6. Stellar Spectra

Excitation and ionization, Saha's equation

Stellar spectral types

Luminosity effects on stellar spectra

Balmer jump, H^-

Occupation numbers: LTE case

Absorption coefficient: $\kappa_\nu = n_i \sigma_\nu$

→ calculation of occupation numbers needed

LTE

each volume element in thermodynamic equilibrium at temperature T(r)

hypothesis: electron-ion collisions adjust equilibrium

difficulty: interaction with non-local photons

LTE is valid if effect of photons is small or radiation field is described by Planck function at T(r)

otherwise: non-LTE

Excitation in LTE

Boltzmann excitation equation

n_{ij}: number density of atoms in excited level i of ionization stage j (ground level: i=1, neutral: j=0)

$$\boxed{\frac{n_{ij}}{n_{1j}} = \frac{g_{ij}}{g_{1j}}\, e^{-E_{ij}/kT}}$$

g_{ij}: statistical weight of level i = number of degenerate states

E_{ij} excitation energy relative to ground state

$g_{ij} = 2i^2$ for hydrogen

$$\log \frac{n_{ij}}{n_{1j}} = \log \frac{g_{ij}}{g_{1j}} - E_{ij}(eV)\frac{5040}{T}$$

The fraction relative to the total number of atoms of in ionization stage j is

$$\frac{n_{ij}}{n_j} = \frac{g_{ij}}{U_j(T)}\, e^{-E_{ij}/kT} \qquad U_j(T) = \sum_i g_{ij} e^{-E_{ij}/kT}$$

U_j (T) is called the partition function

Ionization in LTE: Saha's formula

Goal: generalize Boltzmann equation for ratio of two contiguous ionic species j and j+1

Consider ionization process j → j+1

initial state: n_{1j} & statistical weight g_{1j}

final state: n_{1j+1} + free electron & statistical weight $g_{1j+1}\, g_{El}$

$$n_{1j+1}(v)\; dx\, dy\, dz\; dp_x\, dp_y\, dp_z$$
$$\downarrow$$

number of ions in groundstate with free electron with velocity in (v,v+dv) in phase space

energy of free electron: $E = \dfrac{p^2}{2m}$

$$\int dx\, dy\, dz = \int dV = \Delta V = 1/n_e$$
$$dp_x\, dp_y\, dp_z = 4\pi p^2\, dp = 4\pi m^3 v^2\, dv$$

Ionization in LTE: Saha's formula

$$n_{1j+1}(v)\ dx\,dy\,dz\ dp_x\,dp_y\,dp_z \qquad \int\!\!\int n_{1j+1}(v)dV\,d^3p = n_{1j+1}$$

number of ions in groundstate with free electron with velocity in (v,v+dv) in phase space

Statistical weight for free electon: g_{El}: volume in phase space normalized to smallest possible volume (h^3) for electron:

$$g_{\mathrm{El}} = 2\,\frac{dx\,dy\,dz\ dp_x\,dp_y\,dp_z}{h^3}$$

2 spin orientations

$$\int dx\,dy\,dz = \int dV = \Delta V = 1/n_e$$

$$dp_x\,dp_y\,dp_z = 4\pi p^2\,dp = 4\pi m^3 v^2\,dv$$

Ionization: Saha's formula

$$n_{1j+1}(v)\; dx\, dy\, dz\;\, dp_x\, dp_y\, dp_z$$

$$g_{\mathrm{El}} = 2\,\frac{dx\, dy\, dz\;\, dp_x\, dp_y\, dp_z}{h^3}$$

Substitute into Boltzmann Equation:

$$\boxed{\frac{n_{ij}}{n_{1j}} = \frac{g_{ij}}{g_{1j}}\; e^{-E_{ij}/kT}}$$

$$\frac{n_{1j+1}(v)}{n_{1j}} dV\, dp_x dp_y dp_z = \frac{g_{1j+1}}{g_{1j}} g_{El}\; e^{-(E_j+\frac{1}{2}mv^2)/kT}$$

$$\frac{n_{1j+1}(v)}{n_{1j}} dV\, dp_x dp_y dp_z = \frac{g_{1j+1}}{g_{1j}} 2\, dV\, \frac{dp_x\, dp_y\, dp_z}{h^3}\; e^{-[E_j+\frac{1}{2m}(p_x^2+p_y^2+p_z^2)]/kT}$$

Ionization: Saha's formula

$$\frac{n_{1j+1}(v)}{n_{1j}} dV dp_x dp_y dp_z = \frac{g_{1j+1}}{g_{1j}} 2\, dV \frac{dp_x\, dp_y\, dp_z}{h^3}\, e^{-[E_j + \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)]/kT}$$

To sum over all final state, we integrate over all phase space:

$$n_{1j+1} = n_{1j} \frac{g_{1j+1}}{g_{1j}} 2 \frac{\Delta V}{h^3} e^{-E_j/kT} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2mkT}(p_x^2+p_y^2+p_z^2)}\, dp_x dp_y dp_z}_{(2\pi mkT)^{3/2}}$$

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

$$dp_x\, dp_y\, dp_z = 4\pi p^2\, dp$$

$$\int dV = \Delta V = 1/n_e$$

$$\boxed{\frac{n_{1j+1}\, n_e}{n_{1j}} = 2\, \frac{g_{1j+1}}{g_{1j}} \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\frac{E_j}{kT}}}$$

$$\boxed{\frac{n_{ij}}{n_{1j}} = \frac{g_{ij}}{g_{1j}}\, e^{-E_{ij}/kT}}$$

Saha 1920

Ionization: Saha's formula

Generalize for arbitrary levels (not just ground state):

$$n_j = \sum_{i=1}^{\infty} n_{ij} \qquad n_{j+1} = \sum_{i=1}^{\infty} n_{ij+1}$$

$$n_{ij+1} = n_{1j+1} \frac{g_{ij+1}}{g_{1j+1}} \, e^{-E_{ij+1}/kT}$$

using Boltzmann's equation

$$n_{j+1} = \frac{n_{1j+1}}{g_{1j+1}} \underbrace{\sum_{i=1}^{\infty} g_{ij+1} \, e^{-E_{ij+1}/kT}}$$

$$U_{j+1}(T): \qquad \text{partition function}$$

also $$n_j = \frac{n_{1j}}{g_{1j}} U_j(T)$$ $\Rightarrow$

$$\boxed{\frac{n_{j+1} n_e}{n_j} = 2 \, \frac{U_{j+1}(T)}{U_j(T)} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} e^{-\frac{E_j}{kT}}}$$

Ionization: Saha's formula

Using electron pressure P_e instead of n_e ($P_e = n_e kT$)

$$\frac{n_{j+1}}{n_j} P_e = 2\,\frac{U_{j+1}(T)}{U_j(T)} \left(\frac{2\pi m}{h^2}\right)^{3/2} (kT)^{5/2} e^{-\frac{E_j}{kT}}$$

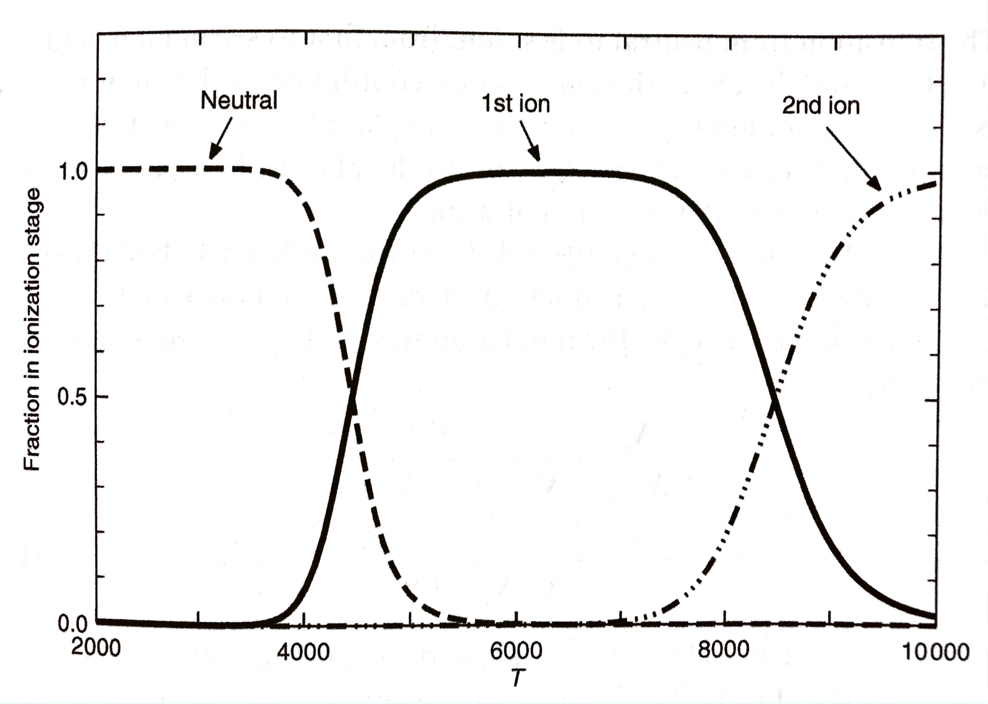
which can be written as:

$$\log_{10} \frac{n_{j+1}}{n_j} = -0.1761 - \log_{10} P_e + \log_{10} \frac{U_{j+1}(T)}{U_j(T)} + 2.5 \log_{10} T - \frac{5040}{T} E_j$$

with P_e in dyne/cm^2 and E_j in eV

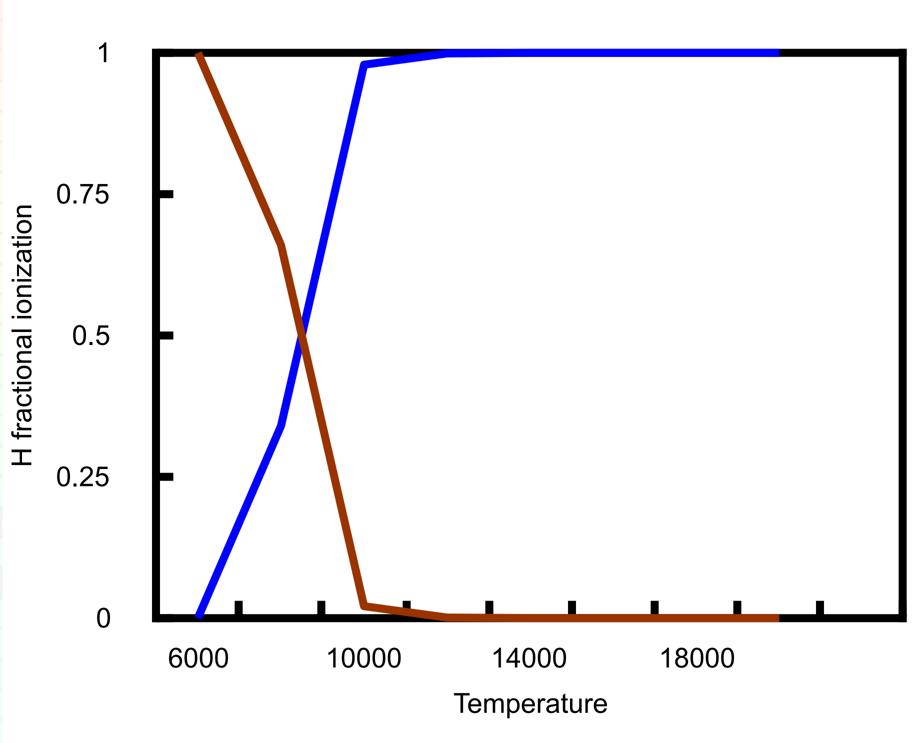
Ionization: Saha's formula

Fe at P_e = 1 dyne/cm^2



Gray 2005

Hydrogen at P_e = 10 dyne/cm^2



On the partition function

Partition function for neutral H atom:

$$U_0(T) = \sum_{i=1}^{\infty} g_{i0}\, e^{-E_{i0}/kT}$$

$E_{i0} \leq E_{ion} = 13.6$ eV

$g_{i0} = 2i^2$

Infinite number of levels → **partition function diverges!**

$$U_0(T) \geq 2\, e^{-E_{\rm ion}/kT} \underbrace{\sum_{i=1}^{\infty} i^2}_{\text{divergent}}$$

Reason: Hydrogen atom level structure calculated as if it were alone in the universe. Not realistic → cut-off needed

On the partition function

$$U_0(T) \geq 2\, e^{-E_{\rm ion}/kT} \underbrace{\sum_{i=1}^{\infty} i^2}_{\text{divergent}}$$

idea: orbit radius $r = r_0\, i^2$

(i is main quantum number)

→ there must be a max i corresponding to the finite spatial extent of atom r_{max}

$$\frac{4\pi}{3} r_{\rm max}^3 = \frac{1}{N} = \frac{4\pi}{3}\left(r_0\, i_{\rm max}^2\right)^3$$

⇒ i_{max}

introduces a pressure dependence of U

An example: pure hydrogen atmosphere in LTE

Given Temperature T and total particle density N: use Saha equation to calculate electron density n_e

$$N = n_e + n_p + \sum_{i=1}^{i_{max}} n_i = n_e + n_p + n_1 \sum_{i=1}^{i_{max}} \frac{g_i}{g_1} e^{-E_{i,1}/kT}$$

From Saha's equation and $n_e = n_p$ (only for pure H plasma):

$$\frac{n_p n_e}{n_1} = 2 \frac{g_p}{g_1} \left(\frac{2\pi m k T}{h^2}\right)^{3/2} e^{-\frac{E_{\rm ion}}{kT}} \qquad g_p = 1 \;\; g_1 = 2$$

$$n_1 = n_e^2 \left(\frac{h^2}{2\pi m k T}\right)^{3/2} e^{\frac{E_{\rm ion}}{kT}}$$

$$\boxed{\frac{n_{1j+1} n_e}{n_{1j}} = 2 \frac{g_{1j+1}}{g_{1j}} \left(\frac{2\pi m k T}{h^2}\right)^{3/2} e^{-\frac{E_j}{kT}}}$$

An example: pure hydrogen atmosphere in LTE

Given Temperature T and total particle density N: use Saha equation to calculate electron density n_e

$$n_1 = n_e^2 \left(\frac{h^2}{2\pi mkT} \right)^{3/2} e^{\frac{E_{\text{ion}}}{kT}}$$

$$N = 2n_e + \underbrace{n_e^2 \left(\frac{h^2}{2\pi mkT} \right)^{3/2} e^{\frac{E_{\text{ion}}}{kT}} \sum_{i=1}^{i_{\max}} i^2 \, e^{-E_{i,1}/kT}}_{\alpha(T)} = 2n_e + n_e^2 \, \alpha(T)$$

$$\Rightarrow \quad n_e = -\frac{1}{\alpha(T)} + \sqrt{\frac{1}{\alpha^2} + N} \quad \Longrightarrow \quad n_p = n_e \quad \Longrightarrow \quad n_1 \rightarrow n_i$$

Recap from last lecture:

$$\frac{n_{1j+1}\, n_e}{n_{1j}} = 2\, \frac{g_{1j+1}}{g_{1j}} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} e^{-\frac{E_j}{kT}}$$

Saha Equation

$$U_j\,(T) = \sum_i g_{ij} e^{-E_{ij}/kT}$$

Partition Function

$$\frac{n_{j+1}\, n_e}{n_j} = 2\, \frac{U_{j+1}(T)}{U_j\,(T)} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} e^{-\frac{E_j}{kT}}$$

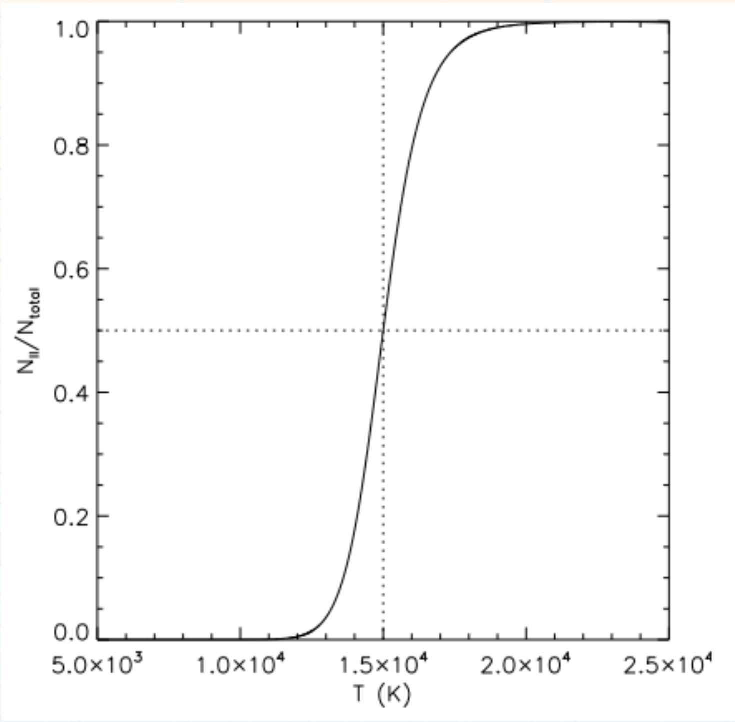
$$\log_{10} \frac{n_{j+1}}{n_j} = -0.1761 - \log_{10} P_e + \log_{10} \frac{U_{j+1}(T)}{U_j\,(T)} + 2.5 \log_{10} T - \frac{5040}{T}\, E_j$$

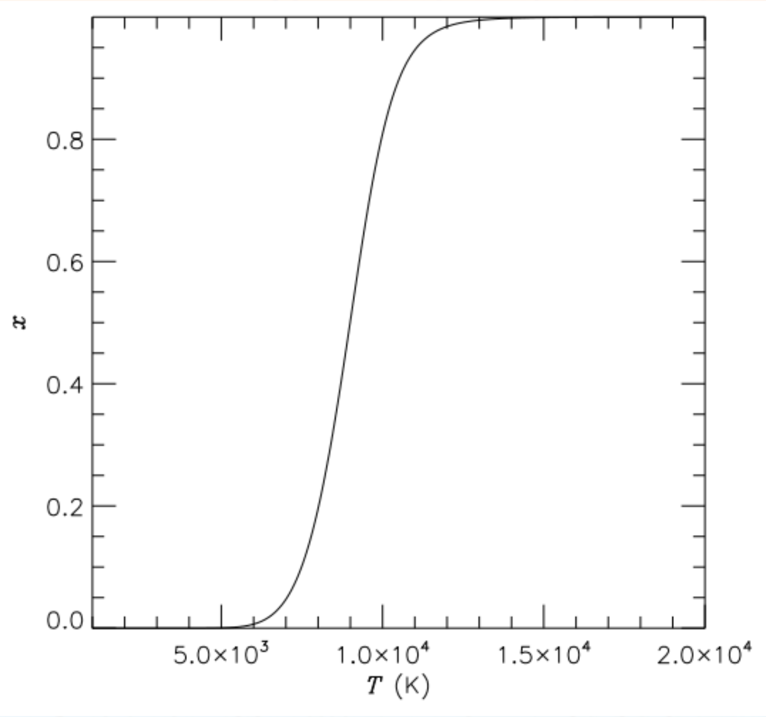
Recap Quiz

- Consider two stars with identical temperatures and surface gravities log(g) = 2.0 and log(g) = 4.5. Which star will have a higher hydrogen ionization fraction and why?

Recap Quiz

- Consider the following two ionization fraction curves plotted as a function of temperature.





What can you conclude about these elements?

The LTE occupation number n_i^*

From Saha's equation:

$$\frac{n_{1j+1}\, n_e}{n_{1j}} = 2\,\frac{g_{1j+1}}{g_{1j}} \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\frac{E_j}{kT}}$$

$$n_{1j} = n_{1j+1}\, n_e\, \frac{g_{1j}}{g_{1j+1}} \frac{1}{2} \left(\frac{h^2}{2\pi mkT}\right)^{3/2} e^{\frac{E_j}{kT}}$$

+ Boltzmann:

$$\frac{n_{ij}}{n_{1j}} = \frac{g_{ij}}{g_{1j}}\, e^{-E_{1i}/kT}$$

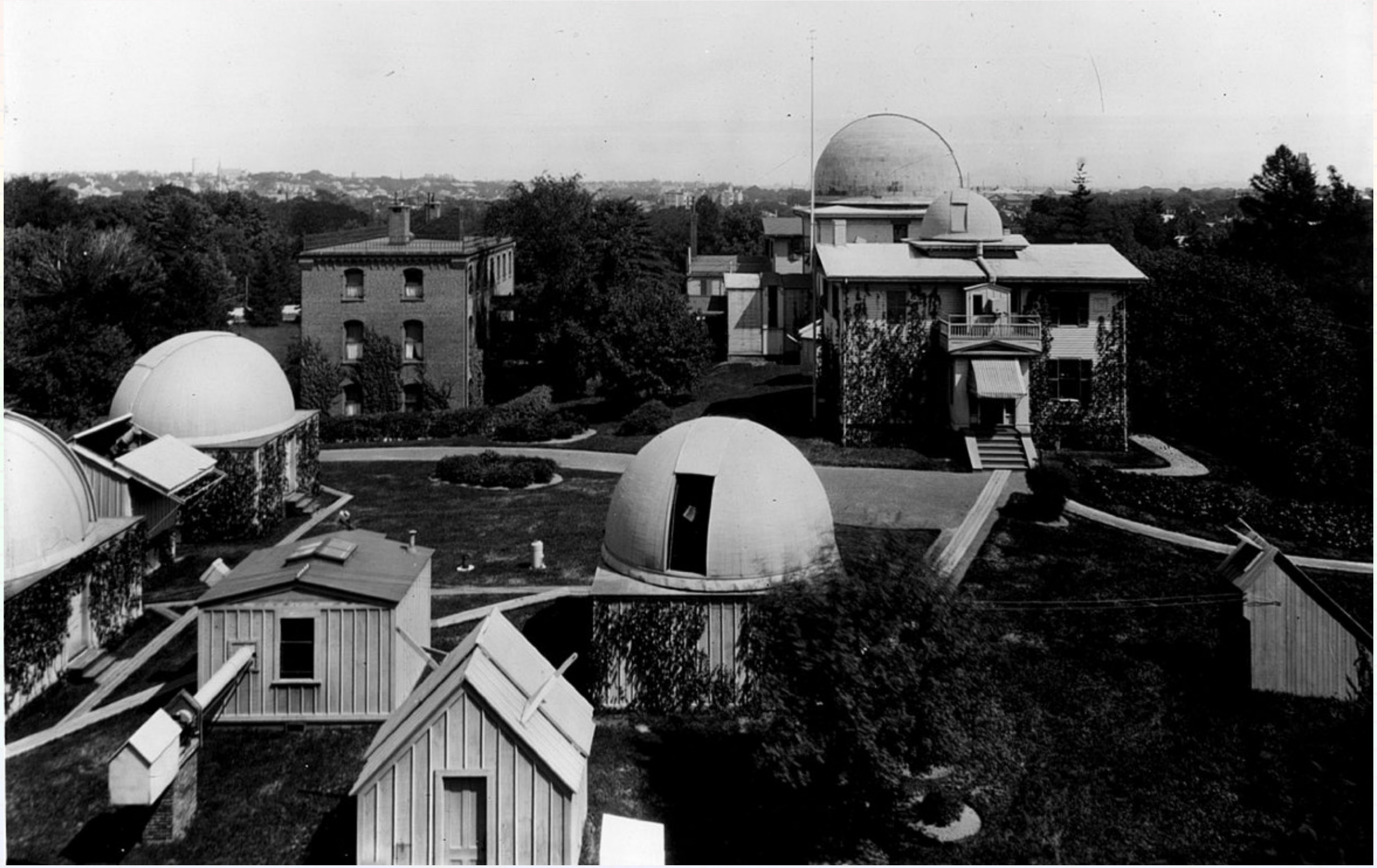
$$\boxed{n_i^* := n_{ij} = n_{1j+1}\, n_e\, \frac{g_{ij}}{g_{1j+1}} \frac{1}{2} \left(\frac{h^2}{2\pi mkT}\right)^{3/2} e^{\frac{E_{ij}}{kT}}}$$

in LTE we can express the bound level occupation numbers as a function of T, n_e and the ground-state occupation number of the next higher ionization stage.

Note n_i^* is the occupation number used to calculate bf-stimulated and bf-spontaneous emission

Stellar Spectroscopy: Some History

Harvard College Observatory, circa 1899



Digital Access to a Sky Century @ Harvard: http://dasch.rc.fas.harvard.edu/project.php

Stellar Spectroscopy: Some History

The pioneers of stellar spectroscopy

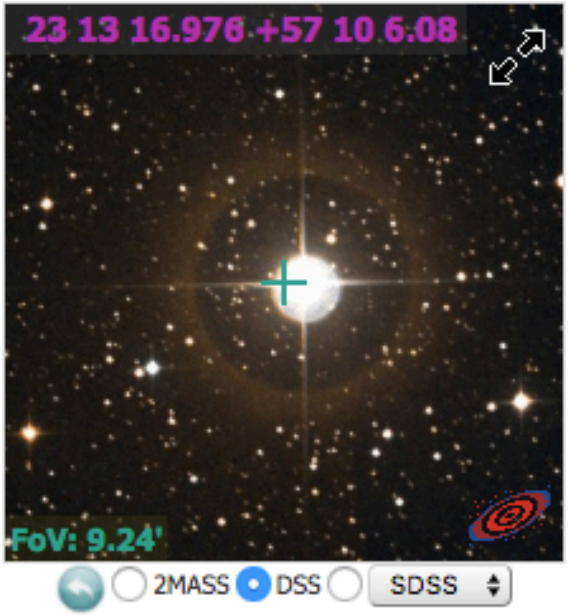

β AURIGÆ DEC 1889
5 10 15 20 25 30

Basic data :

HD 219134 -- Flare Star

Other object types:	* (Ref,HD,...), **PM*** (Ci,LFT,...), ** (**,CCDM,...), **V*** (CSV,NSV), **IR** (IRAS,2MASS), **Fl*** ([GKL99]), **X** (1RXS)
ICRS coord. *(ep=J2000)* :	**348.32073467 +57.16835620 (Optical) [2.90 2.20 90] A 2007A&A...474..653V**
Ecl coord. *(ep=J2000 eq=2000)* :	023.7427522 +54.5466070 [2.90 2.20 90]
FK4 coord. *(ep=B1950 eq=1950)* :	23 10 51.87 +56 53 31.2 [16.65 12.83 0]
Gal coord. *(ep=J2000)* :	109.8985 -03.1986 [2.90 2.20 90]
Proper motions *mas/yr* :	**2075.07 295.45 [0.33 0.25 0] A 2007A&A...474..653V**
Radial velocity / Redshift / cz :	**V(km/s) -18.83 [0.07]** / z(~) -0.000063 [0.000000] / cz -18.83 [0.07] **A 2002ApJS..141..503N**
Parallaxes *(mas)*:	**152.76 [0.29] A 2007A&A...474..653V**
Spectral type:	**K3V B 1989ApJS...71..245K**
Fluxes (8) :	**U 7.460 [0.010] C 1984A&AS...57..357O** **B 6.560 [0.007] C 1993A&AS..100..591O** **V 5.570 [0.009] C 1993A&AS..100..591O** **R 4.76 [~] C 2002yCat.2237....0D** **I 4.23 [~] C 2002yCat.2237....0D** **J 3.86 [~] C 2002yCat.2237....0D** **H 3.40 [~] C 2002yCat.2237....0D** **K 3.25 [~] C 2002yCat.2237....0D**

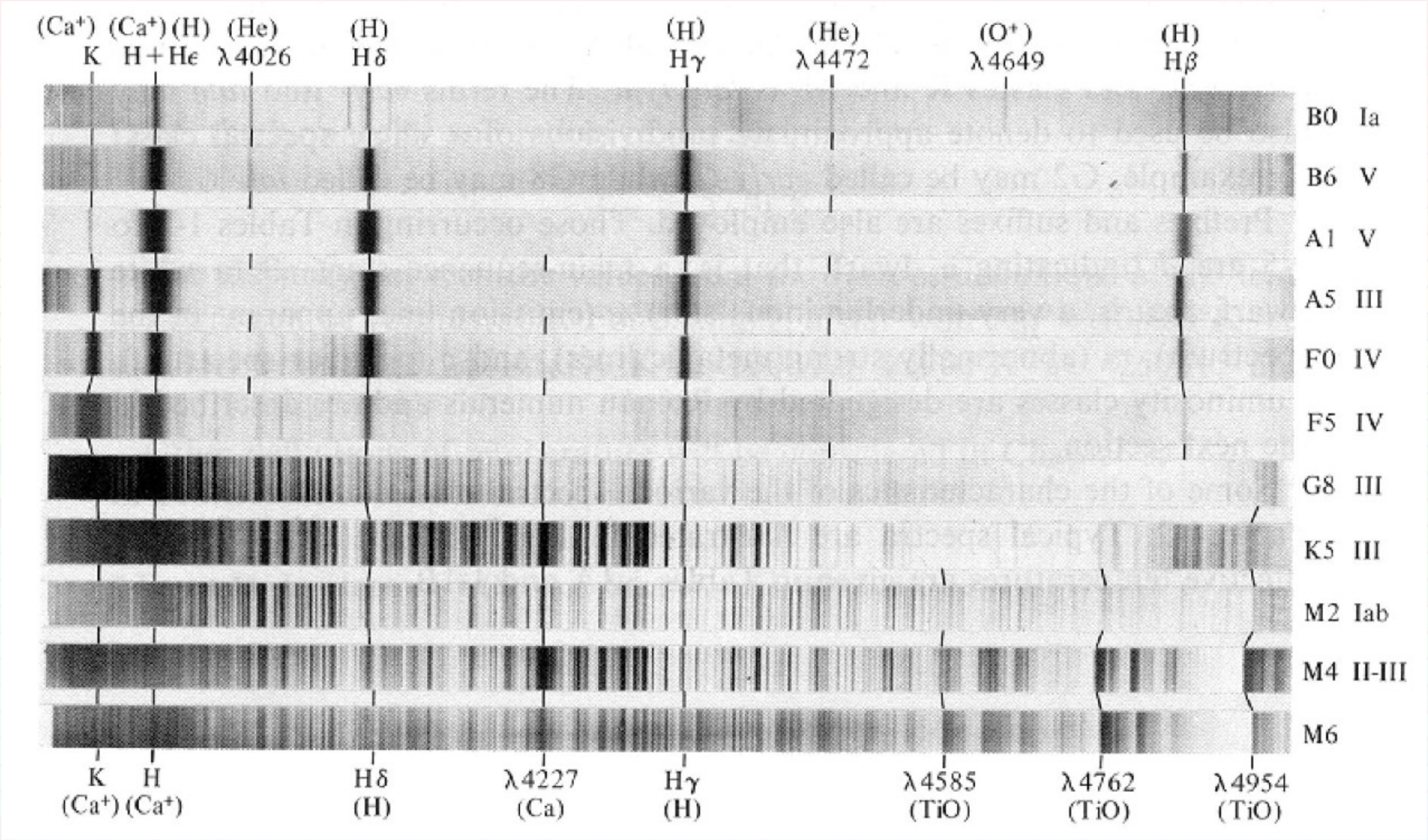
Interactive AladinLite view



Annie Jump Cannon

(1863 - 1941)

Stellar classification and temperature: application of Saha – and Boltzmann formulae

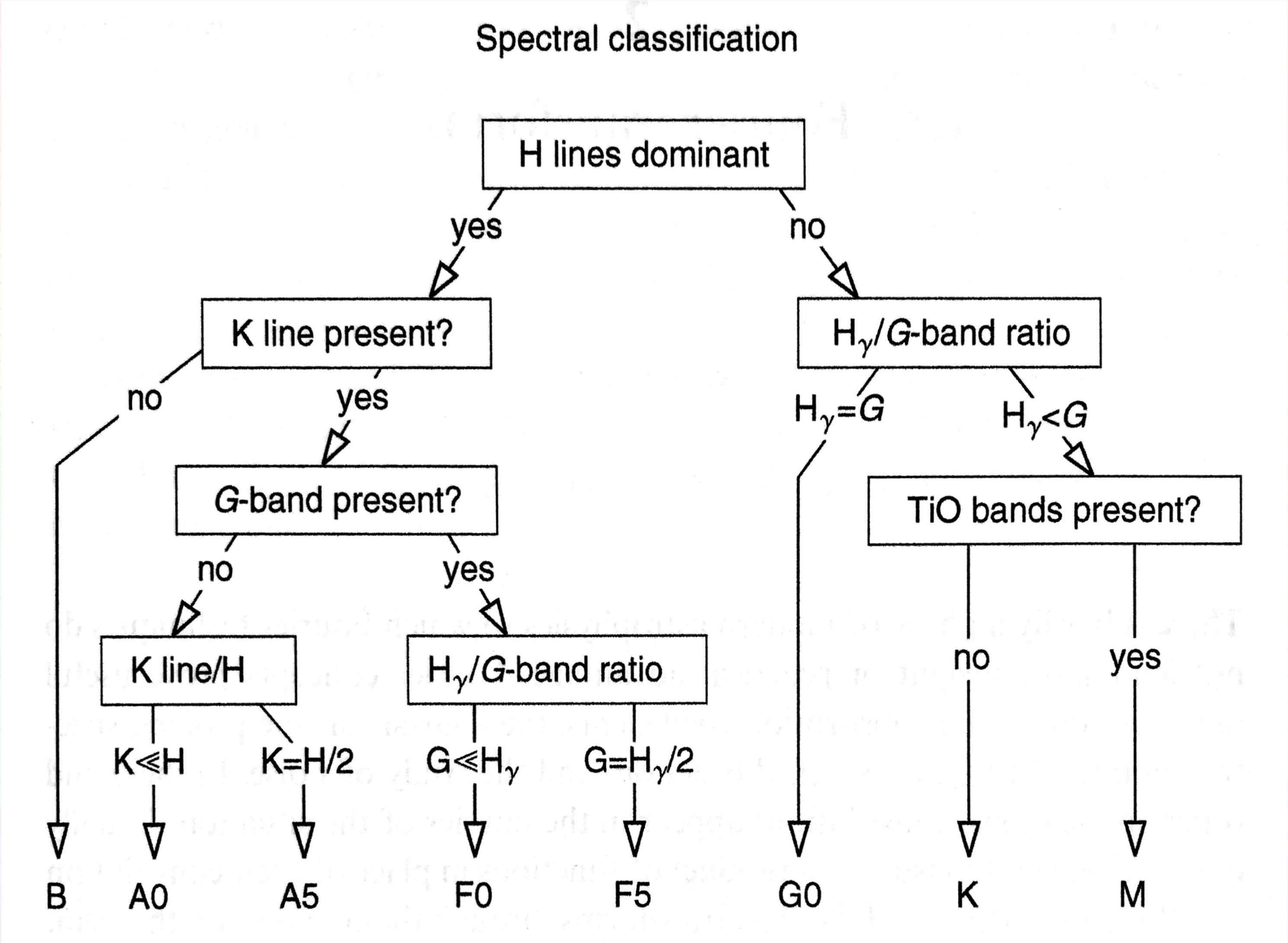


temperature *(spectral type)* & pressure *(luminosity class)* variations + chemical abundance changes.

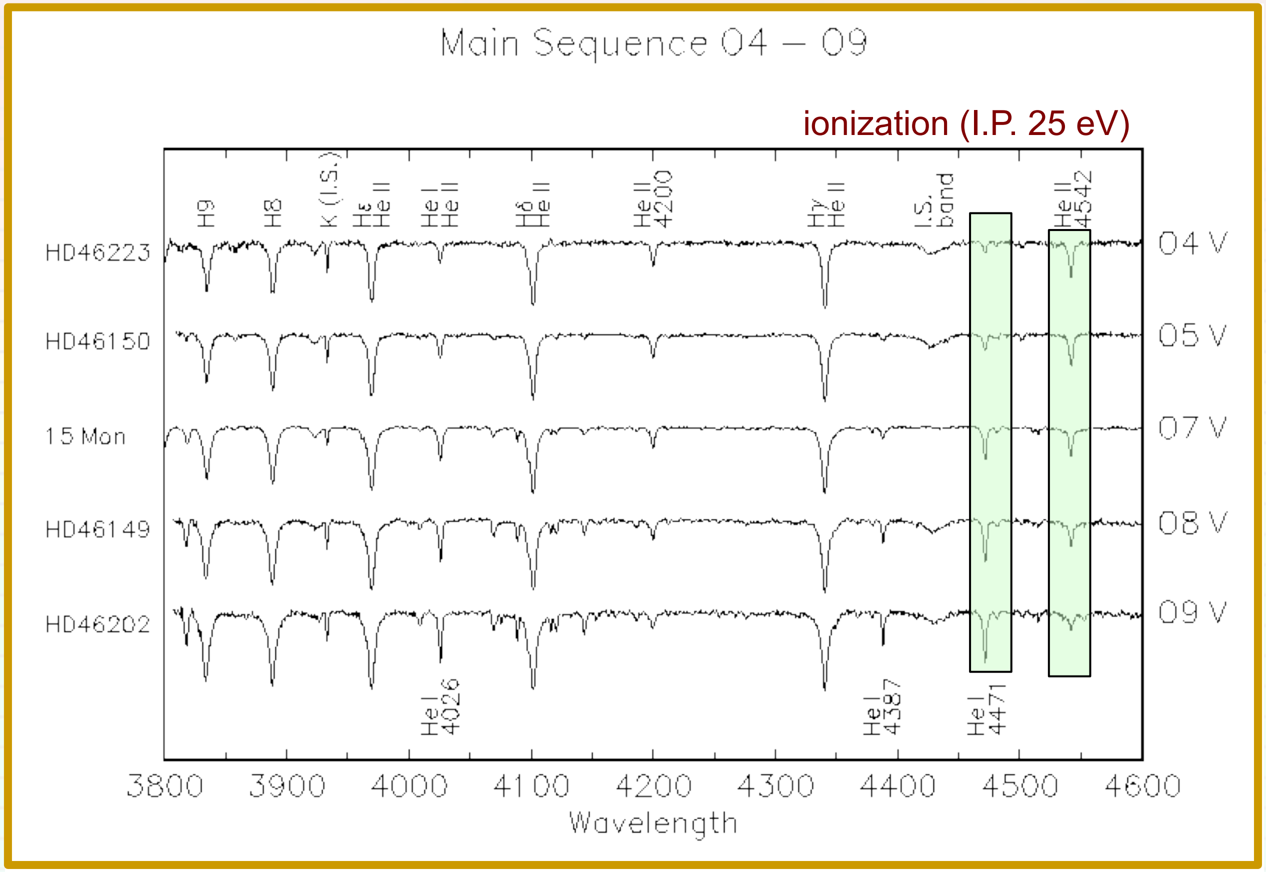
Stellar classification

Type	Approximate Surface Temperature	Main Characteristics
O	> 25,000 K	Singly ionized helium lines either in emission or absorption. Strong ultraviolet continuum. He I 4471/He II 4541 increases with type. *H and He lines weaken with increasing luminosity.* H weak, He I, He II, C III, N III, O III, Si IV.
B	11,000 - 25,000	Neutral helium lines in absorption (max at B2). H lines increase with type. Ca II K starts at B8. *H and He lines weaken with increasing luminosity.* C II, N II, O II, Si II-III-IV, Mg II, Fe III.
A	7,500 - 11,000	Hydrogen lines at maximum strength for A0 stars, decreasing thereafter. Neutral metals stronger. Fe II prominent A0-A5. *H and He lines weaken with increasing luminosity.* O I, Si II, Mg II, Ca II, Ti II, Mn I, Fe I-II.
F	6,000 - 7,500	Metallic lines become noticeable. G-band starts at F2. H lines decrease. *CN 4200 increases with luminosity.* Ca II, Cr I-II, Fe I-II, Sr II.
G	5,000 - 6,000	Solar-type spectra. Absorption lines of neutral metallic atoms and ions (e.g. once-ionized calcium) grow in strength. *CN 4200 increases with luminosity.*
K	3,500 - 5,000	Metallic lines dominate, H weak. Weak blue continuum. *CN 4200, Sr II 4077 increase with luminosity.* Ca I-II.
M	< 3,500	Molecular bands of titanium oxide TiO noticeable. *CN 4200, Sr II 4077 increase with luminosity.* Neutral metals.

Stellar classification



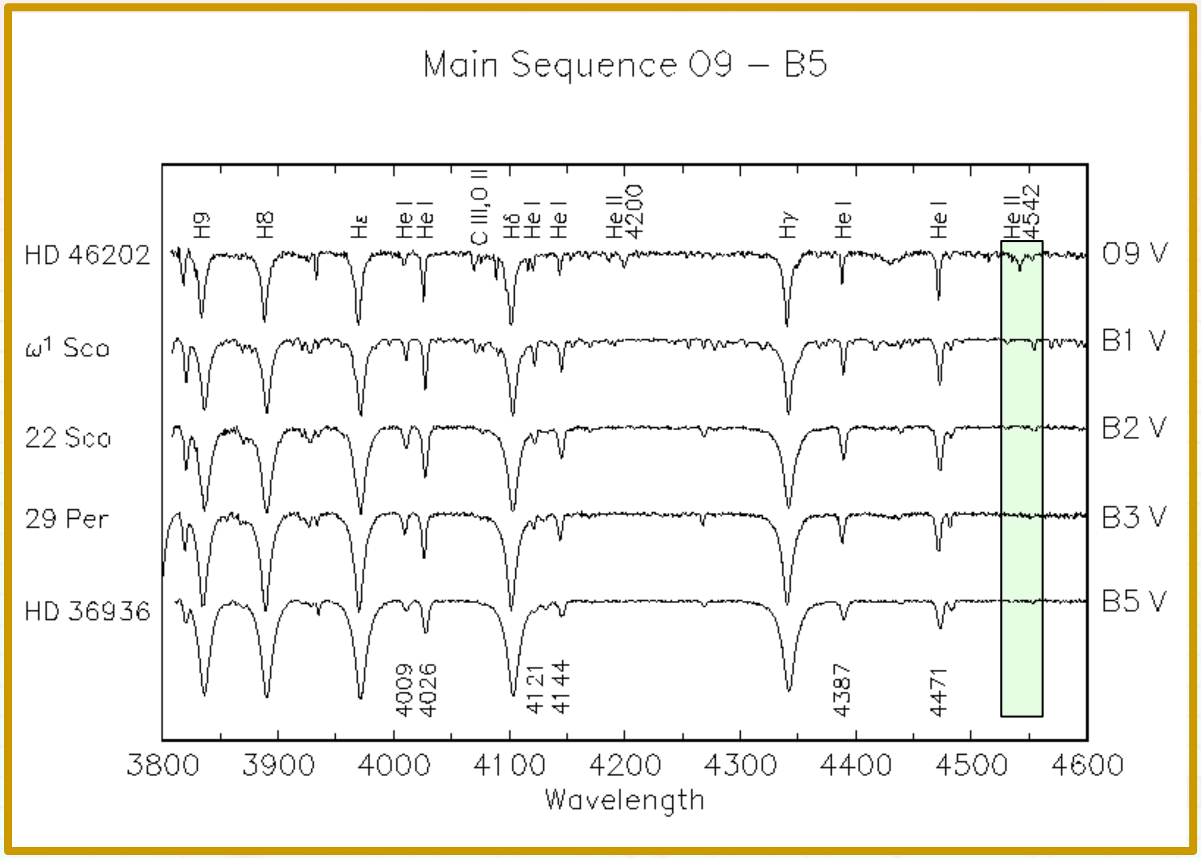
Main Sequence O4-O9



Judged easily by the ratio of the strengths of lines of He I to He II; He I tends to increase in strength with decreasing temperature while He II decreases in strength. The ratio He I 4471 to He II 4542 shows this trend clearly.

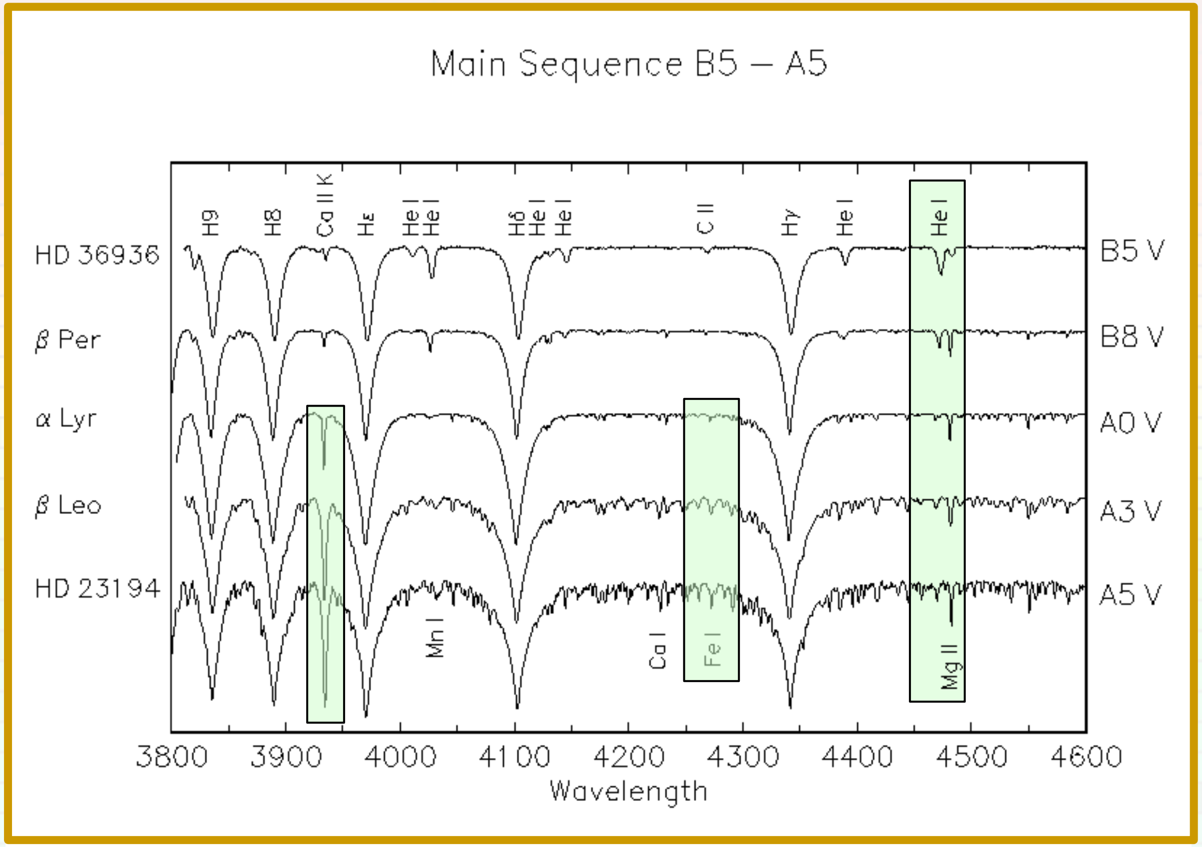
A DIGITAL SPECTRAL CLASSIFICATION ATLAS R. O. Gray http://nedwww.ipac.caltech.edu/level5/Gray/Gray_contents.html

Main Sequence O9-B5



Break between the O and B-type stars is the absence of lines of ionized helium (He II) in the spectra of B-type stars. The lines of He I pass through a maximum at approximately B2, and then decrease in strength towards later (cooler) types. A useful ratio to judge the spectral type is the ratio of He I 4471/Mg II 4481.

Main Sequence B5-A5



Break between the B and A-type stars is the absence of Helium. Ca II K-Line becomes notable in A stars, and metallic lines start to emerge. Very deep hydrogen lines!

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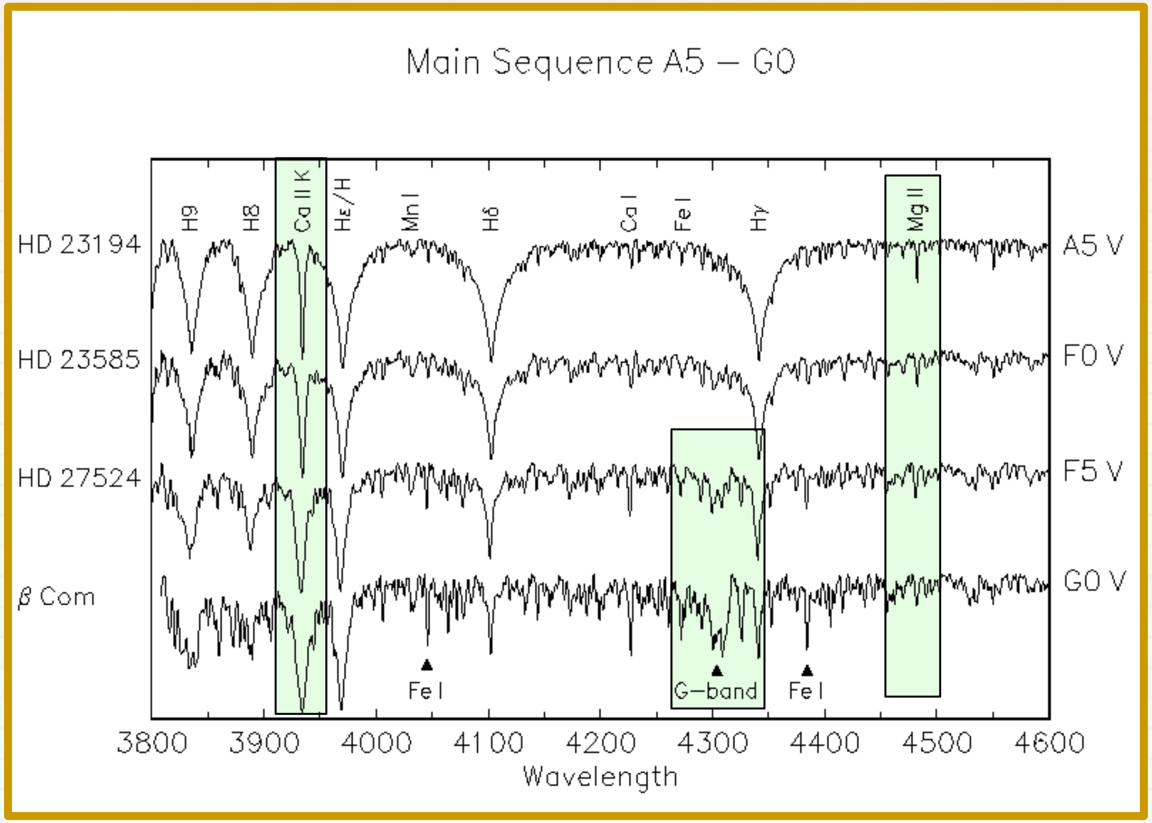
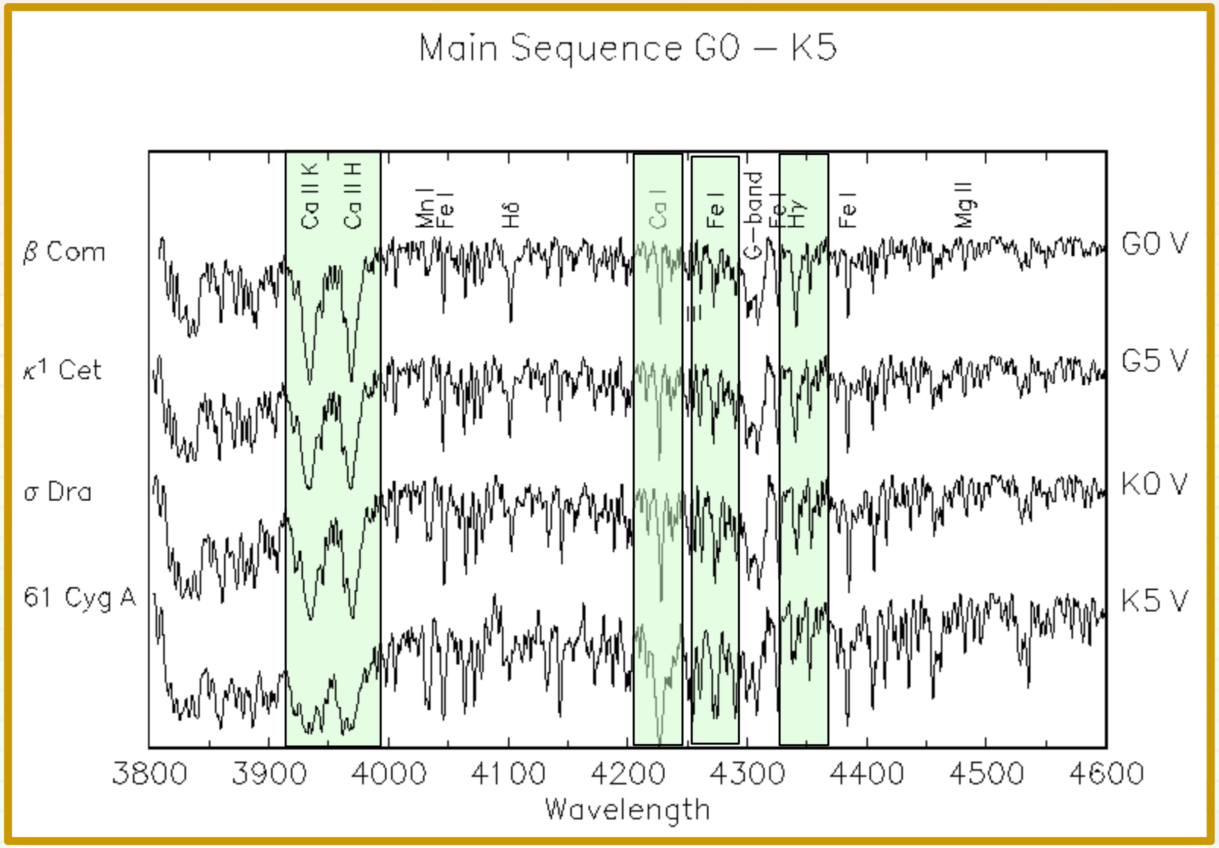
Main Sequence A5-G0



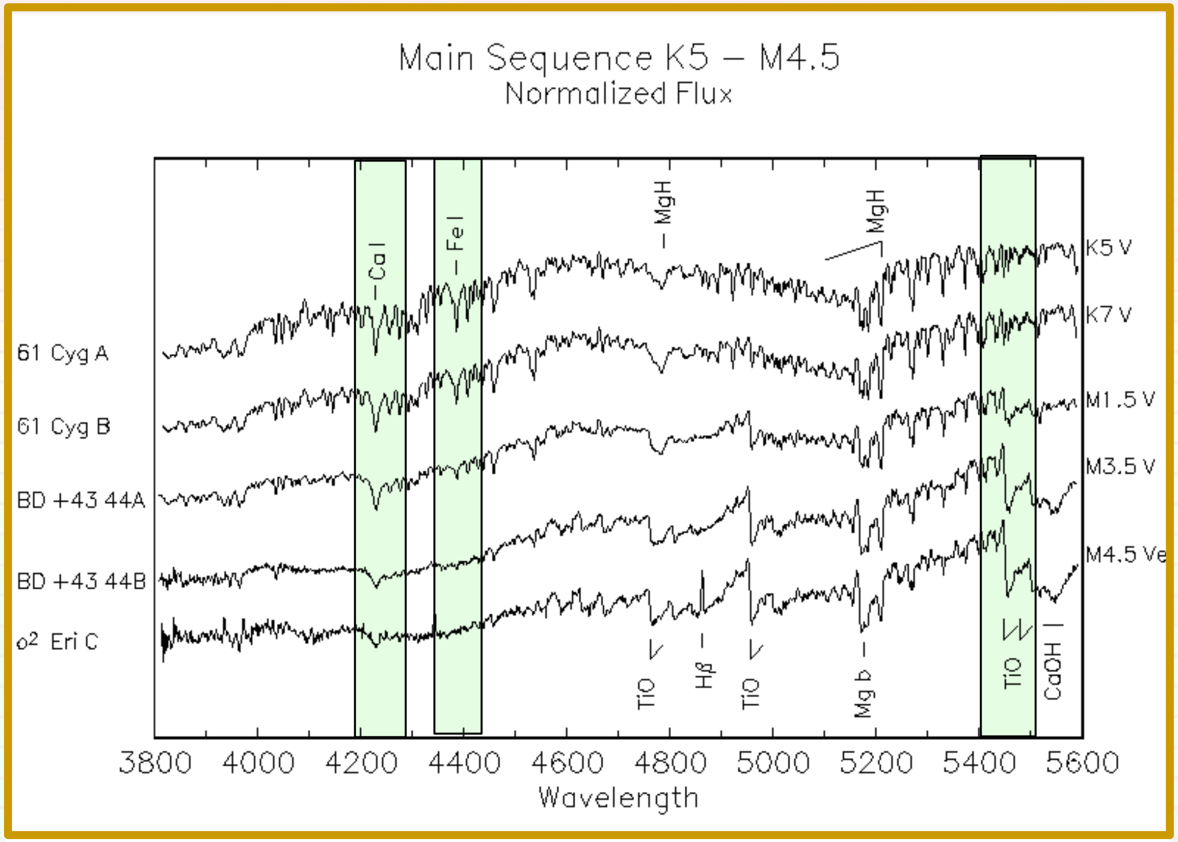
H lines begin to weaken, Ca II K-line continues to strengthen and saturated by late F. Metallic lines grow dramatically. G-band (thousands of closely spaced lines due to the diatomic molecule CH) appears around F2

Main Sequence G0-K5



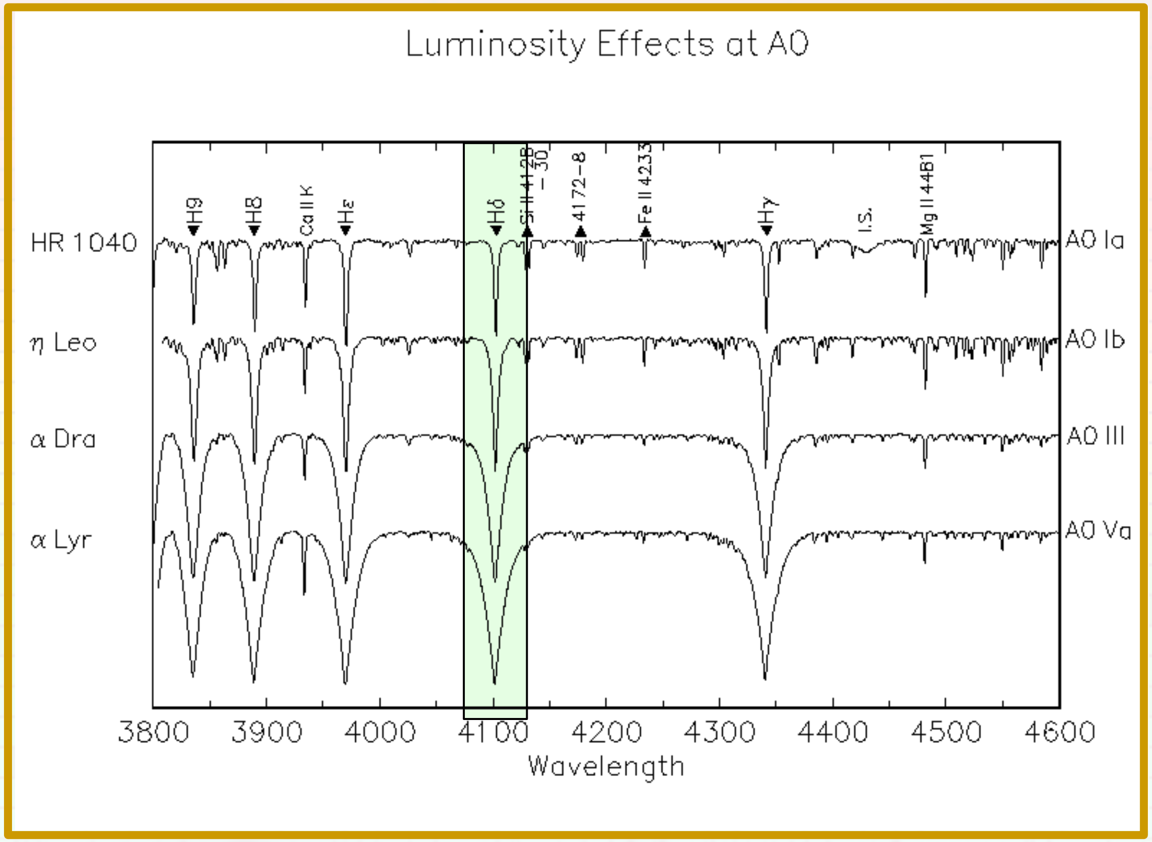
H continues to fade while the metallic-line spectrum continues to increase. The G-band continues to increase in strength until ~K2, and then begins to fade. The Ca I 4227 line grows gradually in strength until early K, and then becomes dramatically stronger by mid-K.

Main Sequence K5-M4.5



In the K-type dwarfs, the spectral type may be estimated from the ratio of Ca I 4227 to Fe I 4383, in the sense that Ca I/Fe I grows toward later types. By M0, bands due to TiO become visible in the spectrum, and these strengthen quite dramatically toward later types; by M4.5 they dominate the spectrum.

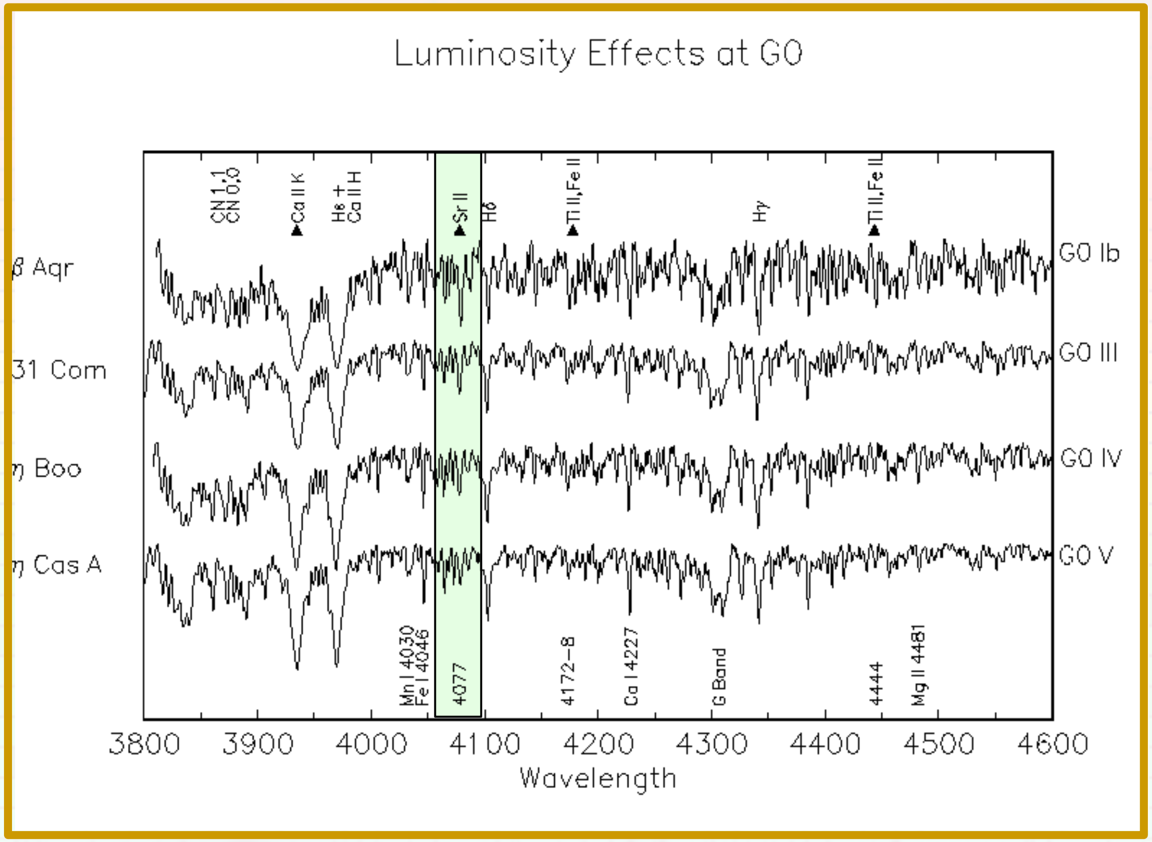
Stellar classification: Luminosity effects



Balmer lines indicate stellar luminosity

A DIGITAL SPECTRAL CLASSIFICATION ATLAS R. O. Gray http://nedwww.ipac.caltech.edu/level5/Gray/Gray_contents.html

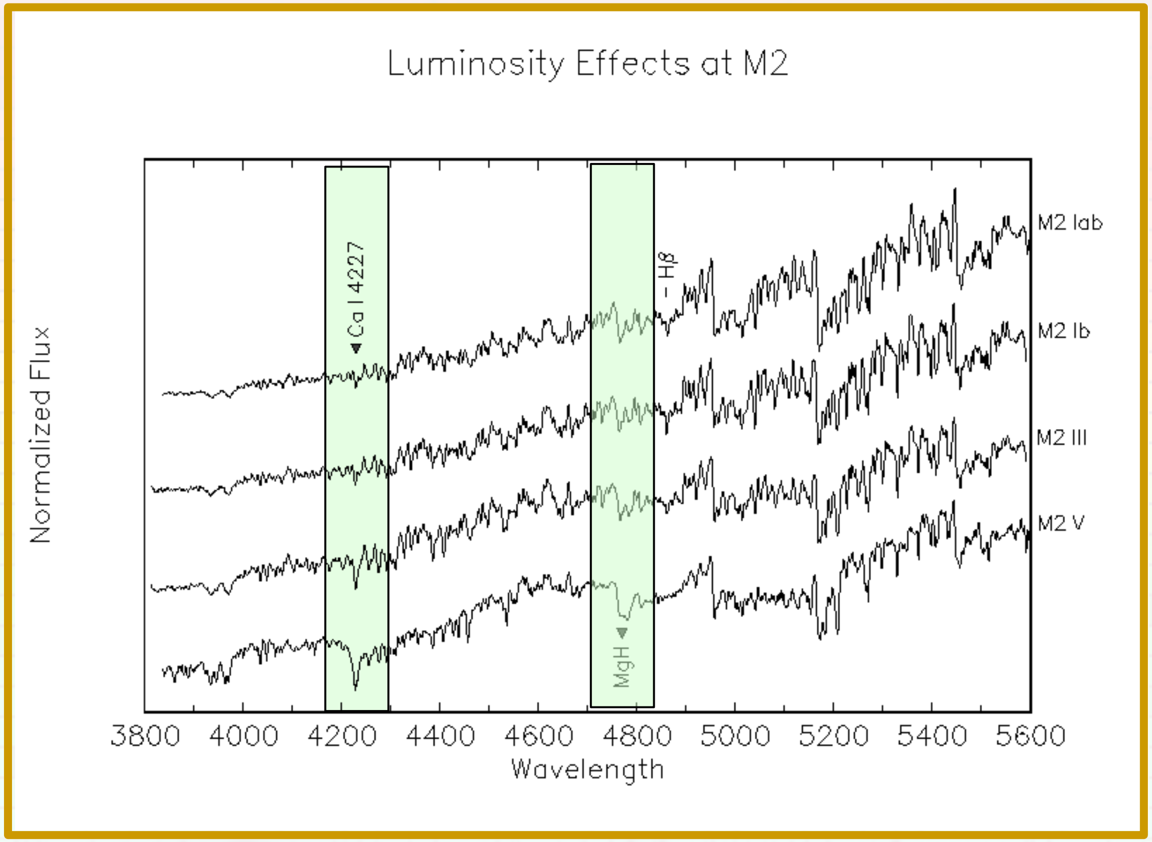
Stellar classification: Luminosity effects



Primary luminosity discriminant at G0 is the strength of the Sr II 4077 line. Of use as well are the blends of Ti II and Fe II which were used in the F-type stars to distinguish luminosity types. Generally, luminosity/gravity effects are more subtle than for early-type stars

A DIGITAL SPECTRAL CLASSIFICATION ATLAS R. O. Gray http://nedwww.ipac.caltech.edu/level5/Gray/Gray_contents.html

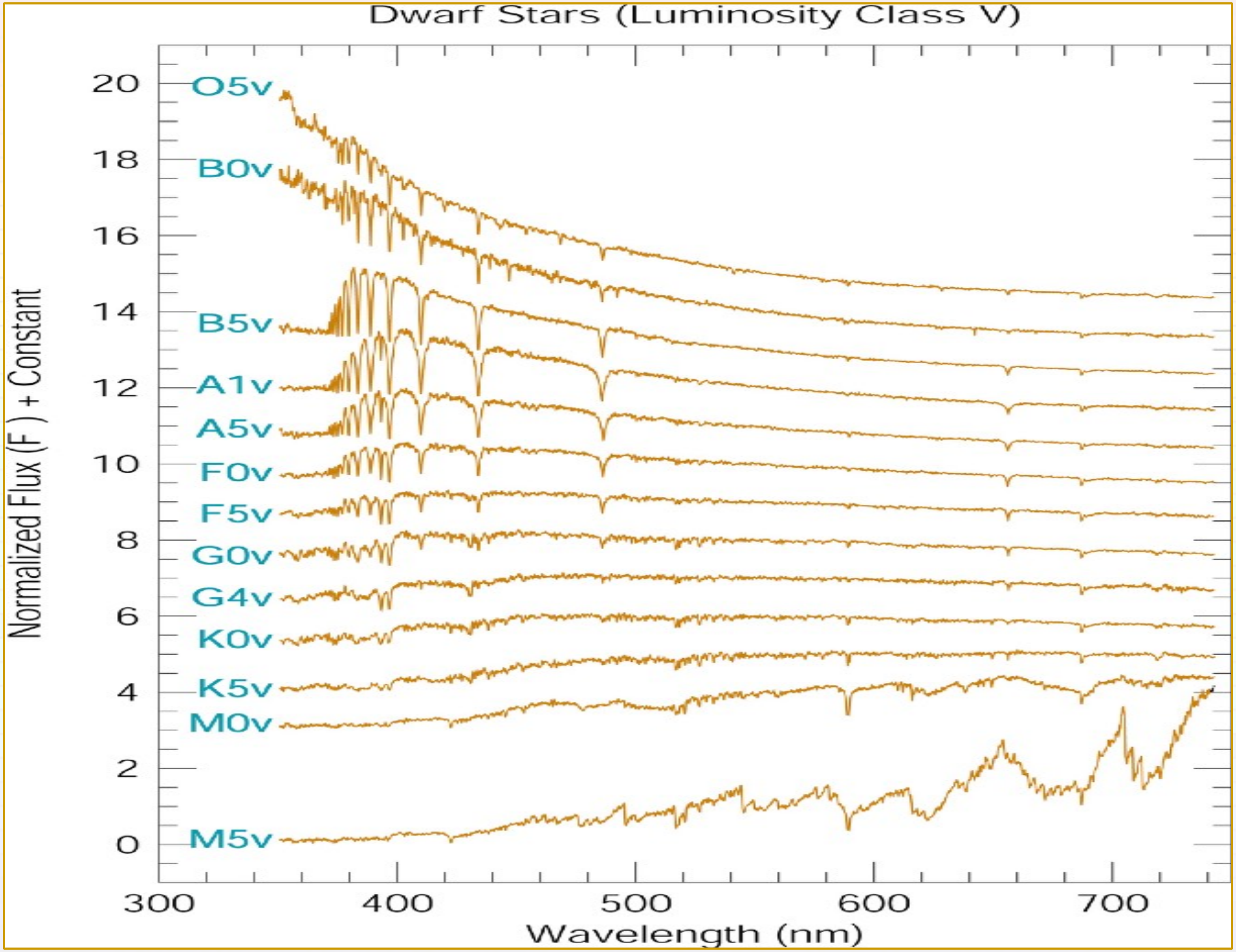
Stellar classification: Luminosity effects



Ca I 4227 line is the most striking luminosity indicator in the M2 stars. The morphology of the MgH/TiO blend near 4770 can be used as well to distinguish luminosity classes; notice that the MgH band dominates this blend in the dwarf star, producing a tooth-shaped feature.

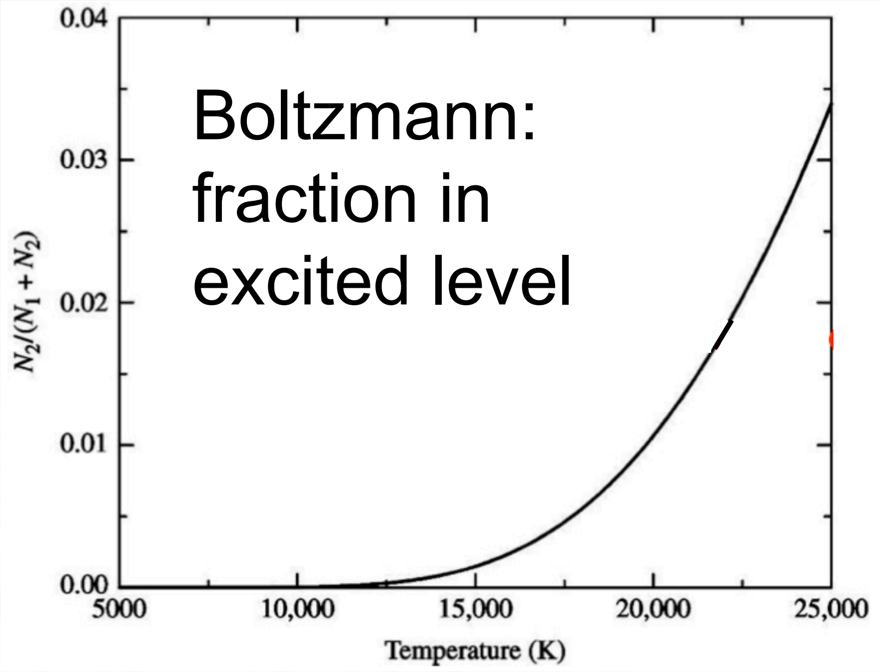
A DIGITAL SPECTRAL CLASSIFICATION ATLAS R. O. Gray http://nedwww.ipac.caltech.edu/level5/Gray/Gray_contents.html

Stellar classification

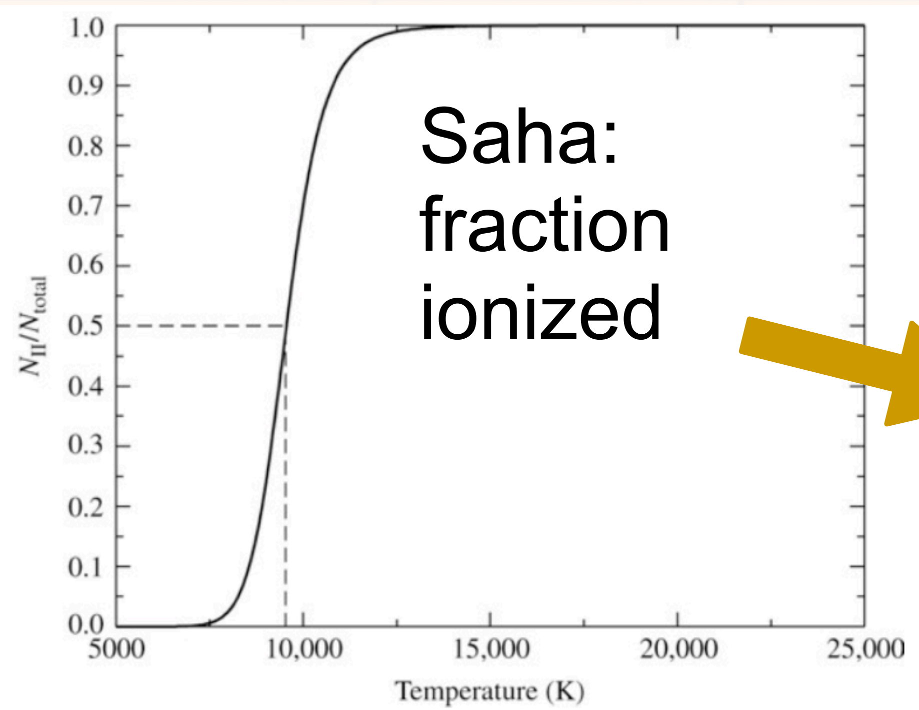


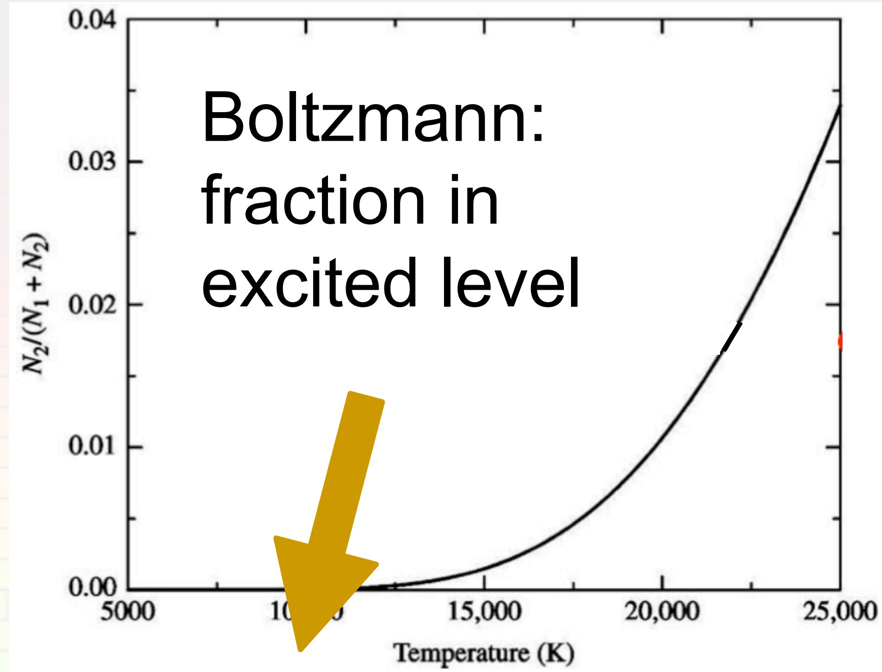
What causes spectral types? Example of Hydrogen

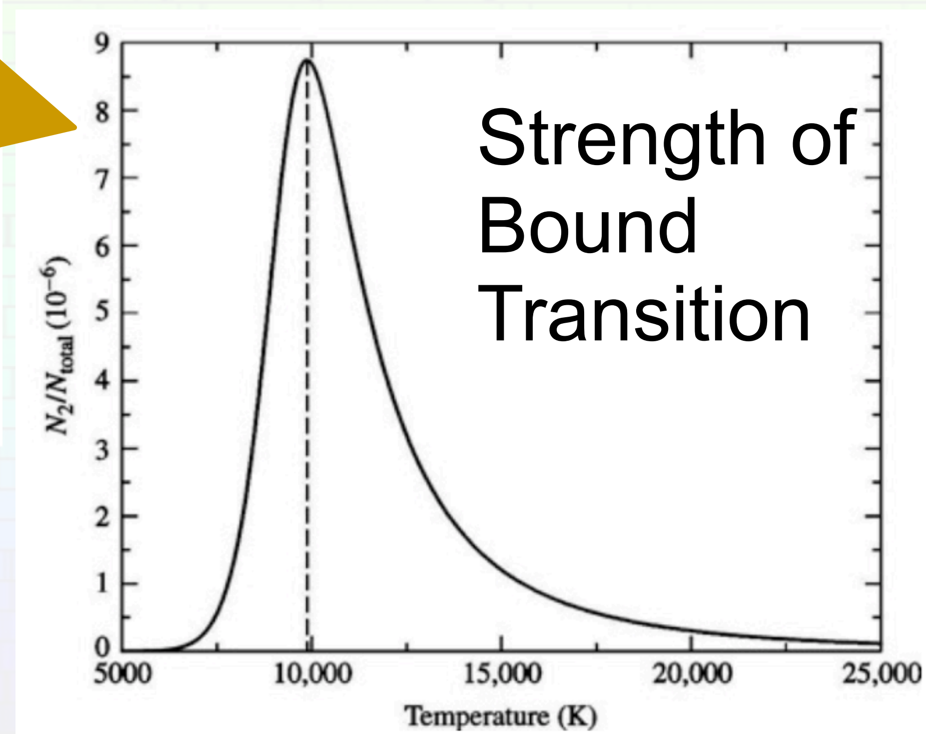
$$\boxed{\frac{n_{ij}}{n_{1j}} = \frac{g_{ij}}{g_{1j}}\, e^{-E_{ij}/kT}}$$



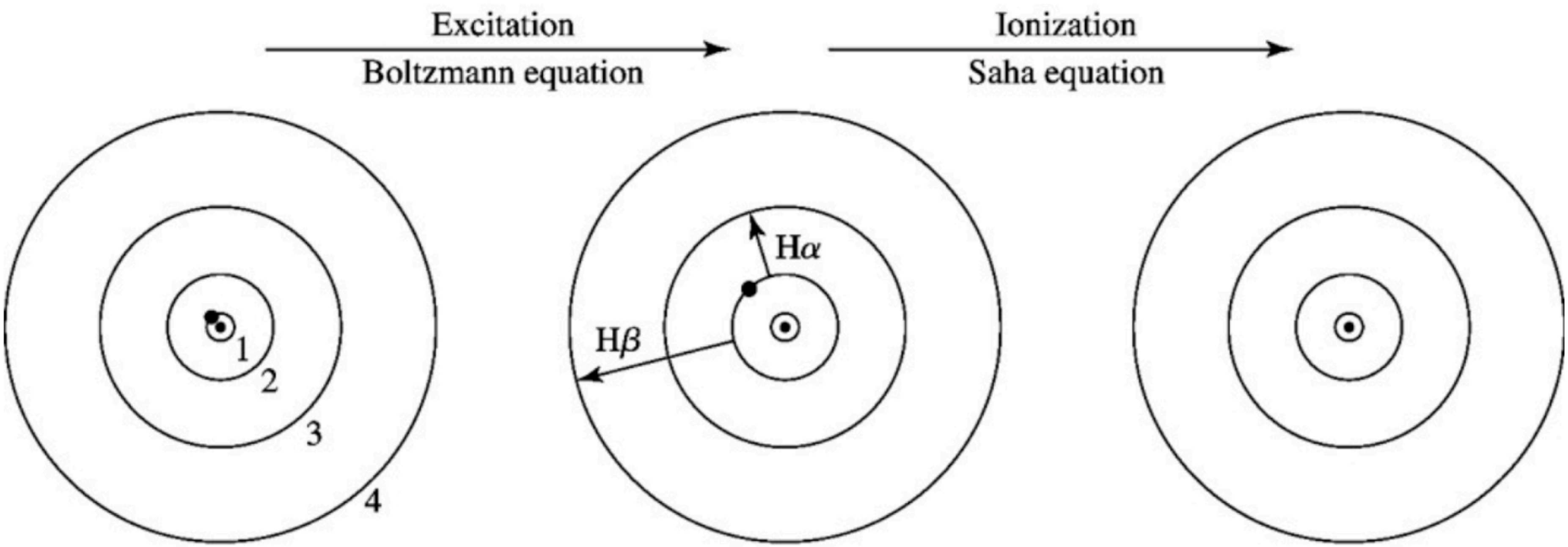
What causes spectral types? Example of Hydrogen







What causes spectral types? Example of Hydrogen



(a) $T < 9900$ K (b) $T = 9900$ K (c) $T > 9900$ K

Hydrogen when T < 9900 K
Majority of electrons in ground state, n=1

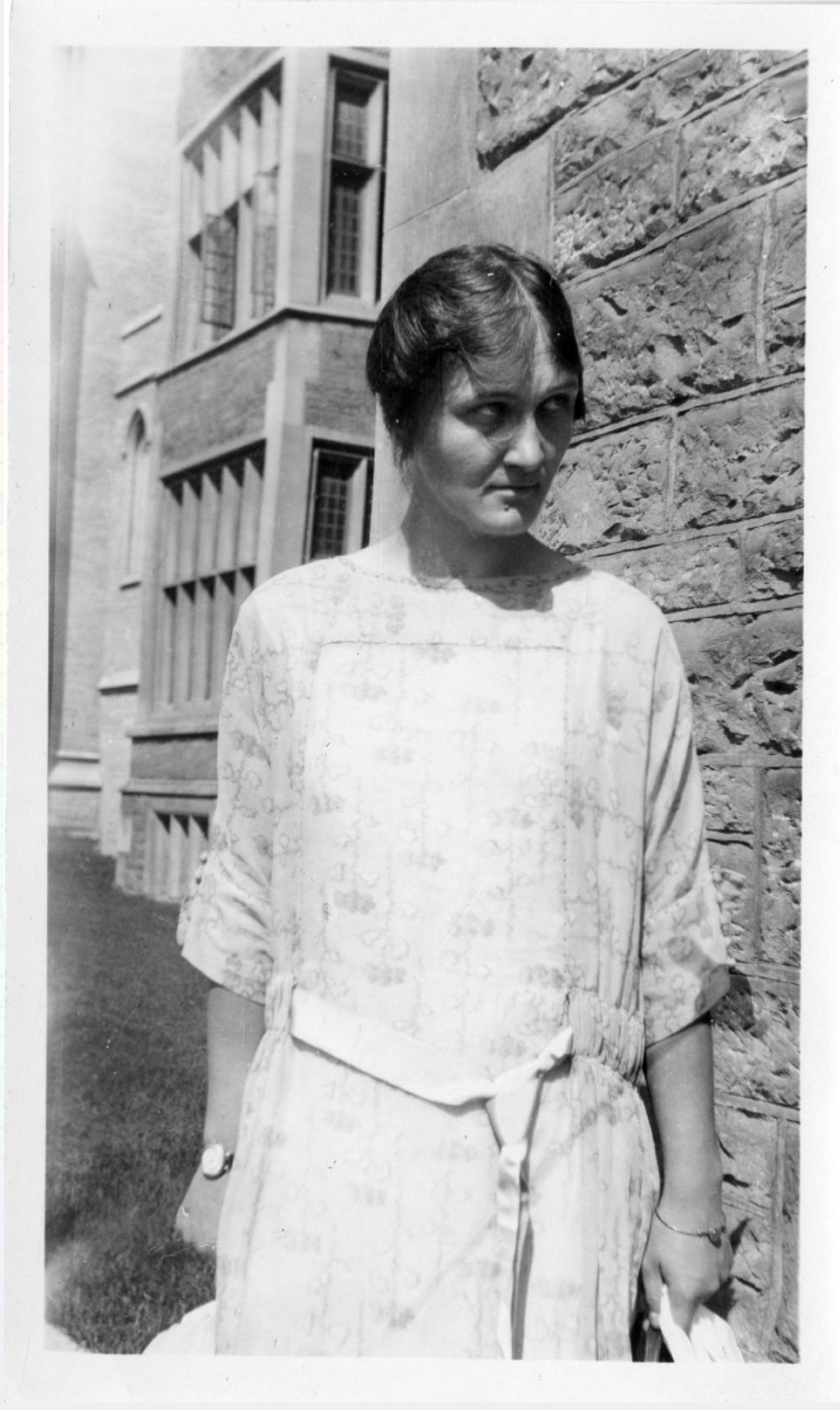
Hydrogen when T > 9900 K
Majority of electrons unbound, ionized hydrogen.

Hydrogen when T = 9900 K
Majority of electrons in first excited, n=2 state, and capable of producing Balmer lines

Alexey Belyanin

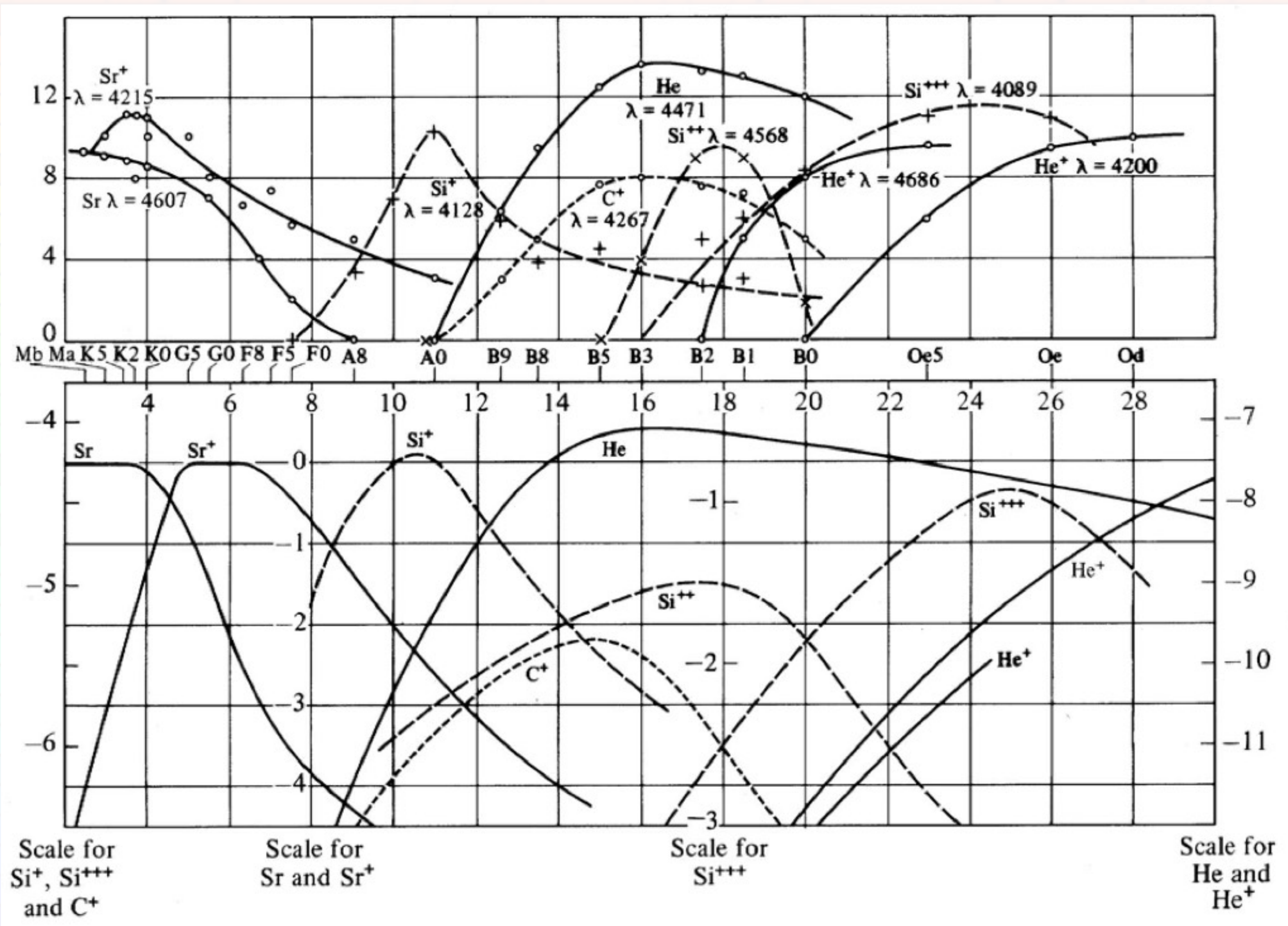
Cecilia Payne-Gaposchkin

(1900-1979)



Stellar classification

Saha's equation → stellar classification (C. Payne's thesis, Harvard 1925)



Variations of observed line strengths with spectral type in the Harvard sequence.

Saha-Boltzmann predictions of the fractional concentration $N_{r,s}/N$ of the lower level of the lines indicated against temperature T (units of 1000 K along the top). The pressure was taken constant at $P_e = N_e\,k\,T$ = 131 dyne cm^{-2}. The T-axis is adjusted to the abscissa of the upper diagram.

ON THE COMPOSITION OF THE SUN'S ATMOSPHERE[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

The *energy of binding* of an electron in different quantum states by *neutral* and *singly ionized* atoms is discussed with the aid of tables of the data at present available. The *structure of the spectra* is next considered, and *tables* of the *ionization potentials* and *the most persistent lines* are given. The *presence* and *absence* of the lines of different elements in the solar spectrum are then simply explained. The *excitation potential*, *E*, for the strongest lines in the observable part of the spectrum is the main factor. Almost all the elements for which this is small show in the sun. There are *very few solar lines* for which *E exceeds 5 volts;* the only strong ones are those of hydrogen.

The *abundance* of the various elements in the solar atmosphere is calculated with the aid of the calibration of Rowland's scale developed last year and of Unsöld's studies of certain important lines. The *numbers of atoms* in the more important energy states for each element are thus determined and found to decrease with increasing excitation, but a little more slowly than demanded by thermodynamic considerations.

The *level of ionization* in the solar atmosphere is such that atoms of *ionization potential 8.3 volts are 50 per cent ionized.*

Tables are given of the *relative abundance* of *fifty-six elements* and *six compounds*. These show that six of *the metallic elements*, *Na*, *Mg*, *Si*, *K*, *Ca*, and *Fe*, contribute 95 per cent of the whole mass. The whole number of metallic atoms above a square centimeter of the surface is 8×10^{20}. Eighty per cent of these are ionized. Their mean atomic weight is 32 and their total mass 42 mg/cm^2. The well-known difference between elements of *even* and *odd* atomic number is conspicuous—the former averaging *ten times as abundant* as the latter. The *heavy metals*, from *Ba* onward, are but little less abundant than those which follow *Sr*, and the hypothesis that the heaviest atoms sink below the photosphere is not confirmed. *The metals from Na to Zn, inclusive*, are *far more common* than the rest. The *compounds* are present in but small amounts, cyanogen being rarer than scandium. Most of those elements which *do not appear* in the solar spectrum should not show observable lines unless their abundance is much greater than is at all probable. There is a chance of finding faint lines of some additional rare earths and heavy metals, and perhaps of boron and phosphorus.

The *abundance of the non-metals*, and especially of hydrogen, is difficult to estimate from the few lines which are available. *Oxygen* appears to be about as abundant by weight as all the metals together. The abundance of *hydrogen* may be found with the aid of Menzel's observations of the flash spectrum. It is finally estimated that the *solar atmosphere contains 60 parts of hydrogen* (*by volume*), *2 of helium*, *2 of oxygen*, *1 of metallic vapors, and 0.8 of free electrons*, practically all of which come from ionization of the metals. This great abundance of hydrogen helps to explain a number of previously puzzling astrophysical facts. The temperature of the reversing layer is finally estimated at 5600° and the pressure at its base as 0.005 atm.

A *letter from Professor Eddington* suggesting that the *departure from the thermodynamic equilibrium* noticed by Adams and the writer is due to a *deficiency* of the number of atoms in the higher excited states is quoted and discussed.

Russell 1929

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The *energy of binding* of an electron in different quantum states by *neutral* and

TABLE XVI

Comparison with Miss Payne's Results

El.	Miss Payne	Table XIV	Diff.	El.	Miss Payne	Table XIV	Diff.
H........	12.9	[11.5]	(+1.4)	*Ca*......	6.7	6.7	0.0
He.......	10.2	....	....	*Ti*......	6.0	5.2	+0.8
Li.......	1.9	2.0	−0.1	*V*.......	4.9	5.0	−0.1
C........	6.4	7.4:	−1.0	*Cr*......	5.8	5.7	+0.1
O........	8.0	9.0:	−1.0	*Mn*.....	6.5	5.9	+0.6
Na.......	7.1	7.2	−0.1	*Fe*......	6.7	7.2	−0.5
Mg......	7.5	7.8	−0.3	*Zn*......	6.1	4.9	+1.2
Al.......	6.9	6.4	+0.5	*Sr*......	3.5	3.3	+0.2
Si.......	7.5	7.3	+0.2	*Ba*......	3.0	3.3	−0.3
K........	5.3	6.8:	−1.5				

than scandium. Most of those elements which *do not appear* in the solar spectrum should not show observable lines unless their abundance is much greater than is at all probable. There is a chance of finding faint lines of some additional rare earths and heavy metals, and perhaps of boron and phosphorus.

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Russell 1929

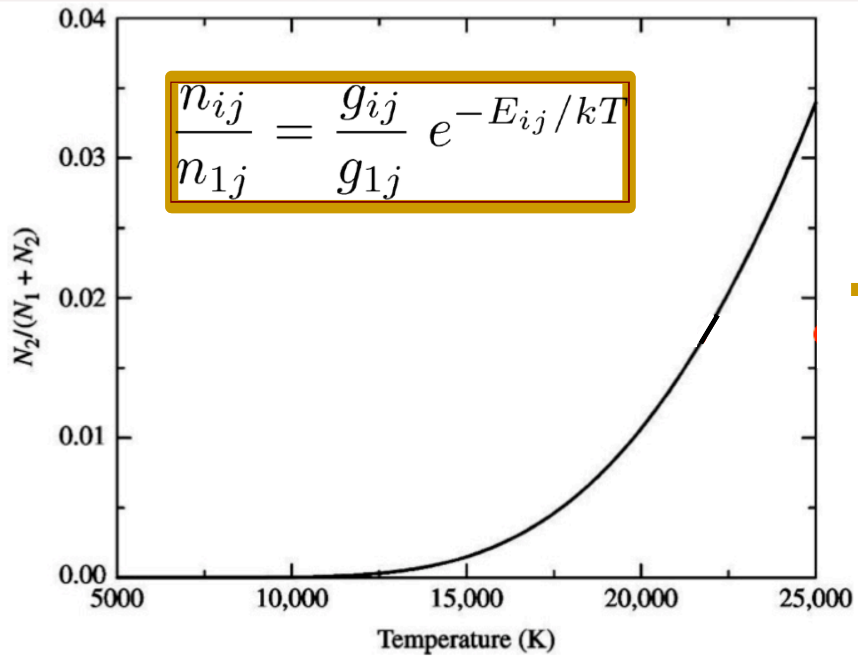
Cecilia Payne-Gaposchkin

(1900-1979)

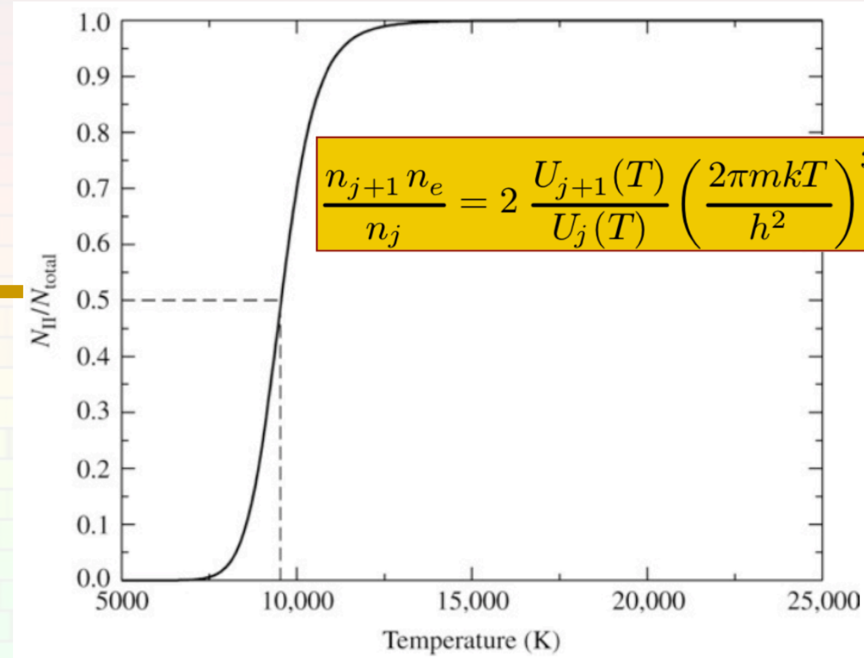
"There is no joy more intense than that of coming upon a fact that cannot be understood in terms of currently accepted ideas."

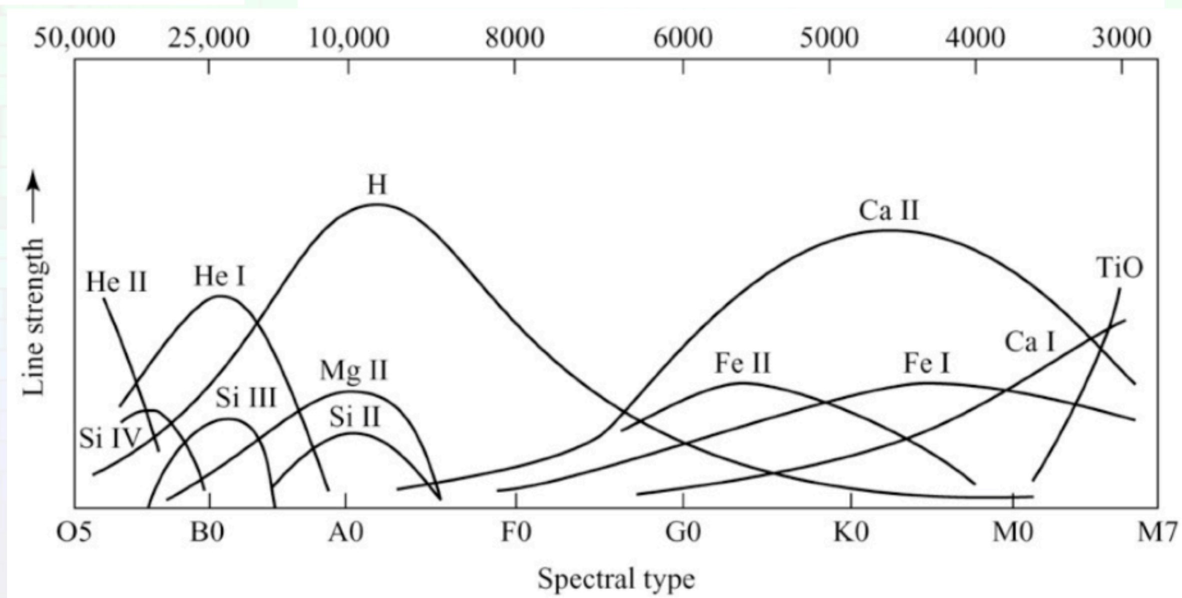
Recap from last lecture:

$$\frac{n_{ij}}{n_{1j}} = \frac{g_{ij}}{g_{1j}}\, e^{-E_{ij}/kT}$$



$$\frac{n_{j+1}\, n_e}{n_j} = 2\,\frac{U_{j+1}(T)}{U_j(T)} \left(\frac{2\pi m k T}{h^2}\right)^{3/2} e^{-\frac{E_j}{kT}}$$





Recap Quiz

- You are predicting the line strength of Hδ and Ca II in an atmosphere with n_H/n_{Ca}=1e6. The fraction of singly ionized to neutral Ca is 0.99, and the fraction of neutral H in the first excited state to the ground state is 1e-9. Which line will be stronger and why?

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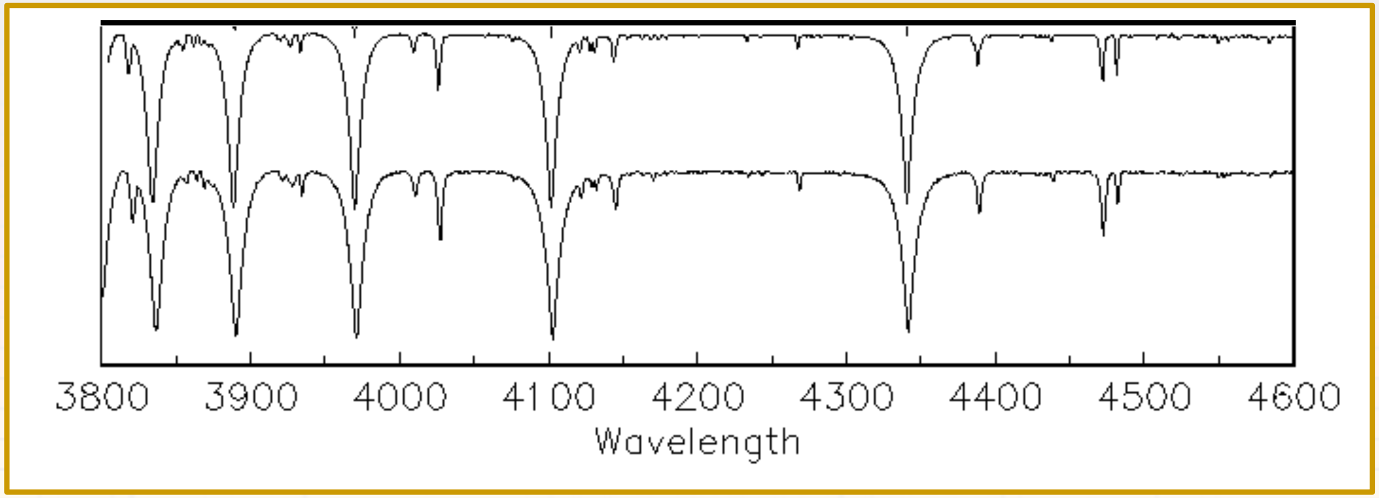
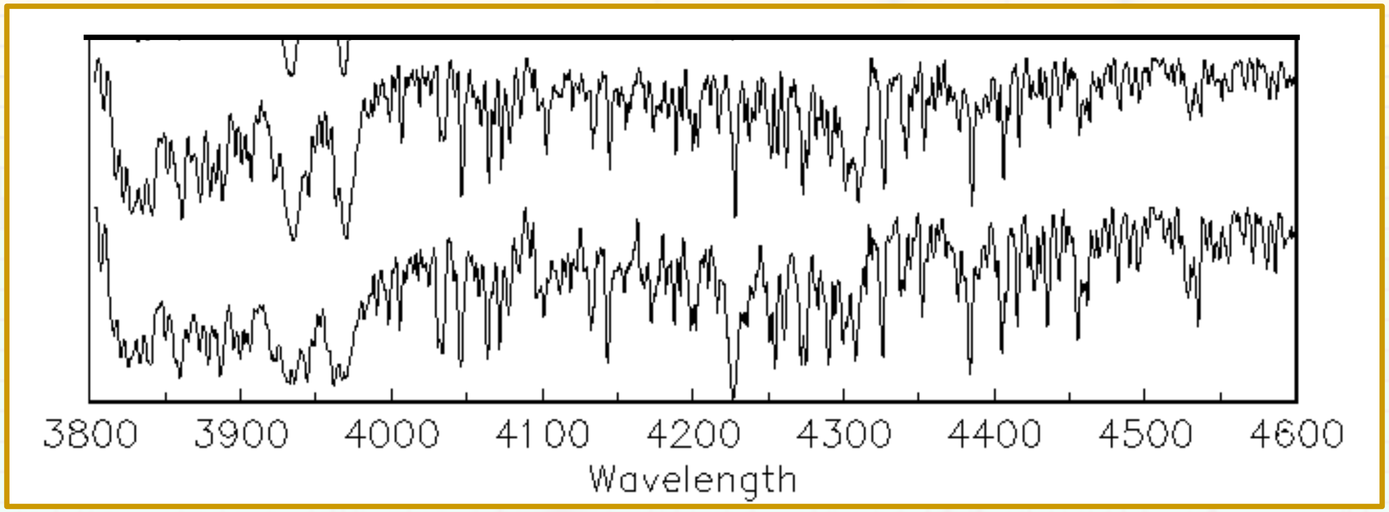
- You are measuring the equivalent width of the Hδ line for two stars with identical effective temperature and determine that EW1>EW2. What can you conclude about the physical properties of the star?


3800
3900
4000
4100
4200
4300
4400
4500
4600
Wavelength

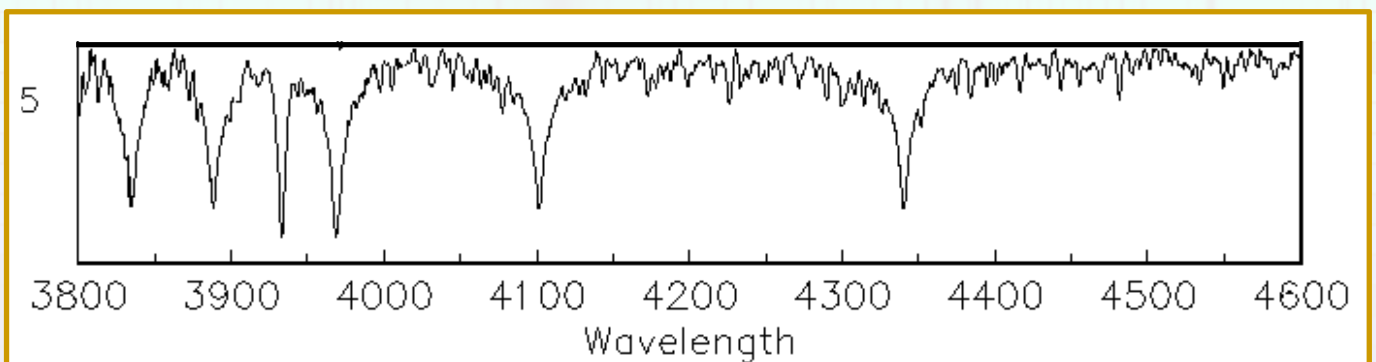
?

?

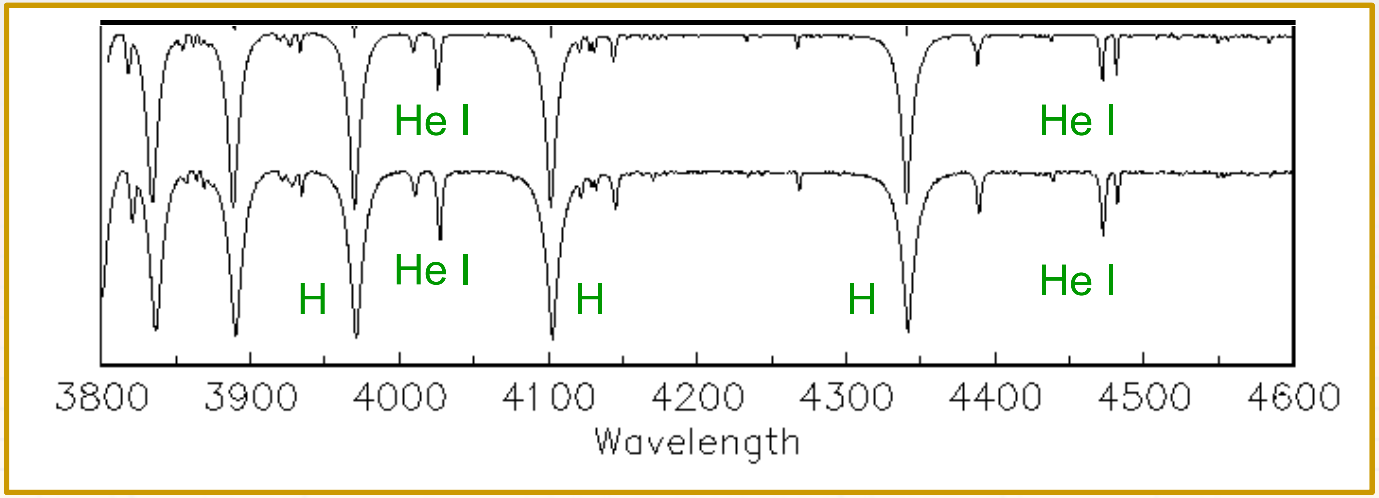

3800
3900
4000
4100
4200
4300
4400
4500
4600
Wavelength

?

?

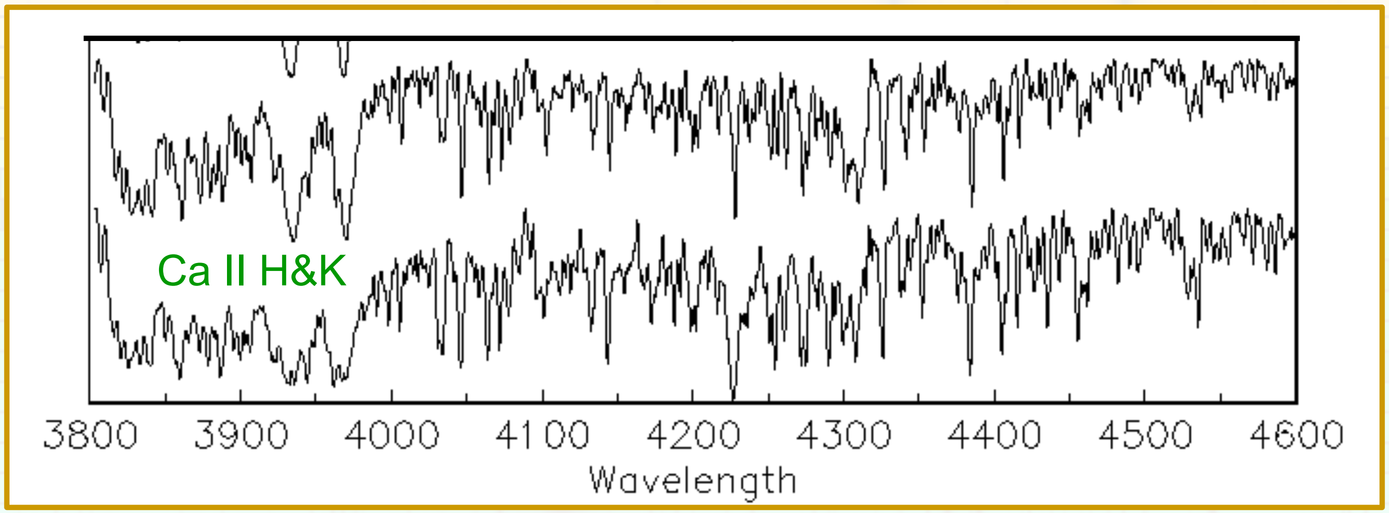

5
3800
3900
4000
4100
4200
4300
4400
4500
4600
Wavelength

?



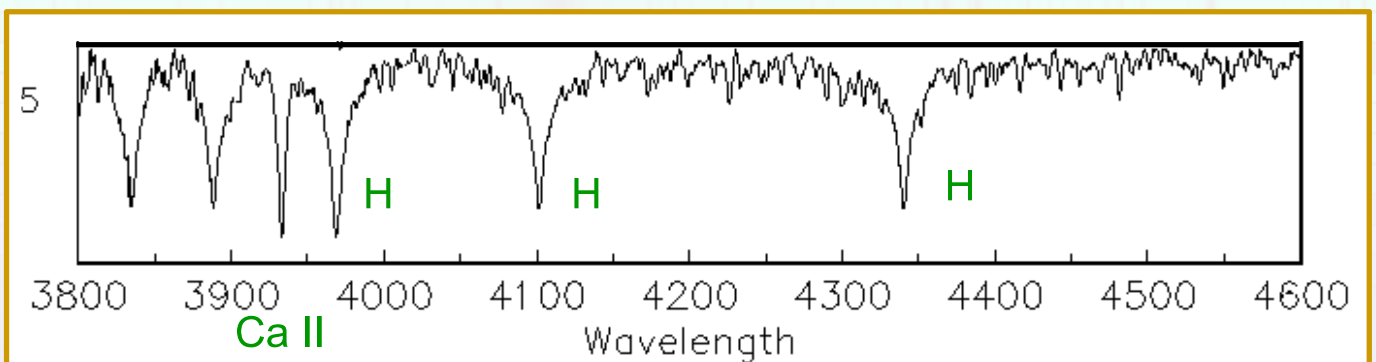
B5III

B5V

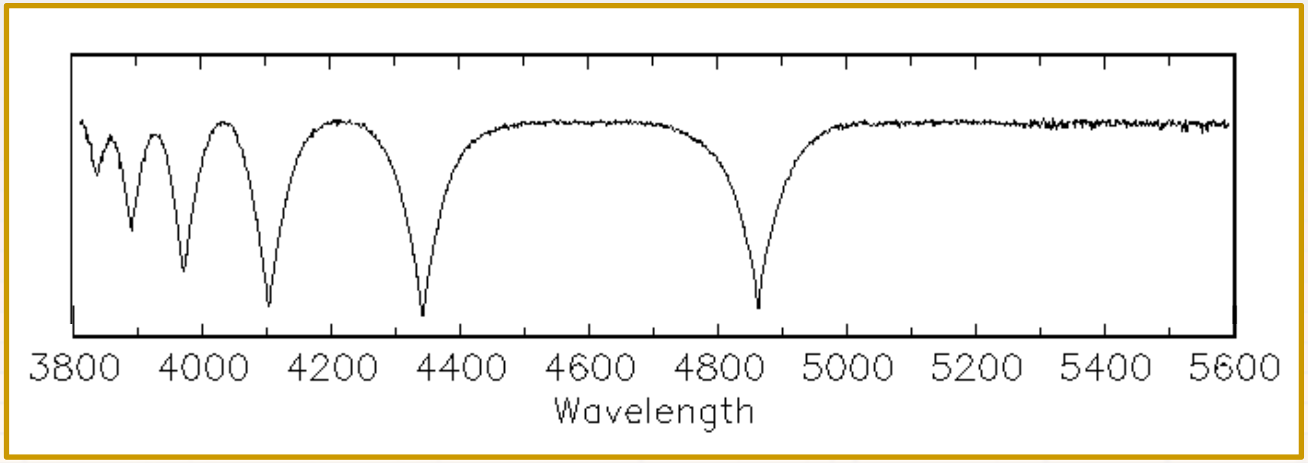


K0V

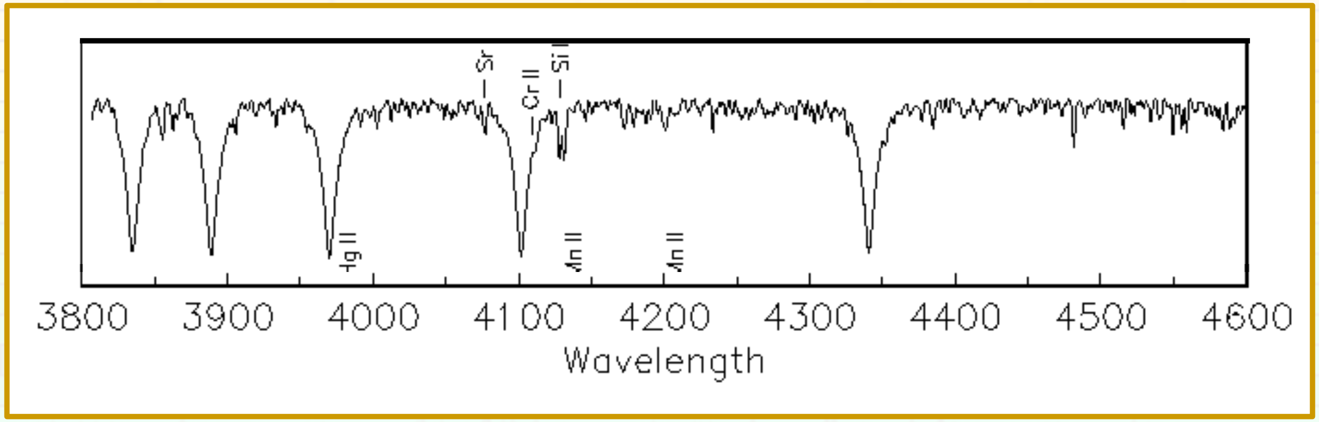
K5V



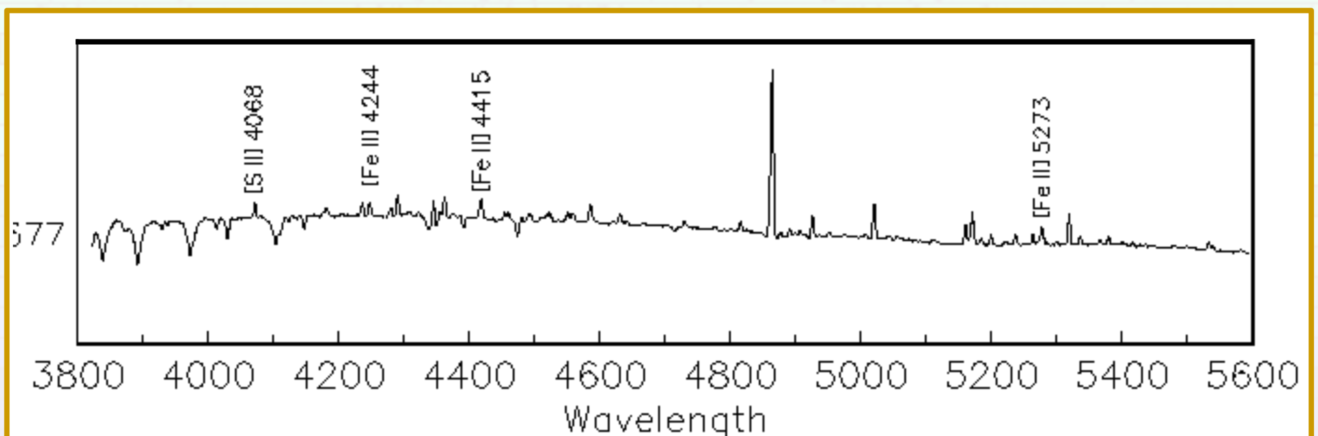
F0V


3800 4000 4200 4400 4600 4800 5000 5200 5400 5600
Wavelength

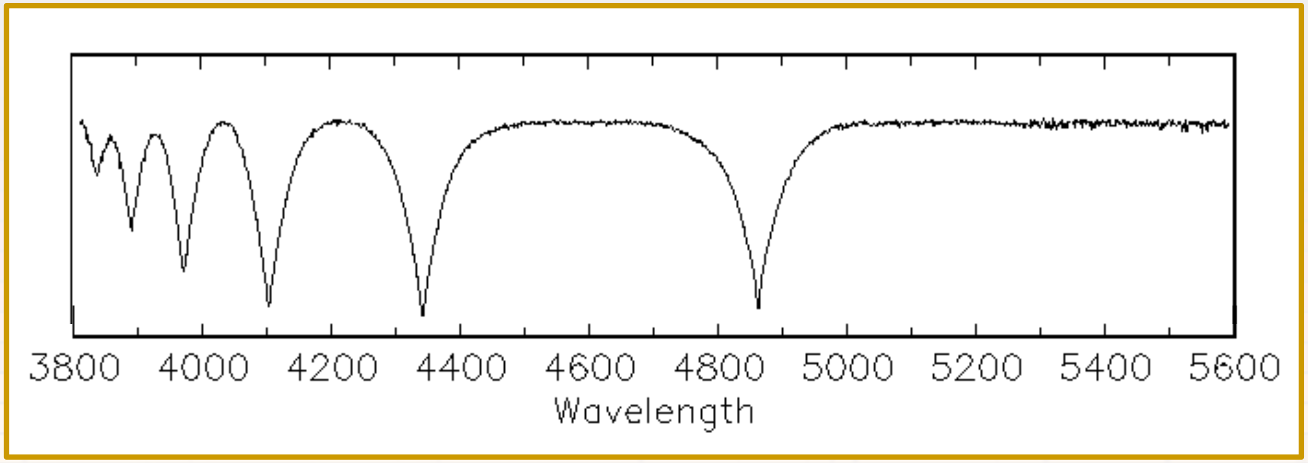
?


Sr
Cr II
3800 3900 4000 4100 4200 4300 4400 4500 4600
Wavelength

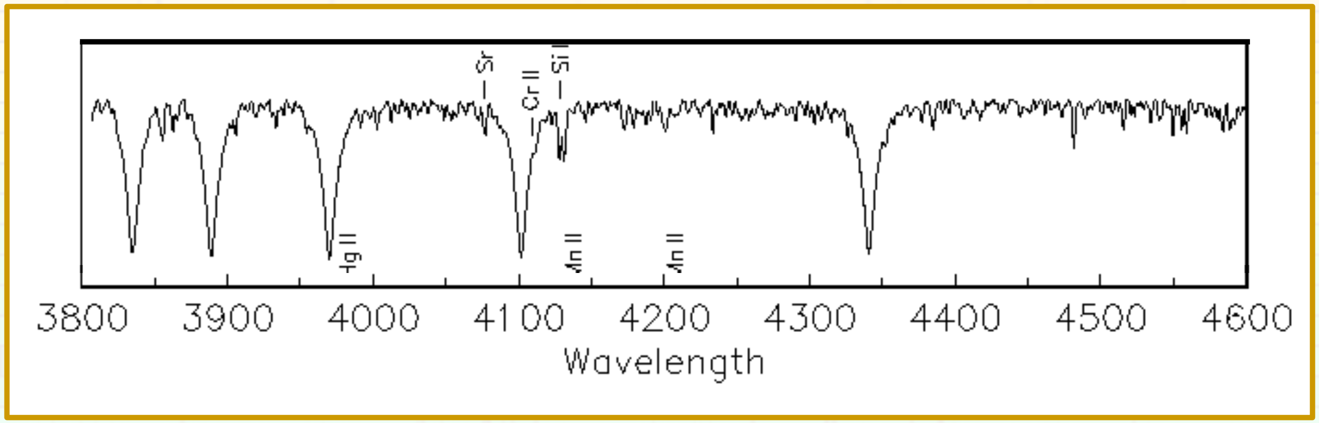
?


[S II] 4068
[Fe II] 4244
[Fe II] 4415
[Fe II] 5273
3800 4000 4200 4400 4600 4800 5000 5200 5400 5600
Wavelength

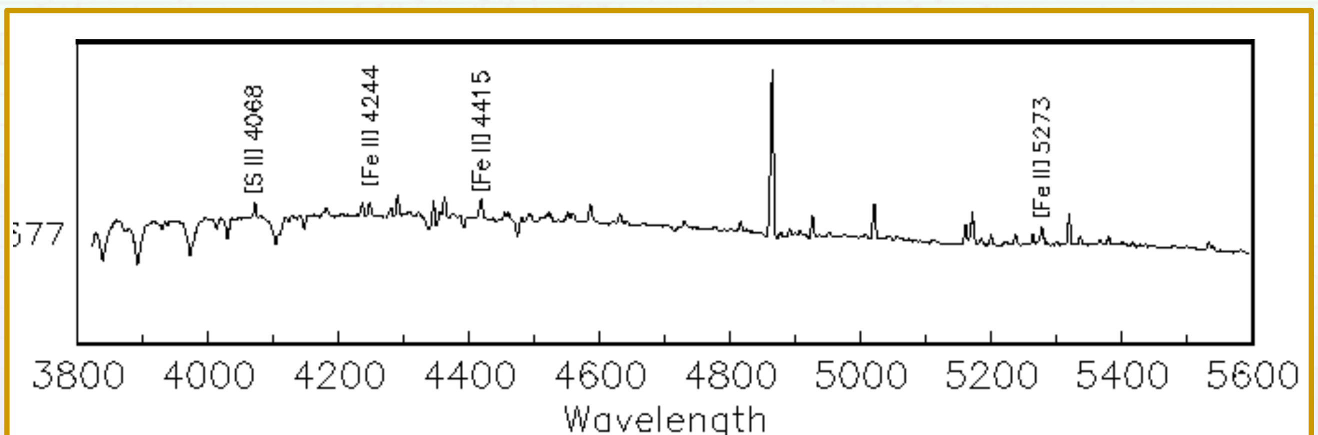
?



White Dwarf

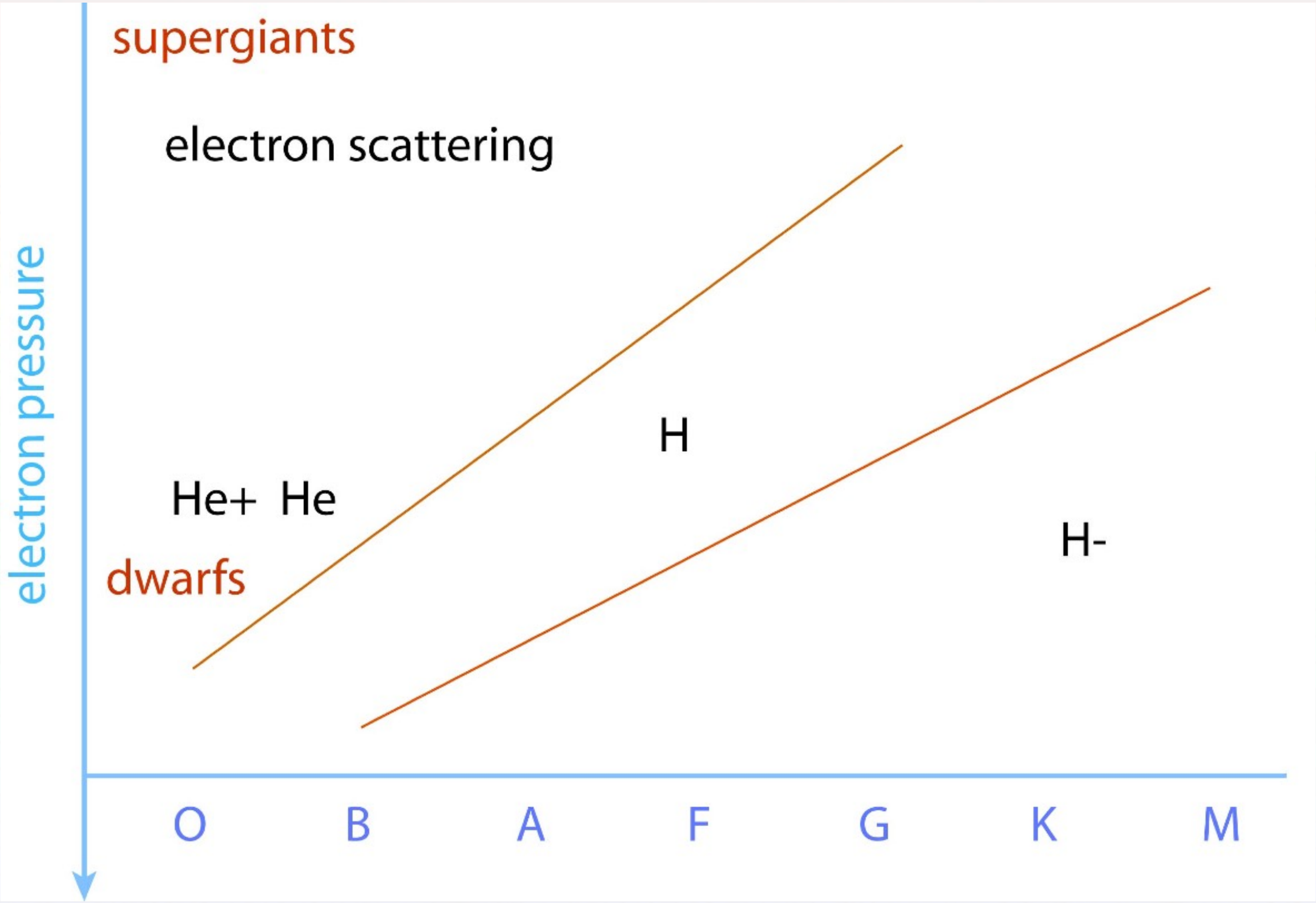


A0VspSiSrCr



B2Ve

Stellar continua: opacity sources



Stellar continua: the Balmer jump

From the ground we can measure part of Balmer (λ < 3646 A) and Bracket (λ > 8207 A) continua, and complete Paschen continuum (3647 – 8206 A)

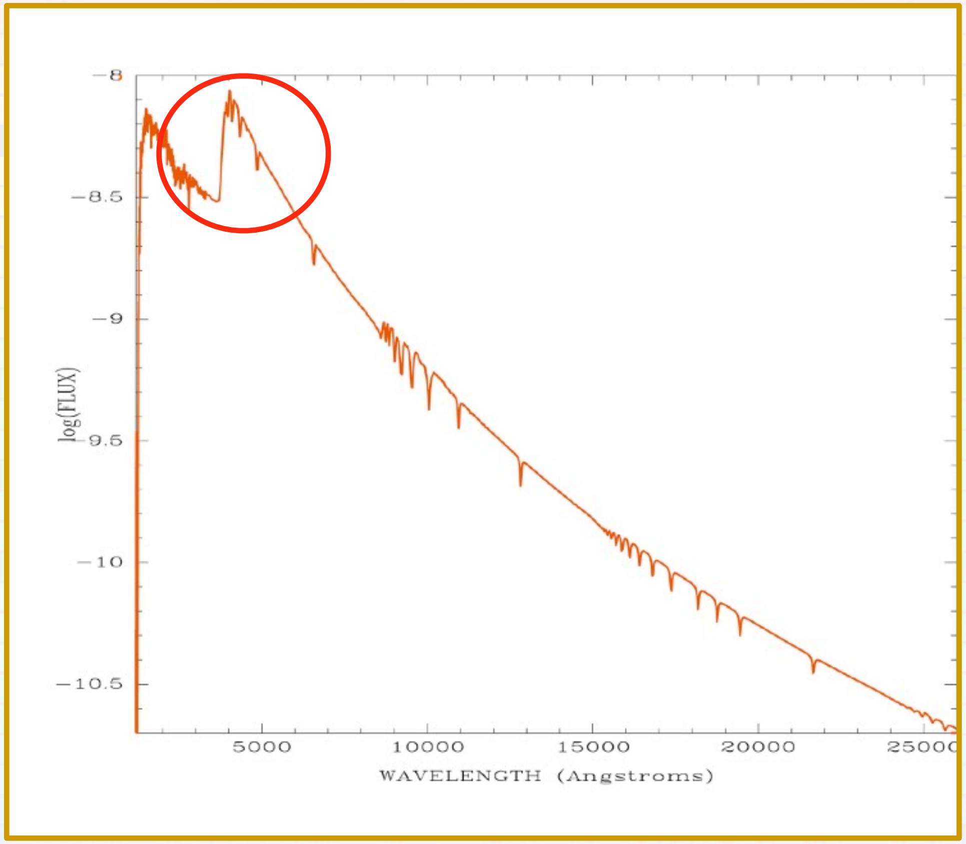
ionization changes are reflected in changes in the continuum flux

for $\lambda > \lambda$(Balmer limit) no n=2 b-f transitions possible → drop in absorption

→ atmosphere is more transparent

→ observed flux comes from deeper hotter layers

→ higher flux → BALMER JUMP



Stellar continua: the Balmer jump

from $$\kappa^{\rm bf} = \sigma_n^{\rm bf}\ N_n \qquad \sigma_n^{\rm bf} \sim \frac{1}{n^5 \nu^3}$$

and Boltzmann's equation $$N_n \sim N_{\rm tot}\ \frac{g_n}{U_n}\ e^{-E_n/kT} = n^2\ N_{\rm tot}\ e^{-E_n/kT}$$

$$\kappa^{\rm bf} \sim \frac{1}{n^3}\ e^{-E_n/kT}$$

$$\frac{\kappa^{\rm bf}(>3650)}{\kappa^{\rm bf}(<3650)} \simeq \frac{8}{27} e^{-(E_3 - E_2)/kT}$$

e.g. 0.0037 at T = 5,000 K

0.033 at T = 10,000 K

if H b-f transitions dominate continuous opacity → the Balmer discontinuity increases with decreasing T → measure T from Balmer jump

Stellar continua: H^-

apply Saha's equation to H^- (H^- is the 'atom' and H^0 is the 'ion'):

$$\frac{N(H^0)\, n_e}{N(H-)} = 4 \times 2.411 \times 10^{21}\, T^{3/2}\, e^{-8753/T}$$

$$\log_{10} \frac{N(H^0)}{N(H-)} = 0.1248 - \log_{10} P_e + 2.5 \log_{10} T - \frac{5040}{T}\, E_I$$

under solar conditions: $N(H^-) / N(H^0) \sim 4 \times 10^{-8}$

at the same time: $N_2 / N(H^0) \sim 1 \times 10^{-8}$ $N_3 / N(H^0) \sim 6 \times 10^{-10}$ (Paschen continuum)

$\Rightarrow$ $N(H^-)/ N(H^0)$: > $N_3 / N(H^0)$: b-f from H^- more important than H b-f in the visible

Stellar continua: H^-

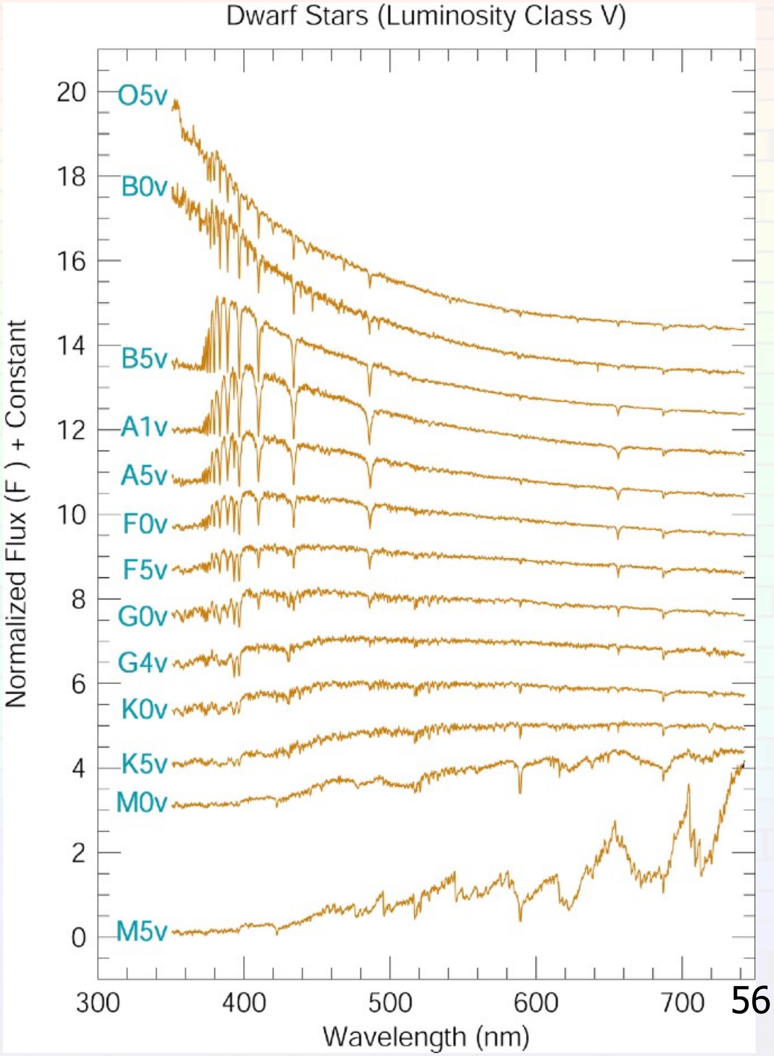
At solar T H^- b-f dominates from Balmer limit up to H^- threshold (16500 A). H^0 b-f dominates in the visible for T > 7,500 K.

Balmer jump smaller than in the case of pure H^0 absorption: instead of increasing at low T, decreases as H^- absorption increases

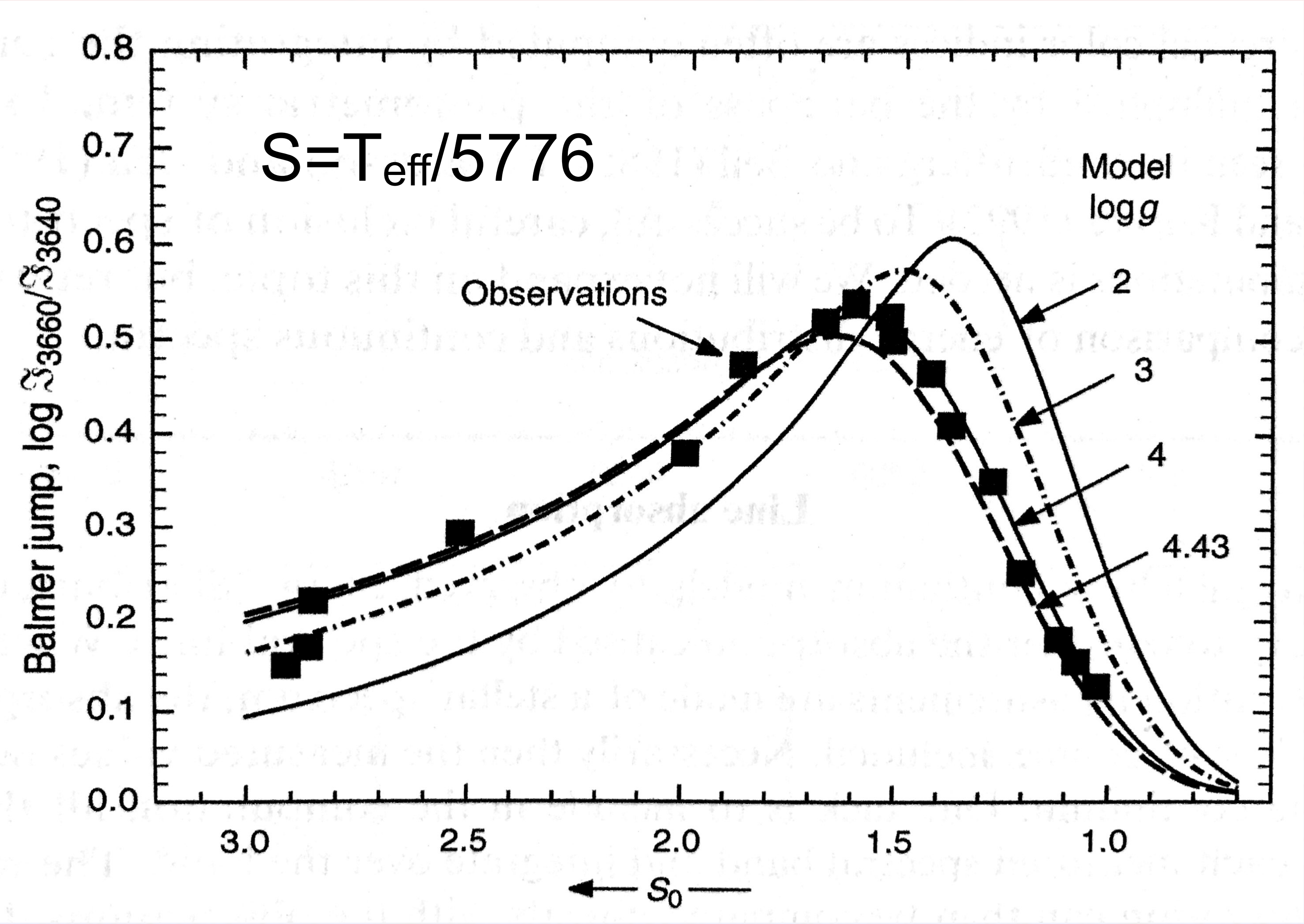
Max of Balmer jump: » 10,000 K (A0 type)

H^- opacity $\propto n_e$ → higher in dwarfs than supergiants

Balmer jump sensitive to both T and P_e (gravity) in A-F stars



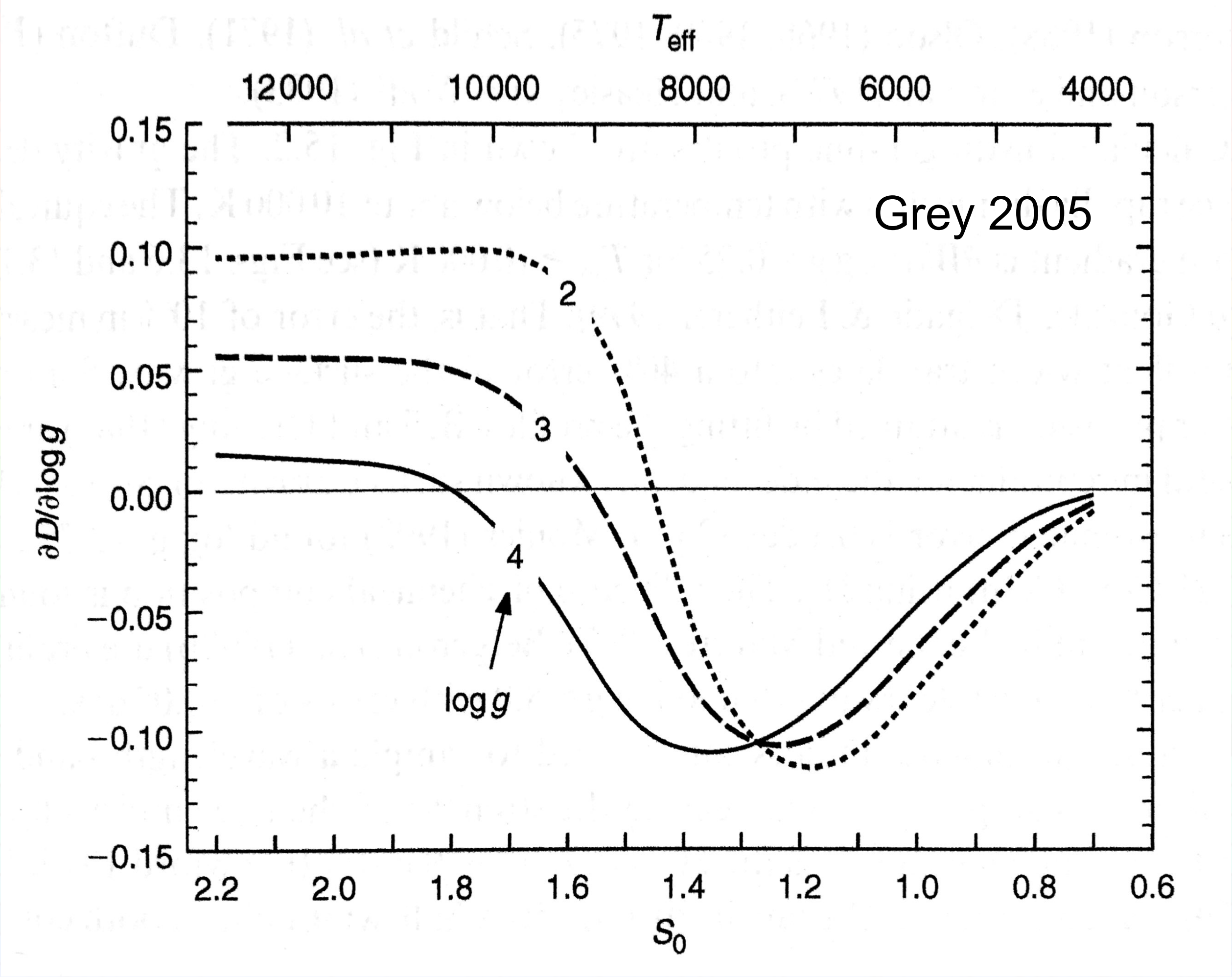
Balmer Jump: T_{eff} and Surface Gravity



Grey 2005

Balmer Jump: Surface Gravity

Balmer Jump
$D = \log F_+ / F_-$



maximum sensitivity for T ~ 7500K, drops quickly for cool stars