

FOUNDATIONS OF HIGHER MATHEMATICS

HOMEWORK 1

Problem 6

Assume that "Joe is a girl" is false and "Joe is ten years old" is false as well. So this means that Joe is a boy and Joe is not ten years old. Thus the true statements are,

- b) If Joe is ten years old, then Joe is a girl.
- c) Joe is ten years old if and only if Joe is a girl.
- d) Joe is not a ten year-old girl.

Problem 13

- a) The contrapositive of, *if* $3 > 1$, *then* $5 > 1$ is, *if* $5 \leq 1$, *then* $3 \leq 1$, so the converse of that is:

$$\text{if } 3 \leq 1, \text{ then } 5 \leq 1$$

- b) The converse of, *if* $3 > 1$, *then* $5 > 1$ is, *if* $5 > 1$, *then* $3 > 1$, so the contrapositive is:

$$\text{if } 3 \leq 1, \text{ then } 5 \leq 1$$

Problem 27

Suppose that each of these statements is true

John is smart.

John or Mary is ten years old.

If Mary is ten years old, then John is not smart.

then...

- a) Mary is ten years old is **false**.
By the contrapositive of the third proposition, if John is smart, then Mary is not ten years old. Since John is smart, we can conclude that Mary is not ten years old.
- b) John is ten years old is **true**.
We can conclude that since John or Mary is ten years old, and Mary is not ten, from part a, John must be ten years old.
- c) Either John or Mary is not ten years old is **true**.
Since we concluded that Mary is not ten years old from part a, this statement is true.

Problem 39

If x is an even integer or $x > 17$, then x is a multiple of 4 and $x \leq 5$.

Negative: x is an even integer or $x > 17$, and x is not a multiple of 4 or $x > 5$.

Contrapositive: If x is not a multiple of 4 or $x > 5$, then x is an odd integer and $x \leq 17$.

Problem 41

Negate the following statements:

- a) All cows eat grass.
No cows eat grass.
- b) There is a horse that does not eat grass.
All horses eat grass.
- c) There is a car that is blue and weighs less than 4,000 pounds.
All cars are not blue or weigh at least 4,000 pounds.
- d) Every math book is either blue or hard to read.
There is a math book that isn't blue and easy to read.
- e) Some cows are spotted.
All cows are spotless.
- f) No car has 15 cylinders.
There is a car with 15 cylinders.
- g) Some cars are old but are still in good running condition.
All cars are new or in bad running condition.

Problem 42

For an integer x to have **property P** symbolically it means,

$$\forall x \forall y (x|ab \rightarrow x|a \vee x|b)$$

So for an integer x to **not** have **property P**,

$$\neg(\forall x \forall y (x|ab \rightarrow x|a \vee x|b)) \Leftrightarrow$$

$$\exists x \exists y (x|ab \wedge x \nmid a \wedge x \nmid b)$$

which means, *For an integer x to not have **property P**, there are integers a and b such that x divides ab , x does not divide a and x does not divide b .*