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Problems: 1, 10, 15, 17, 18, 19 & Group Work 9/11/13

# Foundations of Higher Mathematics
# Homework 1

## Group Work

1. Find the power set of $A = \{1, 2, 3, 4\}$

$$\begin{aligned}\mathcal{P}(A) = \{&\{\varnothing\}, \{1\}, \{2\}, \{3\}, \{4\},\\ &\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\},\\ &\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\},\\ &\{1,2,3,4\}\}\end{aligned}$$

2. What is the cardinality of $\mathcal{P}(A)$?

$$|\mathcal{P}(A)| = 16$$

3. **No**, there is not a set with 12 subsets. As seen above, a set with 4 elements has 16 subsets. A set with 3 elements has 8 elements. We need a set less than 4 elements and more than 3, which clearly makes a set with 12 subsets impossible.

4. If A and B are sets and $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

   a) *Proof 1:* is **not correct**, because if we let $x \in \mathcal{P}(A)$ then $x \notin A$. Correction: $x \subseteq A$

   b) *Proof 2:* is **not correct**, because we only proved the case where $A = \{1, 2\}$ and $B = \{1, 2, 3\}$.

   c) *Proof 3:* is **not correct**, because it only shows a specific example for the power set. This only shows single elements in the power set, not all of the other possibilities.

5. *Modified Proof 1.* Let $x \in P(A)$. Then $x \subseteq A$. Since $A \subseteq B$, $x \subseteq B$. Therefore $x \in \mathcal{P}(A)$, so $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. ■

## Problem 1

**Proposition 2.1.** *If A is a set that has no members and B is a set that has no members, then $A = B$.*

*Proof.* Assume A and B have no members. Then $A = \{\}$ and $B = \{\}$. For all x in A, it is (vacuously) true that it is in B. Thus, $A \subseteq B$. Similarly for B, all elements in B are in A. Hence, $B \subseteq A$. Therefore, $A = B$. ■

## Problem 10

a) $\varnothing \in \{\varnothing\}$ is **true**.

b) $\varnothing \subseteq \{\varnothing\}$ is **true**.

c) $\frac{\pi}{4} \in \{\frac{\pi}{4}\}$ is **true**.

d) $\varnothing \in \{\frac{\pi}{4}\}$ is **false**.

e) $\frac{\pi}{4} \in \{\{\frac{\pi}{4}\}\}$ is **false**.

f) $\{\frac{\pi}{4}\} \subseteq \{\{\frac{\pi}{4}\}\}$ is **false**.

g) $\{\frac{\pi}{4}\} \in \{\{\frac{\pi}{4}\}\}$ is **true**.

h) $\varnothing \subseteq \{\{\frac{\pi}{4}\}\}$ is **true**.

i) $\{\frac{\pi}{4}\} \subseteq \{\frac{\pi}{4}, \{\frac{\pi}{2}\}\}$ is **true**.

## Problem 15

a) $\{x \in \mathbb{R} : x > 3, x^2 < 5, \text{ and } x \neq 4\} = \varnothing$

b) $\{x \in \mathbb{R} : x > 3 \text{ or } -x > 3\}$ is the set of all real numbers excluding 3.

c) $\{x \in \mathbb{R} : x > 3 \text{ and } -x > 3\} = \varnothing$, a number cannot be above and below 3 at the same time.

d) $\{x \in \mathbb{R} : x > 3 \text{ and } -x < 3\}$ is all real numbers greater than 3.

e) $\{x \in \mathbb{R} : x^2 \neq x\}$ is the set of all real numbers, excluding 1 and 0.

## Problem 17

Suppose, A, B, and C are sets. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

*"Proof 1".* Let $x \in A$. Since $A \not\subseteq B$, $x \notin B$. Since $x \notin B$, $x \notin C$. Therefore x is a member of A and x is not a member of C, so $A \not\subseteq C$.

Proof 1 is **incorrect**. The step, Since $x \notin B, x \notin C$ is incorrect. If $x \in B$, then we could conclude $x \notin C$.

*"Proof 2".* Since $B \not\subseteq C$, there exists $x \in B$ such that $x \notin C$. Since $A \not\subseteq B$ there exists $y \in A$ such that $y \notin B$. Therefore, $x \neq y$, and therefore $A \not\subseteq C$.

Proof 2 is **incorrect** about the $x \neq y$, therefore $A \not\subseteq C$ part. $A \not\subseteq C$ does not follow from $x \neq y$.

*"Counterexample".* Let $A = \{1, 2, 3\}$, $B = \{1, 2, 4, 5\}$, $C = \{1, 2, 3, 4\}$

The counterexample is correct. $A \not\subseteq B$ because $3 \in A$ and $3 \notin B$. $B \not\subseteq C$ because $5 \in B$ and $5 \notin C$. The part that shows that the counter example is correct is that, A **is a subset** of C. $1, 2, 3 \in A$ and $1, 2, 3 \in C$.

## Problem 18

See part 4 of the group work.

## Problem 19

See part 5 of the group work.