EECS101: HOMEWORK #7

Due: March 3, 2017

Written Problem

Note that this is the same as the problem on the practice exam. Assume that we have a Lambertian polyhedron illuminated by a collimated source at the same position as the viewer.

a) What is the reflectance map $R(p, q)$ for this case?

Suppose that from an image we have recovered the edges below corresponding to a place in space where 3 planes meet. The angles correspond to angles in the image.

```
  \       C
   \
    \   120 deg.
     \
150   -------------------
deg.  |
      |  90 deg.
 B    |
      |     A
      |
      |
```

b) Draw a diagram in gradient space with points corresponding to the gradient of each plane and their relationship. Assume orthographic projection.

c) Suppose that our imaging system is calibrated so that measured image irradiance equals the corresponding value in the reflectance map ($E(x, y) = R(p, q)$). If we measure image irradiance values of 1.0 for plane A and $\frac{1}{\sqrt{5}}$ for plane B, then determine the irradiance of plane C and the orientation (p, q) for each plane.

Computer Problem

In this problem, you will write a program to generate images of a sphere under orthographic projection using a reflectance model. Consider a representation for a sphere centered on the optical axis with radius r and center $(0, 0, z_0)$

$$z(x, y) = z_0 + \sqrt{r^2 - (x^2 + y^2)} \qquad (x^2 + y^2) \leq r^2 \tag{1}$$

What is the unit surface normal $\hat{N}(x, y)$ to the sphere as a function of x and y? Turn in this answer with your written homework.

We will consider only illumination by point sources. Let $\hat{S}$ denote a unit vector in the direction of the source. We assume that the source is distant relative to the size of the sphere so that for a given source position, the vector $\hat{S}$ is constant across the surface of the sphere. We let $\hat{V}$ denote a unit vector in the direction of the camera. We assume that the camera is distant relative to the size of the sphere so that $\hat{V}$ for this geometry is always $(0, 0, 1)$. The scene radiance L for a Lambertian surface is proportional to $\cos(\theta)$ where θ is the angle between $\hat{S}$ and $\hat{N}$. For a more general surface, we can write

$$L = aL_l + (1-a)L_s \tag{2}$$

for a constant a $(0 \leq a \leq 1)$ where L_l is the scene radiance due to Lambertian reflection

$$L_l = cos(\theta) \tag{3}$$

and L_s is the scene radiance due to specular reflection. We can model L_s using

$$L_s = e^{-(\alpha/m)^2} \tag{4}$$

where m is a constant that is related to the roughness of the surface and α is defined as follows. Let $\hat{H}$ be the unit vector that is the angular bisector of $\hat{V}$ and $\hat{S}$, i.e.

$$\hat{H} = \frac{\hat{V} + \hat{S}}{|\hat{V} + \hat{S}|} \tag{5}$$

Then α is the angle in radians between $\hat{N}$ and $\hat{H}$. $\hat{H}$ is the hypothetical normal to a surface that would give perfect specular reflection in the direction of the camera $\hat{V}$. Thus, α measures how much $\hat{N}$ deviates from this orientation. If α is small, then L_s will be near 1. Otherwise L_s will be small.

Note that the maximum value of L is 1. Your program should generate images of the sphere by evaluating (2) across the surface and scaling the resulting values by 255 to generate numbers that are appropriate for eight bit pixel values. Note that we are using the fact that image irradiance is proportional to scene radiance. Each image will have only one source position $\hat{S}$. For each image, only the vector $\hat{N}$ should change as you compute the image since $\hat{S}$ and $\hat{V}$ are assumed constant. Try different values for $\hat{S}$, m, a, and r to generate images to submit. You are encouraged to experiment with different values for these parameters and to try to understand their role in the image formation process.