Code used to generate the figures of manuscript “Off-fault damage controls near-surface rupture behaviour in soft sediments” (reference number: NCOMMS-24-75893).

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Stress intensity k0 for inhomogeneous stress (two fault segments case, use Fossum and Freund expression and integrate by parts):

In[∘]:= $\texttt{K0a} = \texttt{Assuming}\left[\{L > Lr\}, \frac{\sqrt{\pi}}{2}\,\texttt{Integrate}\left[\frac{\tau 1}{\sqrt{L-x}}, \{x, 0, L1\}\right]\right] + \texttt{Assuming}\left[\{L > Lr\}, \frac{\sqrt{\pi}}{2}\,\texttt{Integrate}\left[\frac{\tau 2}{\sqrt{L-x}}, \{x, L1, L\}\right]\right]$

Out[∘]= $-\left(-\sqrt{L} + \sqrt{L - L1}\right)\sqrt{\pi}\,\tau 1 + \sqrt{L - L1}\,\sqrt{\pi}\,\tau 2$

re-arrange to get

In[∘]:= $\texttt{K0} = \sqrt{\pi L}\,\tau 1\left(1 - (1 - \tau 2/\tau 1)\sqrt{1 - L1/L}\right)$

Out[∘]= $\sqrt{L}\,\sqrt{\pi}\,\tau 1\left(1 - \sqrt{1 - \frac{L1}{L}}\left(1 - \frac{\tau 2}{\tau 1}\right)\right)$

if $\tau 2$=0 then $\texttt{K0a} = \sqrt{\pi L}\,\tau 1\left(1 - \sqrt{1 - L1/L}\right)$

for zero stress on the second segment:

In[∘]:= $\texttt{K0a /. } \tau 1 \to 0$

Out[∘]= $\sqrt{L - L1}\,\sqrt{\pi}\,\tau 2$

get G0 (static energy flow) from K0 for plane strain

In[∘]:= $\texttt{G0} = \frac{\texttt{Re[K0]}^2}{\mu}\,\frac{1 - v}{2}$

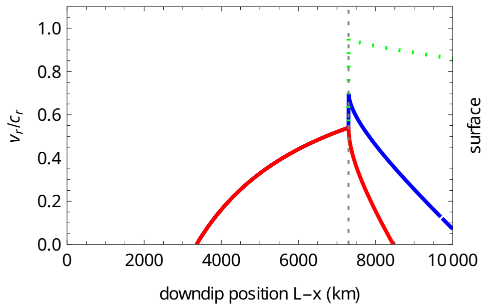
Out[∘]= $\frac{\pi (1 - v)\,\mathrm{Re}\left[\sqrt{L}\,\tau 1\left(1 - \sqrt{1 - \frac{L1}{L}}\left(1 - \frac{\tau 2}{\tau 1}\right)\right)\right]^2}{2\,\mu}$

deep (2700 m)

```
maxL = 10 × 10^3; shallo = 2700;
params := {cr → 1, μ → 20 × 10^9, ν → .3, τ1 → 3 × 10^6};
(* Egcal is dissipation.
  Root value is 1.66 MJ/m2.
   Superfical value is less, with 3 different scenarios: E0, E1 oe E2  *)
Egcal[L_, L1_, add_] = 1.66 × 10^6 + UnitStep[L - L1] * add /. params;
(* vrcal is the rupture velocity.
    vrcal is obtained from equation (2) in https://doi.org/10.1038/s41467-024-47970-6
    also in equation (2) of https://doi.org/10.48550/arXiv.2411.00544 *)
vrcal = cr (1 - (Egcal[L, L1, add] / G0)) /. params;
E0 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → 0} /. params;
E1 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → -.6 × 10^6} /. params;
E2 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → -1.5 × 10^6} /. params;
Print[E0, "J/m2 |", E1, "J/m2 |", E2,
  "J/m2 |", maxL - shallo, "--", 10^4 - maxL - 1400, " m"];
plo2 = Show[Plot[{
    vrcal /. {L1 → maxL - shallo, τ2 → 0, add → -.6 × 10^6} /. params,
    vrcal /. {L1 → maxL - shallo, τ2 → 0, add → -1.5 × 10^6} /. params,
    vrcal /. {L1 → maxL - shallo, τ2 → 0, add → 0} /. params},
   {L, 0, maxL},
   PlotRange → {{0, maxL}, {0, 1.1}},
   Frame → True,
   PlotStyle → {{Thickness[.01], Dashing[{1, 0}], Blue}, {Thickness[.01],
      Dashing[{.005, 0.03}], Green}, {Thickness[.01], Dashing[{1, 0}], Red}},
   (*FrameTicks→{{Automatic,None},
     { {{0,10},{2000,8},{4000,6}, {6000,4}, {8000,2},{10000,0}},None}},*)
   FrameLabel → {"downdip position L-x (km)", "v_r/c_r", None, "surface"},
   ImageSize → 250],
  ListLinePlot[{{maxL - shallo, 0}, {maxL - shallo, 1.1}},
   PlotStyle → {{Thickness[.005], Dashing[{.01, 0.02}], Gray}}]]
```

```
1.66 × 10^6J/m2 |1.06 × 10^6J/m2 |160 000.J/m2 |7300---1400 m
```

Out[ ]=



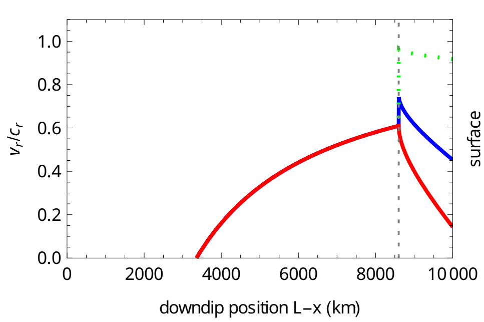
1.0
0.8
0.6
0.4
0.2
0.0
0
2000
4000
6000
8000
10 000
v_r/c_r
surface
downdip position L−x (km)

Intermediate (1400 m)

In[∘]:=
```
maxL = 10 × 10^3; shallo = 1400;
params := {cr → 1, μ → 20 × 10^9, ν → .3, τ1 → 3 × 10^6};
Egcal[L_, L1_, add_] = 1.66 × 10^6 + UnitStep[L - L1] * add /. params;
vrcal = cr (1 - (Egcal[L, L1, add] / G0)) /. params;
E0 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → 0} /. params;
E1 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → -.6 × 10^6} /. params;
E2 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → -1.5 × 10^6} /. params;
Print[E0, "J/m2 |", E1, "J/m2 |", E2,
  "J/m2 |", maxL - shallo, "--", 10^4 - maxL - 1400, " m"];
plo2 = Show[Plot[{
     vrcal /. {L1 → maxL - shallo, τ2 → 0, add → -.6 × 10^6} /. params,
     vrcal /. {L1 → maxL - shallo, τ2 → 0, add → -1.5 × 10^6} /. params,
     vrcal /. {L1 → maxL - shallo, τ2 → 0, add → 0} /. params},
    {L, 0, maxL},
    PlotRange → {{0, maxL}, {0, 1.1}},
    Frame → True,
    PlotStyle → {{Thickness[.01], Dashing[{1, 0}], Blue}, {Thickness[.01],
       Dashing[{.005, 0.03}], Green}, {Thickness[.01], Dashing[{1, 0}], Red}},
    (*FrameTicks→{{Automatic,None},
      { {{0,10},{2000,8},{4000,6}, {6000,4}, {8000,2},{10000,0}},None}},*)
    FrameLabel → {"downdip position L-x (km)", "vr/cr", None, "surface"},
    ImageSize → 250],
   ListLinePlot[{{maxL - shallo, 0}, {maxL - shallo, 1.1}},
    PlotStyle → {{Thickness[.005], Dashing[{.01, 0.02}], Gray}}]]
```

```
1.66 × 10^6J/m2 |1.06 × 10^6J/m2 |160 000.J/m2 |8600---1400 m
```

Out[∘]=



Shallow (300 m, i.e. Lisan formation thickness)

In[∘]:=
```
maxL = 10 × 10^3; shallo = 300;
params := {cr → 1, μ → 20 × 10^9, ν → .3, τ1 → 3 × 10^6};
Egcal[L_, L1_, add_] = 1.66 × 10^6 + UnitStep[L - L1] * add /. params;
vrcal = cr (1 - (Egcal[L, L1, add] / G0)) /. params;
E0 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → 0} /. params;
E1 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → -.6 × 10^6} /. params;
E2 = Egcal[maxL, maxL, add] /. {L1 → maxL - shallo, τ2 → 0, add → -1.5 × 10^6} /. params;
Print[E0, "J/m2 |", E1, "J/m2 |", E2, "J/m2 |",
  maxL - shallo, "--", 10^4 - maxL - 1400, " m |", "nuc dep:", 10 - 3.4];
plo2 = Show[Plot[{
     vrcal /. {L1 → maxL - shallo, τ2 → 0, add → -.6 × 10^6} /. params,
     vrcal /. {L1 → maxL - shallo, τ2 → 0, add → -1.5 × 10^6} /. params,
     vrcal /. {L1 → maxL - shallo, τ2 → 0, add → 0} /. params},
   {L, 0, maxL},
   PlotRange → {{0, maxL}, {0, 1.1}},
   Frame → True,
   PlotStyle → {{Thickness[.01], Dashing[{1, 0}], Blue}, {Thickness[.01],
      Dashing[{.005, 0.03}], Green}, {Thickness[.01], Dashing[{1, 0}], Red}},
   (*FrameTicks→{{Automatic,None},
     { {{0,10},{2000,8},{4000,6}, {6000,4}, {8000,2},{10000,0}},None}},*)
   FrameLabel → {"downdip position L-x (km)", "v_r/c_r", None, "surface"},
   ImageSize → 250],
  ListLinePlot[{{maxL - shallo, 0}, {maxL - shallo, 1.1}},
   PlotStyle → {{Thickness[.005], Dashing[{.01, 0.02}], Gray}}]]
```

1.66 × 10^6J/m2 |1.06 × 10^6J/m2 |160 000.J/m2 |9700---1400 m |nuc dep:6.6

Out[∘]=