The Earthquake Dissipative Engine: Energy Budget and Partition

PART X: XXXX

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Short course on earthquake physics

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27 Oct. - 16:30-18:30 classroom 2G
28 Oct. - 14:30-16:30 classroom 2H
29 Oct. - 10:30-12:30 classroom Lab Paleo
31 Oct. - 14:30-16:30 classroom 2L

Wavefronts from a Uniformly Moving Source

Consider a source (rupture tip) moving along the x-axis with constant velocity V in a medium where waves propagate at speed c (shear wave speed). Let's represent the moving source as a series of diffracting points in time, as proposed by Huygyens.

At time $\tau \leq t$, the source is located at $x = V\tau$ and emits a circular wavelet of radius $c(t-\tau)$. All such circles satisfy

$$F(x,y,t;\tau) = (x - V\tau)^2 + y^2 - c^2(t-\tau)^2 = 0, \tag{1}$$

which defines a one-parameter family of curves (parameter: τ).

Envelope and Tangency Condition

The observable wavefront is the envelope of this family, i.e., the locus of points that belong to two neighboring members of the family simultaneously. Algebraically, the envelope satisfies the double-root conditions

$$F = 0, \qquad \frac{\partial F}{\partial \tau} = 0, \tag{2}$$

where (x,y,t) are held fixed when differentiating with respect to τ.

Differentiating (1) yields

$$\frac{\partial F}{\partial \tau} = -2V(x - V\tau) + 2c^2(t-\tau) = 0, \tag{3}$$

which rearranges to

$$c^2(t-\tau) = V(x - V\tau). \tag{4}$$

Substituting (4) back into (1) gives

$$y^2 = \left(\frac{V^2}{c^2} - 1\right)(x - V\tau)^2. \tag{5}$$

$$y = (x - V\tau)\,\sqrt{\frac{V^2}{c^2} - 1} \tag{6}$$

Is this reminiscent of something right now? You may have spotted it, but we'll come back to this later...

Supersonic Case $V > c$

If $V > c$, then $\dfrac{V^2}{c^2} - 1 > 0$ and (5) describes a pair of straight lines through the instantaneous source position $(x,y) = (Vt, 0)$. Eliminating τ in favor of $x - Vt$ shows these lines make an angle μ with the direction of motion such that

$$\tan\mu = \frac{c}{\sqrt{V^2 - c^2}} \qquad \Rightarrow \qquad \sin\mu = \frac{c}{V}, \tag{7}$$

which is the Mach relation with Mach number $M = V/c > 1$. The wavefront is planar in 2D (Mach lines) or conical in 3D (Mach cone).

Subsonic Case $V < c$

If $V < c$, then $\dfrac{V^2}{c^2} - 1 < 0$ and (5) has no real solutions for a straight-line envelope. There is instead a unique retarded time τ for each point (x,y), meaning the instantaneous wavefront is a rounded, smooth curve that always encloses the source. No shock front forms.

Geometric Interpretation

The distinguishing factor

$$1 - \frac{V^2}{c^2}$$

you may now recognise, is the squared Lorentz contraction!... It changes sign as the source becomes supersonic. When positive (subsonic), the geometry is Euclidean/elliptic and the wavefront is smooth. When negative (supersonic), the quantity becomes imaginary under the square root, whose real projection yields the factor $\frac{V^2}{c^2} - 1$; the geometry becomes hyperbolic and a conical Mach envelope emerges.