Introduction to Machine Vision
(EECS 101)

Homework #1

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Question 1:

Parametric equations:
$x = x_0 + ta$
$y = y_0 + tb$
$z = z_0 + tc$

Constants given:
$x_0 = 0.5$
$y_0 = -1$
$z_0 = 0$
$a = 0$
$b = 1$
$c = -1$
$f' = 1$

Therefore:

$$x = 0.5$$

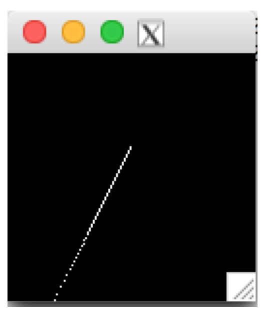
$$y = -1 + t$$

$$z = -t$$

Perspective Projection:

$$x' = \frac{f'x}{z} = \boxed{\frac{0.5}{t}}$$

$$y' = \frac{f'y}{z} = \boxed{\frac{-1+t}{-t}}$$

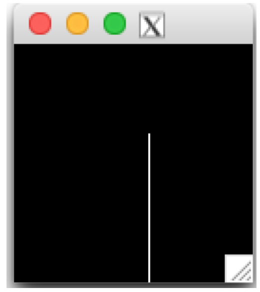


The projection of the line is a line for perspective projection.
The range of t is from -0.333 to ∞ for perspective projection.
As t goes to ∞, x' goes to 0 and y' goes to -1, which is consistent with the image.

Orthographic Projection:

$$x' = x = \boxed{0.5}$$

$$y' = y = \boxed{-1+t}$$



The projection of the line is a line for orthographic projection.
The range of t is from -3 to 5 for orthographic projection.

Question 2:

Parametric equations:
$x = x_1 + ta$
$\hat{x} = x_2 + ta$
$y = y_1 + tb$
$\hat{y} = y_2 + tb$
$z = z_0$
$\hat{z} = z_0$

Constants given:
$x_1 = 0.5$
$x_2 = -0.5$
$y_1 = -1$
$y_2 = -1$
$z_0 = -1, -2, -3$
$a = 1$
$b = 1$
$f' = 1$

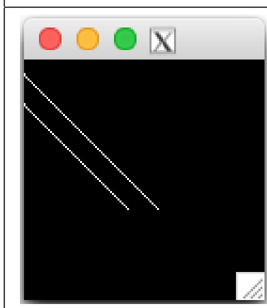
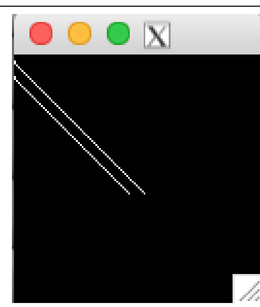
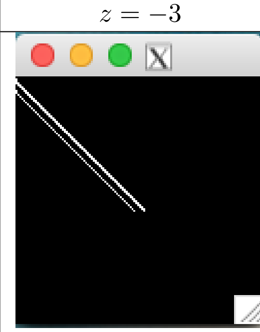
Therefore:

$$x = 0.5 + t$$
$$\hat{x} = -0.5 + t$$
$$y = -1 + t$$
$$\hat{y} = -1 + t$$
$$z = \hat{z} = -1, -2, -3$$

Perspective Projection:

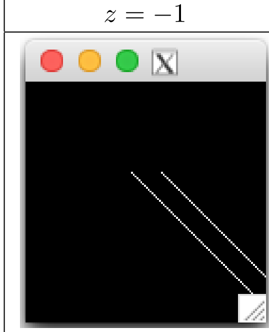
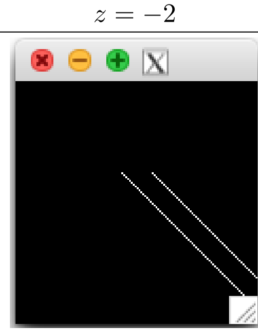
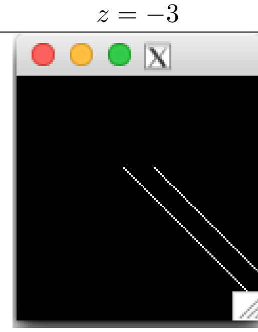
$$x' = \frac{f'x}{z} = \boxed{\frac{0.5+t}{-1}} \qquad \text{when } z = -1$$
$$= \boxed{\frac{0.5+t}{-2}} \qquad \text{when } z = -2$$
$$= \boxed{\frac{0.5+t}{-3}} \qquad \text{when } z = -3$$

$$
\begin{aligned}
\hat{x}' = \frac{f'\hat{x}}{\hat{z}} &= \boxed{\frac{-0.5+t}{-1}} && \text{when } \hat{z} = -1 \\
&= \boxed{\frac{-0.5+t}{-2}} && \text{when } \hat{z} = -2 \\
&= \boxed{\frac{-0.5+t}{-3}} && \text{when } \hat{z} = -3 \\
y' = \frac{f'y}{z} &= \boxed{\frac{-1+t}{-1}} && \text{when } z = -1 \\
&= \boxed{\frac{-1+t}{-2}} && \text{when } z = -2 \\
&= \boxed{\frac{-1+t}{-3}} && \text{when } z = -3 \\
\hat{y}' = \frac{f'\hat{y}}{\hat{z}} &= \boxed{\frac{-1+t}{-1}} && \text{when } \hat{z} = -1 \\
&= \boxed{\frac{-1+t}{-2}} && \text{when } \hat{z} = -2 \\
&= \boxed{\frac{-1+t}{-3}} && \text{when } \hat{z} = -3
\end{aligned}
$$

$z = -1$	$z = -2$	$z = -3$
		

Orthographic Projection:

$$
\begin{aligned}
x' = x &= \boxed{0.5+t} \\
\hat{x}' = \hat{x} &= \boxed{-0.5+t} \\
y' = y &= \boxed{-1+t} \\
\hat{y}' = \hat{y} &= \boxed{-1+t}
\end{aligned}
$$

$z = -1$	$z = -2$	$z = -3$
		

The magnification equation is: $m = \frac{f'}{-z_0}$. Therefore the magnification is the same for both lines since $z = \hat{z} = z_0$. Hence this shows that the lines must be parallel for both perspective and orthographic projection which the images generated shows.
The orthographic projection is a good approximation to the perspective projection for this case because z is constant.
If $z_0 = f'$, then $m = -1$, which means that the perspective image is equal to the orthographic image.

Question 3:

Parametric equations:
$x = x_1$
$\hat{x} = x_2$
$y = y_0 + tb$
$\hat{y} = y_0 + tb$
$z = z_0 + tc$
$\hat{z} = z_0 + tc$

Constants given:
$x_1 = -1$
$x_2 = 1$
$y_0 = -1$
$z_0 = 0$
$b = 0, 1, -1$
$c = 1, -1$
$f' = 1$

Therefore:

$$
\begin{aligned}
x &= -1 \\
\hat{x} &= 1 \\
y = \hat{y} &= -1 \quad \text{when } b = 0 \\
&= -1 + t \quad \text{when } b = 1 \\
&= -1 - t \quad \text{when } b = -1 \\
z = \hat{z} &= t \quad \text{when } c = 1 \\
&= -t \quad \text{when } c = -1
\end{aligned}
$$

Perspective Projection:

$$
\begin{aligned}
x' = \frac{f'x}{z} &= \boxed{\frac{-1}{t}} \qquad \text{when } c = 1 \\
&= \boxed{\frac{1}{t}} \qquad \text{when } c = -1 \\
\hat{x}' = \frac{f'\hat{x}}{\hat{z}} &= \boxed{\frac{1}{t}} \qquad \text{when } c = 1 \\
&= \boxed{\frac{1}{-t}} \qquad \text{when } c = -1
\end{aligned}
$$

$$y' = \hat{y}' = \frac{f'y}{z} = \frac{f'\hat{y}}{\hat{z}}$$

$$= \boxed{\frac{-1}{t}} \qquad \text{when } b = 0 \text{ and } c = 1$$

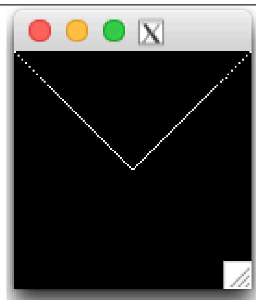
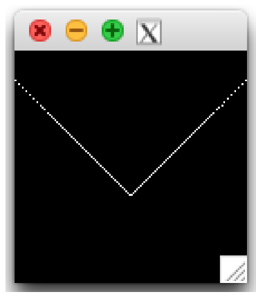
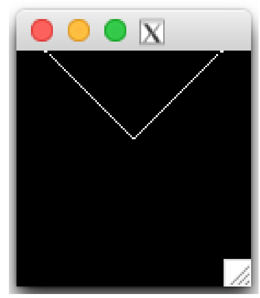
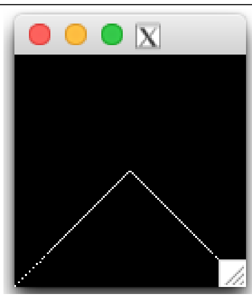
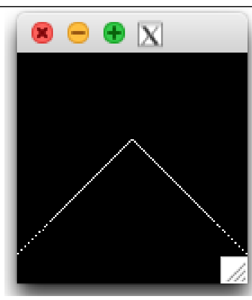
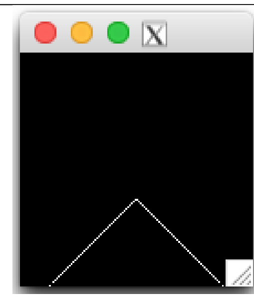
$$= \boxed{\frac{1}{t}} \qquad \text{when } b = 0 \text{ and } c = -1$$

$$= \boxed{\frac{-1+t}{t}} \qquad \text{when } b = 1 \text{ and } c = 1$$

$$= \boxed{\frac{-1+t}{-t}} \qquad \text{when } b = 1 \text{ and } c = -1$$

$$= \boxed{\frac{-1-t}{t}} \qquad \text{when } b = -1 \text{ and } c = 1$$

$$= \boxed{\frac{-1-t}{-t}} \qquad \text{when } b = -1 \text{ and } c = -1$$

	$b = 0$	$b = 1$	$b = -1$
$c = 1$			
$c = -1$			

Orthographic Projection:

$$x' = x = \boxed{-1}$$

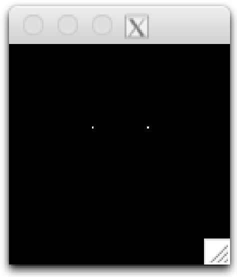
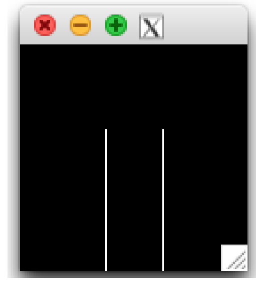
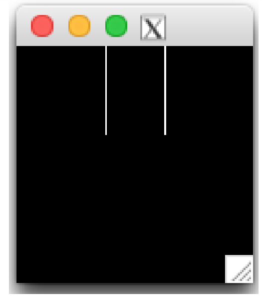
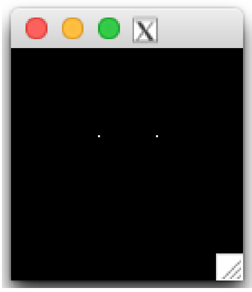
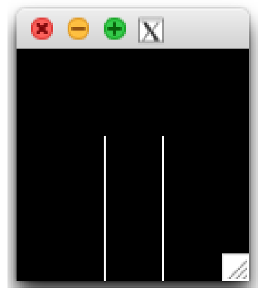
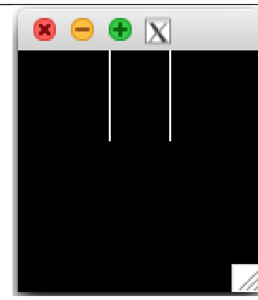
$$\hat{x}' = \hat{x} = \boxed{1}$$

$$y' = \hat{y}' = y = \hat{y}$$

$$= \boxed{-1} \qquad \text{when } b = 0$$

$$= \boxed{-1+t} \qquad \text{when } b = 1$$

$$= \boxed{-1-t} \qquad \text{when } b = -1$$

	$b=0$	$b=1$	$b=-1$
$c=1$			
$c=-1$			

The magnification equation is: $m = \frac{f'}{-z_0}$. The z depends on t so for the perspective image, the lines are not parallel but instead they intersect each other.
As for the orthographic image, z is not dependent so the two lines will be parallel.
Hence the images are consistent.
The orthographic projection is not a good approximation to perspective projection for this case.
As t goes to ∞:

$$x' = \hat{x}' = 0$$

$$y' = \hat{y}' = 0 \quad \text{when } b = 0$$

$$= 1 \quad \text{when } b = 1 \text{ and } c = 1,\ b = -1 \text{ and } c = -1$$

$$= -1 \quad \text{when } b = 1 \text{ and } c = -1,\ b = -1 \text{ and } c = 1$$

The generated images are consistent.

Bonus:

function *clear()*:

- takes the image array and clears it by looping through every pixel and set the value equal to 0, which is equivalent to the pixel being black.

function *header():*

- creates a header for the image needed to create an .ras image
- the header function is different depending on the machine, whether it uses little-endian or big-endian

function *main():*

1. clears the image (setting all the pixels to value of 0)
2. reads the input file (.raw file)

- it reads through every pixel and store it in the image array

3. closes the input file (.raw file)
4. opens or creates the output file (.ras file)
5. writes the header to the file
6. writes the image itself to the file
7. closes the output file (.ras file)