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Math 320
Ch 4: # 10, 11, 13, 19, 20, 25, 34 10/16/13

Foundations of Higher Mathematics
Homework 7

Problem 10

Proof. ($\subseteq$) Let $x \in A \cup B$ and $x \notin A \cap B \cap C$. Then $x \notin A$ or $x \notin B$ or $x \notin C$ by De Morgan's law. Suppose $x \in A$. Then $x \notin B$ or $x \notin C$. It follows that $x \notin B \cap C$, and thus, $x \in A - (B \cap C)$. So, $x \in [A - (B \cap C)] \cup [B - (A \cap C)]$. Now suppose the $x \in B$. Then $x \notin A$ or $x \notin C$. Thus, $x \notin A \cap C$. It follows that $x \in B - (A \cap C)$, and thus $x \in [A - (B \cap C)] \cup [B - (A \cap C)]$.

($\supseteq$) Let $x \in [A - (B \cap C)] \cup [B - (A \cap C)]$. Suppose $x \in A - (B \cap C)$. Then $x \in A$ and $x \notin B$ or $x \notin C$. It follows that $x \in A$ or $x \notin A$. Similarly, $x \notin B$ or $x \in B$. By De Morgan's Law, $x \notin A \cap B \cap C$. Since $x \in A$ not $x \in B$ and $x \notin A \cap B \cap C$, $(A \cup B) - (A \cap B \cap C)$. Now suppose that $x \in B - (A \cap C)$. Then $x \in B$ and $x \notin (A \cap C)$. It follows that $x \notin A$ or $x \notin C$. Since $x \in B$ then $x \in B \cup A$. Also, $x \in B$ or $x \notin B$. Then $x \notin A$ or $x \notin B$ or $x \notin C$ and thus $x \notin A \cap B \cap C$. Therefore, $x \in (A \cup B) - (A \cap B \cap C)$. ■

Problem 11

a) *Proof.* Suppose $S \subseteq T$ and x is in the domain of S. Then for an arbitrary element $y \in S[x]$, $(x, y) \in S$. Since $S \subseteq T$ and $(x, y) \in S$, then $(x, y) \in T$ and $y \in T[x]$. Therefore, $S[x] \subseteq T[x]$. ■

b)
- $(A \times B)[x] = B$ if $x \in A$.

  Proof. ($\subseteq$) If $x \in A$ and $y \in (A \times B)[x]$, then $(x, y) \in (A \times B)$. It follows that $y \in B$ from the definition of the Cartesian Product. Therefore, $(A \times B)[x] \subseteq B$.
  ($\supseteq$) If $x \in A$ and $y \in B$ then $(x, y) \in (A \times B)$. It follows that $y \in (A \times B)[x]$. Therefore, $B \subseteq (A \times B)[x]$. ■

- $(A \times B)[x] = \varnothing$

  Proof. ($\supseteq$) $\varnothing$ is the subset of every set, therefore $\varnothing \subseteq (A \times B)[x]$.
  ($\subseteq$) If $x \notin A$ and $y \in (A \times B)[x]$ Since x, and arbitrary element, is not in the domain A then $(A \times B)[x]$ is not defined and thus, it is vacuously true that $y \in \varnothing$ so $(A \times B)[x] \subseteq \varnothing$. ■

Problem 13

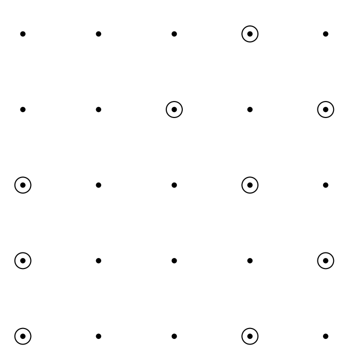
$R[7] = \{7, 14, 21, 28, 35, \ldots\}$

$R[14] = \{14, 28, 42, 56, 70, \ldots\}$

The only $n \in \mathbb{N}$ for which $R[n] = \mathbb{N}$ is $n = 1$.

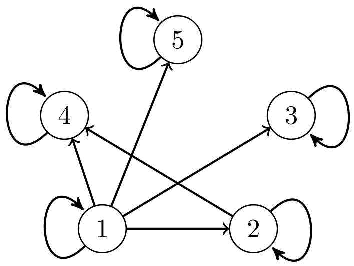
Problem 19

Given the relation $A = \{(1, 1), (1, 2), (1, 3), (3, 4), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4)\}$ the Cartesian graph looks like:



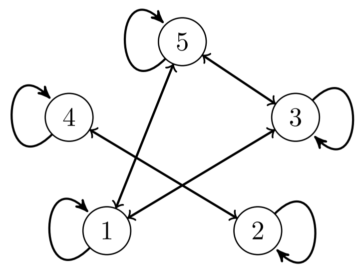
Problem 20a

$R = \{(a, b) \in A \times A : a \text{ divides } b\}$



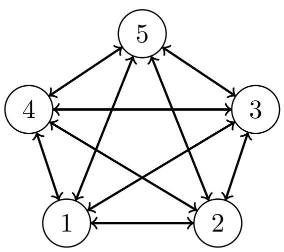
Problem 20c

$E = \{(a, b) \in A \times A : a + b \text{ is even}\}$



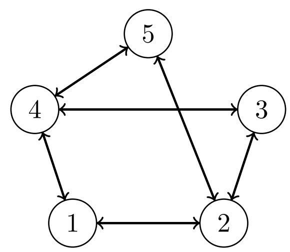
Problem 20b

$U = \{(a, b) \in A \times A : a \neq b\}$



Problem 20d

$O = \{(a, b) \in A \times A : a + b \text{ is odd}\}$



Problem 25

Problem 34

a) **Non Reflexive:** $1 \in \mathbb{R}$ but $1 \cdot 1 \neq 0$ and thus $(1, 1) \notin R_1$
Symmetric: Suppose $(x, y) \in R_1$. Then $xy = 0$ and $yx = 0$. Thus $(y, x) \in R_1$. Therefore, R_1 is Symmetric.
Non Transitive: $1 \cdot 0 = 0$ so $(1, 0) \in R_1$. $0 \cdot 1 = 0$ so $(0, 1) \in R_1$ but $1 \cdot 1 \neq 0$ so $(1, 1) \notin R_1$.

b) **Reflexive:** Suppose $x \in \mathbb{R}$. Then, $|x - x| = 0 < 5$ so $(x, x) \in R_2$.
Symmetric: Suppose $(x, y) \in R_2$. Then $|x - y| < 5$. It follows that, $|-(y - x)| < 5$ and $|y - x| < 5$. Therefore, $(y, x) \in R_2$ so R_2 is symmetric.
Non Transitive: $|10 - 6| < 5$ and $|6 - 2| < 5$ so $(10, 6) \in R_2$ and $(6, 2) \in R_2$ but $|10 - 2| = 8 \not< 5$.

c) **Non Reflexive:** $0 \in \mathbb{R}$ but $0 \cdot 0 = 0$.
Symmetric: Suppose $(x, y) \in R_3$. Thus, $xy \neq 0$. It follows that $yx \neq 0$ so $(y, x) \in R_3$. Therefore, R_3 is symmetric.
Transitive: (Contrapositive) Suppose $(x, z) \notin R_3$. Then $xz = 0$. So either $x = 0$ or $z = 0$. Assume $x = 0$. Then $xy = 0$, and $(x, y) \notin R_3$. So $(x, y) \notin R_3$ or $(y, z) \notin R_3$ by disjunction introduction. Now assume, $z = 0$. It follows that $yz = 0$. So, $(y, z) \notin R_3$ or $(x, y) \notin R_3$. Therefore, by the contrapositive, R_3 is transitive. Suppose $(x, y) \in R_3$ and $(y, z) \in R_3$.

d) **Reflexive:** Let $x \in \mathbb{R}$. Then $x \geq x$ so $(x, x) \in R_4$.
Non Transitive: $2 \geq 1$ so $(2, 1) \in R_4$ but $1 < 2$ so $(1, 2) \notin R_4$.
Transitive: Suppose $(x, y) \in R_4$ and $(y, z) \in R_4$. So $x \geq y$ and $y \geq z$. It follows that, $x \geq y \geq z$ and thus, $x \geq z$. Hence, $(x, z) \in R_4$.

e) **Non Reflexive:** $1 \in \mathbb{R}$ but $1^2 + 1^2 = 2 \neq 1$.
Symmetric: Suppose $(x, y) \in R_5$. So $x^2 + y^2 = 1$. It follows that $y^2 + x^2 = 1$ so $(y, x) \in R_5$
Non Transitive: $1^2 + 0^2 = 1$ so $(1, 0) \in R_5$ and $0^2 + 1^2 = 1$ so $(0, 1) \in R_5$ but $1^1 + 1^1 = 2 \neq 1$ so $(1, 1) \notin R_5$.