THE UNIVERSITY OF THE WEST INDIES

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Examinations of: December ■ /April/May ☐ /July ☐ 2017

Originating Campus: **Cave Hill** ■ **Mona** ■ **St. Augustine** ☐

Mode: **On Campus** ■ **By Distance** ☐

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Science**

Date:	**Tuesday, December 19, 2017**	Time:	**9:00 a.m.**
Duration:	**2 Hours.**	Paper No:	**1 (of 1)**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐
Calculator: *(where applicable)*	**Programmable**	☐	**Non Programmable**	■	**Not required**	☐
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20** ☐	**1-100** ☐

Auxiliary/Other material(s) – Please specify:

Candidates are permitted to bring the following items to their desks: Pencil or pen, Ruler, ID card, Exam card

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Instructions to Candidates: This paper has 5 pages & 5 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

This Examination consists of Two Sections.

Section A is COMPULSORY. Candidates are required to answer TWO questions in Section B.

Calculators are allowed.

SECTION A – COMPULSORY (40 Marks)

QUESTION ONE (40 Marks)

(a) i. Expand $(2m - 3n)^5$ using the Binomial Theorem. **[3]**

ii. What is the row of Pascal's triangle containing the binomial coefficients $\binom{7}{k}$, $0 \le k \le 7$ **[1]**

iii. By constructing a Tree Diagram, determine how many 2-combinations are there of 4 objects (A, B, C, D)? **[2]**

(b) i. Write a formula to determine the number of strings that can be formed by ordering the letters UNUSUALLY. **[1]**

ii. Write a formula to find the number of integer solutions of

$$p_1 + p_2 + p_3 + p_4 = 19$$

subject to $p_1 \geq 0$, $p_2 \geq 1$, $p_3 > 2$, $p_4 > 5$ **[1]**

(c) i. Let f be the Fibonacci function.
Write the Fibonacci sequence $f(0)$ to $f(7)$. **[1]**

ii. Consider the recurrence function

$$T(n) = 9T(n/3) + 9n^3$$

Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \le 1$. **[5]**

(d) Find the generating function for the sequence

$$\{0, 0, 0, 2, 4, 6, 8, 10, \ldots\}$$ **[4]**

(e) Consider the geometric series:

$$3 + 15/9 + 75/81 + 375/729 + \ldots$$

I. Determine a formula for S_n where S_n is the sum of the first n terms of the series? **[4]**

II. What is the limit of S_n **[1]**

(f) Prove whether the following statement is true or false. If the statement is false, give a counterexample. Assume that the functions g and h take on only positive values.

$$\text{If } g(x) = O(h(x)) \text{ and } h(x) = O(g(x)), \text{ then } g(x) = \Theta(h(x))$$ **[4]**

(g) Given G, a phrase-structure grammar.
Let $G = (V, T, S, P)$, where

V = {a, b, A, B, S}, T = {a, b}, S is the start symbol,

P = {S → AbBa, A → bB, Ab → aA, B → ab, AB → b}.

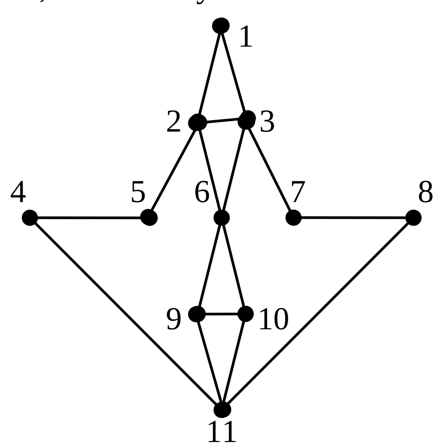
What is the *language generated by G, L(G)*, that is the set of all strings of terminals that are derivable from the starting state *S*. **[3]**

(h) Construct a finite-state machine that models a newspaper vending machine that accepts \$10 and \$20 coins. The machine has a red button (R) that may be pressed at any stage to cancel the purchase and have all collected money returned. The machine also has a blue button (B) that may be pressed at any stage to donate to the vending company, the sum already deposited in the machine. When a donation takes place, no money is returned to the customer; however, the customer may commence another purchase/donation by entering more coins. When \$50 or more is deposited, the excess (over \$50) is returned and the customer immediately receives that day's newspaper (DP).

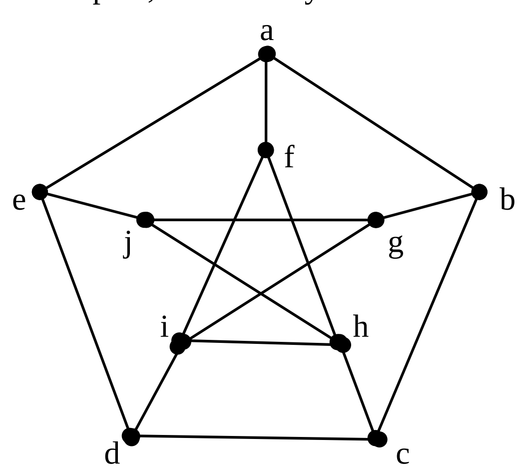
I. Represent the finite-state machine as a **state table**. **[4]**

II. Represent the finite-state machine as a **state diagram**. **[3]**

(i) I. Find an Euler cycle, or state why one does not exist in the graph. **[2]**



II. Find a Hamiltonian path, or state why one does not exist in the graph. **[1]**



SECTION B – DO TWO QUESTIONS (20 Marks)

QUESTION TWO (10 Marks)

(a) In a given city only 3 percent of all men have a haircut each month. Find the probability that among 100 men in that city, at most two of them will have a haircut each month. **[2]**

(b) What is the order of growth of $\sum_{i=1}^{n} i^m$ where m is a positive integer? **[4]**

(c) Consider the arithmetic series:

$$7/3 + 19/3 + 31/3 + 43/3 + \ldots$$

What is the smallest value of n such that $S_n > 200$, where S_n is the sum of the first n terms of the series? **[4]**

QUESTION THREE (10 Marks)

(a) Suppose that five Politicians, Appe, Beau, Culp, Dewy and Ever are members of the committees, C_1, C_2, C_3, C_4, C_5 and C_6.

Politicians	**Committees**
Appe	C_1, C_3
Beau	C_3, C_4
Culp	C_1, C_2, C_5, C_6
Dewy	C_1, C_4
Ever	C_1, C_3, C_4

i. Model the above situation as a matching network **[2]**

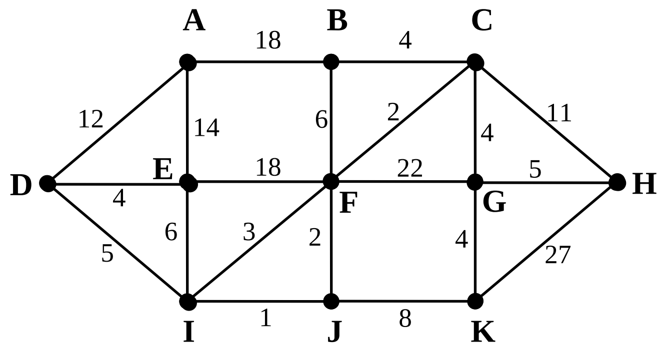
ii. Find a maximal matching **[1]**

iii. Find a way in which each politician can be assigned to a committee such that each committee has no more than one of these five politicians, or use Hall's Theorem to explain why no such complete matching exists. **[3]**

(b) A random experiment consists of rolling an unfair, six-sided die. The number 1 is three times as likely to appear as the number 6. The number 2 is twice as likely to appear as the numbers 3 and 5, and the number 6 is twice as likely as the numbers 2 and 4. Assign probabilities to the six outcomes in the sample space. **[4]**

QUESTION FOUR (10 Marks)

(a) Find a minimal spanning tree for the following graph. **[3]**



(b) If there are 34 students who have completed a Computer Science course and 11 possible grades that could have been attained, use the Pigeonhole Principle to show that there is a grade that at least four students attained. **[2]**

(c) Let $f(n)$ be defined by

$$f(n) = 8f\left(\frac{n}{2}\right) + 3n^5$$

if $n > 1$ and $n = 2^m$, where m is a positive integer.

By using the principles of Recurrence Relation, find a general formula for $f(n)$. **[5]**

QUESTION FIVE (10 Marks)

(a) Given seven courses CS1, CS2, CS3, CS4, M1, M2 and P1, and the following listing which shows courses for which examinations cannot be at the same time due to students' course selections:

Students who pursue Course CS1 also pursue CS3.
Those who pursue Course CS2 also pursue CS4 and CS3.
Students who pursue Course CS4 also pursue CS1, M2 and P1.
Those who pursue Course M1 also pursue M2 and CS3.
Students who pursue Course M2 also pursue CS4, M1 and P1.
Those who pursue Course P1 also pursue M2 and CS4.

i. Construct the graph representing the above information **[2]**

ii. By using graph coloring, schedule the final exams and by determining the chromatic number χ, state the minimum number of time periods necessary for the examinations of the seven courses. **[2]**

(b) Find a closed form expression for the following recurrence relation

$s_0 = 0$

$s_1 = 2$

$s_n = s_{n-1} + 2s_{n-2}$ for $n \geq 2$ **[6]**

END OF QUESTION PAPER