THE UNIVERSITY OF THE WEST INDIES
Mona Campus

Semester I ■ Semester II ☐ Supplemental/Summer School ☐

Mid-Semester Examinations of: October ■ /February/March ☐ /June ☐ 2012/2013

Course Code and Title: **COMP2101/CS20S Discrete Mathematics for Computer Scientists**

Date: **Friday, October 19, 2012** Time: **2:00 p.m.**

Duration: **1 Hour.** Paper No: **1 (of 1)**

Materials required:

Answer booklet: **Normal** ■ **Special** ☐ **Not required** ☐

Calculator: **Programmable** ☐ **Non Programmable** ■ **Not required** ☐
(where applicable)

Multiple Choice answer sheets: **numerical** ☐ **alphabetical** ☐ **1-20** ☐ **1-100** ☐

Auxiliary/Other material(s) – Please specify: None

Candidates are permitted to bring the following items to their desks: Pencil or pen, Ruler, ID card, Exam card

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Instructions to Candidates: This paper has 2 pages & 6 questions.

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

All questions are COMPULSORY.

Calculators are allowed.

1. Assuming that the functions *f*, *g* and *h* take on only positive values,
Prove or disprove the following:
$f(n) = \Theta(h(n))\ and\ g(n) = \Theta(h(n))$, then $f(n) + g(n) + h(n) = \Theta(h(n))$ **[3]**

2. (a) Consider the random experiment of tossing nine fair coins. What is the probability that the number of heads and the number of tails differ by at most 3? **[3]**

   (b) By using the inclusion-exclusion principle
   Calculate the number of bit strings of length 8 that begin with two 0s, have seven consecutive 0s, or end with a 1 bit **[4]**

3. The seven "Double" dominoes of a certain pack are placed in a bag. The Double-Blank is three times as likely to be pulled as the dominoes Double-Ace, Double-Four and Double-Six. The Double-Six is two times as likely to be pulled as Double-Three and Double-Five. Double-Five is two times as likely to be pulled as Double-Deuce.

   i. Assign probabilities to the seven outcomes in the sample space **[5]**

   ii. Suppose that the random variable X, is assigned the value of the digit that appears when the domino is pulled. Therefore Double-Blank is assigned 0, Double-Ace is assigned 2, Double-Deuce is assigned 4, Double-Three is assigned 6, and so on. If the expected value is denoted by
   $E(X) = \sum_{i=0}^{n} p(x_i)X(x_i)$ where $p(x_i)$ is the probability for the event x_i,
   what is the expected value of X? **[2]**

4. (a) In a given town only 2 percent of all robberies will be reported to the police. Find the probability that among 300 robberies in that town, at least three will be reported to the police. **[3]**

   (b) If you study intensely the probability of passing this COMP2101 Mid-term examination is 85%, if you studied lightly the probability of passing the examination is 40%, and if you studied none at all the probability of passing is 5%. The course lecturer is sure that 60% of the students study intensely, 35% of them study lightly and 5% do not study. Given that you pass this Mid-term examination, what is the probability that you studied intensely? **[4]**

5. Consider the recurrence function
$$T(n) = 8T(n/2) + \tfrac{1}{2}n^3$$
Give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Assume that $T(n) = 1$ for $n \leq 1$. **[4]**

6. If there are 71 students who have completed a Computer Science course and 12 possible grades that could have been attained, use the Pigeonhole Principle to show that there is a grade that at least six students attained. **[2]**

END OF QUESTION PAPER