a.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: (*where applicable*)	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐ **1-100**	☐

Auxiliary/Other material(s) – Please specify: None

Candidates are permitted to bring the following items to their desks:

Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) i. Write a formula to determine the number of strings that can be formed by ordering the letters SALESPERSONS. **[1]**

ii. Write a formula to find the number of integer solutions of

$$a_1 + a_2 + a_3 + a_4 = 14$$

subject to $a_1 \geq 0$, $a_2 > 1$, $a_3 > 2$, $a_4 \geq 5$ **[1]**

(b) i. By constructing a Tree Diagram, determine how many 2-combinations are there of 4 objects (A, B, C, D) selecting all 4 objects? **[2]**

ii. Let f be the Fibonacci function.
Write the Fibonacci sequence $f(0)$ to $f(8)$. **[1]**

(c) i. Expand $(3v - 2w)^5$ using the Binomial Theorem. **[3]**

ii. What is the row of Pascal's triangle containing the binomial coefficients $\binom{6}{k}$, $0 \le k \le 6$ **[1]**

iii. Consider the recurrence function

$$T(n) = 3T(n/3) + ½n$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \le 1$.
[4]

(d) i. Consider the geometric series:

$$3 + 15/7 + 75/49 + 375/343 + \ldots$$

I. Determine a formula for S_n where S_n is the sum of the first n terms of the series? **[4]**

II. What is the limit of S_n **[1]**

(e) Find the generating function for the sequence

$$\{0, 0, 0, 1, 2, 3, 4, 5, \ldots\}$$ **[4]**

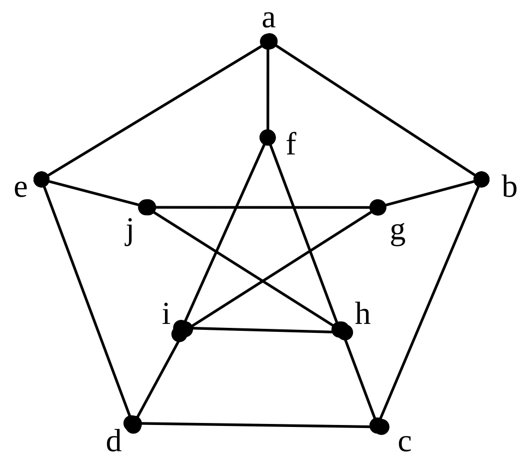
(f) Let $f(n)$ be defined by

$$f(n) = 16 f\left(\frac{n}{2}\right) + 5n^4$$

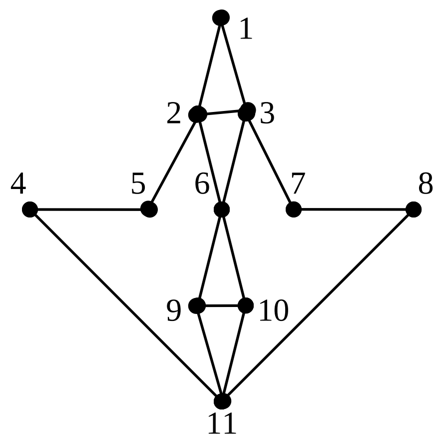
if $n > 1$ and $n = 2^m$, where m is a positive integer.

By using the principles of Recurrence Relation, find a general formula for $f(n)$. **[5]**

(g) i. Find a Hamiltonian cycle, or state why one does not exist in the graph. **[1]**



ii. Find an Euler cycle, or state why one does not exist in the graph. **[2]**



(h) i. Given *G*, a phrase-structure grammar.
Let $G = (V, T, S, P)$, where

V = {a, b, A, B, S}, T = {a, b}, S is the start symbol,

P = {S → AbBa, A → BB, Ab → aA, B → ab, AB → b}.

What is the *language generated by G*, *L(G)*, that is the set of all strings of terminals that are derivable from the starting state *S*. **[3]**

ii. Construct a finite-state machine that models a newspaper vending machine that accepts $10 and $20 coins. The machine has a red button (R) that may be pressed at any stage to cancel the purchase and have all collected money returned. When $50 or more is deposited, the excess (over $50) is returned, and the customer may press the green button (G) to receive that day's newspaper (DP).

I. Represent the finite-state machine as a **state table**. **[4]**

II. Represent the finite-state machine as a **state diagram**. **[3]**

SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) If you studied intensely the probability of passing this COMP2101 final examination is 85%, if you studied lightly the probability of passing the examination is 40%, and if you studied none at all the probability of passing is 0%. The course tutor knows that 65% of the students studied intensely, 30% of them studied lightly and 5% did not find the time to study. What is the probability that you will pass this COMP2101 final examination? **[4]**

(b) Suppose that five University students, Ann, Bea, Cal, Dew and Ele are members of the committees, C_1, C_2, C_3, C_4, C_5 and C_6.

Student	**Committees**
Ann	C_1, C_2, C_4
Bea	C_2, C_4
Cal	C_1, C_3, C_5, C_6
Dew	C_1, C_4
Ele	C_1, C_2

i. Model the above situation as a matching network **[2]**

ii. Find a maximal matching **[1]**

iii. Find a way in which each student can be assigned to a committee such that each committee has no more than one of these five students, or use Hall's Theorem to explain why no such complete matching exists. **[3]**

QUESTION THREE (10 Marks)

(a) Let a,b,c be integers such that $a \geq 1$, $b > 1$ and $c > 0$.
Let $f: N \rightarrow R$ be functions
where N is the set of Natural numbers and R is the set of Real numbers
Given that
$f(1) = c$, $n = b^k$, where k is a positive integer greater than 1.

$$f(n) = a^k f(n/b^k) + c\sum_{i=0}^{k-1} a^i$$

Show that if $a \neq 1$, then $f(n) = \dfrac{c(an^{\log_b a} - 1)}{a-1} = \Theta(n^{\log_b a})$ **[6]**

(b) Consider the arithmetic series:

$$7/2 + 15/2 + 23/2 + 31/2 + \ldots$$

What is the smallest value of n such that $S_n > 600$, where
S_n is the sum of the first n terms of the series? **[4]**

QUESTION FOUR (10 Marks)

(a) By using the inclusion-exclusion principle, give a formula for the number of elements in the union of four sets A_1, A_2, A_3, and A_4 **[1]**

(b) Find a closed form expression for the following recurrence relation

$$s_0 = 0$$
$$s_1 = 1$$
$$s_n = -2s_{n-1} - s_{n-2} \quad \text{for } n \geq 2$$ **[5]**

(c) Given seven courses CS1, CS2, CS3, CS4, M1, M2 and M3, and the following listing which shows courses for which examinations cannot be at the same time due to students' course selections:

Students who pursue Course CS1 also pursue CS3.
Those who pursue Course CS2 also pursue CS1, M2 and M3.
Students who pursue Course CS4 also pursue CS2 and CS3.
Those who pursue Course M1 also pursue M2 and CS3.
Students who pursue Course M2 also pursue M1, M3 and CS2.
Those who pursue Course M3 also pursue M2 and CS2.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and by determining the chromatic number χ, state the minimum number of time periods necessary for the examinations of the seven courses. **[2]**

QUESTION FIVE (10 Marks)

(a) If there are 54 students who have completed a Computer Science course and 11 possible grades that could have been attained, use the Pigeonhole Principle to show that there is a grade that at least five students attained. **[2]**

(b) Prove whether the following statement is true or false. If the statement is false, give a counterexample. Assume that the functions *g* and *h* take on only positive values.

$$\textit{If } g(x) = O(h(x)) \textit{ and } h(x) = O(g(x)), \text{then } g(x) = \Theta(h(x))$$ **[4]**

(c) Represent the following postfix expression as a binary tree.

A B / C D * – E F + * **[2]**

(d) In a given city only 2 percent of all churches have someone who preaches on the bus. Find the probability that among 100 churches in that city, at most two will have someone who preaches on the bus. **[2]**

END OF QUESTION PAPER