Basic Theory: Background

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This document provides basic review of superconducting materials from a materials science & engineering perspective. This entails a very basic review of conductivity, magnetism, and superconductivity theory. The remainder is focused on aspects of specific material systems, such as compositional phases that exhibit superconducting phase, mechanical properties, and processing. High-level review of applications is also provided. The intent of this document is to act as a quick digest for someone who plans to dive deeper into the provided references.

- As we saw in **?@fig-HgSC_HKO**, there are materials where electrical conductivity drops to "exactly"[1] zero.
- How is this achieved?
  - Well at low-temperature we have Bardeen, Cooper, and Schrieffer (BCS) to thank Bardeen, Cooper, and Schrieffer (1957).
- What mechanism did they describe?
  - Describe microscopic superdconducting using quantum theory.
  - **Solution**: Electron Cooper pairs via condesnsate state.
  - Why Pairs? Blame the phonons.

[1] Here I use exactly in that it is zero within measurement precision. If your device can only measure to 10^{-10} then you show a resistivity value on that order.

BCS Theory: Bardeen, Cooper, and Schrieffer theory of low-temperature superconductivity.

Basic Theory: Cooper Pairs @ Low Temperature (1/4)

Mathematical Foundation

Hamiltonian: $H = H_0 + H_{\text{int}}$

- H_0: Kinetic energy term
- H_{int}: Interaction term

BCS Wave Function:

$$|\Psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle \tag{1}$$

- u_k: Probability amplitude for unoccupied state
- v_k: Probability amplitude for occupied state
- $c^\dagger_{k\uparrow}$, $c^\dagger_{-k\downarrow}$: Electron creation operators

Basic Theory: Cooper Pairs @ Low Temperature (2/4)

Role of Phonons

Electrons interact indirectly via phonons, leading to a net attractive force among pairs of e$^-$.

$$V(q,\omega) = \frac{2\omega(q)}{q^2}\chi(q,\omega)$$

- $V(q, \omega)$: Electron-phonon interaction

- $\omega(q)$: Phonon frequency

- $\chi(q, \omega)$: Polarizability

Cooper Pairs

- Formed by two electrons with opposite spins and momenta.
- Exhibit Bose-Einstein-like condensation at low temperatures.

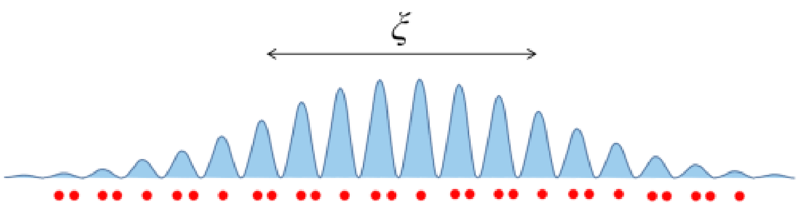

Figure 1: A "static" schematic of real-space cooper pair probablity distribution with coherence length ϵ. Red dots are the distorted lattice positions[2]

Basic Theory: Cooper Pairs @ Low Temperature (3/4)

Energy Gap

$\Delta = 2\,|V|\,\sqrt{N(0)V}$

- Δ: Energy gap, V: Pairing potential, $N(0)$: Density of states at Fermi level/

Critical Temperature T_c

The temperature below which a material becomes superconducting.

$T_c = \frac{1.13\Delta}{k_B}$

- Δ: Energy gag, k_B: Boltzmann constant

[2] Adapted from https://thiscondensedlife.wordpress.com/2015/09/12/draw-me-a-picture-of-a-cooper-pair

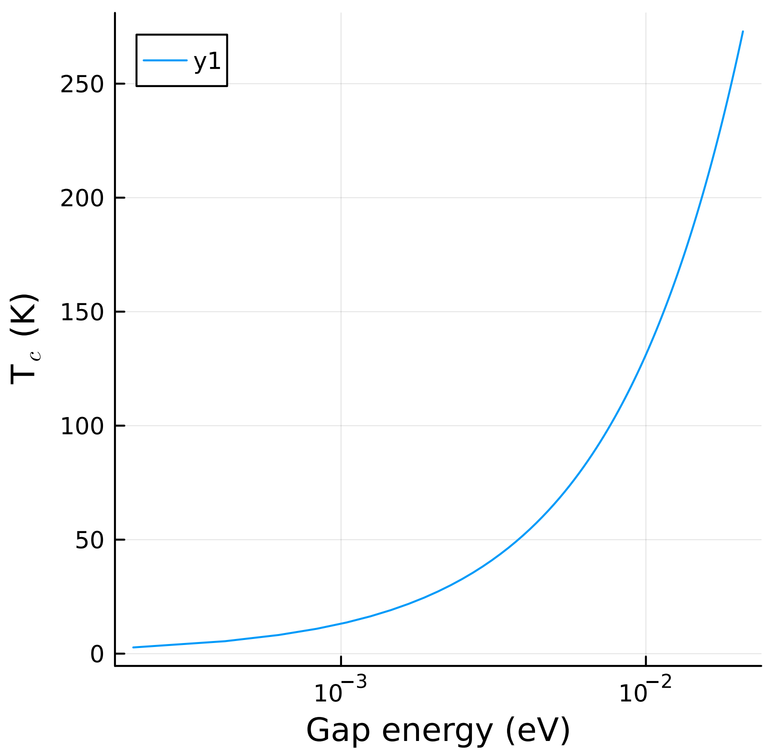

Figure 2: Critical tempature as a function of energy gap.

Basic Theory: Superconducting State Property Predictions

Meissner Effect

The expulsion of magnetic flux lines from the interior of a superconducting material.

London Equations:

$$\vec{J} = -\frac{ne^2}{m}\vec{A} \tag{2}$$

$$\nabla \times \vec{J} = -\frac{ne^2}{m}\vec{B} \tag{3}$$

- $\vec{J}$: Superconducting current density
- $\vec{A}$: Vector potential
- $\vec{B}$: Magnetic field
- n: Density of superconducting carriers
- e: Elementary charge
- m: Electron mass

Basic Theory: Experimental Evidence

- Tunneling experiments
- Specific heat measurements
- Magnetic penetration depth

Bardeen, J., L. N. Cooper, and J. R. Schrieffer. 1957. "Theory of Superconductivity." *Phys. Rev.* 108 (December): 1175–1204. https://doi.org/10.1103/PhysRev.108.1175.