

FOUNDATIONS OF HIGHER MATHEMATICS

JOURNAL 2

Overview

This week we looked at some basic set theory terms and symbolic notation. Some of the terms we discussed were subsets and powersets. We briefly touched on cardinality and countability. When we discussed that $|\mathbb{N}| = |\mathbb{Q}|$ there was quite a commotion in class. Since we just had one day of class there was not too much to write about so I thought I would recap some of the definitions and notation we used this week.

Notation and Definitions

Definition (set). A **set** is a well defined collection of objects.

Notation. If we have an object a and a set A and a is an element of A then we write

$$a \in A$$

Definition (subset). Given sets A and B , B is a **subset** of A if every element of B is in A .

Notation. If A is a **subset** of B we write,

$$A \subseteq B$$

which is equivalent to,

$$\forall x(x \in A \rightarrow x \in B)$$

Definition (power set). The **power set** of A is the set of every subset of A .

Notation. The power set of A is defined as,

$$\mathcal{P}(A)$$

which can be expressed in symbols as,

$$\forall b \exists a \forall x (x \in a \leftrightarrow x \subseteq b)$$

or expressed in set builder notation:

$$\mathcal{P}(A) = \{x \mid x \subseteq A\}$$

Definition (cardinality). The number of elements in a set is called its **cardinality**.

Notation. Given a set A the cardinality of A is,

$$|A|$$