NAME:____

Quiz 2-Homework: Fundamentals of Mathematics (Math 320) Stevenson

Please redo this quiz and do the other set inclusion exercises below. When you start on part (a) you must start by saying: "If $f(x) \in A$ then $f(x) == c + bx + x^2$." Then you must SHOW that this $f(x)$ satisfies the criteria to be in B. Similarly, in part (b) you must start with "If $f(x) \in B$ then $f(x) == a_0 + a_1x + a_2x^2$ such that $\deg(f(x) - x^2 = 0$." Then you must SHOW that this $f(x)$ satisfies the criteria to be in A.

Notice that I added something to the set B. This is because the degree of the zero polynomial $z(x) = 0 + 0x + 0x^2$ is a special case. An optional exercise could be to write a journal about what you think that the degree of the zero polynomial should be. Whatever it is, it should satisfy the rules you know about degree. In particular: $\deg(h(x)g(x)) = \deg h(x) + \deg g(x)$.

1. Let $\mathbf{P}$ be the set of all polynomials with coefficients in $\mathbb{R}$ and in variable x of degree less than or equal to 2. That is,

$$\mathbf{P} = \left\{ f(x) = a_0 + a_1x + a_2x^2 \mid a_0, a_1, a_2 \in \mathbb{R} \right\}.$$

Consider the following two subsets of $\mathbf{P}$:

$$A = \left\{ f(x) \in \mathbf{P} \mid \exists\, c, b \in \mathbb{R} \ s.t. \ f(x) = c + bx + x^2 \right\}$$

$$B = \left\{ f(x) \in \mathbf{P} \mid \ \deg(f(x) - x^2) \leq 1 \ or \ f(x) - x^2 = 0 + 0x + 0x^2 \right\}.$$

   (a) Prove that $A \subset B$.
   (b) Is $B \subset A$? Prove your assertion.
   (Hint: If $f(x) \in \mathbf{P}$ then $f(x) = a_0 + a_1x + a_2x^2$. What does $f(x) - x^2$ look like?)

2. Continuing with $\mathbf{P}$ as above. Consider some more subsets.

$$C = \{f(x) \in \mathbf{P} \mid f'(0) = 0\}$$

$$D = \{f(x) \in \mathbf{P} \mid f(0) = 0\}.$$

$$E = \{f(x) = a_0 \mid a_0 \in \mathbb{R}\}.$$

$$F = \{f(x) \in \mathbf{P} \mid f(0) = f(1) = f(2) = 0\}.$$

$$G = \{f(x) \in \mathbf{P} \mid f''(0) = f'(0) = 0\}.$$

   (a) Give two examples of elements in each set.
   (b) For each of the containments below state whether it is true or false. If true give a proof, if false, give an example of an element in the first set which is not contained in the second set.
      i. $C \subset D$.
      ii. $D \subset C$.

iii. $C \subset E$.
iv. $E \subset C$.
v. $C \subset F$.
vi. $F \subset C$.
vii. $C \subset G$.
viii. $G \subset C$.
ix. $D \subset E$.
x. $E \subset D$.
xi. $D \subset F$.
xii. $F \subset D$.
xiii. $D \subset G$.
xiv. $G \subset D$.
xv. $E \subset F$.
xvi. $F \subset E$.
xvii. $E \subset G$.
xviii. $G \subset E$.
xix. $F \subset G$.
xx. $G \subset F$.